

Register Allocation

CS143

Lecture 16

<https://web.stanford.edu/class/cs143/>

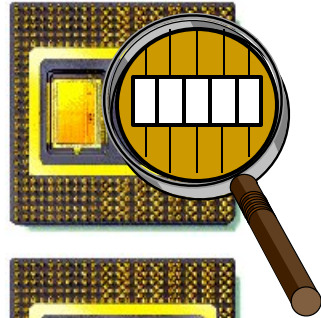
Instructor: Fredrik Kjolstad

Slide design by Prof. Alex Aiken, with modifications

Lecture Outline

- Memory Hierarchy Management
- Register Allocation
 - Register interference graph
 - Graph coloring heuristics
 - Spilling
- Cache Management

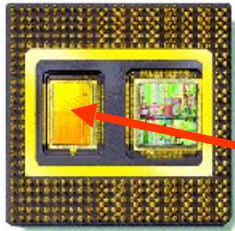
The Memory Hierarchy



Registers

1 cycle

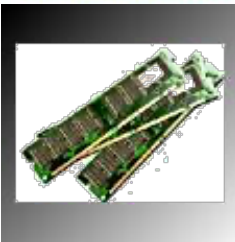
256-8000 bytes



Cache

3 cycles

256k-40MB



Main memory

20-100 cycles

4GB-32+G



Disk

0.5-5M cycles

1-10TB's

Managing the Memory Hierarchy

- Most programs are written as if there are only two kinds of memory: main memory and disk
 - Programmer is responsible for moving data from disk to memory (e.g., file I/O)
 - Hardware is responsible for moving data between memory and caches
 - Compiler is responsible for moving data between memory and registers

Current Trends

- Power usage limits
 - Size and speed of registers/caches
 - Speed of processors
- But
 - The cost of a cache miss is very high
 - Typically requires 2-3 caches to bridge fast processor with large main memory
- It is very important to:
 - Manage registers properly
 - Manage caches properly
- Compilers are good at managing registers

The Register Allocation Problem

- Intermediate code uses unlimited temporaries
 - Simplifies code generation and optimization
 - Complicates final translation to assembly
- Typical intermediate code uses too many temporaries

The Register Allocation Problem (Cont.)

- The problem:

Rewrite the intermediate code to use no more temporaries than there are machine registers
- Method:
 - Assign multiple temporaries to each register
 - But without changing the program behavior

History

- Register allocation is as old as compilers
 - Register allocation was used in the original FORTRAN compiler in the '50s
 - Very crude algorithms
- A breakthrough came in 1980
 - Register allocation scheme based on graph coloring
 - Relatively simple, global and works well in practice

An Example

- Consider the program

```
a := c + d
e := a + b
f := e - 1
```

- Assume **a** and **e** dead after use
 - Temporary **a** can be “reused” after **a + b**
 - Temporary **e** can be “reused” after **e - 1**

- Can allocate **a**, **e**, and **f** all to one register (r_1):

```
r1 := r2 + r3
r1 := r1 + r4
r1 := r1 - 1
```

- A dead temporary is not needed
 - A dead temporary can be reused

The Idea

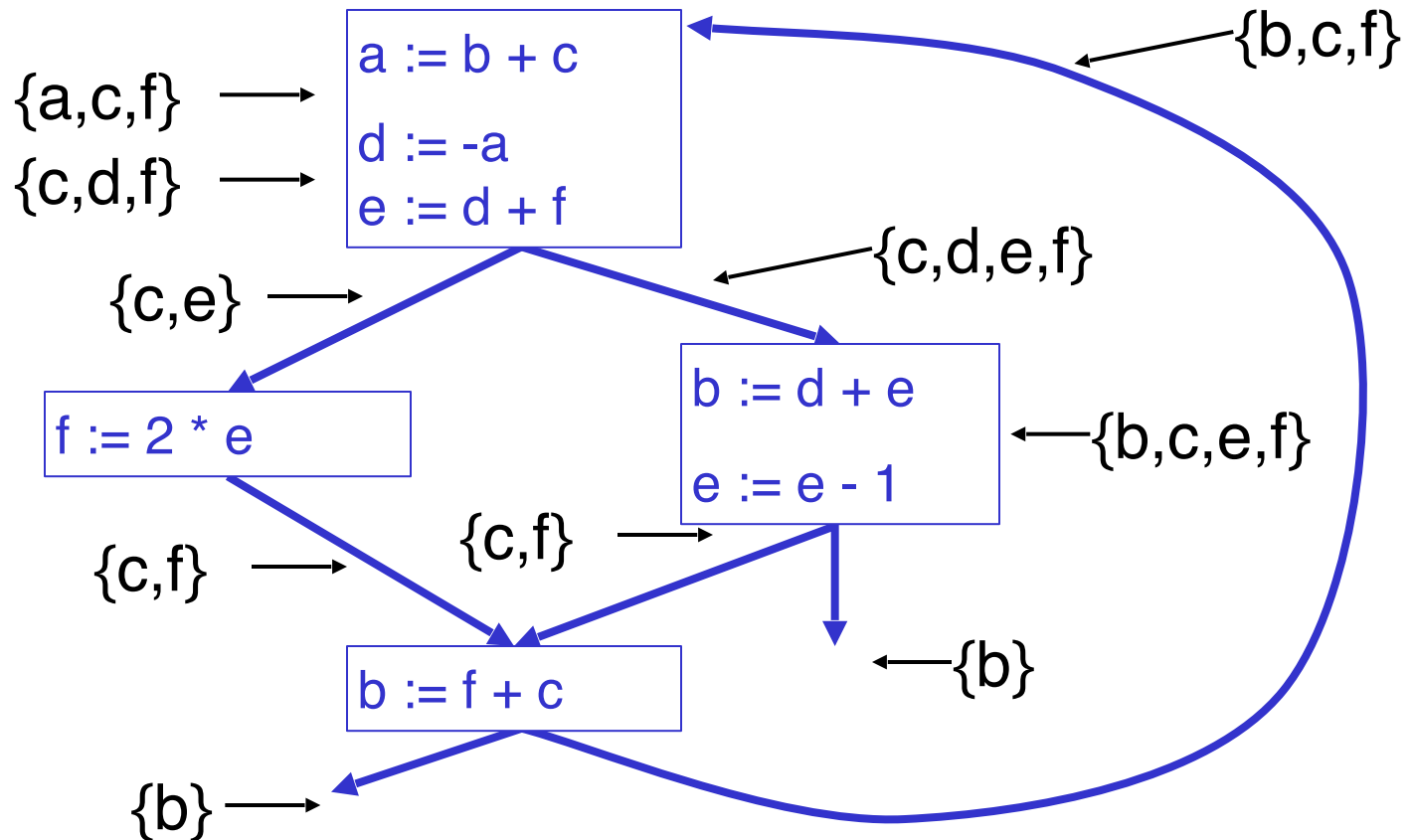
Temporaries t_1 and t_2 can share the same register if at any point in the program at most one of t_1 or t_2 is live .

