

A weighted multiplicative analysis to estimate trends in year-class size of capelin

Geoffrey T. Evans and Brian S. Nakashima

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Relative sizes of year-classes of capelin have previously been estimated as a compromise among several measurements. In this paper, the variability of each type of measurement around the compromise is estimated, and an improved compromise that downweights high-variability types of measurement is produced. For the most recent year-class, where few measurements are as yet available, the compromise estimate is obtained by a method that balances year-class variability and measurement precision.

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G. T. Evans and B. S. Nakashima: Science, Oceans, and Environment Branch, Department of Fisheries and Oceans, PO Box 5667, St John's, Newfoundland, Canada A1C 5X1. Correspondence to G. T. Evans; tel: +1 709 7722090; fax: +1 709 7724188; e-mail: evansgt@dfo-mpo.gc.ca

Introduction

Many measurements have been used, at different times, to indicate the relative size of year-classes of capelin, *Mallotus villosus* (Müller), near the east coast of Newfoundland. None have provided an absolute measure, and almost none have been available for as long as half the length of the total data series. To obtain a single compromise index, to describe the history of relative year-class size over the whole time period for which any measurements were made, Winters (1995) assumed that, for each year-class y and each measurement type i , the measured value $u_{yi} = s_y c_i \varepsilon_{yi}$ is the product of the year-class size s_y , a calibration factor c_i for the measurement type, and a multiplicative lognormal error term. Year-class sizes and calibration factors could then be estimated simultaneously by minimizing

$$\sum (\log u_{yi} - \log s_y - \log c_i)^2. \quad (1)$$

(The one ambiguity can be resolved by scaling so that Πs_y , or the geometric mean year-class size, is 1.)

This analysis treated all indices with equal weight, as if the error term had the same statistical distribution for all measurement types. The only way to downweight an index was to make a subjective decision to omit it from the analysis, often only after long and unsatisfactory

debate. This paper will demonstrate a method for using the observations themselves to identify and downweight high-variability measurements.

The method benefits from having many indices over the life of the year-class (ages 0–5 years), whose independent errors should not reinforce each other. When a year-class is first seen, this benefit does not yet exist, and it behoves us to compare the likely variability of year-class size and of the error in its earliest indices. In this paper, we present a method to take account of the variability in both year-class size and measurement method to improve the estimate of the size of the incoming year-class.

Methods

Table 1 indicates the diversity of measurements that have been made, and the ages and years for which they are available (from which one can generate year-classes). More details on each measurement are given in Anon. (1999) and references therein. Many of the measurements are made only on that part of the year-class that moves inshore to spawn in each year.

Weighted combination of indices

We here wish to take account of conditions that may be true in the real world but are not reflected in expression

Table 1. The abundance indices used in the analysis.

Description	Years	Ages
Eggs on beach as indication of spawner biomass	1990–1999	2–5
Area of schools observed from air	1982–1998	2–5
Catch rate of purse-seine fishery	1981–1993, 1996	2–5
Catch rate of trap fishery	1981–1993	2–5
Catch rate of Russian offshore fishery	1972–1991	2–5
By-catch in bottom-trawl survey (north)	1980–1982, 1987, 1988, 1991–1994	2–4
By-catch in bottom-trawl survey (south)	1985, 1988, 1989, 1991–1993	1–3
Survey of larvae and juveniles offshore	1994–1999	0–1
Larvae on 15 beaches	1987–1993	0
Larvae on one beach	1990–1999	0
Offshore acoustic survey (spring)	1982–1990, 1992, 1996	1–5
Offshore acoustic survey (autumn)	1981, 1983–1994	1–5

(1). First, indices may not all be equally reliable, so we wish to be able to penalize them differently. Second, large deviations may be more common than the lognormal distribution predicts, so squaring the deviation may give large deviations too much influence. Therefore, after the year-class sizes and calibration factors were computed according to expression (1), we computed the variability

$$v_i = \frac{1}{n_i - 1} \sum_{i=1}^{n_i} |\log u_{yi} - \log s_y - \log c_i|^\kappa \quad (2)$$

of each calibrated year-class measurement around the compromise index. Here, n_i is the number of measurements of type i and κ is an index between 1 (double exponential distribution of errors, leading to the median as an estimate of central tendency) and 2 (Gaussian distribution, leading to the mean; v is the variance). No attempt was made to estimate κ (indeed, it is probably wrong to suppose that κ is independent of i); instead we simply explored whether the results depended on the value assumed for κ . Each measurement at each age was considered to be a separate index with its own weight. We then computed the s_y and c_i that minimized

$$\sum \frac{|\log u_{yi} - \log s_y - \log c_i|^\kappa}{v_i} \quad (3)$$

For $\kappa=2$, this is standard reciprocal-variance weighting.

