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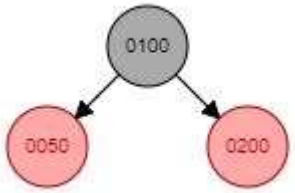
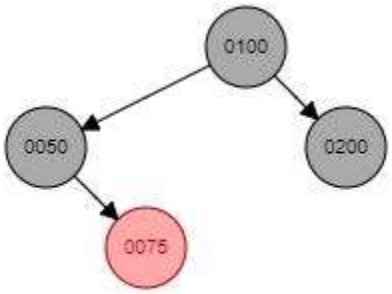
CSCI 2270

Assignment 7

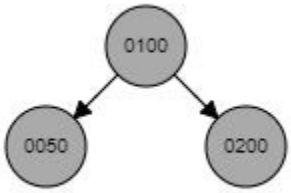
1. Inserting a node, re-balancing the tree, and then deleting the inserted node does not result in the original. Consider this initial tree:



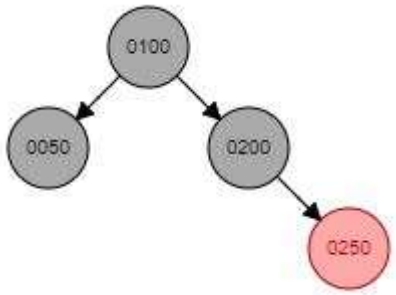
Inserting 75 into this tree results in a violation of Property 4—if a node is red, then both of its children must be black. Both 75 and its parent, 50, are red nodes; 75's uncle, 200, is also red, which is a Case 1 violation. Thus, the parent and uncle become black while the grandparent, 100, becomes red. Because 100 is also the root, this is a violation of Property 2—the root node is black. This means that 100 must also be recolored to be black. This results in a balanced tree.



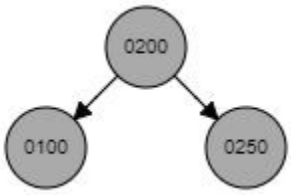
Deleting 75 results in the same tree configuration, but with different colors. Thus, the tree is not the same as it originally was. No rotation is necessary to solve this case.



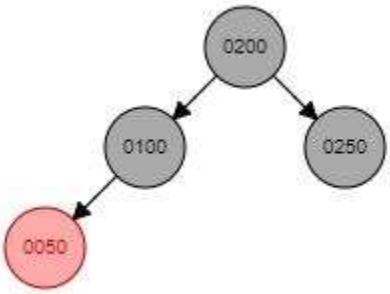
2. Deleting a node with no children from a red-black tree, re-balancing, and then inserting a node with the same key does not result in the same tree. Consider the following example:



Deleting 50 results in an unbalanced tree that violates Property 5—for each node in the tree, all paths from that node to the leaf nodes contain the same number of black nodes. The null node that is in 50's place is marked as double black. In this case, if at least one of the null node's sibling's children is red, the solution is rotation. The sibling is marked as the target around which rotation is performed, and because it is a right child, the rotation is leftward. 200, the sibling, becomes the new root. Then, because the tree is still unbalanced in terms of Property 5—the left path contains one black node and the right path does not—the right child, 250, is recolored to be black. This new tree is now balanced.



Now, when 50 is added back to the tree, it is less than 200 and 100, which means that it becomes the left child of 100, and is marked red. Because there are no additional black nodes, the tree remains balanced.



This tree retains the same number of nodes with the same values, but is clearly different from how it was originally.