Or

If t_1 and t_2 are live at the same time, they cannot share a register

Algorithm: Part I

- Compute live variables for each point:

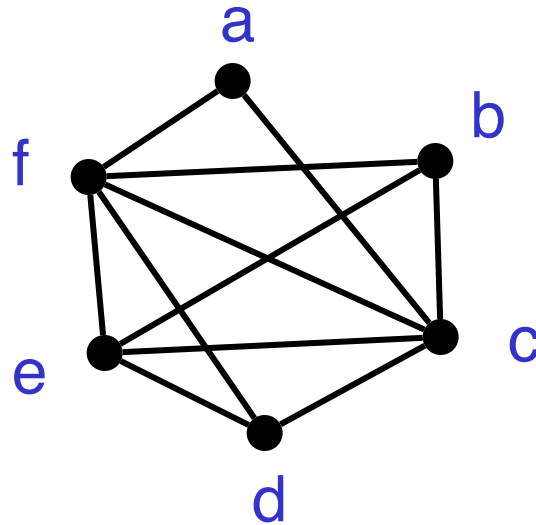


The Register Interference Graph

- Construct an undirected graph
 - A node for each temporary
 - An edge between t_1 and t_2 if they are live simultaneously at some point in the program
- This is the register interference graph (RIG)
 - Two temporaries can be allocated to the same register if there is no edge connecting them

Example

- For our example:



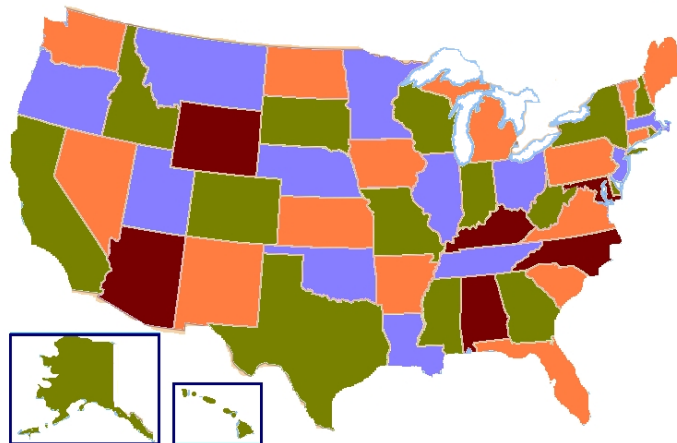
- E.g., **b** and **c** cannot be in the same register
- E.g., **b** and **d** could be in the same register

Notes on Register Interference Graphs

- Extracts exactly the information needed to characterize legal register assignments
- Gives a global (i.e., over the entire flow graph) picture of the register requirements
- After RIG construction the register allocation algorithm is architecture independent

Definitions

- A coloring of a graph is an assignment of colors to nodes, such that nodes connected by an edge have different colors
- A graph is k-colorable if it has a coloring with k colors

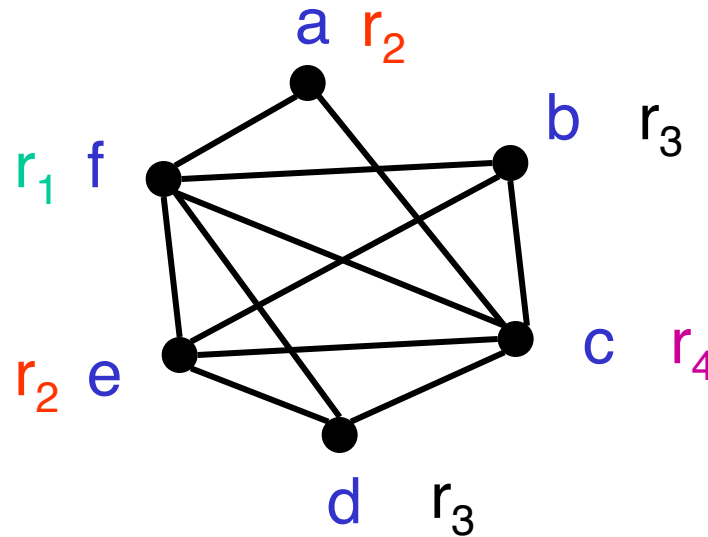


Register Allocation Through Graph Coloring

- In our problem, colors = registers
 - We need to assign colors (registers) to graph nodes (temporaries)
- Let k = number of machine registers
- If the RIG is k -colorable then there is a register assignment that uses no more than k registers

