Data 605 - HW13

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Import Libraries

```
library(tidyverse)
library(pracma)
```

Question 1

Use integration by substitution to solve the integral below. $\int 4e^{-7x}dx$ We will substitute $u = -7x$ differentiating both sides gives $du = -7dx$ or the algebraic equivalent of $dx = \frac{du}{-7}$. Now we integrate with respect to du and then resubstitute $u = -7x$.

$$\int 4\frac{e^u du}{-7} = \frac{-4}{7}\int e^u du = \frac{-4}{7}e^u = \frac{-4}{7}e^{-7x}$$

Question 2

Biologists are treating a pond contaminated with bacteria. The level of contamination is changing at a rate of: $\frac{dN}{dt} = -\frac{3150}{t^4} - 220$ bacteria per cubic centimeter per day, where t is the number of days since treatment began. Find a function N(t) to estimate the level of contamination if the level after 1 day was 6530 bacteria per cubic centimeter.

$$\frac{dN}{dt} = -\frac{3150}{t^4} - 220$$

multiply both sides by dt

$$dN = -\frac{3150}{t^4}dt - 220dt$$

then integrate both sides

$$N(t) = -\frac{1050}{t^3} - 220t + C$$

Substituting in for N(1) = 6530

$$6530 = -1050 - 220 + C$$

Solving for C gives C = 7800 and yields or original function N(t):

$$N(t) = -\frac{1050}{t^3} - 220t + 7800$$

Question 3

Find the total area of the red rectangles in the figure below, where the equation of the line is f(x) = 2x - 9.

The total area is equal to the sum of the area for each rectangle. Each is 1 wide. The centers of the rectangles are at x= 5, 6, 7, 8 so the height of each rectangle is give by f(x).

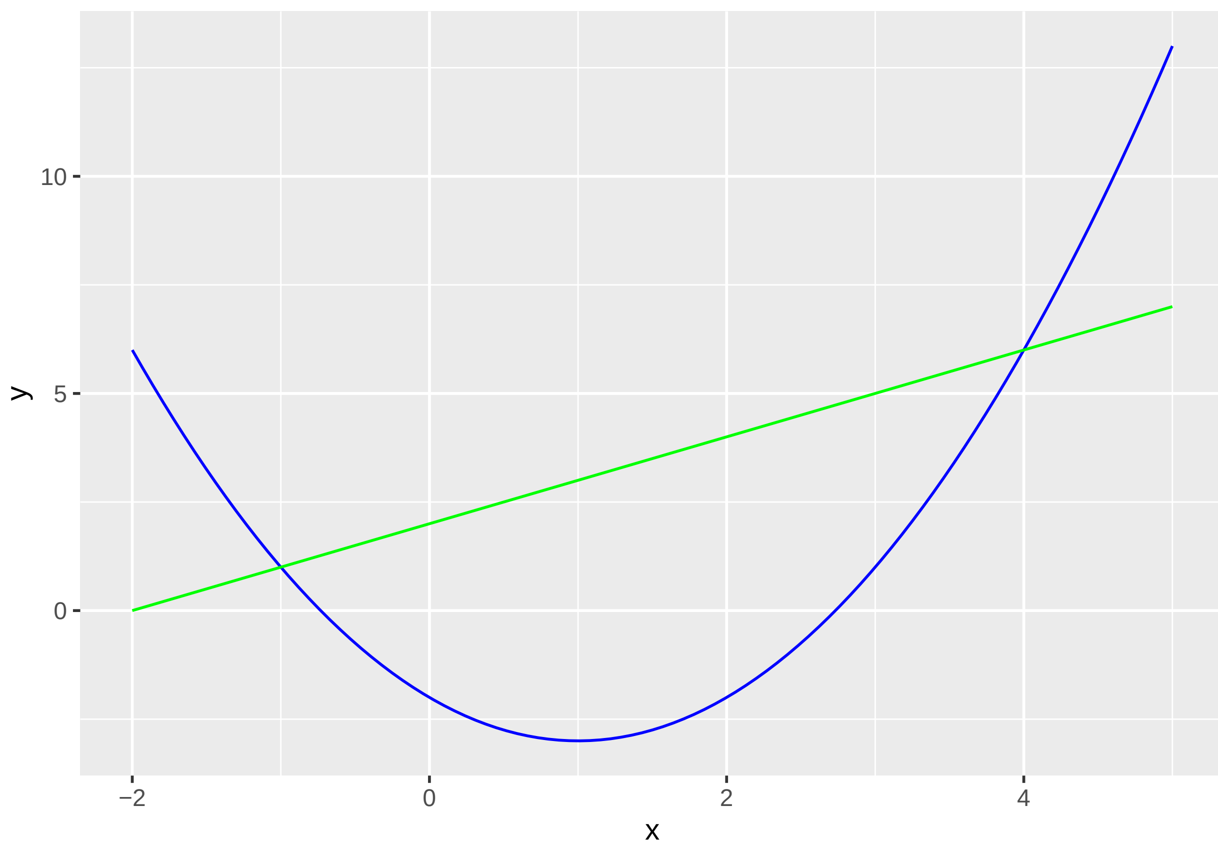
```
f <- function(x){
  2 * x - 9
}
area = (1 * f(5)) + (1 * f(6)) + (1 * f(7)) + (1 * f(8))
area
```

```
## [1] 16
```

