

CHAPTER-2CONCEPT LEARNING & THE GENERAL-TO-SPECIFIC ORDERING

concept learning ~~task~~ means inferring a boolean valued func. from training examples of its i/p & o/p.

of concept learning Task:

- Consider an example where a friend enjoys water sport.

concept learning task is "Enjoy Sport".

| example | Sky | Air Temp | Humidity | Wind | Water | Forecast | Enjoy Sport |
|---------|-------|----------|----------|--------|-------|----------|-------------|
| 1. | sunny | warm | normal | strong | warm | same | yes |
| 2. | sunny | warm | high | strong | warm | same | yes |
| 3. | rainy | cold | high | strong | warm | change | no |
| 4. | sunny | warm | high | strong | cool | change | yes |

Table: 2.1

- The "Attribute" - Enjoy sport rep. whether the person enjoys water sport that day or not.
- Task: To learn to predict "enjoysport" of an arbitrary day based on other values of attributes.
- Hypothesis repn: - Conjunction of constraints on the instance attributes.
- So Hypothesis is a vector having 6 constraints here, i.e., sky, Airtemp, humidity, wind, water & forecast.
 - For each attribute, hypothesis will be:
 - (i) "?" = Any value is acceptable for this attribute ex:- warm.
 - (ii) Specify a single reqd. value ex:- ?
 - (iii) "∅" - No value is acceptable.

- Positive Example: - If an instance x satisfies all constraints of hypothesis 'h' then 'h' classifies x as +ve example $\Rightarrow h(x) = 1$.
- ex: ① (?, cold, High, ?, ?, ?) \Rightarrow on cold day with high humidity
- ② (?, ?, ?, ?, ?, ?) \Rightarrow enjoy sports every day \Rightarrow every day +ve
- ③ (∅, ∅, ∅, ∅, ∅, ∅) \Rightarrow No day is a positive example

→ The "Enjoy Sport" Concept Learning Task:-

- Given:-
Instances X : possible days, each described by attributes
 1. Sky (sunny, cloudy & rainy)
 2. Air Temp (warm & cold)
 3. Humidity (Normal & High)
 4. Wind (strong & weak)
 5. Water (warm & cool)
 6. Forecast (Same & Change)

Hypothesis H : Each hypothesis is described by constraint
on attributes.

"?" → any value acceptable

"X" → no value acceptable.
specific value

Target concept C : Enjoy Sport: $X \rightarrow \{0, 1\}$

Training Examples D : +ve & -ve examples of Target func.

Determine:-

A hypothesis "h" in H such that

$$h(x) = C(x) \quad \forall x \in X.$$

→ Notation:-

- Instances: set of items over which the concept is defined
is called set of instances, X .

- Target Concept: The concept or func. to be learned is
called Target Concept, C .
It can be any boolean valued func.
defined over the instances X .

$$C: X \rightarrow \{0, 1\}$$

Ex:- In "enjoy sport" concept, if
 $C(x)=1 \Rightarrow$ if Enjoy Sport = Yes
 & $C(x)=0 \quad \text{if} \quad \text{No.}$

- Training examples :- when learning a target concept a learner is provided a set of training examples. They consist of an instance x from X , along with its target concept value $c(x)$.
 - If $c(x) = 1 \Rightarrow$ Positive Training example,
 - If $c(x) = 0 \Rightarrow$ Negative "
 - It is written in ordered pairs $(x, c(x))$ for defining a training example.
Here, x = instance & $c(x)$ = Target Concept Value
 - The set of available training set examples are denoted by "D".

- All possible Hypotheses :- - Denoted by H
 - Given a target concept "c"
 - Using a set of Training examples, the learner must hypothesize or estimate "c".
 - So, the set of all possible hypotheses is H in order to identify of target concept.
 - Each hypothesis in set " H " is denoted by " h ". which is a boolean value defined over X .
 $\Rightarrow h: X \rightarrow \{0, 1\}$

- The goal of the learner is to find a hypothesis ' h' such that
$$h(x) = c(x) \forall x \in X$$

→ The Inductive Learning Hypothesis :-

It is a hypothesis which is found to approximate target func. well, over a sufficiently large set of training examples & it will also approximate the target func. well, over other unobserved examples.

Concept learning as search:

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- Concept learning is like a task of searching through a large space of hypotheses (defined by hypothesis repn)
 - Here, goal is to find hypothesis, which fits the best to the training examples.
- ex: - In "Enjoy Sport" learning task, H = Hypotheses set
 X = Instances set

The Attribute Sky has 3 possible values & Remaining Air Temp, Humidity, Wind, Water & forecast has 2 possible values.

Thus there are $3 \times 2 \times 2 \times 2 \times 2 \times 2 = 96$ instances in X .

- coming to Hypotheses there are $5 \times 4 \times 4 \times 4 \times 4 \times 4 = 5120$

"Syntactically distinct Hypotheses" in H .