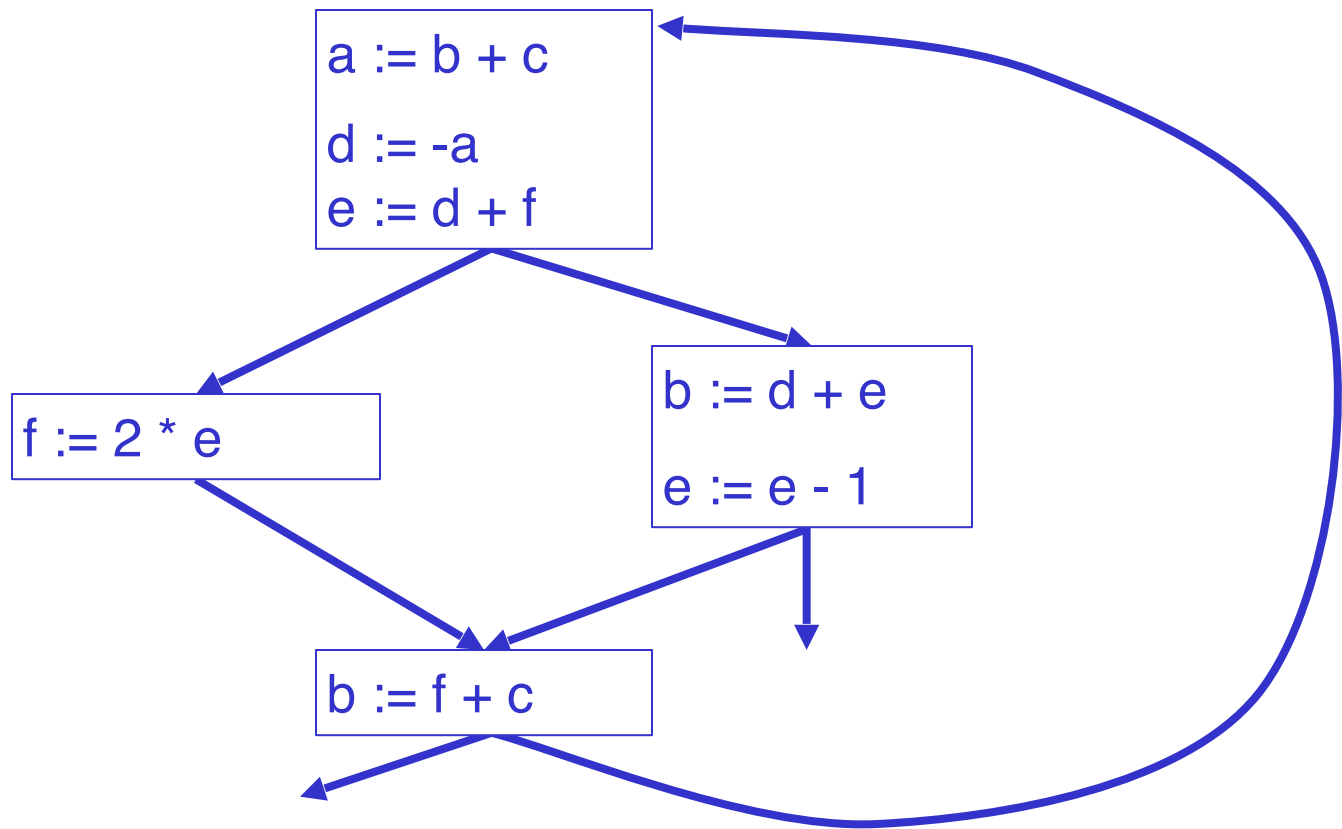
Graph Coloring Example

- Consider the example RIG



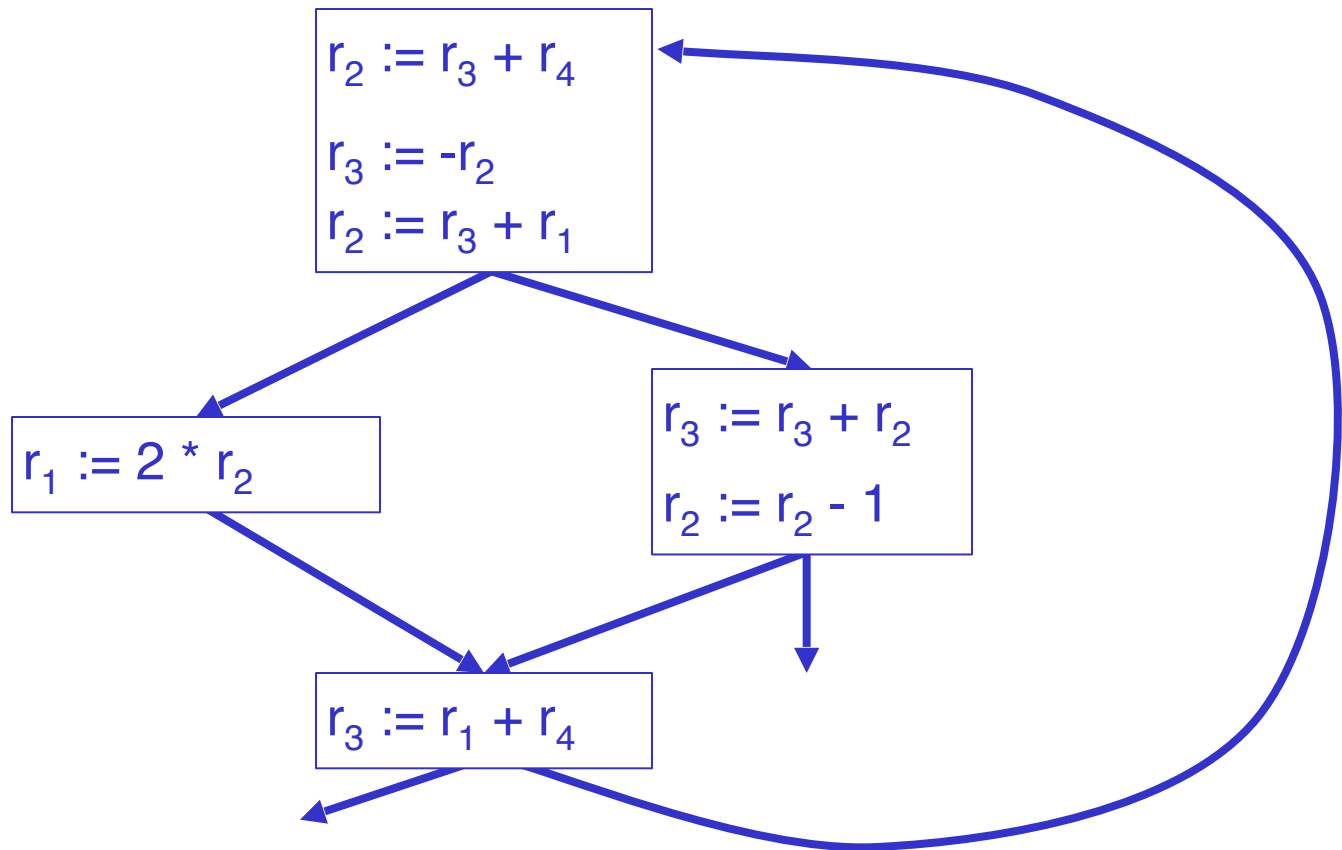
- There is no coloring with less than 4 colors
- There are 4-colorings of this graph

