Assignment3

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1) Formulating Transportation using R

```
library(lpSolveAPI)
library(tinytex)
library(lpSolve)
```

#The data here is to be converted into a table format

```
#cost matrix
cost_1 <- matrix(c(22,14,30,600,100,
                 16,20,24,625,120,
                 80,60,70,"-","-","210/220"),ncol = 5,nrow = 3,byrow = TRUE)
```

```
## Warning in matrix(c(22, 14, 30, 600, 100, 16, 20, 24, 625, 120, 80, 60, : data
## length [16] is not a sub-multiple or multiple of the number of rows [3]
```

```
colnames(cost_1) <- c("Warehouse_1", "Warehouse_2","Warehouse_3","ProductionCost","Production Capacity")

rownames(cost_1) <- c("Plant_A","Plant_B","Demand")

cost_1
```

```
##         Warehouse_1 Warehouse_2 Warehouse_3 ProductionCost Production Capacity
## Plant_A "22"        "14"        "30"        "600"          "100"
## Plant_B "16"        "20"        "24"        "625"          "120"
## Demand  "80"        "60"        "70"        "-"            "-"
```

The Objective function is to Minimize the TC

$$\text{Min } TC = 622x_{11} + 614x_{12} + 630x_{13} + 641x_{21} + 645x_{22} + 649x_{23}$$

Subject to the following constraints : Supply

$$X_{11} + X_{12} + X_{13} >= 100$$

$$X_{21} + X_{22} + X_{23} >= 120$$

Subject to the following constraints : Demand

$$X_{11} + X_{21} >= 80$$

$$X_{12} + X_{22} >= 60$$

$$X_{13} + X_{23} >= 70$$

Non-Negativity Constraints

$$X_{ij} >= 0$$

Where i = 1,2 and j= 1,2,3

```
#The capacity = 220 and Demand = 210.
#A "Dummy" row for Warehouse_4 Is added here
trans.cost_1 <- matrix(c(622,614,630,0,100,
                         641,645,649,0,120,
                         80,60,70,10,220), ncol = 5, nrow = 3, byrow = TRUE)
trans.cost_1
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]  622  614  630    0  100
## [2,]  641  645  649    0  120
## [3,]   80   60   70   10  220
```

```
#Defining names for the rows and columns
colnames(trans.cost_1) <- c("Warehouse_1","Warehouse_2","Warehouse_3","Dummy","Production Capacity")

rownames(trans.cost_1) <- c("Plant_1", "Plant_2","Monthly Demand")
trans.cost_1
```

```
##                Warehouse_1 Warehouse_2 Warehouse_3 Dummy Production Capacity
## Plant_1                622         614         630     0                 100
## Plant_2                641         645         649     0                 120
## Monthly Demand          80          60          70    10                 220
```

```
#costs matrix
costs <- matrix(c(622,614,630,0,
                  641,645,649,0), nrow = 2, byrow = TRUE)
costs
```

```
##      [,1] [,2] [,3] [,4]
## [1,]  622  614  630    0
## [2,]  641  645  649    0
```

We have to know that Supply function cannot be greater than the specified units where as the Demand function can be greater than the specified units.

```
#setting up constraint signs and right-hand sides(supply side)
row.signs <- rep("<=",2)
row.rhs <- c(100,120)

#Demand side constraints#
col.signs <- rep(">=",4)
col.rhs <- c(80,60,70,10)
```

```r
lptrans <- lp.transport(costs, "min", row.signs,row.rhs,col.signs,col.rhs)
```

```r
lptrans$solution
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0   60   40    0
## [2,]   80    0   30   10
```

80 AEDs in Plant 2 - Warehouse_1

60 AEDs in Plant 1 - Warehouse_2

40 AEDs in Plant 1 - Warehouse_3

30 AEDs in Plant 2 - Warehouse_3

The above should be the production in each plant and distribution to the three wholesaler warehouses to minimize the overall cost of production as well as shipping.

```r
lptrans$objval
```

```
## [1] 132790
```

The combined cost of production and shipping for the defibrilators is $132,790

```r
lptrans$duals
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
```

2)Formulate the dual of the transportation problem

Since the primal was to minimize the transportation cost the dual of it would be to maximize the value added(VA).

U'v are the variables here.

```r
cost_2 <- matrix(c(622,614,630,100,"u1",
                   641,645,649,120,"u2",
                   80,60,70,220,"-",
                   "v1","v2","v3","-","-"),ncol = 5,nrow = 4,byrow = TRUE)
colnames(cost_2) <- c("Warehouse_1", "Warehouse_2","Warehouse_3","Production Capacity","Supply(Dual)")

rownames(cost_2) <- c("Plant_A","Plant_B","Demand","Demand(Dual)")
```

$$\text{Max } VA = 100P_1 + 120P_2 + 80W_1 + 60W_2 + 70W_3$$

Total Profit Constraints

$$u_1 + v_1 <= 622$$

$$u_1 + v_2 <= 614$$

$$u_1 + v_3 <= 630$$

$$u_2 + v_1 <= 641$$

$$u_2 + v_2 <= 645$$

$$u_2 + v_3 <= 649$$

These are taken from the transposed matrix of the Primal of the LP. These are unrestricted where