- Every hypotheses with one or more \emptyset symbol rep. empty set of instance \Rightarrow instance is -ve.
- Thus now "Semantically distinct Hypotheses" are

$$\text{only } 1 + (4 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3) = 973$$

General-to-Specific Ordering of Hypotheses:

Def: - Let h_j and h_k be boolean valued func., defined over X . Then ' h_j ' is "more general than- or equal" to h_k ($h_j \geq_g h_k$) iff

$$(\forall x \in X) [(h_k(x) = 1) \rightarrow (h_j(x) = 1)]$$

ex: - $h_1 = \langle \text{sunny}, ?, ?, \text{strong}, ?, ? \rangle$

$h_2 = \langle \text{sunny}, ?, ?, ?, ?, ?, ? \rangle$

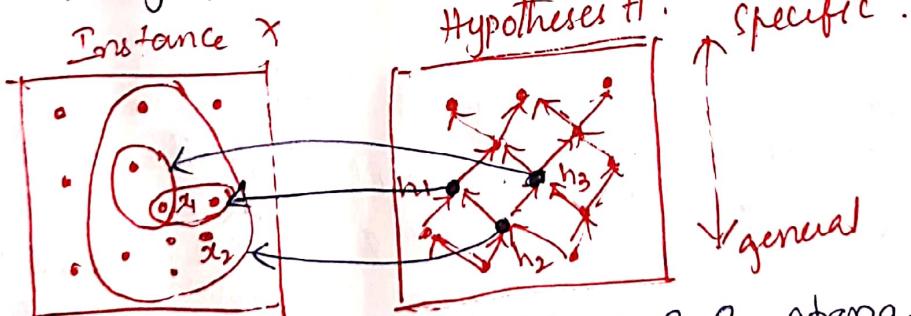
Consider set of instances classified as +ve by h_1 & h_2

As seen, any instance classified by h_1 will also be classified +ve by h_2 (B)
 $\Rightarrow h_2$ is more general than h_1
 $(\because h_2$ imposes fewer constraints on instance, also, it classifies more instances to be +ve)

- In general, for any instance "x" in the set of instances X and any hypothesis "h" in set of hypotheses H , we say, x satisfies h iff $h(x) = 1$.
- Given 2 hypotheses h_k & h_j , if any ^{instance} satisfying h_k also satisfies h_j then h_j is more general or equal to h_k .
- strictly, If $(h_j \geq g h_k) \wedge (h_k \not\geq g h_j)$ then h_j is more general than h_k . (or) ~~but~~
 h_k is more specific than h_j . (Inverse)

→ Example, Assume 3 hypotheses h_1 , h_2 and h_3 from "enjoy sport" example.

~~Also $h_2 \geq g h_1$ since, every instance~~



$$x_1 = \langle \text{sunny, warm, high, strong, cool, same} \rangle \quad h_1 = \langle \text{sunny, ?, ?, ?, strong, ?, ?} \rangle$$

$$x_2 = \langle \text{sunny, warm, high, light, warm, same} \rangle \quad h_2 = \langle \text{sunny, ?, ?, ?, ?, ?, ?} \rangle$$

$$h_3 = \langle \text{sunny, ?, ?, ?, cool, ?} \rangle$$

- Here we see that, $h_2 \geq g h_1$, because every instance that satisfies h_1 also satisfies h_2 ; $h_2 \not\geq g h_3$
 $\Rightarrow \geq g$ relⁿ depends only on instances & not on the classifying them.

- A \Rightarrow_{Rel} defines a partial order on hypothesis space (Rel is Reflexive, Antisymmetric & Transitive).
- The structure is partial order \Rightarrow there may be pairs of hypotheses such as $h_1 \& h_2$ such that $h_1 \not\geq h_2$ and $h_2 \not\geq h_1$.

Fund-S: finding a Maximally Specific Hypothesis:

- It is an appl'n of more general than partial order.
- Here, we begin with most specific possible hypothesis in H & generalize it; when it fails to cover an observed +ve Training example (classifies +ve).

Fund-S Algorithm: -

1. Initialise h to most specific hypothesis in H .
2. For each +ve Training instance x .
 - For each attribute constraint a_i in h , if constraint a_i is satisfied by x then do nothing
 - else Replace a_i in h with next more general constraint satisfied by?
3. O/p hypothesis h .

Example: -

- consider "Enjoy Sport task".

- (i) Initialise h to most specific hypothesis in H .

$$h \leftarrow \{\emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset\}.$$

- (ii) From table 2.1, none of constraints are satisfied by this so it is replaced by next more general constraint that fits this example.

$$h \leftarrow \{\text{Sunny, Warm, Normal, Strong, Warm, Same}\}$$

[It's less specific b'coz all instances are -ve except one +ve Training example, (See Table 2.1 to relate)]

→ The second training example forces algo. to further generalise h , which can be done by substituting any attribute value in ' h ' by a '?'.

$$h \leftarrow \langle \text{sunny}, \text{warm}, ?, \text{Strong}, \text{warm}, \text{Same} \rangle$$

→ Upon encountering a -ve Training example (3rd one) find s algo simply ignores it. So no revision of ' h ' is needed.

