Parser Overall Picture

First overall parser

top down parser in more details

The Formal Language Hierarchy (Chomsky Hierarchy)

Type 3: Regular Languages → Finite Automata, Regular Expressions

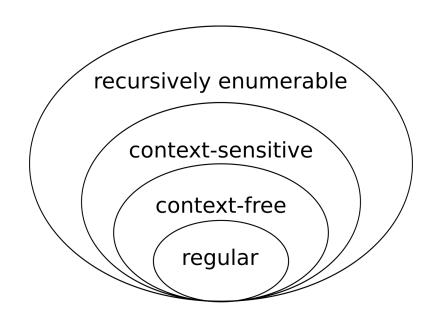
- Lexical Analysis (Flex/Lex)

Type 2: Context-Free Languages → Pushdown Automata, CFG

- Syntax Analysis (Bison/Yacc, Recursive Descent)

Type 1: Context-Sensitive Languages → Linear Bounded Automata

- Some semantic checks



https://en.wikipedia.org/wiki/Chomsky hierarchy

Type 0: Recursively Enumerable Languages → Turing Machines

- General computation

Context free grammars (CFG)

A CFG consists of

- A set of terminals T
- A set of non-terminals N
- A start symbol S (a non-terminal)
- A set of productions

Generating strings:

identify the start symbol S (a non-terminal)

- This represent the set of all strings in L(S)
- Choose a grammar $\alpha \rightarrow \beta$
- rewrite α with β

repeat this until no more nonterminals

Why Context-Free Grammars (CFG) for Parsing?

- Regular expressions can't handle nested structures
- CFGs naturally express programming language syntax
- Efficient parsing algorithms exist for CFGs

CFG example

 $S \rightarrow aS$

 $S \to Sb$

 $S \to cS \,$

 $S \rightarrow \epsilon$

Derivation of string acb

 $S \; (use \; S \rightarrow aS)$

 \rightarrow aS

(use $S \rightarrow Sb$)

 \rightarrow aSb

(use $S \rightarrow cS$)

 \rightarrow acSb

(use $S \rightarrow \epsilon$)

 \rightarrow acb

Final String

Regular expression equal to this?

CFG for L = $\{a^nb^nc^n: n > 0\}$?

Why Context-Free Grammars (CFG) for Parsing?

Regular expressions can't handle nested structures

CFGs naturally express programming language syntax

Efficient parsing algorithms exist for CFGs

Grammar Normal Forms-Chomsky Normal Form (CNF)

Every production is either:

 $A \rightarrow BC$ (two non-terminals)

 $A \rightarrow a$ (single terminal)

 $S \rightarrow \epsilon$ (only start symbol can derive empty)

```
Original:
```

```
E \rightarrow E + E \mid E * E \mid (E) \mid id
CNF:
E \rightarrow IE' \mid LY \mid LX \mid id
Y \rightarrow XE'
X \rightarrow ER
E' \rightarrow PE \mid ME \mid PC \mid MC
C \rightarrow EE'
L \rightarrow (R \rightarrow) \qquad P \rightarrow + \qquad M \rightarrow * \qquad I \rightarrow id
```

Simplified Grammar for Parsing

- Simplify parsing algorithms
- Enable theoretical proofs
- Standardize grammar analysis

Real-World Grammar Examples

```
# Python Simple statement grammar
stmt: simple_stmt | compound_stmt
simple_stmt: expr_stmt | return_stmt
compound_stmt: if_stmt | while_stmt | def_stmt
if_stmt: 'if' test ':' suite ('elif' test ':' suite)* ['else' ':' suite]
```

```
json: object | array
object: '{' pairs '}'
pairs: pair | pair ',' pairs
pair: STRING ':' value
array: '[' elements ']'
elements: value | value ',' elements
value: STRING | NUMBER | object | array | 'true' | 'false' | 'null'
```

Parser Landscape Overview

Top-Down Parsers (LL Family)

Recursive Descent (hand-written)

∠ LL(1), LL(k) (predictive)

Top-Down: Start from root, build parse tree downward

Leftmost derivation

LL: Often hand-written, educational

Bottom-Up Parsers (LR Family)

∠ LR(0), SLR(1), LR(1), LALR(1)

Bottom-Up: Start from leaves, build parse tree upward

Rightmost derivation in reverse

LR: Most parser generators (more powerful)

LL vs LR - Concrete Example

Goal: Parse "2 + 3 * 4"

LL Process:

- $E \rightarrow E + T$ (predict)
- \rightarrow T + T (expand E \rightarrow T)
- \rightarrow F + T (expand T \rightarrow F)
- \rightarrow 2 + T (match '2')
- \rightarrow 2 + T * F (expand T \rightarrow T * F)
- \rightarrow 2 + F * F (expand T \rightarrow F)
- \bullet \rightarrow 2 + 3 * F (match '3')
- \rightarrow 2 + 3 * 4 (match '4')

Goal: Parse "2 + 3 * 4"

LR Process (shift/reduce):

• 2 + 3 * 4 (shift '2') • F + 3 * 4 (reduce: $2 \rightarrow F$) • T + 3 * 4 (reduce: $F \rightarrow T$) • E + 3 * 4 (reduce: $T \rightarrow E$) • E + 3 * 4 (shift '+') • E + 3 * 4 (shift '3') • E + F * 4 (reduce: $3 \rightarrow F$) • E + T * 4 (reduce: $F \rightarrow T$) • E + T * 4 (shift '*') • E + T * 4 (shift '4') • E + T * F (reduce: $4 \rightarrow F$) • E + T (reduce: $T * F \rightarrow T$)

• E (reduce: $E + T \rightarrow E$)

Works like a Pattern Recognition

```
Goal: Parse "2 + 3 * 4"
```

LL Process:

```
    E → E + T (predict)
    → T + T (expand E → T)
    → F + T (expand T → F)
    → 2 + T * F (expand T → T * F)
    → 2 + F * F (expand T → F)
    → 2 + 3 * F (match '3')
    → 2 + 3 * 4 (match '4')
```

Goal: Parse "2 + 3 * 4"

LR Process (shift/reduce):

```
2+3*4 (shift '2')
F+3*4 (reduce: 2 → F)
T+3*4 (reduce: F → T)
E+3*4 (shift '+')
E+3*4 (shift '+')
E+7*4 (reduce: 3 → F)
E+T*4 (reduce: F → T)
E+T*4 (shift '*')
E+T*5 (reduce: A → F)
E+T*F (reduce: A → F)
E+T* (reduce: F → T)
E+T* (reduce: F → T)
```

Works like a Pattern Recognition

Both approaches typically build ASTs bottom-up:

LL: Build subtrees during recursive returns

LR: Build nodes during reduce actions

Example AST construction in both

```
// LL Recursive Descent
Expr* parse expression() {
   Expr* left = parse_term();
   if (current_token == PLUS) {
       get token();
        Expr* right = parse expression();
       return new AddExpr(left, right); // Build on return
   return left;
// LR Bison
expression: term PLUS term { \$\$ = \text{new AddExpr}(\$1, \$3); }
            term { $$ = $1; }
```

LL(1) vs LR(1) Comparison

Predictive Top-Down

Top-down, Leftmost derivation

1 token lookahead

Implemented via Recursive procedures

Must eliminate left recursion

Shift-Reduce Bottom-Up

1 token lookahead

Implemented with State machines + stack

How to simply implement parser

Table driven LL(1)

Recursive descent

```
def parse_expression(self):
    left = self.parse_term()
    while self.current_token in ('+', '-'):
        op = self.current_token
        self.advance()
        right = self.parse_term()
        left = BinaryOp(left, op, right)
        return left
```

```
// Parse table driving recursive calls
void parse() {
   stack.push(START SYMBOL);
   while (!stack.empty()) {
        symbol = stack.top();
       if (is terminal(symbol)) {
           match(symbol);
            stack.pop();
       } else {
            rule = parsing_table[symbol][lookahead];
            stack.pop();
            push rule on stack(rule); // in reverse
order
```

Introduction to Top-Down Parsing

Characteristics:

- Parse tree constructed from the top
- From left to right
- Terminals seen in order of appearance

Today's Focus:

- Syntax-Directed Translation
- Recursive Descent Parsing
- LL(1) predictive top-down parsing
- Parser tools (Bison examples

Parse tree

a graphical representation of a derivation.

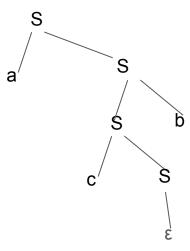
S (use $S \rightarrow aS$)

 \rightarrow aS (use S \rightarrow Sb)

 \rightarrow aSb (use S \rightarrow cS)

 \rightarrow acSb (use S \rightarrow ϵ)

 \rightarrow acb Final String



exercises

CFG for all strings over {a,b} which contain the substring "baba"

Draw parse tree for ababab

CFG for all strings over {a,b} which starts with **ba** ends with **ba**

Draw parse tree for baaabba

Ambiguity

A CFG is ambiguous:

- If for some string s,
 - the grammar has > 1 different parse tree

Arithmetics

$$E \rightarrow E + E \mid E - E \mid E * E \mid E / E \mid id$$

Dangling else

$$E \rightarrow if E then E \mid if E then E else E \mid OTHER$$

$$E \rightarrow E + E \mid E - E \mid E * E \mid E / E \mid id$$

Rewrite grammar (right associative):

• $E \rightarrow id \mid id + E \mid id - E \mid id * E$

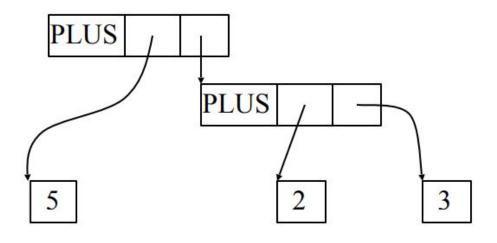
Or declare precedence and associativity(in yacc):

```
%right '='
%left '+' '-
%left '*' '/'
```

Abstract syntax tree

So far a parser traces the derivation of a sequence of tokens

- The rest of the compiler needs a structural representation of the program
- Abstract syntax trees
- Like parse trees but ignore some details
- Abbreviated as AST

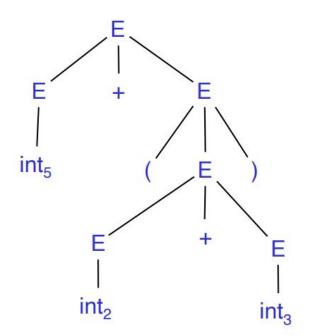


Grammar: $E \rightarrow int \mid (E) \mid E + E$

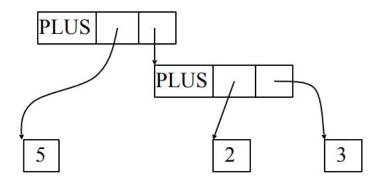
String: 5 + (2 + 3)

After lex: int5 '+' '(' int2 '+' int3 ')'

During parsing we build a parse tree ...



Abstract syntax tree



Also captures the nesting structure

- But abstracts from the concrete syntax
- => more compact and easier to use
- An important data structure in a compiler

captures the nesting structure

• But too much info – Parentheses – Single-successor nodes

https://web.stanford.edu/class/cs143/lectures/lecture06.pdf

Semantic actions

This is what we'll use to construct ASTs

Each grammar symbol may have attributes

For terminal symbols (lexical tokens) attributes can be calculated by the lexer

Each production may have an action

- X → Y1 ... Yn { action }
 - action can refer to/compute symbol attributes

Semantic Actions: Adding Behavior to Grammar Rules

What are semantic actions?

Code that runs when a grammar rule is applied

 Used to build ASTs, compute values, check types, generate code Bison- Simple Calculator Example:

```
E \rightarrow E1 + E2 \quad \{ \$\$ = \$1 + \$3; \} \quad // \text{ Compute value} E \rightarrow \text{number} \quad \{ \$\$ = \$1; \} \quad // \text{ Pass number value up} E \rightarrow (E1) \quad \{ \$\$ = \$1; \} \quad // \text{ Pass through parentheses}
```

- \$\$ = result for the left-hand side (what we're building)
- \$1, \$2, \$3 = values from the 1st, 2nd, 3rd symbols on right-hand side

```
expr ::= expr + term
                                                expr ::= term { + term }*
       | term
                                                // parse
                                                void expr() {
                                                    term();
// parse expr ::= ...
                                                    while (next symbol is PLUS) {
void expr() {
                                                        getNextToken();
   expr();
   if (current token is PLUS) {
                                                        term();
       getNextToken();
       term();
```

```
// parse
// factor ::= int | id | ( expr )
void factor() {
   switch(nextToken) {
   case INT:
                                                               Next: How to process terminals,
       process int constant;
                                                                non-terminals
       getNextToken();
       break;
   case ID:
                                                               How to eliminate recursion and left
       process identifier;
                                                                factoring problem
       getNextToken();
       break;
                                                          E.g. left factoring
   case LPAREN:
       getNextToken();
                                                          ifStmt ::= if ( expr ) stmt
       expr();
       getNextToken();
                                                                  if (expr) stmt else stmt
```

Annotating grammar with actions

Consider the grammar

$$E \rightarrow int \mid E + E \mid (E)$$

For each symbol X define an attribute X.val

- For terminals,
 - val is the associated lexeme
- For non-terminals,
 - val is the expression's value (and is computed from values of subexpressions)

