Lecture 1: From Classical Bits to Quantum Qubits

Welcome to Quantum Computing!

What is Quantum Computing?

- A new way of processing information
- Based on the laws of quantum mechanics
- Not just "faster computers" but **different kind** of computation

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How quantum computing works?

```
Classical Computing:                    Quantum Computing:
• Uses bits (0 or 1)                    • Uses qubits (can be 0, 1, or both)
• Deterministic operations              • Probabilistic operations
• Independent bits                      • Entangled qubits
• Like reading a book                   • Like experiencing VR
```

Why do we need Quantum Computing?

Are there real quantum computers?

Who is doing quantum computation?

What is this course about?

- Quantum computing
- Quantum information, quantum logic, gates, circuits
- Quantum algorithms, design principals, complexities
- Applications of quantum algorithms to data analysis, optimization, ai
- Comparison to non-quantum (classical) algorithms

Weekly topics

- Mathematical foundations
- Core quantum algorithms
- More advanced algorithms
- Applications
- Quantum error correction

Grading

- 40% 3-5 assignments including python implementations (you can work as groups of 1-3 people)
- 20% Midterm
- 40% Final

Textbooks & Resources

No required textbook. But, lecture notes are mostly based on

- "Quantum Computation and Quantum Information" by Nielsen & Chuang (primary theoretical reference)
- "An Introduction to Quantum Computing" by Kaye, Laflamme, Mosca (CS-focused theory)
- "Quantum Computing for Computer Scientists" by Yanofsky & Mannucci (CS-focused and more basics)
- "Quantum Computing: A Gentle Introduction" by Eleanor G. Rieffel and Wolfgang H. Polak.

Online available notes-books:

- A Course on the Theory of Quantum Computing by John Watrous, see also his IBM-Qiskit Textbook (free online: qiskit.org/textbook)
- Quantum country by Andy Matuschak and Michael Nielsen see also his great introduction to neural networks
- Pennylane tutorials and documentation
- Quantum Algorithm Zoo

From Classical Bits to Quantum Qubits

What is a bit?

- Smallest unit of classical information
- Can be either **0** or **1**
- Like a light switch: ON or OFF

Examples of Bits:

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Computer memory: 0 or 1
Light bulb: ON or OFF
Coin: Heads or Tails
```

Mathematical Representation:
We can represent a bit as a vector:

$$0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad 1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Introducing Quantum Bits (Qubits)

The Quantum Surprise:
A qubit can be **0, 1, or BOTH at the same time!**

Analogy: Spinning Coin

