PROBABILITY

Vocabulary

Event is an outcome that can occur when something happens by chance. For example, tossing a coin and getting a head.

Experiment is a procedure that can be repeated and has a finite set of outcomes. For example, a coin can be tossed 100 times and each time the outcome can be a head or a tail.

Sample Space is the finite set of outcomes of an experiment. For example, if a person is asked to pick a number from 1 to 10, the sample space will be S = {1,2,3,4,5,6,7,8,9,10}.

Probability of an event is the proportion of times an event occurs when the experiment is repeated independently under the same conditions. Denoted by **Pr(A).** For example, if a coin is tossed, the probability of getting a head is ½ or 0.5 or 50%.

Independent Events are events where the outcome of one event does not affect that of the other event. For example, if a dice is rolled twice, the outcome of the second roll is independent of the outcome of the first roll.

Dependent Events are events where the outcome of one event impacts the outcome of the second. For example, draw a card from a deck of 52 cards and suppose the card is a King. Now, without replacing the drawn card, draw another card. This time, there are 51 cards to draw from.

Discrete Probability is the subset of probability that is related to categorical data.

Random Variable is a variable whose value depends on the outcome of a random event. The probability distribution is provided by a function f(x) which provides the value for each possible value of the random variable x.

Monte Carlo Simulation is the technique of modelling the probability of different outcomes by repeating a random process a large enough number of times that the results are similar to what would be observed if the process were repeated forever. For example, toss a coin 1 time and the probability of getting a head is 50%. To determine the probability of getting a head if the coin is tossed forever, we can toss the coin 10000 times, determine the outcomes and calculate the probability. It will be ~50%.

Probability Distribution for a variable describes the probability of observing each possible outcome for that variable.

Conditional Probability is the probability of an event given that another event has already occurred. For example, a card is drawn from a deck and it is a King. Now, without replacing the card, determine the possibility that the next drawn card is also a King.

Event A = Card 1 is King
Event B = Card 2 is King
Pr(Event B | Event A) = 3 / 51

Permutations are all possible ways of choosing outcomes in a particular order. In this case, (a,b) is different from (b,a).

Combinations are all possible ways of choosing outcomes when the order does not matter. In this case the outcome (a,b) is the same as (b,a).

Multiplication Rule

Pr(A and B) = Pr(A) * Pr(B|A)

For example, in a game of Blackjack, the probability of scoring a straight 21 by drawing an ace followed by a face card can be calculated as below.

A = draw an Ace
Pr(A) = No. of Aces / Total Number of Cards = 4 / 52 = 1/13..

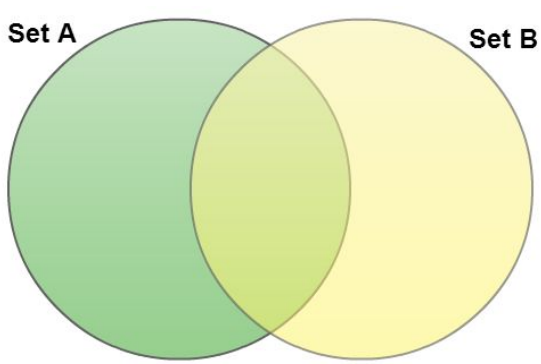
B = draw a face card
Pr(B | A) = Number of Face Cards / Number of Cards left in the deck = 16/51.

Therefore, Pr(scoring a straight 21 in Blackjack) = Pr(A) . Pr(B|A) = 1/13 * 16/51 = ~0.025 or 2.5%.

Note: For independent events, Pr(A and B) = Pr(A).Pr(B).

Addition Rule

Pr(A or B) = Pr(A) + Pr(B) - Pr(A and B)



For example, in this Venn Diagram, if we add A and B, we are calculating the intersection of A and B twice. Hence, we exclude one intersection.

Probability Distribution Function

For any random variable X, the probability distribution function can be defined as F(a) = Pr(x <= a) for all values of a.

Central Limit Theorem

The distribution of the sum of a random variable is approximated by a normal distribution.

Expected Value

The average of many draws of a random variable.

For any event with two outcomes a and b with probability p and 1 - p respectively, the expected value can be calculated as below.

Expected value of one random variable: **E[X] = μ = (a * p) + (b * (1 - p))**

Expected value of sum of n draws of a random variable: **n * ((a * p) + (b * (1 - p)))**

Standard Error

The standard deviation of many draws of a random variable.

For any event with two outcomes a and b with probability p and 1 - p respectively, the standard error can be calculated as below.

Standard Error of one random variable: **SE[X] = σ = | b - a | * sqrt(p * (1 - p))**

Standard Error of sum of n draws of a random variable: **sqrt(n) * | b - a | * sqrt(p * (1 - p))**

Statistical Properties of Averages

- Random variable times a constant
  - The expected value of a random variable multiplied by a constant is that constant times its original expected value.

$$E[aX] = a * E[X] = a * \mu$$

  - The standard error of a random variable multiplied by a constant is that constant times its original standard error.

$$SE[aX] = a * SE[X] = a * \sigma$$

- Average of multiple draws of a random variable
  - The average value of the average of multiple draws from an urn is the expected value of the urn (μ).
  - The standard error of the multiple draws from an urn is the standard deviation divided by the square root of the number of draws ($\sigma / \sqrt{n}$).
- The sum of multiple draws of a random variable
  - The expected value of the sum of n draws of a variable is n times the original expected value.

$$E[nX] = n * E[X] = n * \mu$$

  - The standard error of the sum of n draws of a random variable is $\sqrt{n}$ times its original standard error.

$$SE[nX] = \sqrt{n} * SE[X] = \sqrt{n} * \sigma$$

- The sum of multiple different random variables
  - The expected value of the sum of different random variables is the sum of the individual expected values for each random variable.

$$E[X1+X2+\cdots+Xn] = \mu1+\mu2+\cdots+\mu n$$

  - The standard error of the sum of different random variables is the square root of the sum of the squares of the individual expected values for each random variable.

$$SE[X1+X2+\cdots+Xn] = \sqrt{(\sigma_1^2+\sigma_2^2+\cdots+\sigma_n^2)}$$

- Transformation of random variables
  - If X is a random variable and a and b are non-random constants, aX + b is also a random variable.

Law of Large Numbers

As n increases, the standard error of the average of the random variable decreases. In other words, when n is large, the average of the draws converges to the average of the urn.

It only applies when n is very large and the events are independent.