We annotate the grammar with actions:

```
E → int { E.val = int.val }

| E1 + E2 { E.val = E1.val + E2.val }

| (E1) { E.val = E1.val }
```

String: 5 + (2 + 3)

Tokens: int5 '+' '(' int2 '+' int3 ')'

Productions Equations $E \rightarrow E1 + E2$ E.val = E1.val + E2.val

E1 \rightarrow int5 E1.val = int5.val = 5

 $E2 \rightarrow (E3)$ E2.val = E3.val

 $E3 \rightarrow E4 + E5$ E3.val = E4.val + E5.val

E4 \rightarrow int2 E4.val = int2.val = 2

E5 \rightarrow int3 E5.val = int3.val = 3

Building a Calculator with Semantic Actions

See an Example with bison: infix notation calculator Infix Calc (Bison 3.8.1)

- yyparse() returns a value of 0 if the input it parses is valid
- yyparse() calls a routine yylex() everytime it wants to obtain a token from the input.

Notes on Semantic actions

- Semantic actions specify a system of equations
- Declarative Style
- Order of resolution is not specified
- The parser figures it out
- Imperative Style
- The order of evaluation is fixed
- Important if the actions manipulate global state

- We'll explore actions as pure equations
- Style 1
- But note bison has a fixed order of evaluation for actions
- Example:

$$E3.val = E4.val + E5.val$$

- Must compute E4.val and E5.val before E3.val
- We say that E3.val depends on E4.val and E5.val

Dependency graph

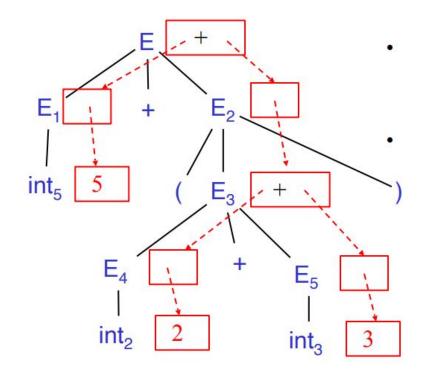
Each node labeled E has one slot for the val attribute

An attribute must be computed after all its successors in the dependency graph have been computed

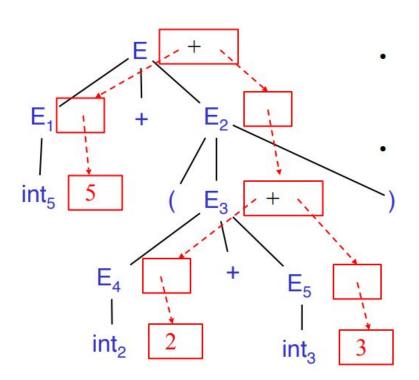
In this example attributes can be computed bottom-up

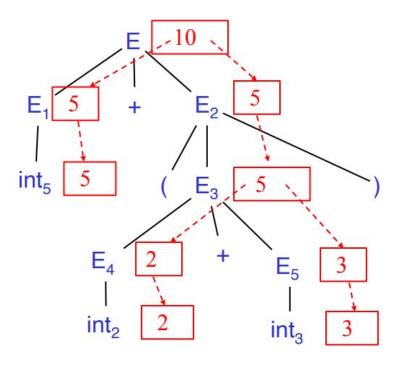
Such an order exists when there are no cycles

Cyclically defined attributes are not legal



Dependency graph





Two Types of Attribute Flow

Synthesized Attributes (Bottom-Up)

Flow in parse tree: Children → Parent

Similar to: Function return values

Use for: Expression values, building AST nodes

```
// Children compute, parent uses results
expr : expr '+' term { $$ = $1 + $3; }
// $1 and $3 from children
```

Inherited Attributes (Top-Down)

Flow in parse tree:

Parent → Children, or Sibling → Sibling

Similar to: Function parameters

Use for: Scope information, type context

```
// Parent passes info down to children
decl : TYPE IDENTIFIER {
    $$ = new_declaration($1, $2);
    // Pass type info down for variable usage
}
```

90% of the time, you'll use synthesized attributes!

Example: A line calculator

Each line contains an expression

$$E \rightarrow int \mid E + E$$

Each line is terminated with the = sign

$$L \rightarrow E = | + E =$$

- In second form the value of previous line is used as starting value
- A program is a sequence of lines

$$P \to \epsilon \mid P \mid L$$

Each E has a synthesized attribute val

- Calculated as before

Each L has an attribute val

$$L \rightarrow E = \{ L.val = E.val \}$$

| + E = { L.val = E.val + L.prev }

- We need the value of the previous line
- We use an inherited attribute L.prev

Real Example: Line Calculator with Memory

```
Problem: Build a calculator that remembers previous result

Input: Output:
5 = 5
+ 3 = 8
+ 2 = 10
```

Solution using Inherited Attributes

- previous flows down from program to line (inherited)
- Line result flows up to program (synthesized)

Example: A line calculator

Each line contains an expression

$$E \rightarrow int \mid E + E$$

Each line is terminated with the = sign

$$L \rightarrow E = | + E =$$

- In second form the value of previous line is used as starting value
- A program is a sequence of lines

$$P \to \epsilon \mid P \mid L$$

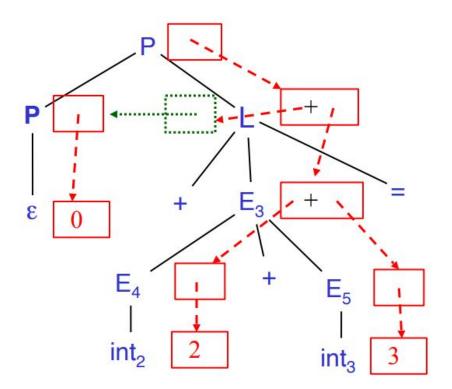
Each P has a synthesized attribute val

– The value of its last line

```
P \rightarrow \varepsilon \{ P.val = 0 \}
| P1 L { L.prev = P1.val;
| P.val = L.val }
```

- Each L has an inherited attribute prev
- L.prev is inherited from sibling P1.val

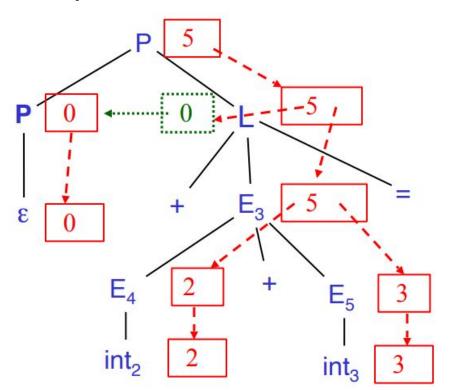
Example of Inherited Attributes

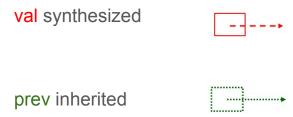




All can be computed in bottom-up order

Example of Inherited Attributes





All can be computed in bottom-up order

Semantic actions (syntax-directed translation)