```
Classical coin:          Heads OR Tails
Quantum coin (spinning): Heads AND Tails simultaneously
Only when you stop it (measure) does it become one or the other
```

Mathematical Representation:
A qubit state is:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

where:

- $|0\rangle$ and $|1\rangle$ are basis states
- α and β are **complex numbers**
- $|\alpha|^2 + |\beta|^2 = 1$ (total probability = 1)

Understanding Probability Amplitudes

Classical Probability:
If a coin has 50% chance heads, 50% tails:

- Probability of heads = 0.5
- Probability of tails = 0.5
- Total = 1

Quantum Probability Amplitudes:
For a qubit: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

- **Amplitude** for $|0\rangle$ is α (a complex number)
- **Probability** of measuring $|0\rangle$ is $|\alpha|^2$
- **Probability** of measuring $|1\rangle$ is $|\beta|^2$
- $|\alpha|^2 + |\beta|^2 = 1$

Example:
If $\alpha = \frac{1}{\sqrt{2}}$ and $\beta = \frac{1}{\sqrt{2}}$:

- Probability of 0 = $\left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$
- Probability of 1 = $\left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}$

Polarization - A Physical Qubit

Polarized Sunglasses Experience:

- When you tilt your head while wearing polarized sunglasses...
- Some light gets through, some doesn't
- The sunglasses act as a **filter** for light direction

What is Light Polarization?
Light is an electromagnetic wave that vibrates in different directions:

Polarizer as a Filter:

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Unpolarized light → Polarizer → Polarized light in one direction
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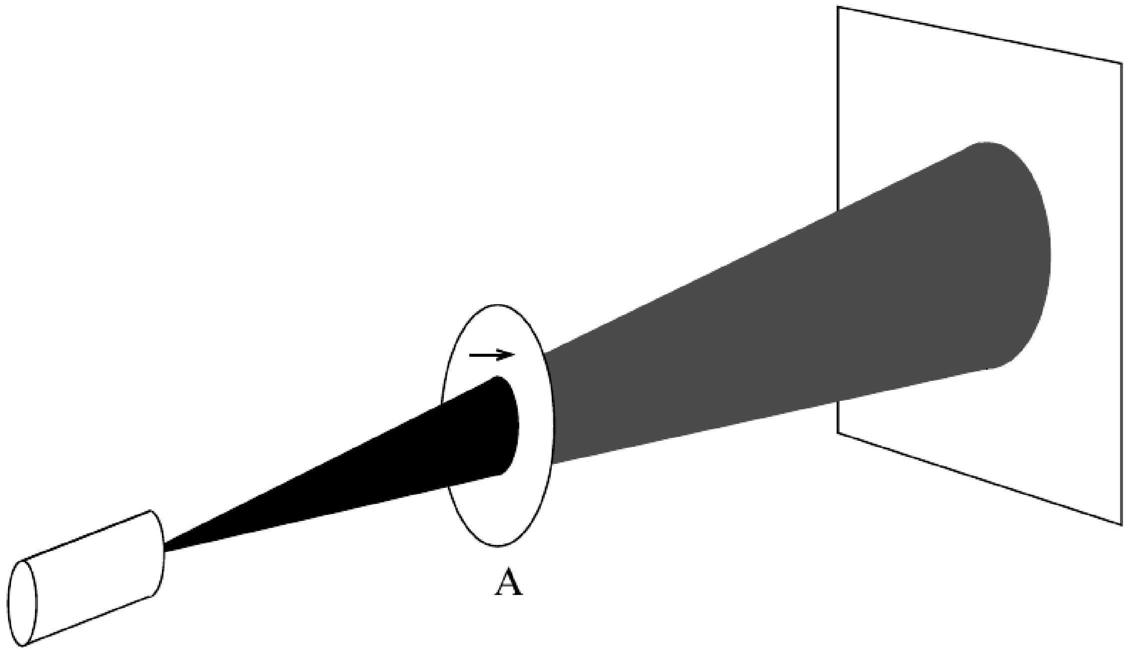