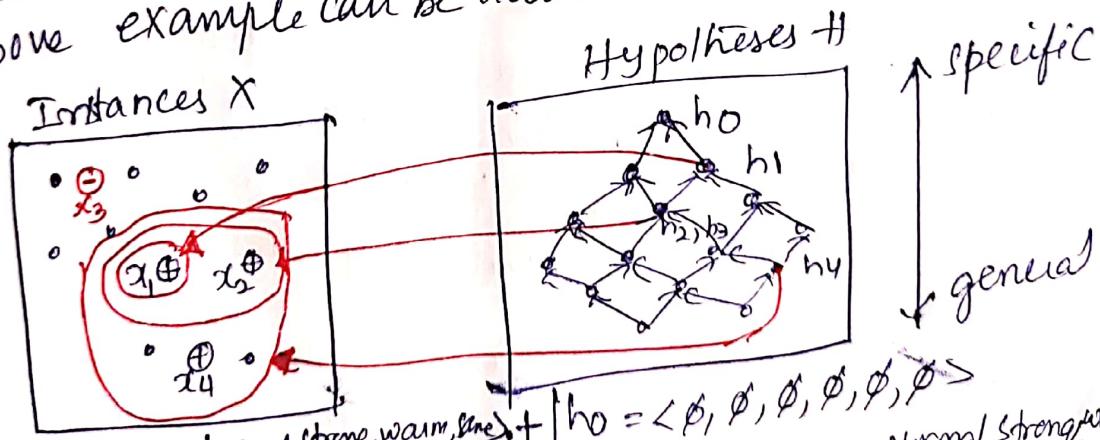
→ the 4th example (+ve) leads to further generalisation of ' h '.

$$h \leftarrow \langle \text{sunny}, \text{warm}, ?, \text{strong}, ?, ? \rangle$$

→ In this Algo., "More general than" partial ordering is thus used to organise the search for an acceptable hypothesis.

→ At each step, the hypothesis is generalised as far as necessary to cover the new +ve examples thus making the hypothesis more consistent with Training examples. Hence it is named as "Ford-S" Algorithm.

→ Above example can be illustrated further with foll. fig.



$$x_1 = \langle \text{sunny}, \text{warm}, \text{normal}, \text{Strong}, \text{warm}, \text{Same} \rangle + h_0 = \langle ?, ?, ?, ?, ?, ? \rangle$$

$$x_2 = \langle \text{sunny}, \text{warm}, \text{high}, \text{Strong}, \text{warm}, \text{Same} \rangle + h_1 = \langle \text{sunny}, \text{warm}, \text{Normal}, \text{Strong}, \text{warm}, \text{Same} \rangle$$

$$x_3 = \langle \text{rainy}, \text{cold}, \text{high}, \text{Strong}, \text{warm}, \text{cha } \rangle - h_2 = \langle \text{sunny}, \text{warm}, ?, \text{strong}, \text{warm}, \text{Same} \rangle$$

$$x_4 = \langle \text{sunny}, \text{warm}, \text{high}, \text{Strong}, \text{cool}, \text{cha } \rangle + h_3 = \langle \text{sunny}, \text{warm}, ?, \text{strong}, \text{warm}, \text{Same} \rangle$$

$$x_4 = \langle \text{sunny}, \text{warm}, ?, \text{strong}, ?, ?, ? \rangle + h_4 = \langle \text{sunny}, \text{warm}, ?, \text{strong}, ?, ?, ? \rangle$$

- "Find-S" is guaranteed to o/p most specific hypothesis within H that is consistent with +ve training examples. Finally, it is also consistent with -ve training examples provided the correct target concept is contained in H & also provided the training examples are correct. (9)
- few voids in this Algo. in the form of Questions are:
 - * Has learner converged to correct target concept?
 - * why prefer most specific hypothesis?
 - * Are Training examples consistent?
 - * what if there are several maximally specific consistent hypotheses?

Version spaces and "The Candidate_Elimination Algorithm."

- Helps to address limitations of Find-S Algo.
- "Find-S" Algo gives as an o/p, only single hypothesis from H that is consistent with Training examples.
- "Candidate_Elimination" Algo on other hand, o/p's a set of "all hypotheses" that are consistent with Training examples.
- It also uses "More_general_than" partial ordering, but produces o/p without explicitly enumerating all of its members. Hence it is more compact rep'n of set of consistent Hypotheses.
- Both "Find-S" & "Candidate_Elimination" Algos. perform poorly when noisy Training data is given.
- Appl'n of "Candidate_Elimination" Algo:
 - Learning regularities in chemical mass Spectroscopy
 - Learning Control rules for heuristic search

→ Representation:

- Consistent:

A hypothesis, h , is said to be consistent with a set of training examples ' D ' iff $h(x) = c(x)$ for each example $\langle x, c(x) \rangle$ in D .

$$\text{consistent } (h, D) \equiv (\forall \langle x, c(x) \rangle \in D) h(x) = c(x)$$

NOTE :

An example ' x ' satisfies a hypothesis ' h ' when $h(x) = 1$ irrespective of x is +ve or -ve example of Target Concept vs. consistent.
 x is consistent with h is not only dependent on target concept but also its compulsory that $h(x) = c(x)$.