Question 4

Find the area of the region bounded by the graphs of the given equations. $f(x) = x^2 - 2x - 2$, g(x) = x + 2

```
ggplot(data.frame(x = c(-2,5)), aes(x=x)) +
  stat_function(fun= function(x){x^2 - 2*x - 2}, color = "blue") +
  stat_function(fun= function(x){x + 2}, color = "green")
```



Since the g(x) is greater than f(x) through the interval [-1, 4], the area bounded by the curves is given by: $\int_a^b g(x) - f(x)\,dx$ then our area is given by: $\int_{-1}^4 -x^2 + 3x + 4$ The integral is equal to $\frac{-x^3}{3} + 3\frac{x^2}{2} + 4x$ and evaluated between 4 and -1 gives:

```r
f <- function(x){
  -1/3 * x^3 + 3/2 * x^2 + 4 * x
}
area = f(4) - f(-1)
area
```

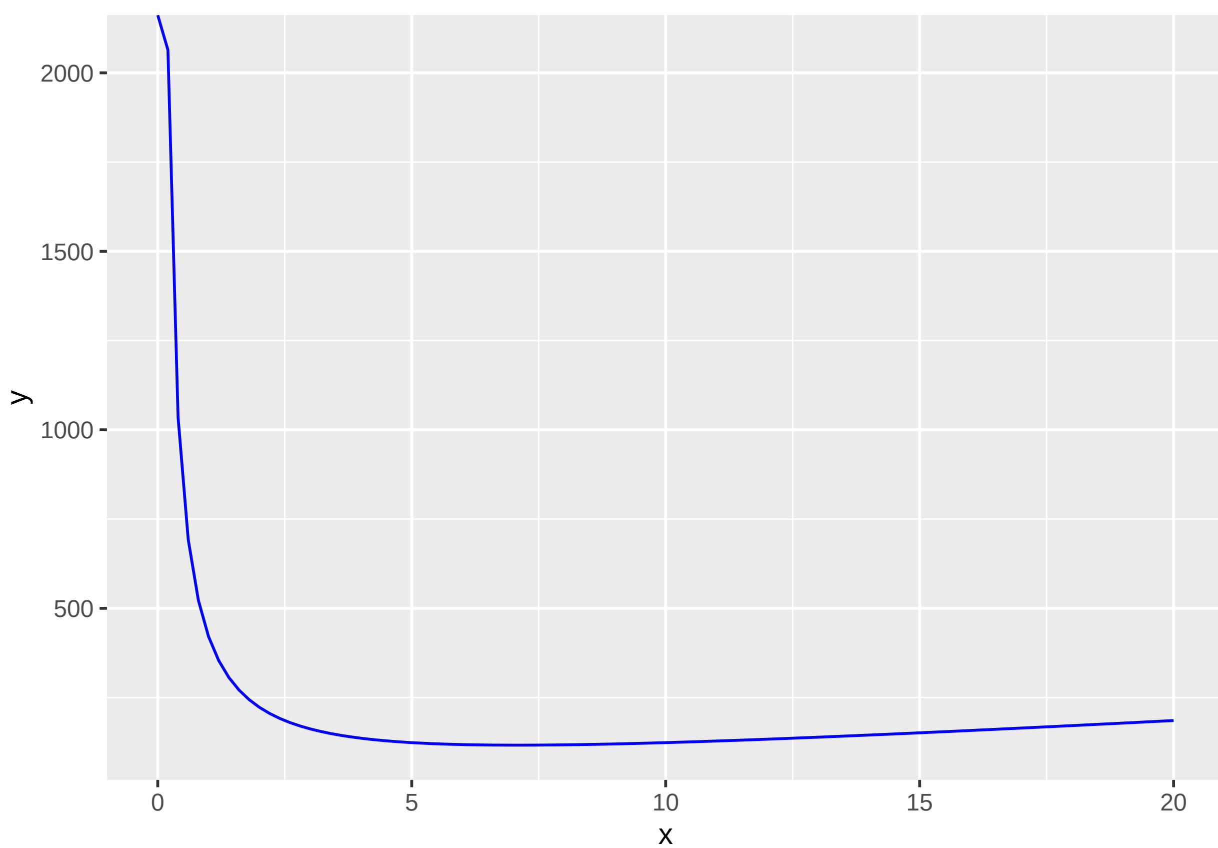
```
## [1] 20.83333
```

Question 5

A beauty supply store expects to sell 110 flat irons during the next year. It costs $3.75 to store one flat iron for one year. There is a fixed cost of $8.25 for each order. Find the lot size and the number of orders per year that will minimize inventory costs.

cost = 8.25 * orders + 3.75 * 110/orders

```r
ggplot(data.frame(x = c(0,20)), aes(x=x)) +
  stat_function(fun= function(x){8.25*x + (3.75 * 110)/x}, color = "blue")
```




```r
cost <- function(x){