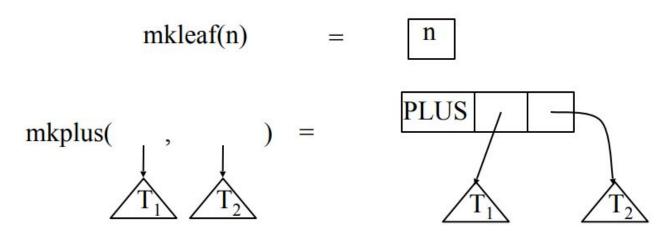
Semantic actions can be used to build ASTs

- And many other things as well
- Also used for type checking, code generation, computation, ...
- Process is called syntax-directed translation (SDT)
- Substantial generalization over CFGs

SDT: constructing an AST

We first define the AST data type

 Consider an abstract tree type with two constructors:



https://web.stanford.edu/class/cs143/lectures/lecture06.pdf

We define a synthesized attribute ast

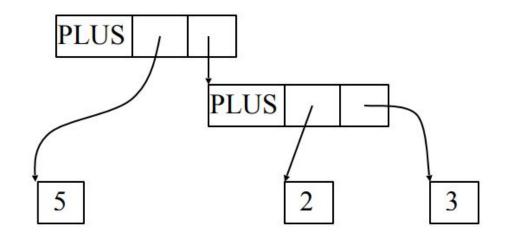
- Values of ast values are ASTs
- We assume that int.lexval is the value of the integer lexeme
- Computed using semantic actions

```
E → int E.ast = mkleaf(int.lexval)
| E1 + E2 E.ast = mkplus(E1.ast, E2.ast)
| ( E1 ) E.ast = E1.ast
```

Consider the string int5 '+' '(' int2 '+' int3 ')'

• A bottom-up evaluation of the ast attribute:

E.ast = mkplus(mkleaf(5), mkplus(mkleaf(2), mkleaf(3))



A simple intro to Bison examples (we will see more after LR(1))

YACC (Yet Another Compiler Compiler) was a widely used parser generator in the Unix environment, recently supplanted by the GNU Bison parser which is generally compatible.

Bison is designed to automatically invoke Flex as needed,

so it is easy to combine the two into a complete program

Bison is optimized for LR(1)

```
The file structure similar to lex:

%{
    /*C preamble code, #include etc*/

%}
   /* declarations*/
    %%
    /*grammar rules*/
    %%
    /*C epilogue code*/
```

```
응 {
#include <stdio.h>
응 }
%token TOKEN_INT
%token TOKEN_PLUS
%token TOKEN_MINUS
%token TOKEN_MUL
%token TOKEN_DIV
%token TOKEN_LPAREN
%token TOKEN_RPAREN
%token TOKEN_SEMI
%token TOKEN ERROR
```

```
응응
program : expr TOKEN SEMI;
expr : expr TOKEN_PLUS term
| expr TOKEN_MINUS term
| term
term : term TOKEN_MUL factor
| term TOKEN_DIV factor
| factor
factor: TOKEN MINUS factor
| TOKEN_LPAREN expr TOKEN_RPAREN
| TOKEN INT
응응
int yywrap() { return 0; }
```

```
#include <stdio.h>
extern int yyparse();
int main()
   if (yyparse() == 0)
       printf("Parse successful!\n");
   else
       printf("Parse failed.\n");
```

The resulting code creates a single function **yyparse()** that returns an integer:

zero indicates a successful parse, one indicates a parse error, and two indicates an internal problem such as memory exhaustion

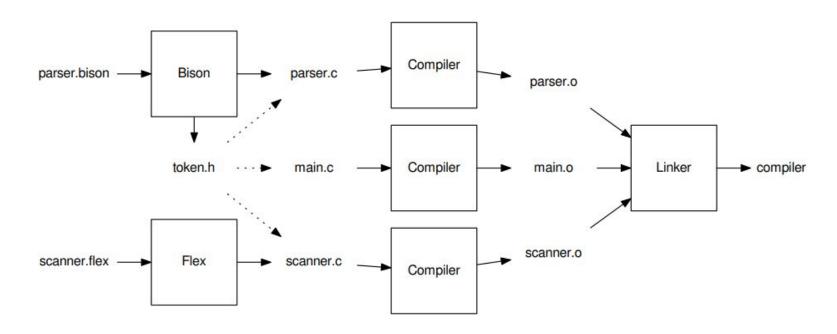
See examples:

https://www.gnu.org/software/bison/manual/html_n ode/Examples.html

Python ply example:

https://www.dabeaz.com/ply/example.html

Build Procedure for Bison and Flex Together



Building AST by using parser tools

```
// Define what our AST nodes look like
typedef struct ASTNode {
    enum { NUMBER, BINOP } type;
    union {
                                      // For numbers
        int value;
       struct {
                                      // For binary operations
            struct ASTNode* left;
            char operator;
            struct ASTNode* right;
        } binop;
    };
  } ASTNode;
  // function prototypes
  ASTNode* create number(int value);
  ASTNode* create_binop(ASTNode* left, char op, ASTNode* right);
  ASTNode* create variable(char* name);
  void free ast(ASTNode* node);
  void print ast(ASTNode* node, int indent);
```

```
#include <stdio.h>
#include <stdlib.h>
#include <string.h>
ASTNode* create number(int value) {
   ASTNode* node = malloc(sizeof(ASTNode));
   node->type = AST NUMBER;
   node->number value = value;
   return node;
ASTNode* create_binop(ASTNode* left, char op, ASTNode* right) {
   ASTNode* node = malloc(sizeof(ASTNode));
   node->type = AST BINOP;
   node->binop.left = left;
   node->binop.operator = op;
   node->binop.right = right;
   return node;
ASTNode* create variable(char* name) {
   ASTNode* node = malloc(sizeof(ASTNode));
   node->type = AST VARIABLE;
   node->variable name = strdup(name); // Copy the string
   return node;
```