- Version Space:

It is denoted as $VS_{H, D}$ w.r.t hypothesis space H & training examples D .

It is a subset of Hypotheses from H consistent with Training examples in D .

→ The "List-Then-Eliminate" Algorithm:

It is used to rep. the Version space by simply listing all of its members.

Algo:-

1. Version space \leftarrow a list containing every hypothesis in H

2. For each Training example, $\langle x, c(x) \rangle$

- remove from "version space" any hypotheses ' h ' for which $h(x) \neq c(x)$

3. O/p. of the list of hypotheses in Version space

→ Limitation:-

- * It can be applied on finite Hypotheses space H ,
* Exhaustive enumeration of all ' h ' in H is reqd.

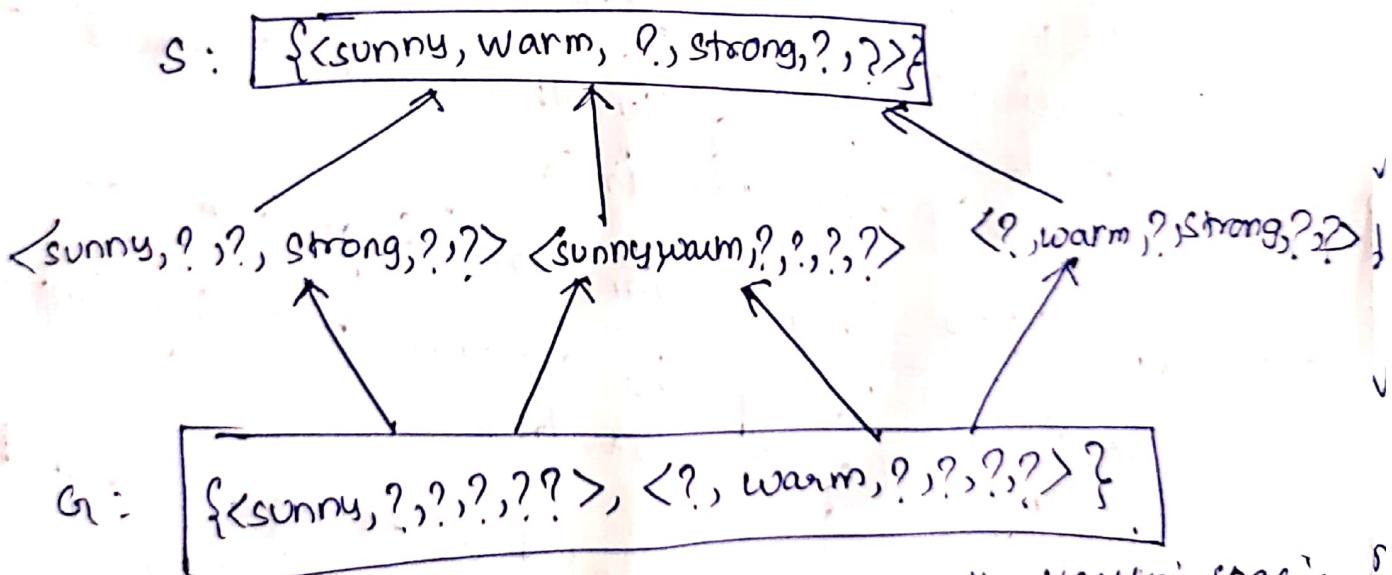
→ Advantages:-

- * guarantees to P all hypotheses consistent
with training data.

→ More compact representation for Version spaces:-

- Here, Version space is rep. by most general & least general members, thus setting boundaries for version space. They form general boundary set & specific boundary set.
- For this, we use example of "enjoy sport task" & its o/p of "Find-S" algorithm, which is:

$$h = \langle \text{sunny}, \text{warm}, ?, \text{strong}, ?, ?, ? \rangle$$



- In "candidate & elimination" Algo., the version space is rep. by storing only its most general members, " G " & and most specific members " S ".

- General Boundary " G ": - G w.r.t Hypotheses space H & training data D is set of maximally general members of H consistent with D

$$G = \{g \in H \mid \text{consistent}(g, D) \wedge (\exists g' \in H) [(g' \supseteq g) \wedge \text{consistent}(g', D)]\}$$

- specific boundary's:
 S w.r.t H & D is set of minimally general members of H
 Consistent with D

- Version space rep" theorem:-

Let X be arbitrary set of instances
 Let H be set of boolean valued hypotheses defined
 on X
 arbitrary Target concept defined

Let H be set of a book. Let C_0, C_1 be an arbitrary Target Concept defined over X .

Let H be an arbitrary hypothesis class.
 Let $c: X - \{0, 1\} \rightarrow \{0, 1\}$ be an arbitrary function.
 Let $C: X - \{0, 1\} \rightarrow \{0, 1\}$ be an arbitrary function.
 Let $c: X - \{0, 1\} \rightarrow \{0, 1\}$ be an arbitrary function.