Example Review



Example After Register Allocation

- Under this coloring the code becomes:



Computing Graph Colorings

- How do we compute graph colorings?
- It isn't easy:
 1. This problem is very hard (NP-hard). No efficient algorithms are known.
 - Solution: use heuristics
 2. A coloring might not exist for a given number of registers
 - Solution: later

Graph Coloring Heuristic

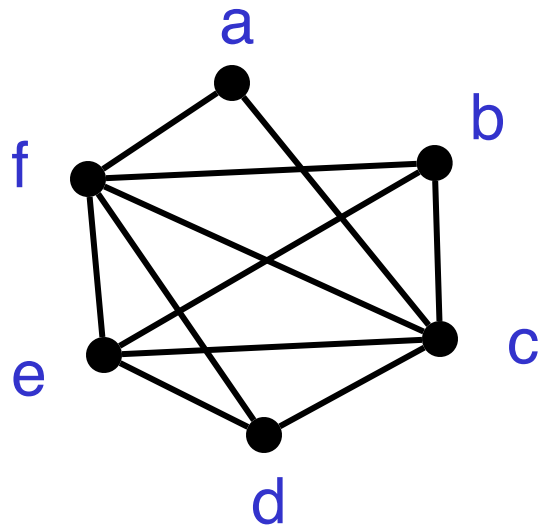
- Observation:
 - Pick a node t with fewer than k neighbors in RIG
 - Eliminate t and its edges from RIG
 - If resulting graph is k -colorable, then so is the original graph
- Why?
 - Let c_1, \dots, c_n be the colors assigned to the neighbors of t in the reduced graph
 - Since $n < k$ we can pick some color for t that is different from those of its neighbors

Graph Coloring Heuristic

- The following works well in practice:
 - Pick a node t with fewer than k neighbors
 - Put t on a stack and remove it from the RIG
 - Repeat until the graph has one node
- Assign colors to nodes on the stack
 - Start with the last node added
 - At each step pick a color different from those assigned to already colored neighbors

Graph Coloring Example (1)

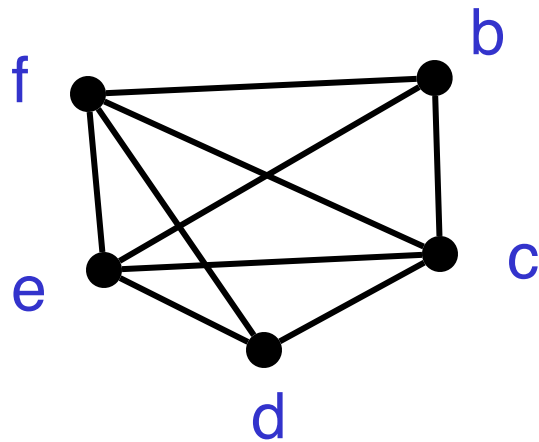
- Start with the RIG and with $k = 4$:



Stack: $\{\}$

- Remove **a**

Graph Coloring Example (2)

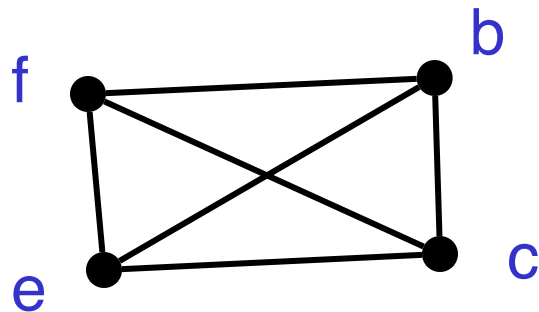


Stack: {a}

- Remove d

Graph Coloring Example (3)

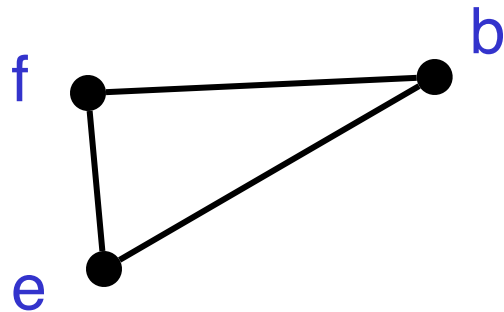
- Note: all nodes now have fewer than 4 neighbors



Stack: {d, a}

- Remove c

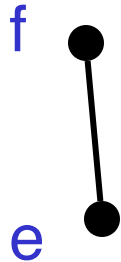
Graph Coloring Example (4)



Stack: {c, d, a}

- Remove b

Graph Coloring Example (5)



Stack: {b, c, d, a}

- Remove e

Graph Coloring Example (6)

f ●

Stack: {e, b, c, d, a}

- Remove f

Graph Coloring Example (7)