#include "ast.h"

```
void print ast(ASTNode* node, int indent) {
   if (node == NULL) return;
   for (int i = 0; i < indent; i++) printf(" ");</pre>
    switch (node->type) {
        case AST NUMBER:
            printf("Number: %d\n", node->number_value);
            break:
        case AST BINOP:
            printf("BinOp: %c\n", node->binop.operator);
            print ast(node->binop.left, indent + 1);
            print ast(node->binop.right, indent + 1);
            break;
        case AST VARIABLE:
            printf("Variable: %s\n", node->variable_name);
            break;
```

```
%{
#include "ast.h"
                                                                      Usage in Bison
%}
%union {
     int int value;
     char* string value;
     ASTNode* node value;
%token <int value> NUMBER
%token <string value> IDENTIFIER
%type <node value> expr term factor
%%
         expr '+' term { $$ = create_binop($1, '+', $3); }
expr '-' term { $$ = create_binop($1, '-', $3); }
term { $$ = $1; }
expr:
term : term '*' factor { $$ = create_binop($1, '*', $3); }
term '/' factor { $$ = create_binop($1, '/', $3); }
                   \{ \$\$ = \$1; \}
         factor
factor : NUMBER
                             { $$ = create_number($1); }
           IDENTIFIER { $$ = create_variable($1); }
'(' expr ')' { $$ = $2; }
```

```
class ASTNode:
                                                                class Variable(ASTNode):
    """Base class for all AST nodes"""
                                                                    def init (self, name):
   def repr (self):
                                                                        self.type = "VARIABLE"
       return self. str ()
                                                                        self.name = name
class Number(ASTNode):
                                                                    def str (self):
   def init (self, value):
                                                                        return f"Variable({self.name})"
       self.type = "NUMBER"
       self.value = value
                                                                # Constructor functions (for consistency with other
   def str (self):
                                                                implementations)
       return f"Number({self.value})"
                                                                def create number(value):
class BinOp(ASTNode):
                                                                    return Number(value)
   def init (self, left, op, right):
       self.type = "BINOP"
                                                                def create binop(left, op, right):
       self.left = left
                                                                    return BinOp(left, op, right)
       self.op = op
       self.right = right
                                                                def create variable(name):
                                                                    return Variable(name)
   def str (self):
       return f"BinOp({self.op}, {self.left}, {self.right})"
```

```
# Pretty printer
def print_ast(node, indent=0):
    if node is None:
        return

spaces = " " * indent

if node.type == "NUMBER":
        print(f"{spaces}Number: {node.value}")
    elif node.type == "BINOP":
```

print(f"{spaces}BinOp: {node.op}")
print_ast(node.left, indent + 1)
print_ast(node.right, indent + 1)

print(f"{spaces}Variable: {node.name}")

elif node.type == "VARIABLE":

Use in PLY

```
import ply.yacc as yacc
from ast import *
# Parser rules with AST construction
def p expr plus(p):
    'expr : expr PLUS term'
    p[0] = create_binop(p[1], '+', p[3])
def p expr minus(p):
    'expr : expr MINUS term'
    p[0] = create_binop(p[1], '-', p[3])
def p_expr_term(p):
    'expr : term'
    p[0] = p[1]
def p_term_times(p):
    'term : term TIMES factor'
    p[0] = create_binop(p[1], '*', p[3])
```

```
def p term divide(p):
    'term : term DIVIDE factor'
    p[0] = create_binop(p[1], '/', p[3])
def p term factor(p):
    'term : factor'
    p[0] = p[1]
def p factor number(p):
    'factor : NUMBER'
    p[0] = create_number(p[1])
def p factor variable(p):
    'factor : IDENTIFIER'
    p[0] = create_variable(p[1])
def p factor parens(p):
    'factor : LPAREN expr RPAREN'
    p[0] = p[2]
```

Ocaml

```
(* Algebraic data type for AST nodes *)
type ast_node =
    Number of int
    BinOp of ast_node * char * ast_node
    Variable of string
(* Constructor functions *)
let create_number value = Number value
let create binop left op right = BinOp (left, op, right)
let create_variable name = Variable name
```

```
(* In parser.mly *)
%{
open Ast
%}
%token <int> NUMBER
%token <string> IDENTIFIER
%token PLUS MINUS TIMES DIVIDE LPAREN RPAREN
%start main
%type <Ast.ast node> main expr term factor
%%
main:
   expr { $1 }
expr:
   expr PLUS term { create_binop $1 '+' $3 }
   expr MINUS term { create_binop $1 '-' $3 }
                  { $1 }
   term
term:
   term TIMES factor { create_binop $1 '*' $3 }
   term DIVIDE factor { create_binop $1 '/' $3 }
   factor
                      { $1 }
factor:
   NUMBER
            { create number $1 }
   IDENTIFIER { create_variable $1 }
   LPAREN expr RPAREN { $2 }
```

```
// Define what our AST nodes look like
typedef struct ASTNode {
                                                                           Result for "2 + 3 * 4"
   enum { NUMBER, BINOP } type;
   union {
       int value;
                                    // For numbers
       struct {
                                    // For binary operations
           struct ASTNode* left;
           char operator;
           struct ASTNode* right;
       } binop;
   };
  } ASTNode;
                                                                             BinOp: +
  // function prototypes
                                                                               Number: 2
expr : expr '+' term { \$\$ = \text{create binop}(\$1, '+', \$3); }
      expr '-' term { $$ = create_binop($1, '-', $3); }
                                                                               BinOp: *
                      \{ \$\$ = \$1; \}
      term
                                                                                  Number: 3
term : term '*' factor { $$ = create_binop($1, '*', $3); }
                                                                                  Number: 4
      factor
                \{ \$\$ = \$1; \}
factor : NUMBER { $$ = create number($1); }
        '(' expr ')' { $$ = $2; }
```

Semantic Actions: Key Takeaways

```
When to Use Semantic Actions:
     Building ASTs (most common)
     expr : expr '+' term \{ \$\$ = \text{new AddNode}(\$1, \$3); \}
    Type Checking
     assignment : ID '=' expr { check types($1.type, $3.type); }
    Symbol Table Management
     declaration : TYPE ID { add to symbol table($2, $1); }
     Code Generation
     expr : expr '+' term \{ \$\$ = generate \ add(\$1, \$3); \}
```

In Project, you need to write actions for smth like this

```
Goal ::= MainClass ( ClassDeclaration )* <EOF>
MainClass ::= "class" Identifier "{" "public" "static" "void" "main" "(" "String" "[" "]" Identifier ")"
"{" Statement "}" "}"
ClassDeclaration ::= "class" Identifier ( "extends" Identifier )? "{" ( VarDeclaration )* (
MethodDeclaration )* "}"
VarDeclaration ::= Type Identifier ";"
MethodDeclaration ::= "public" Type Identifier "(" ( Type Identifier ( "," Type Identifier )* )? ")" "{"
( VarDeclaration )* ( Statement )* "return" Expression ";" "}"
Type ::= "int" "[" "]"
    "boolean"
    "int"
    Identifier
Statement ::= "{" ( Statement )* "}"
    "if" "(" Expression ")" Statement "else" Statement
    "while" "(" Expression ")" Statement
    "System.out.println" "(" Expression ")" ";"
                                                                 Minijava grammar (written in BNF)
    Identifier "=" Expression ";"
                                                                The MiniJava Language Specification Grammar
    Identifier "[" Expression "]" "=" Expression ";"
```

```
Expression ::= Expression ( "&&" | "<" | "+" | "-" | "*" ) Expression
    Expression "[" Expression "]"
    Expression "." "length"
    Expression "." Identifier "(" ( Expression ( ", " Expression )* )? ")"
    <INTEGER LITERAL>
    "true"
    "false"
    Identifier
    "this"
   "new" "int" "[" Expression "]"
   "new" Identifier "(" ")"
    "!" Expression
   "(" Expression ")"
Identifier ::= <IDENTIFIER>
```