Let $C: X \rightarrow \{0, 1\}^S$ be an arbitrary set of training examples $\{(x, c(x))\}$, well defined,

Let D be an abstract set such that S & G are well defined.

$\nexists x, H, C \& D$ such that $\{ \forall c \in S \} (\exists g \in G) (g \geq_g h \geq_g s) \}$,

$$VS_{H,D} = \{ h \in H \mid \exists s \in S \text{ such that } h = s \}$$

proof: - satisfying RHS is in $V_{S^H, D}$.
statisfies RHS

* Every h satisfying RHS is a member of $V\mathcal{S}_{H,D}$ satisfies RHS.

- * Every h in $S_{H,D}$ satisfies κ 's
- * Every member of $V_{S_{H,D}}$ satisfies κ 's
must be proved.

* Eve. This must be proved.
It's an arbitrary member of \mathcal{G}
" " \mathcal{H}

$$n \geq h \geq s$$

such that $g \geq g$, $h \geq g$ & $\sum_{i=1}^n x_i = 1$ must be satisfied by all x_i . It also be satisfied by all x_i .

\Rightarrow s must be satisfied by
 $\Rightarrow D$ should also be satisfied by
examples in D

$\Rightarrow b$ must be " "
 $b \geq s \Rightarrow$ it should also have +ve examples in D_i - libya -

$\Rightarrow h \geq s \Rightarrow$ it should be satisfied by all +ve examples in D
 $\therefore h \geq s \Rightarrow$ it should be satisfied by a -ve example in D

As per defⁿg Gr, q can't be satisfied by
also " " "

$\vdash \text{def}^n g G$, g can't be s as s is a variable
 $\vdash g \geq g$ $\vdash h \Rightarrow h$ also " " " "
 $\vdash g \geq g$ $\vdash h \Rightarrow h$ is a member of V_{S_H} , \exists

$\vdash q \geq g \ h \Rightarrow h \text{ also } " \ D \Rightarrow \text{his membership} \in S_H, \vec{D}$
 $\Rightarrow h \text{ is consistent with } D$

"Candidate_Elimination" Learning Algorithm:-

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1. Initialise G_0 to set of maximally general Hypotheses $\{ \cdot \}$
 2. " " " " " " specific " "
 3. For each training example ' d ', do
 - If d is positive example
 - (i) Remove from ' G ' any hypothesis inconsistent with d .
 - (ii) For each hypothesis ' s ' in S that's inconsistent with ' d '.
 - (a) Remove ' s ' from ' S '.
 - (b) Add to ' S ' all minimal generalisations ' h ' of ' s ' such that
 - (i) h is consistent with d
 - (ii) some members of G is more general than h .
 - (c) Remove from S , any hypothesis i.e remove general than another hypothesis in S .
 - If d is Negative example
 - (i) Remove from ' S ' any hypothesis inconsistent with d .
 - (ii) For each hypothesis ' g ' in G i.e not consistent with ' d '.
 - (a) Remove ' g ' from ' G '.
 - (b) Add to ' G ' all minimal generalisations ' h ' of ' g ' such that
 - (i) h is consistent with d .
 - (ii) some members of S is more specific than h .
 - (c) Remove from G , any hypothesis i.e less general than another hypothesis in G .

Example :-

→ Initialise G & S

$$G_0 \leftarrow \{<?, ?, ?, ?, ?, ?, ?>\} \text{ Most general hypothesis}$$

$$S_0 \leftarrow \{\emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset\} \text{ " specific " , " " }$$

→ Candidate Elimination Trace :-

Training Examples :

1. $\langle \text{sunny}, \text{warm}, \text{Normal}, \text{strong}, \text{warm}, \text{same}, \text{Enjoy sport} = \text{yes} \rangle$.

2. $\langle \text{sunny}, \text{warm}, \text{high}, \text{strong}, \text{warm}, \text{same}, \text{Enjoy sport} = \text{yes} \rangle$.

- S_0 & G_0 are initial boundary sets.
- Training Examples 1 & 2 force S boundary to become more general, as in "Find S " Algo.
- They don't have any effect on G boundary.

$$S_0: \boxed{\{\emptyset, \emptyset, \emptyset, \emptyset, \emptyset, \emptyset\}}$$

$$S_1: \boxed{\{\text{sunny}, \text{warm}, \text{Normal}, \text{strong}, \text{warm}, \text{same}\}}$$

$$S_2: \boxed{\{\text{sunny}, \text{warm}, ?, \text{strong}, \text{warm}, \text{same}\}}$$