- Now start assigning colors to nodes, starting with the top of the stack

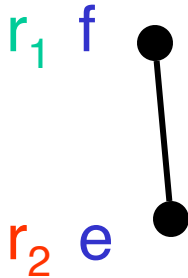
Stack: {f, e, b, c, d, a}

Graph Coloring Example (8)

r_1 f ●

Stack: {e, b, c, d, a}

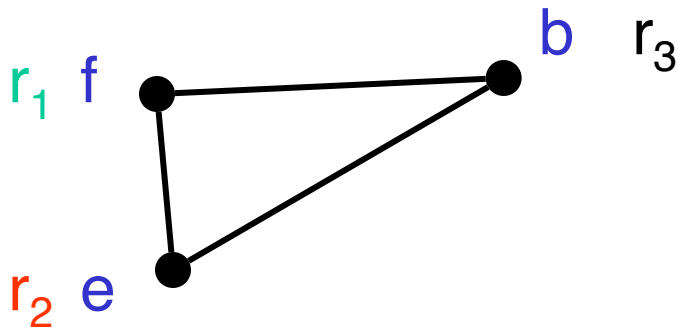
Graph Coloring Example (9)



Stack: { b , c , d , a }

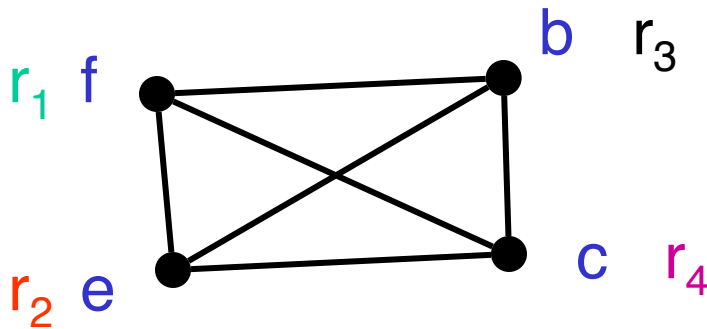
- e must be in a different register from f

Graph Coloring Example (10)



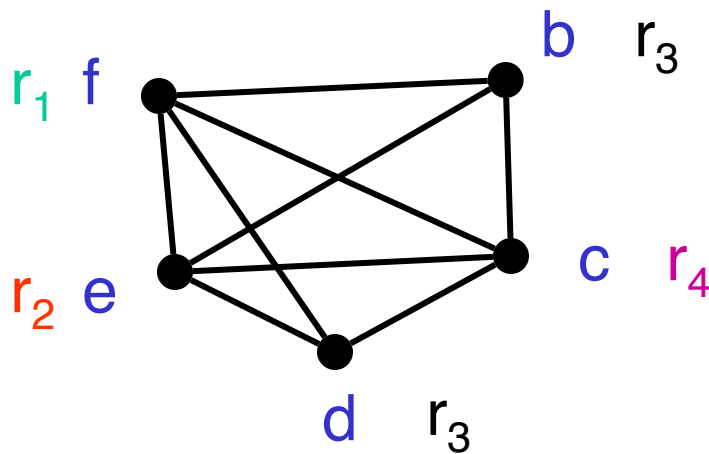
Stack: { c , d , a }

Graph Coloring Example (11)



Stack: { d , a }

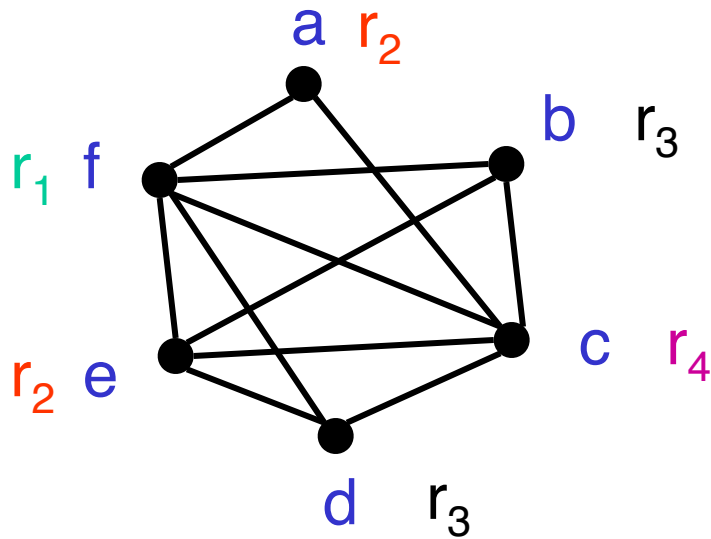
Graph Coloring Example (12)



Stack: { a }

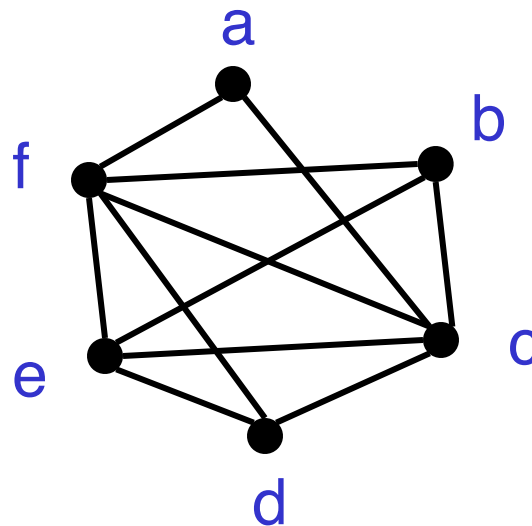
- d can be in the same register as b

Graph Coloring Example (13)



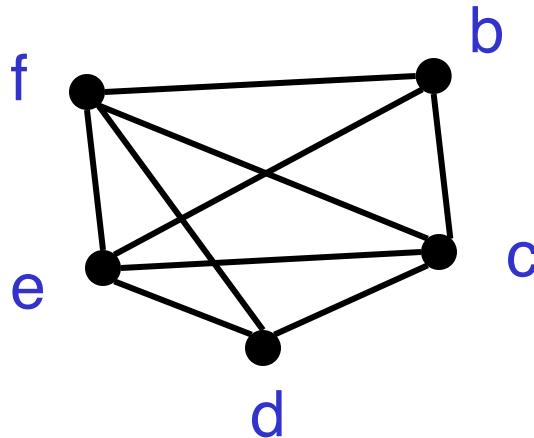
What if the Heuristic Fails?