Minijava grammar (written in BNF)
https://www.cambridge.org/resources/052182060X/
https://www.cs.purdue.edu/homes/hosking/502/project/grammar.html

Summary of SDT

We can specify language syntax using CFG

- A parser will answer whether $s \in L(G)$
- ... and will trace a parse tree
- in whose productions we build an AST
- that we pass on to the rest of the compiler

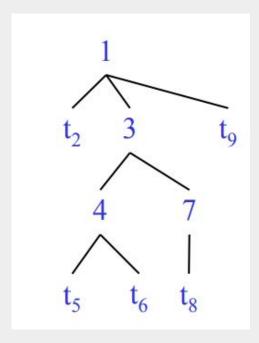
Intro to top down parsing

The parse tree is constructed

- From the top
- From left to right

Terminals are seen in order of appearance in the token stream:

t2 t5 t6 t8 t9



```
E \rightarrow T \mid T + E
T \rightarrow int \mid int * T \mid (E)
```

Token stream is: (int5)

Start with top-level non-terminal E

- Try the rules for E in order

If there is a mismatch backtrack

$$E \rightarrow T \mid T + E$$

$$T \rightarrow int \mid int * T \mid (E)$$

• Mismatch: int is not (

 $E \rightarrow T \mid T + E$

 $T \rightarrow int \mid int * T \mid (E)$

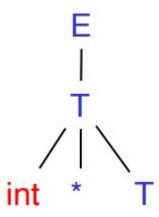
backtrack

Token stream is: (int5)

Start with top-level non-terminal E

– Try the rules for E in order

If there is a mismatch backtrack

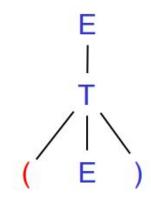


https://web.stanford.edu/class/cs143/lectures/lecture06.pdf

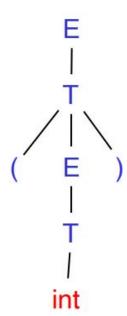
$$E \rightarrow T \mid T + E$$
 $T \rightarrow int \mid int * T \mid (E)$

Token stream is: (int5)

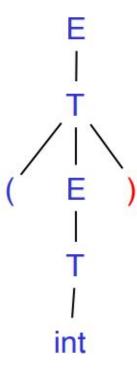
- Match: (
- Advance input
 - Move to int5



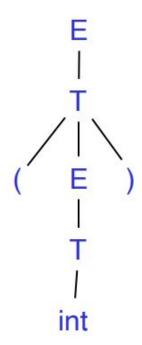
- Match: int5
- Advance input
 - o Move to)



- Match:)
- Advance input
 - Move



• End of input, accept!



Writing a limited recursive descent parser

Let TOKEN be the type of tokens

Special tokens INT, OPEN, CLOSE, PLUS,
 TIMES

Let the global **next** point to the next token

Define boolean functions that check the token string for a match of

A given token terminal

```
bool term(TOKEN tok) {
    return *next++ == tok;
}
```

• The nth production of S:

```
bool Sn() { ... }
```

• Try all productions of S:

```
bool S() { ... }
```

Writing a limited recursive descent parser

```
    For production E → T
    bool E1() {
        return T();
}
    For production E → T + E
    bool E2() {
        return T() && term(PLUS) && E();
}
```

```
For all productions of E (with backtracking)
bool E(){
   TOKEN *save = next;
   return (next = save, E1()) || (next = save, E2());
}
```

```
Functions for non-terminal T
bool T1() { return term(INT); }
bool T2() { return term(INT) && term(TIMES) && T(); }
bool T3() { return term(OPEN) && E() && term(CLOSE); }
bool T(){
   TOKEN *save = next;
   return (next = save, T1()
       || (next = save, T2())
       || (next = save, T3());
```

Notes on writing recursive descent parser

To start the parser

- Initialize next to point to first token
- Invoke E()
- Notice how this simulates the example parse
- Easy to implement by hand
- But not completely general
- Cannot backtrack once a production is successful
- Works for grammars where at most one production can succeed for a non-terminal

Example

```
E \rightarrow T \mid T + E
                                                                                  (int)
T \rightarrow int \mid int * T \mid (E)
bool term(TOKEN tok) { return *next++ == tok; }
bool E1() { return T(); }
bool E2() { return T() && term(PLUS) && E(); }
bool E() {TOKEN *save = next;
     return (next = save, E1()) || (next = save, E2()); }
bool T1() { return term(INT); }
bool T2() { return term(INT) && term(TIMES) && T(); }
bool T3() { return term(OPEN) && E() && term(CLOSE); }
bool T() { TOKEN *save = next;
     return (next = save, T1())
      || (next = save, T2())
      || (next = save, T3()); }
```

Programming examples

```
// Direct translation of grammar rules
ASTNode* parse expression() {
    ASTNode* left = parse term();
   if (current token == PLUS) {
        get next token();
        ASTNode* right = parse expression();
        return create binop node('+', left, right);
    return left;
ASTNode* parse term() {
    ASTNode* left = parse factor();
   if (current token == TIMES) {
        get next token();
        ASTNode* right = parse term();
        return create binop node('*', left, right);
    return left;
```

Programming examples

```
# Object-oriented with exception handling
def parse expression(self):
    left = self.parse term()
    while self.current token.type in ('PLUS', 'MINUS'):
        op = self.current token
        self.advance()
        right = self.parse term()
        left = BinOp(left, op, right)
    return left
def parse term(self):
    left = self.parse factor()
    while self.current token.type in ('TIMES', 'DIVIDE'):
        op = self.current token
        self.advance()
        right = self.parse factor()
        left = BinOp(left, op, right)
    return left
```

Programmin g examples

```
(* Pattern matching and recursive structure *)
let rec parse expression tokens =
  let left, tokens = parse term tokens in
  match tokens with
  (PLUS, ) :: rest ->
     let right, rest = parse_expression rest in
     Binop(Plus, left, right), rest
  (MINUS, ) :: rest ->
     let right, rest = parse_expression rest in
     Binop(Minus, left, right), rest
  -> left, tokens
and parse term tokens =
  let left, tokens = parse_factor tokens in
 match tokens with
  (TIMES, _) :: rest ->
     let right, rest = parse term rest in
     Binop(Times, left, right), rest
  (DIVIDE, ) :: rest ->
     let right, rest = parse term rest in
     Binop(Divide, left, right), rest
  _ -> left, tokens
```