$$G_0, G_1, G_2: \boxed{\{<?, ?, ?, ?, ?, ?, ?>\}}$$

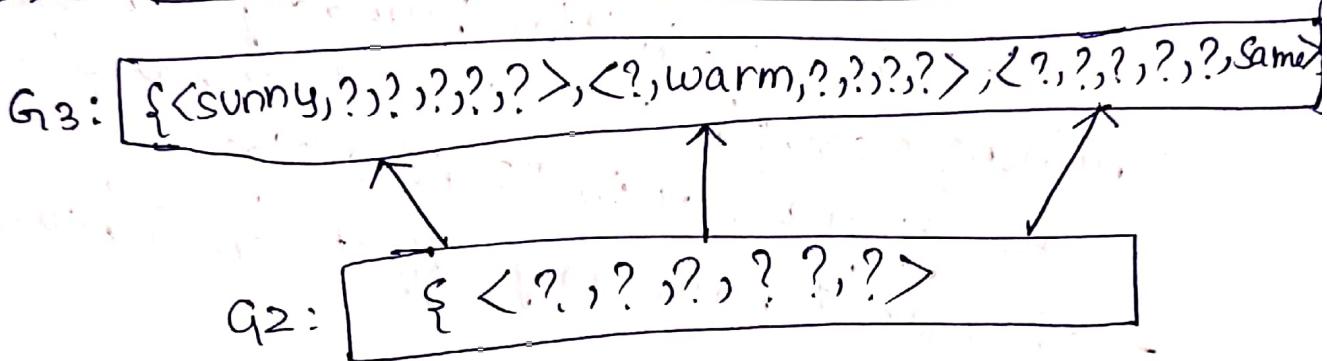
→ candidate-elimination Trace 2:-

Training Example:

3. $\langle \text{Rainy}, \text{cold}, \text{High}, \text{strong}, \text{warm}, \text{change} \rangle$,
 Enjoy sport = No

∴ it's a -ve example, it will now force G_2 boundary to be specialised to G_3 . & there will be several alternative maximally general hypotheses that are included.

$S_2, S_3 : \{\langle \text{sunny}, \text{warm}, ?, \text{strong}, \text{warm}, \text{same} \rangle\}$



→ candidate-elimination Trace 3:-

Training Example:

4. $\langle \text{sunny}, \text{warm}, \text{high}, \text{strong}, \text{cool}, \text{change} \rangle$,
 Enjoy sport = Yes.

∴ +ve Training example, it generalizes 'S' boundary from S_3 to S_4 .

Also one member of G_3 should be deleted since

it is no longer more general than S_4

boundary.

S3: $\{ \langle \text{sunny}, \text{warm}, ?, \text{strong}, \text{warm}, \text{same} \rangle \}$

S4: $\{ \langle \text{sunny}, \text{warm}, ?, \text{strong}, ?, ?, ? \rangle \}$

G4: $\{ \langle \text{sunny}, ?, ?, ?, ?, ? \rangle \langle ?, \text{warm}, ?, ?, ?, ? \rangle ? \}$

G3: $\{ \langle \text{sunny}, ?, ?, ?, ?, ? \rangle \langle ?, \text{warm}, ?, ?, ?, ? \rangle \langle ?, ?, ?, ?, ?, \text{same} \rangle ? \}$

Final version :-

- Set S4 & G4 delimit version space of all hypotheses consistent with set of incrementally observed training examples.
- The foll. learned version space is independent of the seq. in which training examples present.
- S & G boundaries move monotonically closer, delimiting smaller & smaller version space of candidate hypotheses.

S4: $\{ \langle \text{sunny}, \text{warm}, ?, \text{strong}, ?, ?, ? \rangle \}$

$\langle \text{sunny}, ?, ?, \text{strong}, ?, ?, ? \rangle \langle \text{sunny}, \text{warm}, ?, ?, ?, ? \rangle \langle ?, \text{warm}, ?, \text{strong}, ?, ? \rangle$

G4: $\{ \langle \text{sunny}, ?, ?, ?, ?, ? \rangle \langle ?, \text{warm}, ?, ?, ?, ? \rangle \}$

Remarks On Version space & "Candidate-Elimination":

- "Version space" learned by the Candidate-Elimination algorithm will converge towards the hypothesis that correctly describes the target concept, provided
 - (i) There are no errors in the training examples.
 - (ii) There is some hypothesis in H that correctly describes the target concept.
- Also if training data contains errors, Algo. will remove correct target concept from Version space & may create an empty version space.
- If the learner is allowed to conduct experiments in which it chooses next instance (a.k.a. Query) then it obtains correct classification for this instance from an external oracle (ex: Nature or Teacher).