$$u_k, v_l$$

where u=1,2 and v=1,2,3

```
#Objective function

f.obj <- c(100,120,80,60,70)

#transposed from the constraints matrix in the primal
f.con <- matrix(c(1,0,1,0,0,
                  1,0,0,1,0,
                  1,0,0,0,1,
                  0,1,1,0,0,
                  0,1,0,1,0,
                  0,1,0,0,1), nrow = 6, byrow = TRUE)

f.dir <- c("<=",
           "<=",
           "<=",
           "<=",
           "<=",
           "<=")

f.rhs <- c(622,614,630,641,645,649)
lp("max",f.obj,f.con,f.dir,f.rhs)
```

```
## Success: the objective function is 139120
```

```
lp("max",f.obj,f.con,f.dir,f.rhs)$solution
```

```
## [1] 614 633   8   0  16
```

Z=139,120 and variables are:

$$u_1 = 614$$

$$u_2 = 633$$

$$v_1 = 8$$

$$v_3 = 16$$

So Z = $139,120 and variables are

$$u_1 = 614$$

which represents Plant A

$$u_2 = 633$$

which represents Plant B

$$v_1 = 8$$

which represents Warehouse_1

$$v_2 = 16$$

which represents Warehouse_3

3) Economic Interpretation of the dua:

minimal Z(Primal) = 132790 maximum Z(Dual) = 139120. We understood that we should not be shipping from Plant(A/B) to all the three Warehouses. Shipping should be done from

$$60X_{12}$$

which is 60 Units from Plant A to Warehouse 2.

$$40X_{13}$$

which is 40 Units from Plant A to Warehouse 3.

$$80X_{13}$$

which is 60 Units from Plant B to Warehouse 1.

$$30X_{13}$$

which is 60 Units from Plant B to Warehouse 3.

We will Max the profit from each distribution to the respective capacity.

$$u_1^0 - v_1^0 \leq 622$$

then we subtract

$$v_1^0$$

to the other side to get

$$u_1^0 \leq 622 - v_1^0$$

To compute it would be $614 <= (-8+622) which is correct. we would continue to evaluate these equations:

$$u_1 \leq 622 - v_1 => 614 \leq 622 - 8 = 614 => correct$$

$$u_1 \leq 614 - v_2 => 614 \leq 614 - 0 = 614 => correct$$

$$u_1 \leq 630 - v_3 => 614 \leq 630 - 16 = 614 => correct$$

$$u_2 \leq 641 - v_1 => 633 \leq 614 - 8 = 633 => correct$$

$$u_2 \leq 645 - v_2 => 633 \leq 645 - 0 = 645 => Incorrect$$

$$u_2 \leq 649 - v_3 => 633 \leq 649 - 16 = 633 => correct$$

From the above using the Sensitivity and Duality, the shadow price can be tested. Change 100 to 101 and 120 to 121 in our LP Transport.

```
row.rhs1 <- c(101,120)
row.signs1 <- rep("<=",2)
col.rhs1 <- c(80,60,70,10)
col.signs1 <- rep(">=",4)
row.rhs2 <- c(100,121)
row.signs2 <- rep("<=",2)
col.rhs2 <- c(80,60,70,10)
col.signs2 <- rep(">=",4)

lp.transport(costs,"min",row.signs,row.rhs,col.signs,col.rhs)
```

```
## Success: the objective function is 132790
```

```r
lp.transport(costs,"min",row.signs1,row.rhs1,col.signs1,col.rhs1)
```

```
## Success: the objective function is 132771
```

```r
lp.transport(costs,"min",row.signs2,row.rhs2,col.signs2,col.rhs2)
```

```
## Success: the objective function is 132790
```

The 'min' of the specific function is taken here and we observe that the number goes down by 19. This indicates the shadow price is 19, that was found from the primal and adding 1 to each of the Plants. Here, Plant B does not have a shadow price.

From the dual variable

$$v_1$$

where Marginal Revenue <= Marginal Cost. The equation was

$$u_2 \leq 645 - v_2 => 633 \leq 645 - 0 = 645 => Incorrect$$

and this was found by using

$$u_1^0 - v_1^0 \leq 622$$

then we subtract

$$v_1^0$$

to the other side to get

$$u_1^0 \leq 622 - v_1^0$$

The economic interpretation of the dual follows the universal rule of profit maximization which is MR >= MC

```r
lp("max", f.obj,f.con, f.dir,f.rhs)$solution
```

```
## [1] 614 633   8   0  16
```

Warehouse1 >= Plant1 + 621 i.e. MR1 >= MC1

Marginal Revenue which is the revenue generated for each additional unit sold relative to Marginal Cost (MC). This is the change in cost at Plant 1 by inducing an increase in the supply function should be greater than or equal to the revenue generated for each additional unit distributed to Warehouse 1.

$$60X_{12}$$

which is 60 Units from Plant A to Warehouse 2.

$$40X_{13}$$

which is 40 Units from Plant A to Warehouse 3.

$$80X_{13}$$

which is 60 Units from Plant B to Warehouse 1.

$$30X_{13}$$

which is 60 Units from Plant B to Warehouse 3.

Out of 6, 5 of them had MR <= MC. Here we need MR=MC. Plant B to Warehouse_2 does not satisfy the requirement. Hence, there will not be any AED device shipment.