Page 1, Problem 1a.

1) one-way ANOVA model

$Y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$ $\quad \varepsilon_{ij} \sim N(0, \sigma^2_\varepsilon)$

Priors

$\pi(\mu) \sim U(0,1)$

$\pi(\alpha_i) \sim N(0, \sigma^2_\alpha)$

$\pi(\sigma^2_\varepsilon) \sim IG(a, b)$

a, b, σ^2_α known.

Conditionals

$\mu \mid \vec{Y}, \alpha, \varepsilon \sim N(\mu_n,$

Full Conditional Distributions

we can also say,

$Y \mid \mu, \alpha, \sigma^2 \sim N(\mu + \alpha_i, \sigma^2)$

we want posteriors for the following.

$\{ \vec{Y} \mid \mu, \alpha, \sigma^2 \}$ Likelihood

$\mu \mid \vec{Y}, \alpha, \sigma^2$

$\alpha_i \mid \vec{Y}, \mu, \sigma^2$

$\sigma^2_\varepsilon \mid \vec{Y}, \mu, \alpha$

have these for our Gibbs Sampler

• $\mu \mid \vec{Y}, \alpha, \sigma^2 \sim N\left(\mu_n + \alpha_i, \frac{\sigma^2}{n}\right)$

given the flat prior and the normal conjugate prior.

end up with

$\phi^{(s)} = \{ \mu^s, \alpha^s, \sigma^{2s} \}$.

Page 2, Problem 1 parts a, b, c, and d.

- $\alpha_i \mid \vec{y}, \mu, \sigma_e^2 \sim \mathcal{N}(\alpha, \sigma_\alpha^2)$, again, given the normal prior and evidence.

- $\sigma_e^2 \mid \vec{y}, \mu, \alpha_i \sim$ Inv. Gamma (

given, $\sigma_e^2 \sim IG(a, b)$

we can say,

$$\sigma_e^2 \mid \vec{y}, \mu, \alpha_i \sim \text{Inv. Gamma}\left(a + \frac{n}{2},\ b + \frac{\sum (y_i - \mu)^2}{2}\right)$$

given the normality of the evidence again.

b) Done in later

c) What conditions lead to slow convergence for μ and α_i?

d) $\eta_i = \mu + \alpha_i$ ~ centering

two parameter cases

comparing

$\phi = \{\mu, \alpha\}$

$\Gamma = \{\mu, \eta\}$

Run Gibbs for each, and compare

2

Page 3, problem 1 part d, problem 2 parts a and b.

$p(\mu^{s+1} | \alpha_i^s, \vec{y}) \sim N(\mu_n + \alpha_i, \frac{\sigma^2}{n})$

$p(\alpha^s | \mu^{s+1}, \vec{y}) \sim N(\alpha^s, \sigma_n^2)$

$Y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$ $\quad i = 1, \dots, 5$

$j = 1$

$\sigma_\varepsilon^2 = \sigma_\alpha^2 = 1.$

$y | \mu, \alpha, \sigma^2 \sim N(\mu + 1, \sigma_\varepsilon^2)$

Say we let $\mu_0 = 0$,

$\sigma^2 = 1$.

Then,

$y | \mu, \alpha, \sigma^2 \sim N(1, 1)$

2) Hoff - 6.2

a) Done in Code

b) Mixed Model

$z = p \cdot N(\theta_1, \sigma_1^2) + (1-p) \cdot N(\theta_2, \sigma_2^2)$

$p \sim \text{Beta}(a, b)$ $\quad \frac{1}{\sigma_i} \sim \text{Gamma}(\nu_0/2, \frac{\nu_0 \sigma_0^2}{2})$

$\theta_i \sim N(\mu_0, \tau_0^2)$

Page 4, Problem 2 part b.

Obtain conditionals for;

$(x_1, \dots, x_n)$

ρ

θ_1

θ_2

σ_1^2

σ_2^2

- $(p(\vec{x} \mid \rho, \theta_1, \theta_2, \sigma_1^2, \sigma_2^2)$

  $= N(\theta_{1|2}, \sigma^2_{1|2})$

- $p(\theta_1 \mid \rho, \vec{x}, \theta_2, \sigma_1^2, \sigma_2^2)$

  $\sim N\left(\theta_{1|2}, \frac{\sigma^2_{1|2}}{2}\right)$

- $p(\theta_2 \mid \rho, \vec{x}, \theta_1, \sigma_1^2, \sigma_2^2)$

  $\sim N\left(\theta_{1|2}, \frac{\sigma^2_{1|2}}{2}\right)$

- $p(\sigma_1^2 \mid \rho, \vec{x}, \theta_1, \theta_2, \sigma_2^2)$

  $\sim \text{Inv. Gamma}\left(a + \frac{n}{2}, b + \frac{\sum (x_i - \mu)^2}{2}\right)$

- $p(\sigma_2^2 \mid \rho, \vec{x}, \theta_1, \theta_2, \sigma_1^2)$

  $\sim \text{Inv. Gamma}\left(a + \frac{n}{2}, b + \frac{\sum (x_i - \mu)^2}{2}\right)$

- $p(\rho \mid \vec{x}, \theta_1, \theta_2, \sigma_1^2, \sigma_2^2)$

  $\sim \text{Beta}\left(a + \sum x_i, b + n - \sum x_i\right)$

4