DB3 - E:C23

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C23

Find the eigenvalues, eigenspaces, algebraic and geometric multiplicities for:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

To solve for the eigenvalues we solve: $det(\lambda I - A) = 0$

$$det(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}) \ = 0$$

$$det(\begin{bmatrix} \lambda - 1 & -1 \\ -1 & \lambda - 1 \end{bmatrix}) \ = 0$$

$(\lambda - 1)^2 - 1 = 0$. Simplified we have: $\lambda(\lambda - 2) = 0 \rightarrow \lambda = 0,\ 2$ with algebraic multiplicities of 1.

For $\lambda = 0$:

$$(\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix})\ \vec{x} = \vec{0}$$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \vec{x} = \vec{0}$$

The RREF of this matrix results from performing R2 = R2 - R1 and R1 = R1 * -1

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \vec{x} = \vec{0}$$

Multiplying these matrices $\rightarrow x_1 + x_2 = 0 \rightarrow x_1 = -x_2$ Setting $x_2 = s$ yields the eigenspace for $\lambda = 0$:

$$E_0 = \text{Span}\left(s \times \begin{bmatrix} 1 \\ -1 \end{bmatrix} \forall s \in \mathbf{R}\right)$$

For $\lambda = 2$:

$$(\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix})\ \vec{x} = \vec{0}$$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \vec{x} = \vec{0}$$

The RREF of this matrix results from performing R2 = R2 + R1

$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \vec{x} = \vec{0}$$

Multiplying these matrices $\rightarrow x_1 - x_2 = 0 \rightarrow x_1 = x_2$ Setting $x_2 = s$ yields the eigenspace for $\lambda = 2$:

$$E_2 = \text{Span}\left(s \times \begin{bmatrix} 1 \\ 1 \end{bmatrix} \forall s \in \mathbf{R} \right)$$

The geometric multiplicity is equal to the dim of both E_0 and $E_2 = 1$

Confirming the values in R

```
A = matrix(c(1,1,1,1),2)
ev = eigen(A)
print(ev$values)
```

```
## [1] 2 0
```

```
print(ev$vectors)
```

```
##           [,1]       [,2]
## [1,] 0.7071068 -0.7071068
## [2,] 0.7071068  0.7071068
```