- What if all nodes have k or more neighbors ?
- Example: Try to find a 3-coloring of the RIG:



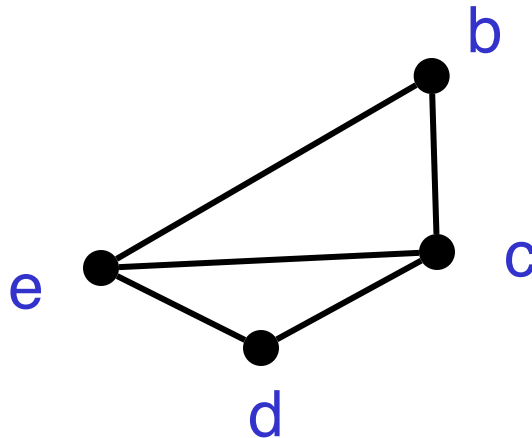
What if the Heuristic Fails?

- Remove **a** and get stuck (as shown below)
- Pick a node as a candidate for spilling
 - A spilled temporary “lives” in memory
 - Assume that **f** is picked as a candidate



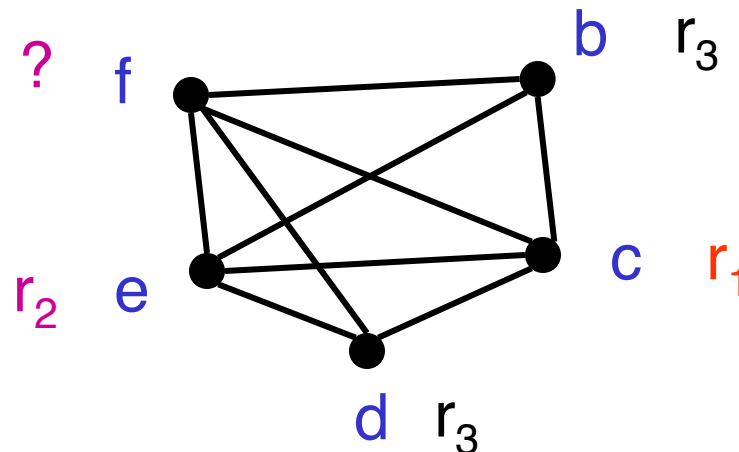
What if the Heuristic Fails?

- Remove **f** and continue the simplification
 - Simplification now succeeds: **b, d, e, c**



What if the Heuristic Fails?

- Eventually we must assign a color to **f**
- We hope that among the 4 neighbors of **f** we use less than 3 colors \Rightarrow optimistic coloring

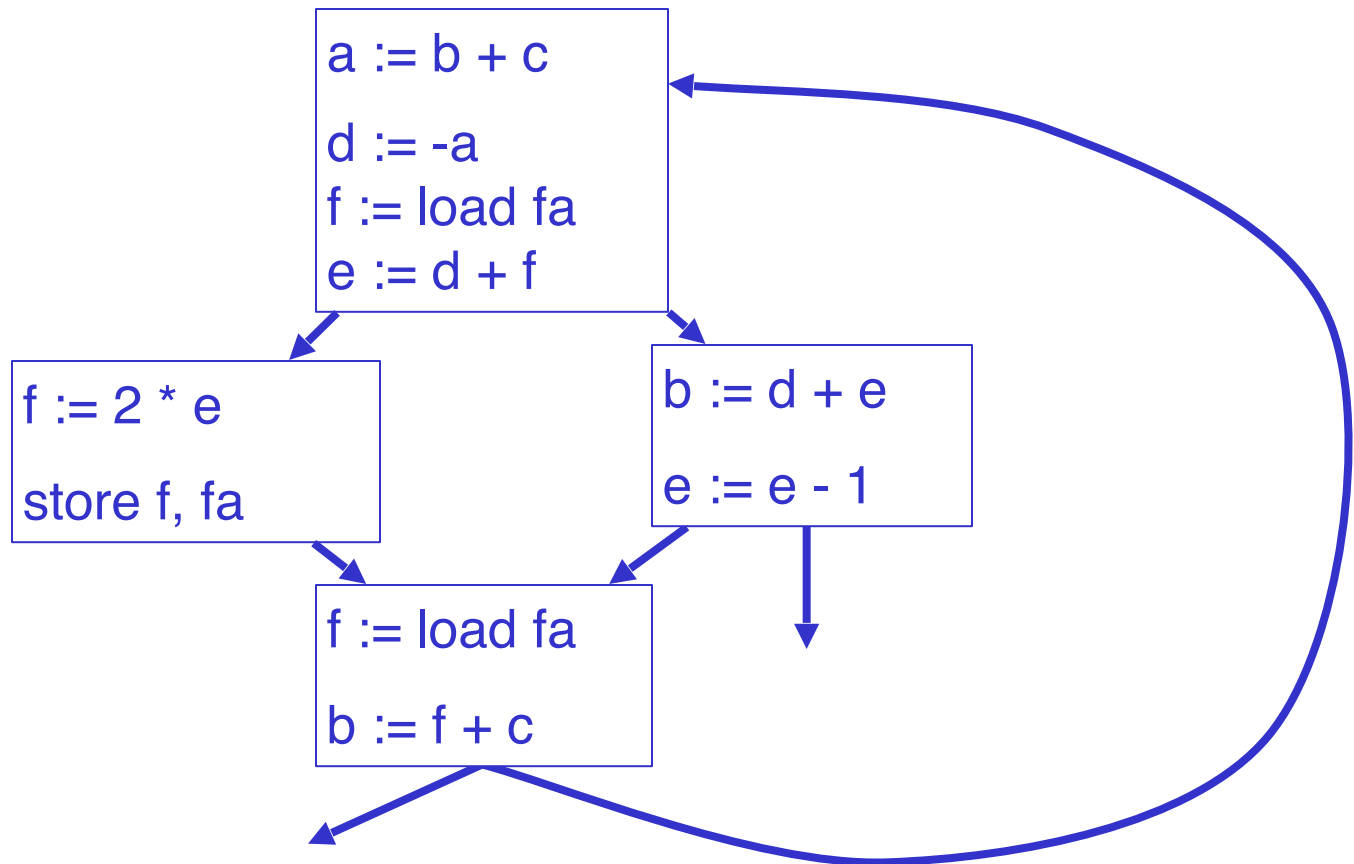


Spilling

- If optimistic coloring fails, we spill f
 - Allocate a memory location for f
 - Typically in the current stack frame
 - Call this address fa
- Before each operation that reads f , insert
 $f := \text{load } fa$
- After each operation that writes f , insert
 $\text{store } f, fa$