Example:-

For "enjoy sport" task, the "Query strategy" would be to choose an instance that would be classified +ve by some of these hypotheses, but negative by others such as:

(sunny, warm, Normal, light, warm, same)
- With optimal query strategy, size of Version space is reduced by half with each new example. The correct target concept can therefore be found with only $\lceil \log_2 |V_{st}| \rceil$ experiments

- It is possible to classify certain examples, with same degree of confidence as if the target concept had been uniquely identified, even if it uses partially learned concept.
- Consider an instance A, which was not amongst the training examples, the learner will classify it as +ve. since, condition will be met iff instance satisfies every member of s. (with definition of more general than).
- ^{why} For instance B, it will be classified Negative by every hypothesis in version space; given a partially learned concept. (Just check if instance satisfies none of members of G).
- For instance C, Half of version space hypotheses classify it +ve & $\frac{1}{2}$ as -ve. Hence Learner can't classify this instance confidently.
- In case of instance D, it is classified +ve by 2 of version space hypotheses & -ve by other 4 \Rightarrow Negative classification it is.

| <u>Instance</u> | <u>Sky</u> | <u>AirTemp</u> | <u>Humidity</u> | <u>Wind</u> | <u>Water</u> | <u>Forecast</u> | <u>Enjoy Sport</u> |
|-----------------|------------|----------------|-----------------|-------------|--------------|-----------------|--------------------|
| A | sunny | warm | normal | strong | cool | change | ? |
| B | rainy | cold | " | light | warm | same | ? |
| C | sunny | warm | " | " | " | " | ? |
| D | sunny | cold | " | strong | " | " | ? |

Inductive Bias:-

- "candidate elimination" Algo converges iff. accurate training examples & its initial hypothesis space contains target concept.
- A Biased Hypothesis space:
 - Assume that hypothesis space contains unknown target concept.
 - In "Enjoy Sport" example, hypothesis space was restricted to include only conjunctions of attribute values.
 - Becoz of this restriction, hypothesis space is unable to rep. even simple disjunctive target concepts such as "Sky = sunny or Sky = cloudy".

| Ex | sky | Air Temp | Humidity | Wind | Water | Forecast | Enjoy Sport |
|----|--------|----------|----------|--------|-------|----------|-------------|
| 1 | Sunny | warm | Normal | Strong | Cool | Change | Yes |
| 2 | Cloudy | warm | " | " | " | " | Yes |
| 3 | Rainy | warm | " | " | " | " | No |

There are no hypotheses which are consistent with these 3 examples above.

- The maximally specific hypothesis from H is s_2 : (? , warm, Normal, Strong, Cool, change)

- problem:- The learner is biased to consider only conjunctive hypotheses.

An Unbiased Learner:-

Powerset:- A hypothesis space capable of representing every "Teachable concept" i.e. Capable of rep. every possible subset of instances of X .
 \Rightarrow Set of all subsets of X is called powerset of X .

Ex: - X for "Enjoy sports" is 9^6 .

(30)

$$\text{So, Distinct subsets} = 2^{1 \times 1} = 2^6 = 10^{28}.$$

- ④ Reformulate "Enjoy sports" Task in unbiased way,
lets define new hypothesis space H' .
 H' rep. every subset of instances \Rightarrow corresponds to power set of X .
lets allow arbitrary disjunctions, conjunctions & negations.
 \Rightarrow "sky=sunny" or "sky=cloudy" can be defined as
 $\langle \text{sunny}, ?, ?, ?, ?, ? \rangle \vee \langle \text{cloudy}, ?, ?, ?, ?, ? \rangle$
- ④ - If a "Candidate-Elimination" Algo is used then
S boundary is disjunction of observed +ve example whereas
G boundary will be negated disjunction of -ve examples
so, we have to present every single instance in X
as Training example, instead of S & G.

→ Futility of Bias-Free learning:

* Fundamental Property of Inductive Inference

- A learner that makes no a priori assumptions regarding the identity of the target concept has no rational basis for classifying any unseen instances.
- Inductive learning requires some form of prior assumptions or Inductive bias to classify new instances.
- Consider an arbitrary learning algo. 'L'. & an arbitrary set of Training data $D_C = \{ \langle x, c(x) \rangle \}$
Here c = Some arbitrary Target concept.
- After Training, 'L' classifies a new instance x_i .
- Let $L(x_i, D_C)$ denotes that classification (+ve or -ve).
- Inductive Inference step performed by L is:
$$(D_C \wedge x_i) \rightarrow L(x_i, D_C)$$

where \Rightarrow symbol means Inductively inferred,
 $\Rightarrow y \Rightarrow z \Rightarrow x$ is inductively inferred from y .

- Assume $L = \text{candidate_Elimination Algo}$
 $D_c = \text{Training data of "enjoy sport" Task}$.
 Here we should also define inductive bias of L .