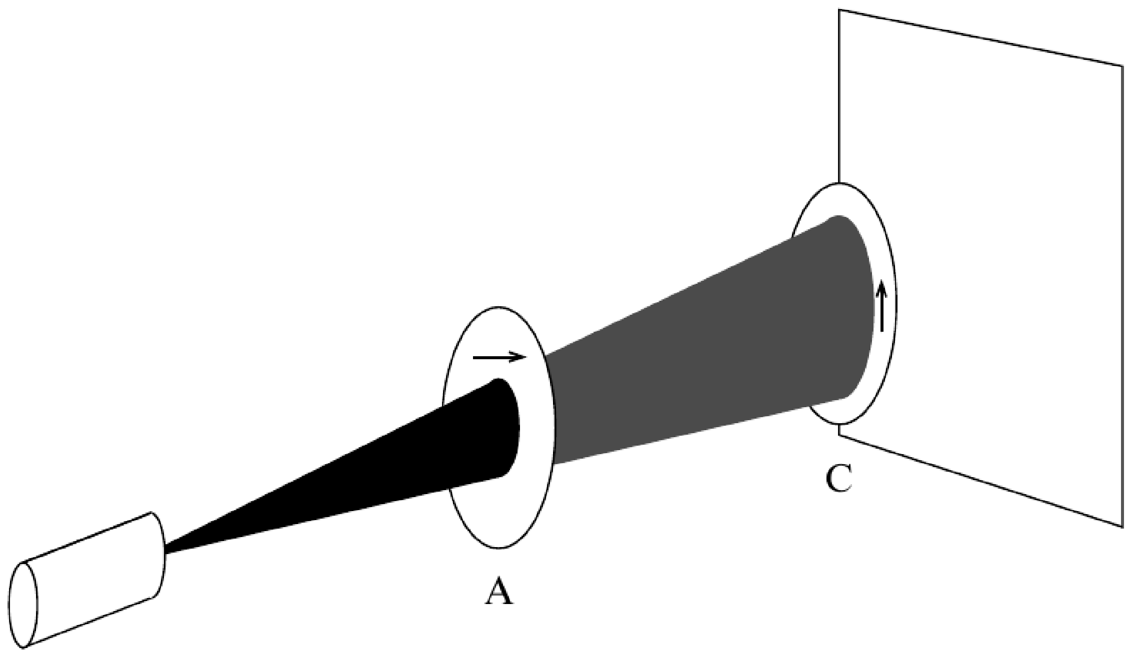
Example: Horizontal Polarizer

Only lets through light vibrating horizontally(50% light pass through):



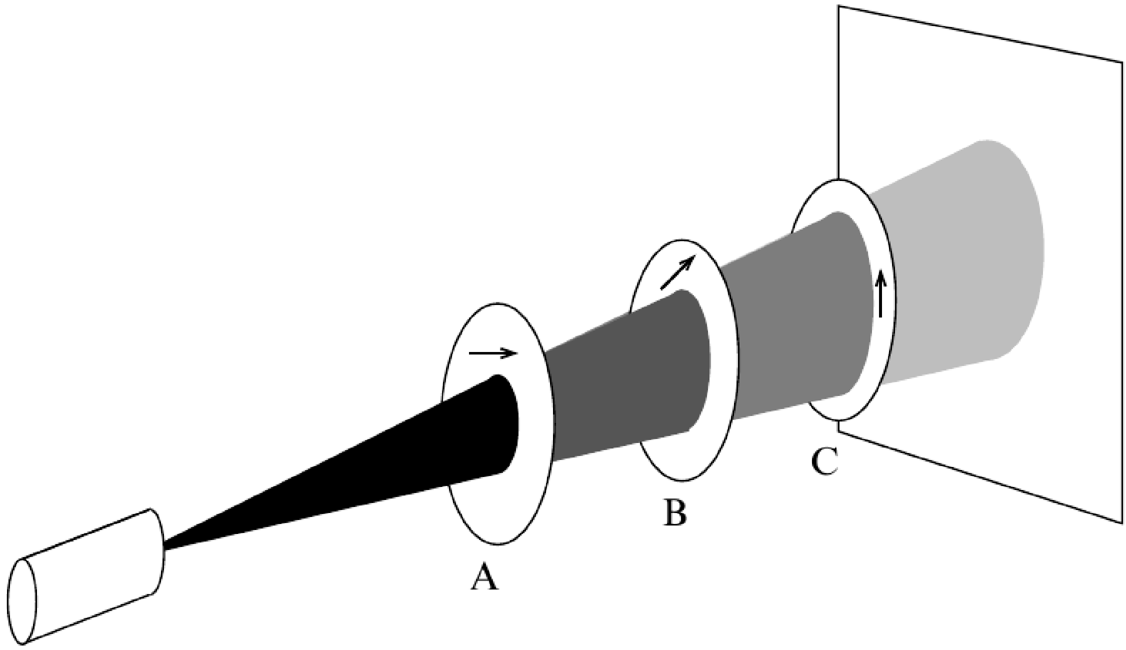
Example: two orthogonal polarizer

No light passes through (all photons are blocked).



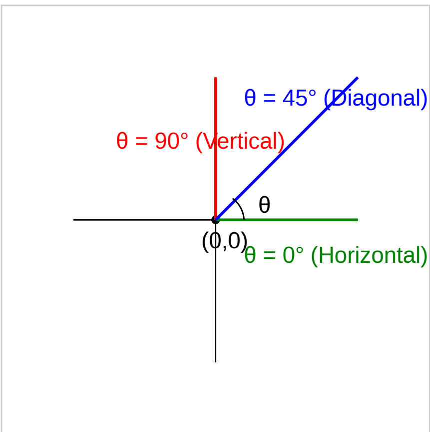
Example: inserting polarizer-B

some light passes through.



Coordinate System for Polarization

Mathematical Representation:
Horizontal polarization = $|H\rangle$
Vertical polarization = $|V\rangle$



Angle θ determines polarization direction:

- θ = 0°: Horizontal polarization $|H\rangle$
- θ = 90°: Vertical polarization $|V\rangle$
- θ = 45°: Diagonal polarization $|D\rangle$

Single Polarizer

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Sunlight (unpolarized) → Polarizer H → Only H polarized light
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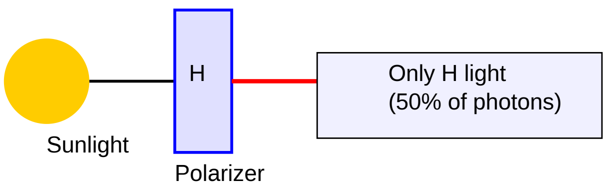

What happens to individual photons?

- Photon is smallest unit of light
- Each photon either PASSES or is ABSORBED
- Probability depends on its initial polarization

For unpolarized light:

- Probability to pass H polarizer = 50%
- Probability to be absorbed = 50%

Crossed Polarizers

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Light → Polarizer H → Polarizer V → Darkness!
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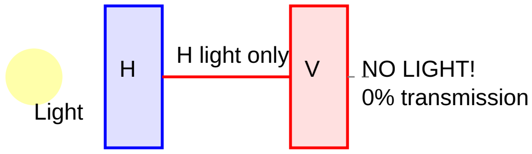

Why no light?

1. First polarizer: Only H photons pass
2. Second polarizer (V): Blocks all H photons
3. Result: No light gets through

Mathematically:
After first polarizer: $|H\rangle$
Probability through V polarizer = $|\langle V|H\rangle|^2 = 0$

