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Advanced Programming

State Monad

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SOFTWARE QUALITY RESEARCH

4A58: live exercise solving

3A52: exercise session as usual

Does FP Eliminate State?

Picture: You after four weeks of ADPRO

- **Variables** and fields are state
- Program **stack&heap** are state
- **Databases** are state
- **Program counter** is state!
- Variable **assignments** set state
- Variable **accesses** read state
- **Loops** must change state
- **Exceptions** modify program counter and stack
- We have **disallowed** almost all of these?

- In FP we often make **state explicit**:
  - Converted loops to **recursive functions**
  - A function is a **state transform**
  - **Arguments** are the state explicitly
  - **No other implicit**, hidden state that can be **changed** by others
- Today, a pure **pattern** for state transforms:
  - A more **recipe-like** way to encode state
  - Allows to hide the state, make it **implicit** like in imperative programming
  - Still the state is **encapsulated** in a well defined value
  - Still **no other** encapsulated state that can be **changed** by others

- `RNG`: a random **integer generator**; we maintain the state
- `Rand[A]`: a random **A generator**; we hide the RNG in the state
- `State[S,A]`: a general pattern for **stateful computations** producing A, where the state is of type S

AGENDA

var

State[S,A]

A Typical Stateful Imperative API

```scala
var rng = new scala.Util.Random

// returns a random number form 0 to 5
def rollDie: Int = rng.nextInt(6)
```

- We call `rollDie` and observe a value `5` the first time, and `0` the second time
- Mentimeter 6547 6694: What is the result of `rollDie + rollDie`?
- What does it tell us about referential transparency of `rollDie`?
- `rng` is an external, implicit state, that can be changed by others
- To make `rollDie` referentially transparent, make the state explicit

Converting RNG to explicit state

- We had: `RNG.nextInt: () =>Int`
- Lets **return new state explicitly**, instead of modifying old (RT)

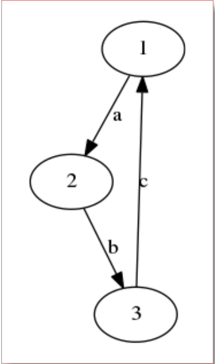
```
trait RNG:
  def nextInt: (Int, RNG)

object RNG:
  def nextInt(rng: RNG): (Int, RNG) =
    rng.nextInt
```

- **In general** a function: `State =>(Output,State)`
- **Wrap** this as `case class State[S,+A] (run: S =>(A,S) )`
- So `RNG` becomes `State[RNG,Int] (run = RNG.nextInt)`
- **Intuition 1**: `Automaton` or `Transition` would be better names than `State`
- **Intuition 2**: `step` would be a better name than `run`

Consider a Simple Automaton

Stateful **by definition**



```
var state = 1
while true do
  state match
    case 1 => print("a"); state = 2
    case 2 => print("b"); state = 3
    case 3 => print("c"); state = 1
```

```
var x = 0
while true do
  print(x)
  x+=1
```

```
def step(state: Int): (String, Int) =
  state match
    case 1 => ("a", 2)
    case 2 => ("b", 3)
    case 3 => ("c", 1)

def loop(state: Int = 1) =
  val (output, state1) = step(state)
  print(output)
  loop(state1)
```

- This automaton as an **instance of `State`**: `State[Int,String](step)`
- More precisely the instance is: `State[Int,String](run =step)`
- Convert the second while loop to a similar instance of `State` (5min)

States vs Streams

- We can **unroll** (unfold) a state machine from an initial state, producing a word of actions (a lazy list)
- **Discuss:** What is the lazy list from our first automaton?
- **Discuss:** What is the lazy list from our second automaton?
- **Mentimeter 6547 6694:** another stream
- An **exercise** implementing this mapping as a function
- **Laziness** of streams is useful here, why?

Anything stateful maps to the state pattern

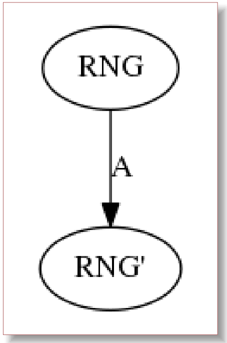
Recap

- Random number generators (state: `RNG` seed)
- Websites with modality (session state)
- Database backed applications (DB state)
- Communication protocols (protocol state)
- etc.

Random Number Generator as an Instance of State

```scala
type Rand[A] = State[RNG,A]
```

- `RNG` is the state of the random generator (usually some large number encapsulated)
- The textbook gives a simple implementation of `RNG` based on multiplication with large primes module 64 bits
- `Rand[A]` is a **computation** that we can run, then it will produce a random `A` and a new state `RNG`
- Another useful intuition: `Rand[A]` is **a generator of random `A`'s**
- Or even just a **"random `A`"**


$$\begin{tikzcd} \text{RNG} \arrow[d, "A"] \\ \text{RNG}' \end{tikzcd}$$

How do I use this generator of random numbers?

```scala
type Rand[A] = State[RNG, A]
val r: Rand[Int] = ...
val (i, r1) = r.run(SimpleRNG(42))
```

- `SimpleRNG` is the book’s concrete implementation of the `RNG` trait
- `42` is the initial seed (state)
- `(r1,i)` is a new state and a random number
- **Question:** How do I get the next random number?
- **Question:** What happens if I call `r.run` again?

What can we do with Automata/State?

State is a monad, similar key operations as for `List`, `Option`, and `Stream`

```
def map[S, A, B](s: State[S, A])(f: A =>B): State[S, B]
```

Can use this to generate even numbers:

```
val even: Rand[Int] = map[Int](r) { n =>n * 2 }
```

Automata can be composed [1/2]

`flatMap` can be used to compose generators:

```
def flatMap[S, A, B](s: State[S, A])(f: A =>State[S, B]): State[S, B]
```

Function `f` takes values produced by `s` and uses them to construct a new automaton.

In the context of our rand:

```
def flatMap[A, B](r: Rand[A])(f: A =>Rand[B]): Rand[B] =
  flatMap[A, B](r: State[RNG, A])(f: State[RNG, B]): State[RNG, B]
```

`flatMap` can compose generators (compute a random size list of random even integers):

```
val int: Rand[Int] =... (assume you have it)
def ints(n: Int): Rand[List[Int]] =... (assume creates a random list of given length)
val ns: Rand[List[Int]] = int.flatMap { n =>ints(n) }
```

The state RNG passed **implicitly**; size generated with different state than each number

Automata can be composed [2/2]

The `map2` function computes a zipping of two automata over the same state space:

```
map2[S, A, B, C](sa: State[S, A])(sb: State[S, B])(f: (A, B) =>C): State[S, C]
```

Could be used to create a product automaton synchronizing two computations, then `C` is `(A,B)`.

More fun in exercises :)

Next Week

- Next week we will design a parallel computation library, in purely functional style
- In two weeks, we will use the generators of random numbers to implement a modern testing framework
- So: keep reading the chapters and solve the exercises!