Spilling Example

- This is the new code after spilling f

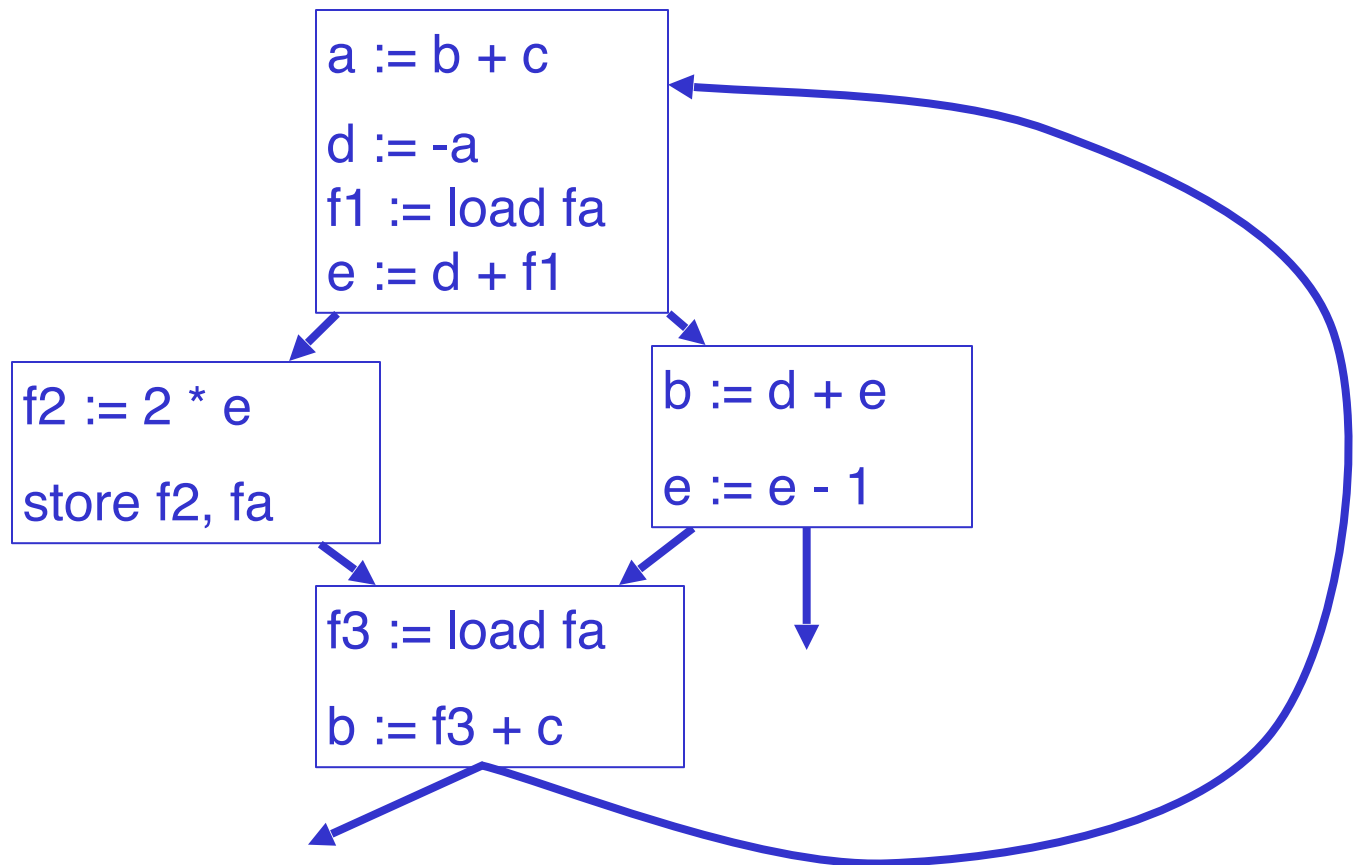


A Problem

- This code reuses the register name **f**
- Correct, but suboptimal
 - Should use distinct register names whenever possible
 - Allows different uses to have different colors