  8.25*x + (3.75 * 110)/x
}
cost_deriv <- function(x){
  8.25 - (3.75 * 110) * x^(-2)
```

```
}
#using the bisection method to find the root
root <- bisect(cost_deriv, 0, 15)
root
```

```
## $root
## [1] 7.071068
##
## $f.root
## [1] 1.776357e-15
##
## $iter
## [1] 55
##
## $estim.prec
## [1] 8.881784e-16
```

```
#since the root is not an integer we need to evaluate at the closest integer and choose the lower cost
cost(7)
```

```
## [1] 116.6786
```

```
cost(8)
```

```
## [1] 117.5625
```

7 orders minimizes inventory costs.

Question 6

Use integration by parts to solve the integral below. $\int ln(9x) * x^6 dx$

Integration by parts formula:

$$\int udv = uv - \int vdu$$

If we set u = ln(9x) and $dv = x^6$ then:

$$\int ln(9x) * x^6 dx = ln(9x) * \frac{x^7}{7} - \int \frac{x^7}{7x} = ln(9x) * \frac{x^7}{7} - \frac{x^7}{49} + C$$

Question 7

Determine whether f(x) is a probability density function on the interval 1, e^6 . If not, determine the value of the definite integral. $f(x) = \frac{1}{6x}$

```
integrate(function(x) {1/(6*x)}, lower=1, upper=exp(6))
```

```
## 1 with absolute error < 9.3e-05
```

This is a probability density function over the interval because the area under the curve sums to 1.