The Magic Middle Polarizer!

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Light → H → 45° → V → SOME LIGHT!
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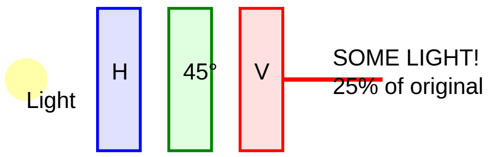

Counterintuitive Result:
Adding a third polarizer at 45° **allows light through**!

Explanation with Qubits:

1. After H polarizer: $|H\rangle$
2. 45° polarizer creates **superposition**: $|D\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$
3. Now has both H and V components!
4. Some can pass final V polarizer

Probability = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ of original

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Superposition in Polarization

Diagonal Polarization as Superposition:

$$|D\rangle = \frac{1}{\sqrt{2}}|H\rangle + \frac{1}{\sqrt{2}}|V\rangle$$

What does this mean?

- Photon is in BOTH H AND V states simultaneously
- When measured by H polarizer: 50% chance H, 50% chance V
- When measured by V polarizer: 50% chance H, 50% chance V

Introducing Quantum Spin

Electrons Have "Spin"

- Not actually spinning like a top
- Intrinsic property like mass or charge
- Always has same magnitude but can point in different directions

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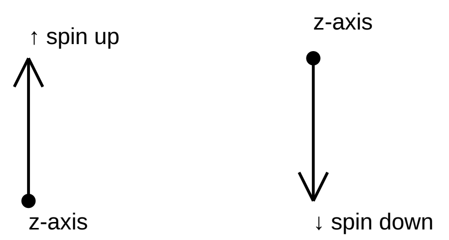
Spin Directions:

- Up along z-axis: $|\uparrow_z\rangle$ or $|0\rangle$
- Down along z-axis: $|\downarrow_z\rangle$ or $|1\rangle$



Stern-Gerlach Experiment (1922)

The Setup:

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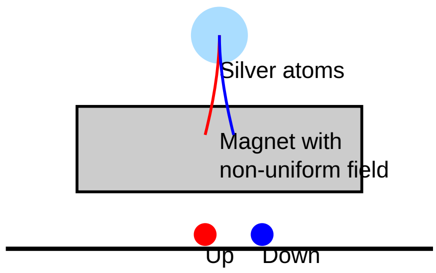
Silver atoms (with one unpaired electron)
        ↓
Strong magnet with non-uniform field
        ↓
Detector screen
```