There are dangers with weighting in this way: one index might appear especially good by accident, or two indices might covary because of some unrecognized dependence. Therefore, we tend to trust estimates of very high variabilities, but distrust estimates of very low ones. This asymmetric trust is expressed by considering one-third of the indices that had the smallest computed v_i , and assigning them the largest v_i from that set. For the same reason, we did not iterate the procedure to convergence.

Estimate of most recent year-class size

Suppose we have an index u whose value in the most recent year is U . We wish to know the year-class size s in the latest year, or better, the probability distribution function (pdf) $P(s=S|u=U)$. By Bayes formula,

$$P(s=S|u=U) = P(u=U|s=S)P(s=S)/P(u=U) \quad (4)$$

Now, given that $s=S$ and because $u=s\epsilon$, the statements $u=U$ and $\epsilon\epsilon=U/S$ are identical, so they must have the same probability. The factor $q=\epsilon\epsilon$ is the catchability, the calibration factor between index and year-class size for a particular year. If we are further willing to assume that this catchability, though a random variable, is statistically independent of year-class size, then

$$P(u=U|s=S) = P(q=U/S|s=S) = P(q=U/S) \quad (5)$$

and therefore

$$P(s=S|u=U) \propto P(q=U/S) P(s=S) \quad (6)$$

In words, the question “is a particular size of the year-class probable?” breaks into two questions: “is it probable given previous experience of what year-class sizes have been?” and “is it probable given the available index and previous experience of what its errors (catchabilities) have been?” From our analysis of previous year-classes, we have empirical distributions for year-class size and catchability. The product of the two distributions is then the best estimate of the recruitment pdf, combining both pieces of information (Evans, 2000). The mean or median of the pdf could be used as a point estimate, but it is more informative to present the whole probability distribution as an indication of the uncertainty.

