Homework #3

STA 630, Spring 2025

Due: March 24, Monday

Problem 1 (20 pts)

Consider the multivariate normal model with known variance-covariance matrix Σ and unknown mean μ. Derive the posterior distribution of μ (show all the algebric steps to get to the final distribution)

Problem 2 (25 pts)

Problem 7.1 from Hoff.

- (a) 10 pts
- (b) 15 pts

Problem 3 (25 pts)

Importance sampling Devise and implement an importance sampler for estimating the expected value of a mixture of two beta distributions (e.g., $0.3 \times Beta\ (5, 2) + 0.7 \times Beta(2, 8)$). Use the same sample to estimate the probability that this random variable is included in the interval [0.45, 0.55].

Problem 4 (30 pts)

Consider a univariate normal model with mean μ and variance σ^2. Suppose we use a $Beta(2, 2)$ prior for μ (somehow we know μ is between zero and one) and a log-normal $LN(1, 10)$ prior for σ^2 (recall that if a random variable X follows $LN(m, v)$ then $\log X$ follows $N(m, v)$). Assume a priori that μ and σ^2 are independent. Implement a Metropolis-Hastings algorithm to evaluate the posterior distribution of μ and σ^2. Remember that you have to jointly accept or reject μ and σ^2. Also compute the posterior probability that μ is bigger than 0.5.

Here are the data:

2.3656491	2.4952035	1.0837817	0.7586751	0.8780483	1.2765341
1.4598699	0.1801679	-1.0093589	1.4870201	-0.1193149	0.2578262

Problem 5 (Bonus: 20 pts)

Prove that the following algorithm is equivalent to the Accept-Reject algorithm:

1. Generate $X \sim g$;
2. Generate $U \mid X = x \sim U_{[0;Mg(x)]}$;
3. Accept $Y = X$ if $U \leq f(x)$;
4. Return to 1. otherwise.

(Hint: prove that $P(Y < y) = P(X < x \mid U < f(x))$.)