When Recursive Descent Does Not Work

```
Consider a production S \rightarrow S a
bool S1() { return S() && term(a); }
bool S() { return S1(); }
```

- S() goes into an infinite loop
- A left-recursive grammar has a non-terminal S S \rightarrow + S α for some α
- Recursive descent does not work in such cases

Elimination of Left Recursion

Consider the left-recursive grammar

$$S \rightarrow S \alpha \mid \beta$$

S generates all strings starting with a β and followed by a number of α

with **right-recursion**

$$S \rightarrow \beta S'$$

$$S' \rightarrow \alpha \ S' \mid \epsilon$$

More on elimination rule for left recursion

In general

Rewrite as

$$S \rightarrow S \alpha_1 \mid ... \mid S \alpha_n \mid \beta_1 \mid ... \mid \beta_m$$

$$S \rightarrow \beta_1 S' \mid \dots \mid \beta_m S'$$

All strings derived from **S** start with one of $\beta 1,...$, βm and continue with several instances of

 $S' \rightarrow \alpha_1 \; S' \; | \; ... \; | \; \alpha_n \; S' \; | \; \epsilon$

Example

- 1. $P \rightarrow E$
- 2. $E \rightarrow E + T$
- 3. $E \rightarrow T$
- 4. $T \rightarrow id$
- 5. $T \rightarrow int$

- 1. $P \rightarrow E$
- 2. $E \rightarrow T E'$
- 3. E' \rightarrow + T E'
- 4. $E' \rightarrow \epsilon$
- 5. $T \rightarrow id$
- 6. $T \rightarrow int$

Indirect left recursion

The grammar

$$S \rightarrow A \alpha \mid \delta$$

$$A \rightarrow S \beta$$

is also left-recursive because $S \rightarrow + S \beta \alpha$

This left-recursion can also be eliminated

A general algorithm (skipped)

```
impose an order on the nonterminals, A_1, A_2, \ldots, A_n
for i \leftarrow 1 to n do;
     for j \leftarrow 1 to i - 1 do;
          if \exists a production A_i \rightarrow A_j \gamma
               then replace A_i \rightarrow A_j \gamma with one or more
                      productions that expand A_j
     end:
     rewrite the productions to eliminate
           any direct left recursion on Ai
end:
```

■ FIGURE 3.6 Removal of Indirect Left Recursion.

General algorithm for top down parsing

 on the mismatch, it must undo the actions

```
root \leftarrow node for the start symbol, S;
focus ← root:
push(null);
word \leftarrow NextWord():
while (true) do:
   if (focus is a nonterminal) then begin;
       pick next rule to expand focus (A \to \beta_1, \beta_2, ..., \beta_n);
       build nodes for \beta_1, \beta_2 \dots \beta_n as children of focus;
       push(\beta_n,\beta_{n-1},...,\beta_2);
       focus \leftarrow \beta_1;
   end:
   else if (word matches focus) then begin;
       word \leftarrow NextWord():
       focus \leftarrow pop()
   end:
   else if (word = eof and focus = null)
       then accept the input and return root;
       else backtrack:
end:
```

Engineering a Compiler by Cooper and Torczon, 2nd Ed. ch. 3

■ FIGURE 3.2 A Leftmost, Top-Down Parsing Algorithm.

Backtrack free parsing

The major source of inefficiency in the leftmost, top-down parser arises from its need to backtrack

on the mismatch, it must undo the actions

0	Goal	\rightarrow	Expr	6	Term'	\rightarrow	× Factor Term'
1	Expr	\rightarrow	Term Expr'	7		1	÷ Factor Term'
2	Expr'	\rightarrow	+ Term Expr'	8		1	ϵ
3		1	- Term Expr'	9	Factor	\rightarrow	<u>(</u> Expr <u>)</u>
4		1	ϵ	10		1	num
5	Term	\rightarrow	Factor Term'	11		1	name

Using a lookahead symbol

- For this grammar, the parser can avoid backtracking by
 - o considering both the focus symbol and the next input symbol(lookahead symbol).
- Using a one symbol lookahead, the parser can disambiguate all of the choices that arise in parsing the right-recursive expression grammar.
 - the grammar is backtrack free with a lookahead of one symbol

0	Goal	\rightarrow	Expr	6	$Term' \rightarrow$	× Factor Term'
1	Expr	\rightarrow	Term Expr'	7	l I	÷ Factor Term'
2	Expr'	\rightarrow	+ Term Expr'	8	1	ϵ
3		Ĺ	- Term Expr'	9	$Factor \rightarrow$	<u>(</u> Expr <u>)</u>
4		I	ϵ	10	l I	num
5	Term	\rightarrow	Factor Term'	11	l i	name

For each grammar symbol α ,

define the set $first(\alpha)$ as the set of terminal symbols that can appear as the first word in some string derived from α

- If α is either a terminal, ε, or eof,
 - then first(α) is { α }.
- For a nonterminal A,
 - o **first(A)** contains the complete set of terminal symbols that can appear as the leading symbol in a sentential form derived from A.

example

	num	name	+	_	×	÷	<u>(</u>	<u>)</u>	eof	ϵ
FIRST	num	name	+	-	Х	÷	<u>(</u>	<u>)</u>	eof	ϵ

	Expr	Expr'	Term	Term'	Factor
FIRST	(,name,num	+,-, ϵ	(,name,num	X, \div , ϵ	(,name,num

Computing First sets

```
For Terminals:
   For each terminal a \in \Sigma: FIRST(a) = {a}
For Non-Terminals:
Repeat:
   For each rule X \rightarrow Y1Y2...Yk in a grammar G:
         if a is in FIRST(Y1) OR a is in FIRST(Yn) and Y1...Yn-1 \Rightarrow \epsilon
           then add a to FIRST(X)
         if Y1...Yk \Rightarrow \varepsilon
           then add \varepsilon to FIRST(X).
until no more changes occur
```

https://www3.nd.edu/~dthain/compilerbook/chapter4.pdf

 $first(\varepsilon) = \{\varepsilon\}$

matches no word returned by the scanner.

parser needs to know which words can appear as the leading symbol after a valid application of rule 4

the set of symbols that can follow an Expr'