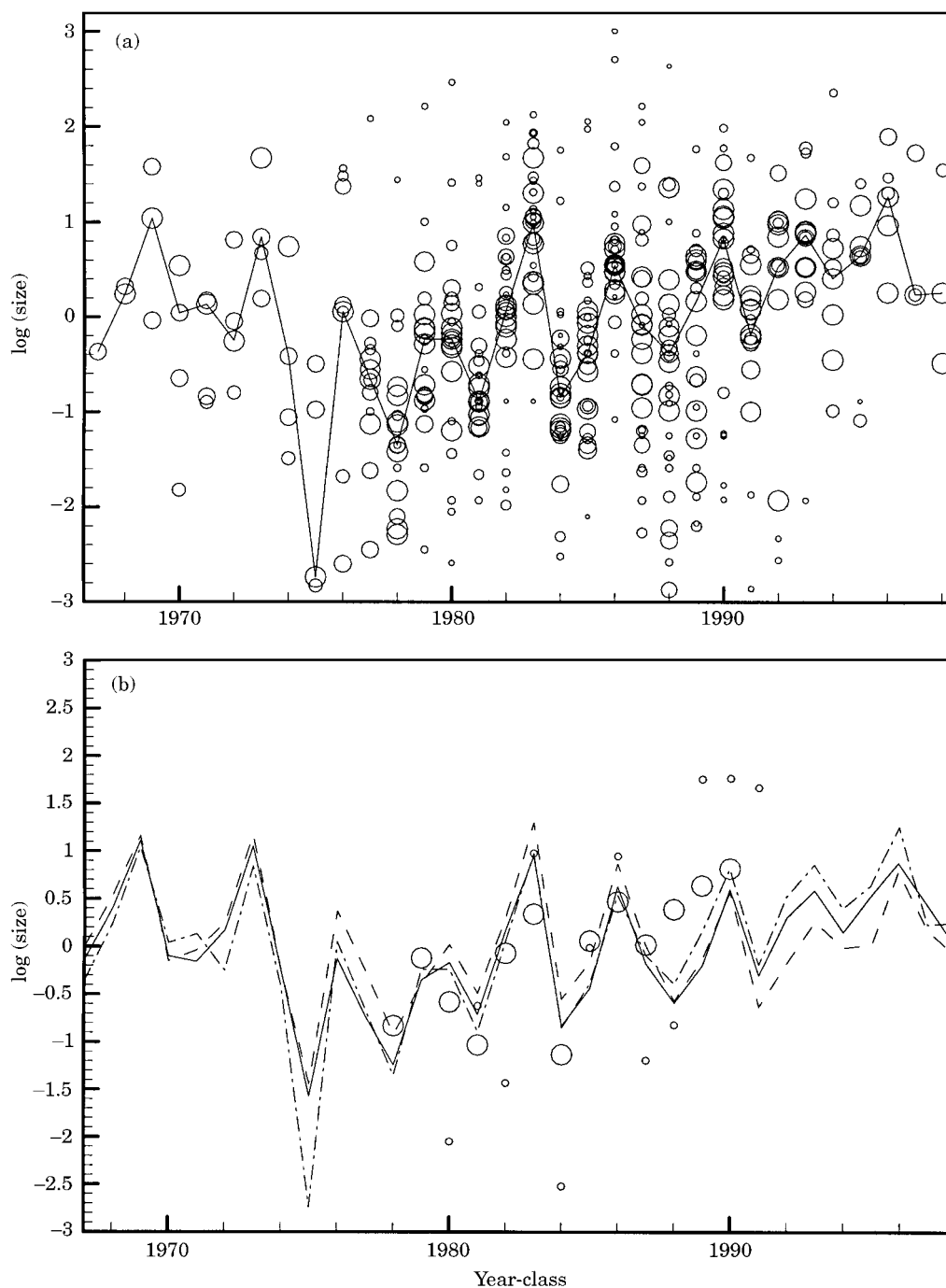


Figure 1. (a) The collection of all indices, with symbol size proportional to the computed weight. The line connects the weighted mean compromise indices for each year. (b) Two of the indices [trap catch rates at ages 2 (small circles) and 3 (large circles)] and with added lines for the unweighted mean (dashed line) and weighted median (dash-dot line).

Results

Figure 1a shows all the indices available in 2000 to assess the Newfoundland capelin stock. The symbol

sizes are proportional to their computed weights. Figure 1b shows just two indices, the catch rates of the inshore capelin trap fishery at ages 2 (low weight) and 3 (high weight, tending to stay closer to the compromise

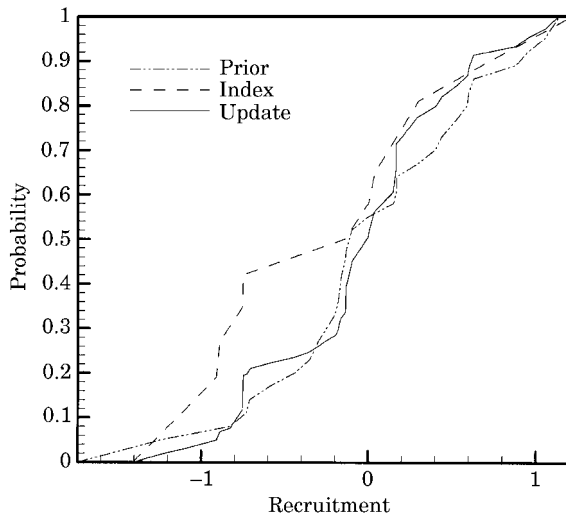


Figure 2. Estimated probability distributions for year-class size in the absence of a new index (dash-dot line), for the size that the new index indicates (dashed line), and for the updated estimate of year-class size (solid line).

line). The relative weights would make sense if most capelin mature at ages 3 or 4, and if the fraction that matures at age 2 is small and very variable, so that the total number migrating inshore at age 2 depends also on factors other than year-class size. Figure 1b also shows that there is little difference between weighted and unweighted treatments, or between treatments assuming Gaussian and double exponential distributions for the log of the error term. Closer examination suggests that, compared with the original unweighted estimates, the weighted estimates show more of an increase since the mid-1970s, and that median-weighted estimates show a greater increase still; but the differences are small compared with the scatter in the data.

Figure 2 shows the two (cumulative) probability distributions on the right-hand side of expression (6), and the compromise product distribution. (The Figure actually shows $P(U/q \leq S)$ instead of $P(q \leq U/S)$

so that it can be displayed on the same axis.) The product of the two distributions is closer to the prior distribution, which is reasonable because there were 30 years of prior estimates of year-class size and only 12 calibrations for the available age-0 indices, and the variances of the two priors are about the same.

Discussion

The biggest barrier to using the weighting method is the requirement that the errors of different indices be independent (or the lack of sufficient information from which to estimate covariances). If two indices have an undetected positive correlation, then they will agree more than others, and the method will interpret this agreement as small variation around the true value, i.e. low variance measurements that should have high weight. Sometimes the same sample is used to infer age compositions for two different indices, which is one source of dependence. Similarly, indices with an undetected negative correlation will be downweighted more than they should be. This could happen if the age at which a year-class tends to spawn varies between year-classes.

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