Actual Result:
Only **TWO** discrete spots appear!

- One spot for spin UP
- One spot for spin DOWN

What Stern-Gerlach Shows Us

Key Discovery:

1. **Quantization:** Spin only takes discrete values (not continuous)
2. **Measurement:** Apparatus "measures" spin along vertical axis
3. **Collapse:** Measurement forces atom into UP or DOWN state

First SG Apparatus:

- Measures spin along z-axis
- Splits beam into $|\uparrow_z\rangle$ and $|\downarrow_z\rangle$
- Select one path (e.g., $|\uparrow_z\rangle$ only) for next experiment

Sequential Stern-Gerlach Experiments

Experiment 1: SG-z → SG-z

```
Atoms → SG-z (select ↑_z) → SG-z → All ↑_z
```

Result: 100% up. Once measured up, stays up if measured same way.

Experiment 2: SG-z → SG-x → SG-z

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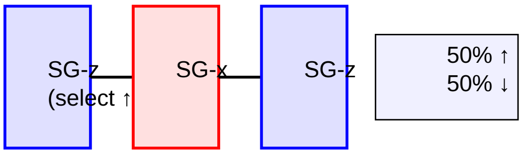
Atoms → SG-z (↑_z) → SG-x → SG-z → 50% ↑_z, 50% ↓_z
```

Surprise: After SG-x, we lose information about z-spin!



Understanding the SG-x Apparatus

What is SG-x?

- Measures spin along x-axis
- Splits into $|\uparrow_x\rangle$ and $|\downarrow_x\rangle$

Mathematical Relationship:

$$|\uparrow_z\rangle = \frac{1}{\sqrt{2}}|\uparrow_x\rangle + \frac{1}{\sqrt{2}}|\downarrow_x\rangle$$

$$|\downarrow_z\rangle = \frac{1}{\sqrt{2}}|\uparrow_x\rangle - \frac{1}{\sqrt{2}}|\downarrow_x\rangle$$

Interpretation:
$|\uparrow_z\rangle$ is a **superposition** of x-spin states!

Analysis of SG-z → SG-x → SG-z

Step by Step:

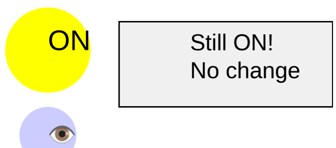
1. Start with $|\uparrow_z\rangle$
2. SG-x measurement: Forces into either $|\uparrow_x\rangle$ or $|\downarrow_x\rangle$ (50% each)
3. Let's say we get $|\uparrow_x\rangle$
4. Now: $|\uparrow_x\rangle = \frac{1}{\sqrt{2}}|\uparrow_z\rangle + \frac{1}{\sqrt{2}}|\downarrow_z\rangle$
5. Final SG-z: 50% chance ↑_z, 50% chance ↓_z

Key Insight:
The middle measurement **destroys** original information

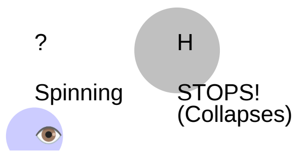
- Like the middle polarizer creates new possibilities

Quantum vs Classical Measurement

Classical Measurement:



Quantum Measurement:



Connection to Computation

Stern-Gerlach as Quantum Gates:

- SG-z: Measures in computational basis {|0⟩, |1⟩}
- SG-x: Measures in Hadamard basis {|+⟩, |-⟩}
- Where: $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

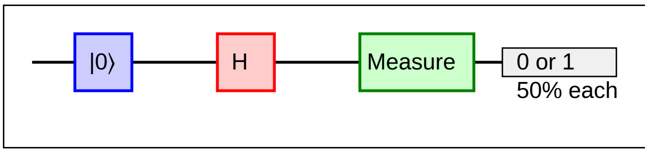
Sequence SG-z → SG-x → SG-z is like:

1. Prepare |0⟩
2. Apply Hadamard gate (creates superposition)
3. Measure in computational basis

The Hadamard Gate:

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$H|0\rangle = |+\rangle, \quad H|1\rangle = |-\rangle$$



Summary - Key Quantum Concepts

1. Superposition:

- Quantum systems can be in multiple states at once
- $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

2. Measurement Collapse:

- Measurement forces system into definite state
- Probability given by $|\alpha|^2$ and $|\beta|^2$

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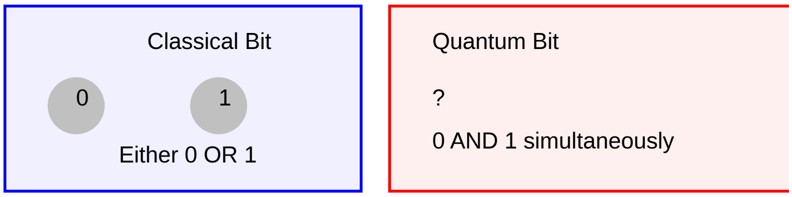
3. Basis Change:

- Same state looks different in different bases

- $|0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$

4. Quantum Information:

- More powerful than classical information
- Enables new algorithms (we'll learn these next!)



Exercises

Problem 1: Polarizer Math
Light passes through polarizers at 0°, 30°, and 90°. What fraction emerges?
Hint: Use Malus's Law: $I = I_0 \cos^2(\theta)$

Problem 2: Quantum State
A qubit is in state: $|\psi\rangle = \frac{3}{5}|0\rangle + \frac{4i}{5}|1\rangle$
a) Verify this is a valid quantum state
b) What is probability of measuring |0⟩?
c) What is probability of measuring |1⟩?

Problem 3: Stern-Gerlach
If we start with $|\uparrow_z\rangle$ and measure:
SG-z → SG-x → SG-y → SG-z
What is the probability of getting ↑_z at the end?

Next Lecture Preview

Coming Up:

1. **Dirac Notation:** The language of quantum mechanics
2. **Quantum Gates:** How to manipulate qubits
3. **Entanglement:** The "spooky" quantum correlation
4. **Quantum Circuits:** Building quantum algorithms

Reading:

- Textbook Chapter 1: Introduction to Quantum Computing
- Qiskit Documentation: Basic quantum concepts

Key Takeaway: Quantum mechanics gives us new computational resources

- superposition and entanglement
- that don't exist in classical computing.

Understanding these through simple examples (polarization, Stern-Gerlach) is the first step toward quantum algorithms!