0	Goal	\rightarrow	Expr	6	$Term' \rightarrow$	× Factor Term'
1	Expr	\rightarrow	Term Expr'	7	1	÷ Factor Term'
2	Expr'	\rightarrow	+ Term Expr'	8	1	ϵ
3		I	- Term Expr'	9	$Factor \rightarrow$	(Expr)
4		I	ϵ	10		num
5	Term	\rightarrow	Factor Term'	11		name

Engineering a Compiler by Cooper and Torczon, 2nd Ed. ch. 3

Follow sets of nonterminals

0	Goal	\rightarrow	Expr	6	Term'	\rightarrow	× Factor Term'
1	Expr	\rightarrow	Term Expr'	7			÷ Factor Term'
2	Expr'	\rightarrow	+ Term Expr'	8			ϵ
3		1	- Term Expr'	9	Factor -	\rightarrow	(Expr)
4		1	ϵ	10		1	num
5	Term	\rightarrow	Factor Term'	11			name

	Expr	Expr'	Term	Term'	Factor
FOLLOW	eof, <u>)</u>	eof, <u>)</u>	eof,+,-, <u>)</u>	eof,+,-, <u>)</u>	eof,+,-,x,÷, <u>)</u>

Computing Follow Sets for a Grammar G

```
FOLLOW(A) is the set of terminals that can come after non-terminal A
FOLLOW(S) = \{\$\} where S is the start symbol.
Repeat:
   if A \rightarrow \alpha B\beta then:
        add FIRST(\beta) (excepting \epsilon) to FOLLOW(B).
   if A \rightarrow \alphaB or FIRST(\beta) contains \epsilon then:
        add FOLLOW(A) to FOLLOW(B).
until no more changes occur.
```

https://www3.nd.edu/~dthain/compilerbook/chapter4.pdf

Using first and follow

For productions

$$\operatorname{FIRST}^{+}(A \to \beta) = \begin{cases} \operatorname{FIRST}(\beta) & \text{if } \epsilon \notin \operatorname{FIRST}(\beta) \\ \operatorname{FIRST}(\beta) \cup \operatorname{FOLLOW}(A) & otherwise \end{cases}$$

In backtrack free grammar, any nonterminal A $A \rightarrow \beta_1 \mid \beta_2 \mid \cdots \mid \beta_n$

$$\operatorname{FIRST}^+(A \to \beta_i) \cap \operatorname{FIRST}^+(A \to \beta_j) = \emptyset, \ \forall \ 1 \le i, j \le n, \ i \ne j.$$

0	Goal	\rightarrow	Expr	6	Term'	\rightarrow	× Factor Term'
1	Expr	\rightarrow	Term Expr'	7		1	÷ Factor Term'
2	Expr'	\rightarrow	+ Term Expr'	8		1	ϵ
3		Ĺ	- Term Expr'	9	Factor	\rightarrow	(Expr)
4		I	ϵ	10		1	num
5	Term	\rightarrow	Factor Term'	11		1	name

only productions 4 and 8 have different first+ and first sets

	Production	FIRST set	FIRST ⁺ set
4	$\textit{Expr}' \rightarrow \epsilon$	$\set{\epsilon}$	$\{\epsilon, \text{eof}, \underline{)}\}$
8	$\mathit{Term}' \to \epsilon$	$\{\epsilon\}$	$\{\epsilon$, eof, +, -, $\underline{)}\}$

The common prefix

Because productions 11, 12, and 13 all begin with name, they have identical first+ sets. When the parser tries to expand an instance of Factor with a lookahead of name, it has no basis to choose among 11, 12, and 13.

Eliminating common prefixes (left factoring)

transform these productions to create disjoint first+ sets.

Take a non terminal

$$A \rightarrow \alpha\beta1 \mid \alpha\beta2 \mid \cdot \cdot \cdot \cdot \mid \alpha\betan$$
$$\mid \gamma1 \mid \gamma2 \mid \cdot \cdot \cdot \cdot \mid \gammaj$$

Rewrite the original as

$$A \rightarrow \alpha B \mid \gamma 1 \mid \gamma 2 \mid \cdot \cdot \cdot \cdot \mid \gamma j$$

$$B \rightarrow \beta 1 \mid \beta 2 \mid \cdot \cdot \cdot \cdot \mid \beta n$$

Summary

Backtrack-free grammars lend themselves to simple and efficient parsing with a recursive descent

By using first, first+ and follow sets we can generate **predictive top** down-parser(LL(1) parser)

An example predictive LL(1) with recursive descent parser

Three helper functions are needed:

scan_token()

returns the next token on the input stream.

putback_token(t)

puts an unexpected token back on the input stream.

expect_token(t)

calls scan token to retrieve the next token.

1.
$$P \rightarrow E$$

2.
$$E \rightarrow T E'$$

3.
$$E' \rightarrow + T E'$$

4. E'
$$\rightarrow \epsilon$$

5.
$$T \rightarrow F T'$$

6. T'
$$\rightarrow$$
 * F T'

7.
$$T' \rightarrow \epsilon$$

8.
$$F \rightarrow (E)$$

9.
$$F \rightarrow int$$

```
token_t t = scan_token();
7. T' \rightarrow \epsilon
                                                        if (t == TOKEN PLUS) {
8. F \rightarrow (E)
                                                             return parse T() && parse E prime();
9. F \rightarrow int
                                                        else{
                                                             putback token(t);
                                                             return 1;
                                                     https://www3.nd.edu/~dthain/compilerbook/chapter4.pdf
```

int parse_P(){

int parse E(){

int parse_E_prime(){

return parse E() && expect token(TOKEN EOF);

return parse_T() && parse_E_prime();

1. $P \rightarrow E$

2. $E \rightarrow T E'$

4. E' $\rightarrow \epsilon$

5. $T \rightarrow F T'$

3. E' \rightarrow + T E'

6. T' \rightarrow * F T'

```
1. P \rightarrow E
                                                               int parse_F(){
                                                                  token t t = scan token();
2. E \rightarrow T E'
                                                                  if (t == TOKEN_LPAREN) {
3. E' \rightarrow + T E'
                                                                       return parse_E() && expect_token(TOKEN_RPAREN);
4. E' \rightarrow \epsilon
5. T \rightarrow F T'
                                                                  else if (t == TOKEN_INT) {
6. T' \rightarrow * F T'
                                                                       return 1;
7. T' \rightarrow \epsilon
8. F \rightarrow (E)
                                                                  else
9. F \rightarrow int
                                                                       printf("parse error: unexpected token %s\n",
                                                                               token_string(t));
 int parse_T_prime(){
                                                                       return 0;
     token_t t = scan_token();
     if (t == TOKEN_MULTIPLY)
          return parse_F() && parse_T_prime();
     else
          putback_token(t);
          return 1;
                                                          https://www3.nd.edu/~dthain/compilerbook/chapter4.pdf
```

Next week table driven LL(1) parser & bottom-up parser