Data 609 - HW 2

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Ex. 1

Show $x^2 + e^x + 2x^4 + 1$ is convex.

Solution:

Since the non negative weighted sums of convex functions are also convex (from 2.1.3) I must show that the 3 separate functions of $f_1 = x^2$, $f_2 = e^x$ and $f_3 = 2x^4 + 1$ are convex. I then have $\alpha f_1 + \beta f_2$ with $\alpha, \beta = 1 \geq 0$. This definition of non negative weighted sums can then be recursively applied to a combination of any number of convex functions which in this case is 3.

From 2.1.2 we have the definition of a convex function where a function is convex if:

$f(\alpha x + \beta y \leq \alpha f(x) + \beta f(y)\ \forall x, y$ and s.t.

$\alpha \geq 0, \beta \geq 0, \alpha + \beta = 1$

Show $f_1 = x^2$ is convex:

$(\alpha x + \beta y)^2 \leq \alpha x^2 + \beta y^2 \implies$ expanding and moving all variables to one sides

$\alpha^2 x^2 + 2\alpha\beta xy + \beta^2 y^2 - \alpha x^2 - \beta y^2 \leq 0$ now factoring terms gives

$\alpha x^2(\alpha - 1) + 2\alpha\beta xy + \beta y^2(\beta - 1) \leq 0$ substituting in for 1 which by definition is equal to $\alpha + \beta \implies$

$\alpha\beta x^2 + 2\alpha\beta xy + \alpha\beta y^2 \leq 0$ factoring yields

$\alpha\beta(2xy - x^2 - y2) \leq 0 \implies -\alpha\beta(x - y)^2 \leq 0 \implies$

$\alpha\beta(x - y)^2 \geq 0$ which must be true because by definition $\alpha, \beta \geq 0$ and $(x - y)^2 \geq 0\ \forall x, y \in R$

Show $f_2 = e^x$ is convex: See figure 2.1 on p.21

Show $f_3 = 2x^4 + 1$ is convex:

f_3 is convex because its the composition of a affine function of the form $Ax + b$ with $g = x^4$ which is strictly increasing.

Ex. 2

Show that the mean of the exponential distribution $p(x) = \lambda e^{-\lambda x}, x \geq 0 (\lambda > 0)$ is $\mu = 1\lambda$ and variance is $\sigma^2 = 1\lambda^2$

The mean and variance are given by the moment generating function. For the exponential distribution the moment is given by:

$M(t) = \frac{\lambda}{\lambda - t}$

Therefore, $M'(t) = \frac{\lambda}{(\lambda - t)^2}$ and the mean $\mu = M'(0) = \frac{1}{\lambda}$.

The variance is given by $M''(0) - M'(0)^2$ Since, $M''(t) = \frac{2\lambda}{(\lambda - t)^3}$ the variance is $\frac{1}{\lambda^2}$

Ex. 3

It is estimated that there is a typo in every 250 data entries in a database, assuming the number of typos can obey the Poisson distribution. For a given 1000 data entries,

The probability of exactly 4 typos:

```
dpois(x=4, lambda=4)
```

```
## [1] 0.1953668
```

The probability of no typo at all:

```
dpois(x=0, lambda=4)
```

```
## [1] 0.01831564
```

Draw 1000 samples with $\lambda = 4$ and show their histogram.

```
Poisson <- rpois(1000, lambda=4)
hist(Poisson)
```