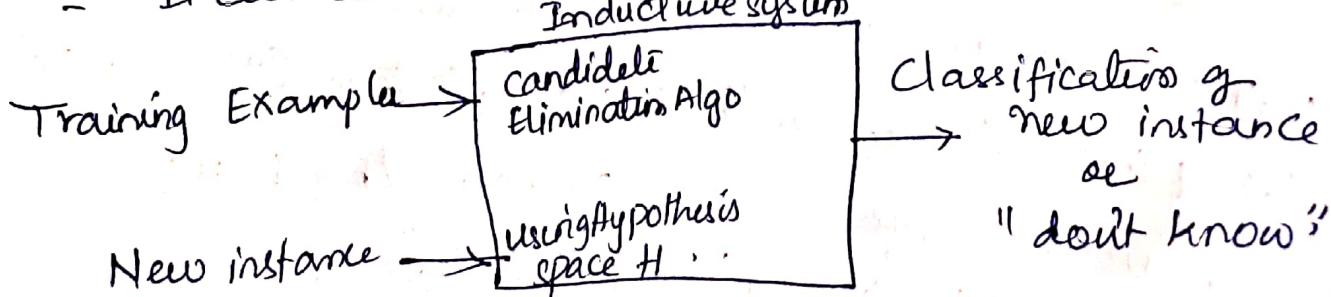
Definition:-

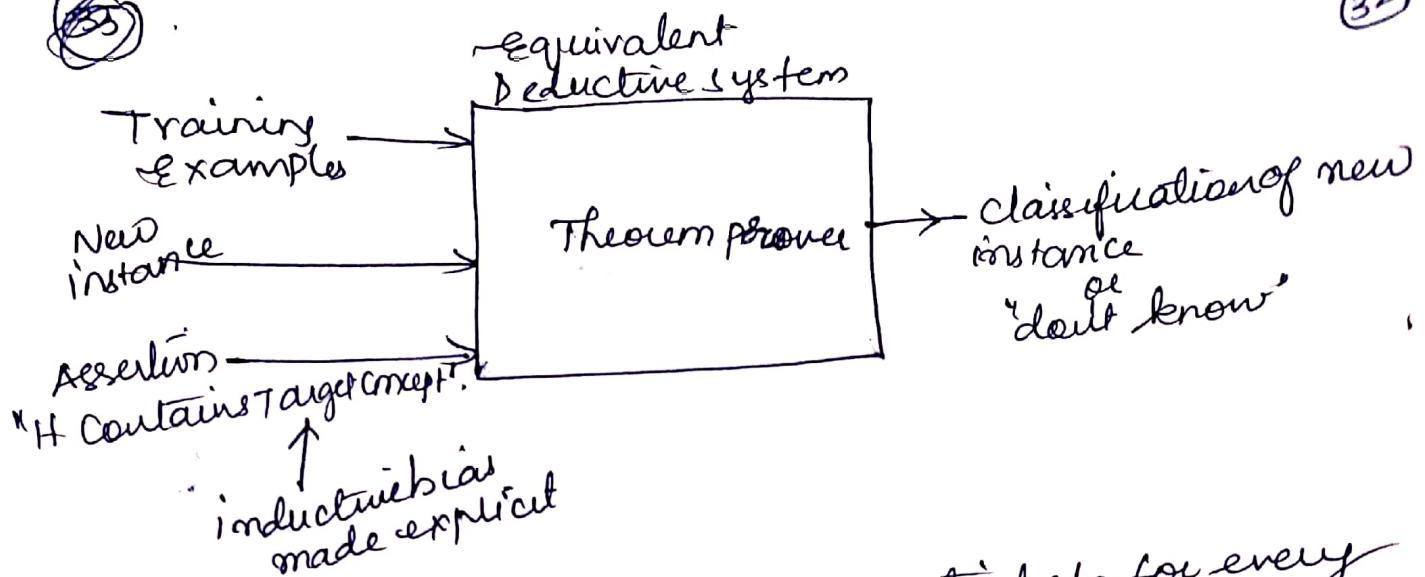
inductive bias of L is any minimal set of Assertions 'B' such that for any target concept $C \in$ corresp. Training examples D_c

$$(\forall x_i \in X) [(B \wedge D_c \wedge x_i) \vdash L(x_i; D_c)]$$

[Note :- $y \vdash z \Rightarrow z$ follows deductively from y
 i.e. z is provable from y]

- Given D_c , use candidate_elimination Algo.
- compute Version Space VS_{H, D_c}
- classify new instance x_i (+ve or -ve)
- Inductive bias : Assumption that $C \in H$
- The classification $L(x_i; D_c)$ follows deductively from $B = \{C \in H\}$ along with Data D_c & description instance x_i
- so, if we assume $C \in H$ then it follows deductively that $C \in VS_{H, D_c}$. Also, classification $L(x_i; D_c)$ is to be unanimous vote of all hypothesis in VS_{H, D_c}
- $L(x_i) = L(x_i; D_c)$
- It can be described by foll. fig:





These 2 sys. will produce identical o/p for every possible i/p set of train set & new instances set X.

→ Inductive Bias Advantages:-

- (i) provides non-procedural means of characterizing their policy for generalizing beyond observed data
- (ii) Allows comparison of diff. learners accdg to strength of inductive bias

→ The 3 Algo. Examples from Weakest to strongest bias are:-

(i) Rule-Learner :-

- Simply storing each observe Training example in memory
- It has no inductive bias
- Instances are classified by looking them up in memory. If found, the stored classification is returned else sys. refuses to classify the new instance.

(ii) Candidate-Elimination :-

- New instances are classified only when all members of the current version space agree on the classification
- It refuses to classify new instance. It has strong inductive bias

(iii) Find-S :-

- Finds most specific hypotheses consistent with the training examples. Then it uses this hypothesis to classify the subsequent instance. It has a stronger inductive bias