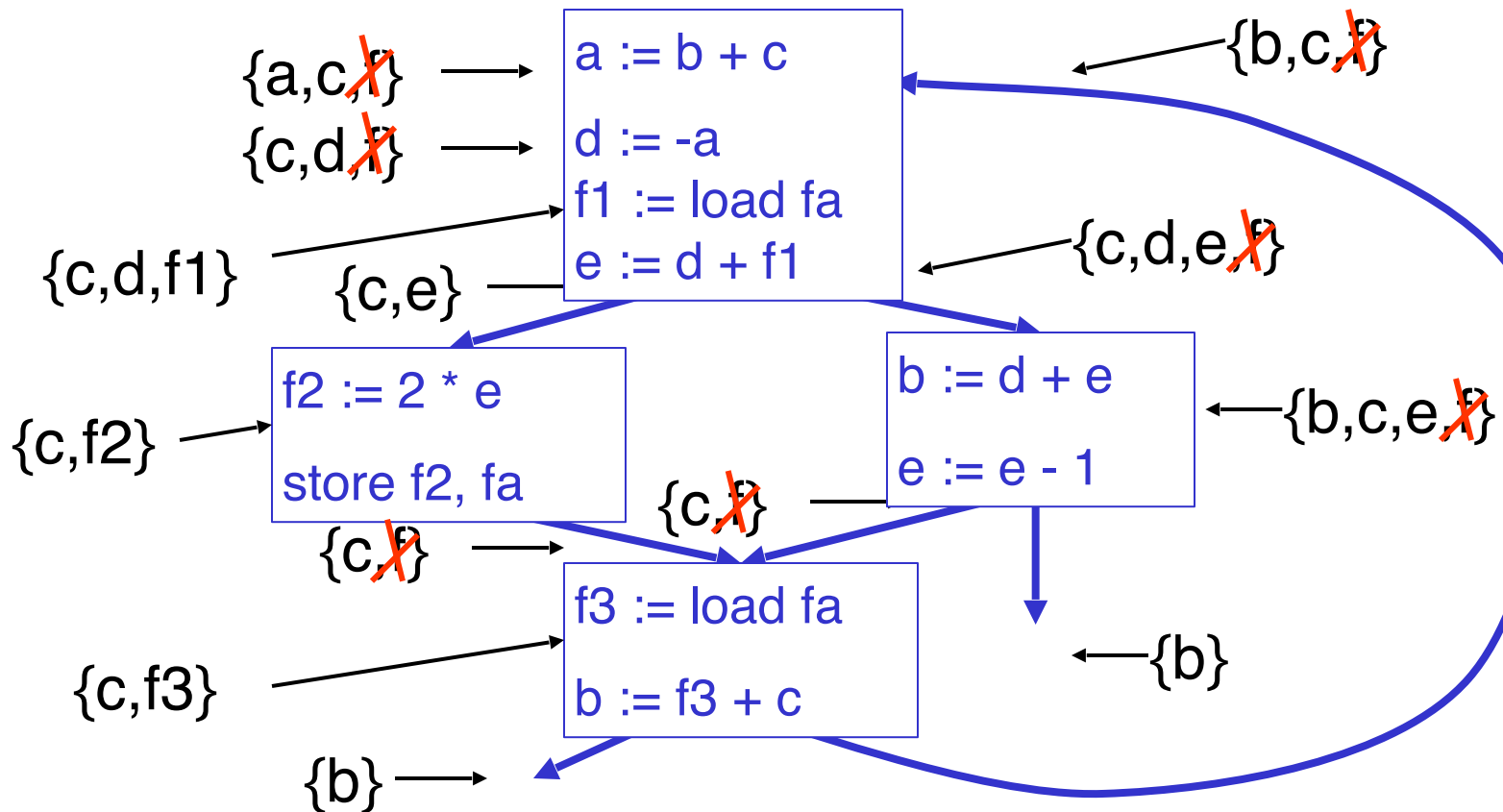
Spilling Example

- This is the new code after spilling **f**



Recomputing Liveness Information

- The new liveness information after spilling:

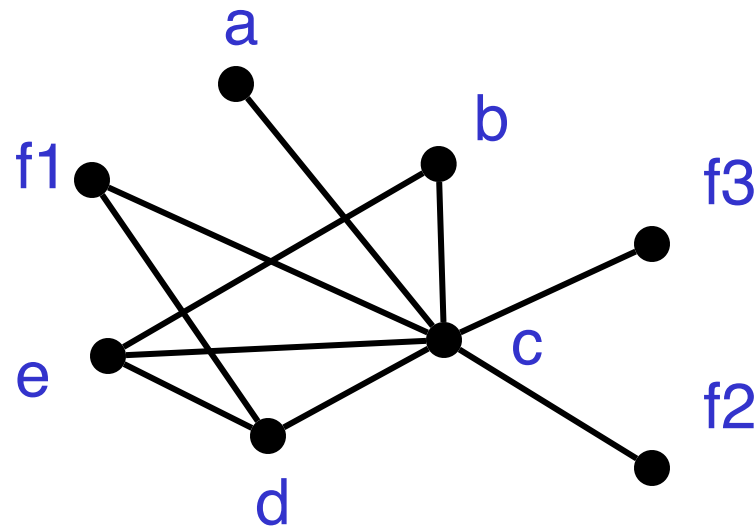


Recomputing Liveness Information

- New liveness information is almost as before
 - Note **f** has been split into three temporaries
- **fi** is live only
 - Between a **fi := load fa** and the next instruction
 - Between a **store fi, fa** and the preceding instr.
- Spilling reduces the live range of **f**
 - And thus reduces its interferences
 - Which results in fewer RIG neighbors

Recompute RIG After Spilling

- Some edges of the spilled node are removed
- In our case **f** still interferes only with **c** and **d**
- And the resulting RIG is 3-colorable



Spilling Notes

- Additional spills might be required before a coloring is found
- The tricky part is deciding what to spill
 - But any choice is correct
- Possible heuristics:
 - Spill temporaries with most conflicts
 - Spill temporaries with few definitions and uses
 - Avoid spilling in inner loops

Caches

- Compilers are very good at managing registers
 - Much better than a programmer could be
- Compilers are not good at managing caches
 - This problem is still left to programmers
 - It is still an open question how much a compiler can do to improve cache performance
- Compilers can, and a few do, perform some cache optimizations

Cache Optimization

- Consider the loop

```
for(j := 1; j < 10; j++)  
  for(i := 1; i < 1000; i++)  
    a[i] *= b[i]
```
- This program has terrible cache performance
 - Why?

Cache Optimization (Cont.)

- Consider the optimized loop:
 for(i := 1; i < 1000; i++)
 for(j := 1; j < 10; j++)
 a[i] *= b[i]
 - Computes the same thing
 - But with much better cache behavior
 - Might actually be more than 10x faster
- A compiler can perform this optimization
 - called loop interchange

Conclusions

- Register allocation is a “must have” in compilers:
 - Because intermediate code uses too many temporaries
 - Because it makes a big difference in performance
- Register allocation is more complicated for CISC machines