Homework #4

STA 630, Spring 2025

Due: April 11, Friday

Problem 1 (60 pts)

Consider the balanced, additive, one-way ANOVA model:

$$Y_{ij} = \mu + \alpha_i + \varepsilon_{ij}, \quad i = 1, \ldots, I, \quad j = 1, \ldots, J, \tag{1}$$

where $\varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2)$ i.i.d, with $\mu \in \mathbb{R}$, $\alpha_i \in \mathbb{R}$, and $\sigma_\varepsilon^2 > 0$. We adopt a prior structure that is the product of independent conjugate priors, wherein μ has a flat prior, $\alpha_i \sim N(0, \sigma_\alpha^2)$ i.i.d., and $\sigma_\varepsilon^2 \sim IG(a, b)$. Assume that σ_α^2, a, and b are known.

1. (10 pts) Derive the full conditional distributions for μ, α_i, and σ_ε^2 necessary for implementing the Gibbs sampler in this problem.

2. (10 pts) What is meant by *convergence diagnosis*? Describe some tools you might use to assist in this regard. What might you do to improve a sampler suffering from slow convergence?

3. (10 pts) Suppose for simplicity that σ_ε^2 is also known. What conditions on the data or the prior might lead to slow convergence for μ and the α_i? (Hint: What conditions weaken the identifiability of the parameters?)

4. (20 pts) Now let $\eta_i = \mu + \alpha_i$ so that η_i *centers* α_i. Then we can consider two possible parameterizations: (i) (μ, α) and (ii) (μ, η). Generate a sample of data from likelihood (1), assuming $I = 5$, $J = 1$, and $\sigma_\varepsilon^2 = \sigma_\alpha^2 = 1$. Write a program to investigate the sample cross-correlations and autocorrelations produced by Gibbs samplers operating on parameterizations (i) and (ii) above. Which performs better?

5. (Bonus: 10 pts)Rerun the program for the case $\sigma_\varepsilon^2 = 1$ and $\sigma_\alpha^2 = 10$. Now which parameterization performs better? What does this suggest about the benefits of hierarchical centering reparameterizations?

Problem 2 (60 pts)

Problem 6.2 from Hoff.

- (a) 10 pts
- (b) 20 pts
- (c) 20 pts
- (d) Bonus: 10 pts