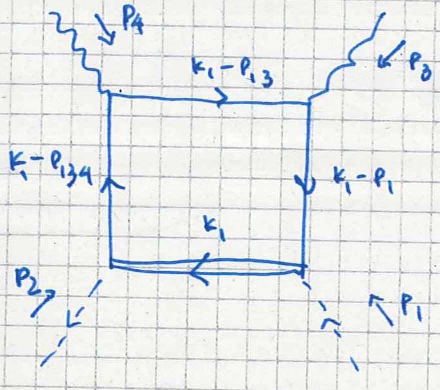
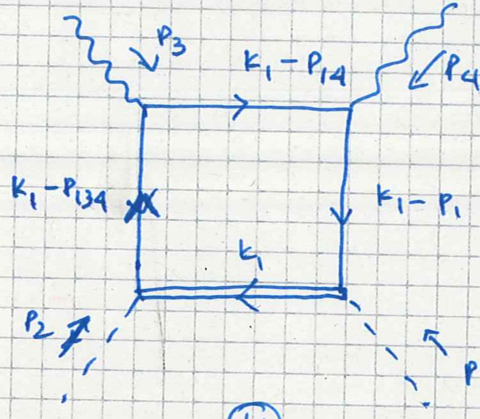



(d1)




(d2)

[notation: $p_{ijk\ldots} = p_i + p_j + \ldots$

e.g. $p_{1123} = p_1 + p_1 + p_2 + p_3$]

Kreimer ↘

We choose to start the trace from the vertex of $p_3$ :

$$d_1 = C \cdot S_1 \cdot \frac{\mathrm{Tr}\left[ \hat{\varepsilon}_3 \gamma_7 (\hat{k} - \hat{p}_{13}) \hat{\varepsilon}_4 \gamma_7 (\hat{k} - \hat{p}_{134}) \gamma_6 (\hat{k} + m) \gamma_7 (\hat{k} - \hat{p}_1) \right]}{(k^2 - m^2)\,(k - p_1)^2\,(k - p_{13})^2\,(k - p_{134})^2}$$

$$d_2 = C \cdot S_2 \cdot \frac{\mathrm{Tr}\left[ \hat{\varepsilon}_3 \gamma_7 (\hat{k} - \hat{p}_{134}) \gamma_6 (\hat{k} + m) \gamma_7 (\hat{k} - \hat{p}_1) \hat{\varepsilon}_4 \gamma_7 (\hat{k} - \hat{p}_{14}) \right]}{(k^2 - m^2)\,(k - p_1)^2\,(k - p_{14})^2\,(k - p_{134})^2}$$

with $C = i^4 \left(-\frac{i}{2}\right)^2 \left(\frac{i}{2}\right)^2 g_w^2 \, \delta_{m_1 m_1} \, \lambda'(f_6) \, \bar{\lambda}'(f_3)$

and $S_1$ , $S_2$ the SU(2) "color" factor of each diagram.

and $\gamma_6 = 1 + \gamma_5$ $\gamma_7 = 1 - \gamma_5$ [notation: $\hat{p} \equiv \not{p} = p^\mu \gamma_\mu$].

Now

(1) We use anticommuting $\gamma_5$ . The mass term in the numerator vanishes, and we end up with traces of the form

$$\mathrm{Tr}\left[ \gamma_6 \cdot \underbrace{\cdots\cdots}_{6\ \gamma\text{-matrices}} \right]$$

we focus only on $\gamma_5$ term.

(2) The numerator is rank 4 in $k$ (at most), so we can write it as

$$N = k^\alpha k^\beta k^\gamma k^\delta X_{\alpha\beta\gamma\delta} + k^\alpha k^\beta k^\gamma \Delta_{\alpha\beta\gamma} + k^\alpha k^\beta \Delta_{\alpha\beta} + k^\alpha Y_\alpha + k^2 k^\alpha \Delta_\alpha$$

The traces $\sim k^\alpha k^\beta k^\gamma k^\delta$ after tensor reduction will be $\sim$ contracted indices and will drop out because of antisymmetry of $\gamma_5$

[i.e. $\mathrm{tr}[\gamma_5 \underbrace{\epsilon}_{3} \hat{k} \underbrace{\gamma}_{4} \hat{k}\, \hat{k}\, \hat{k}] = 0$ ].

The traces $\sim k^\alpha Y_\alpha$ do not vanish but are $O(p^3)$ so we drop them.

In general, a diagram of the form

$$\frac{k^2 k^\alpha \Delta_\alpha(p) + k^\alpha k^\beta \Delta_{\alpha\beta}(p^2) + k^\alpha k^\beta k^\gamma \Delta_{\alpha\beta\gamma}(p)}{(k^2-m^2) \prod_i (k+q_i)^2}$$

momentum expansion

$$= \frac{(k^2 k^\alpha \Delta_\alpha + k^\alpha k^\beta k^\gamma \Delta_{\alpha\beta\gamma})}{(k^2-m^2)} \cdot \frac{1}{(k^2)^3}\left[1 - 2\frac{k^\delta Q_\delta}{k^2}\right] + \frac{k^\alpha k^\beta \Delta_{\alpha\beta}}{(k^2-m^2)(k^2)^3} + O(p^3).$$

where $Q^\mu = \sum_i q_i^\mu$

$$\underbrace{= \cdots}_{\text{tensor reduction}} = \frac{1}{(k^2-m^2)(k^2)^2} \cdot \frac{1}{d}\left[\Delta_{\alpha\alpha} - 2 Q^\alpha \cdot \Delta_\alpha - 2\frac{\Delta_{\alpha Q \alpha} + \Delta_{\alpha\alpha Q} + \Delta_{Q\alpha\alpha}}{d+2}\right]$$

with $\Delta_{\alpha\alpha Q} \equiv \Delta_{\alpha\beta\gamma}\, g^{\alpha\beta} Q^\gamma$ and similarly for $\Delta_{\alpha Q \alpha}$ $\Delta_{Q\alpha\alpha}$

For d1 $\quad \Delta_a = \mathrm{Tr}[\hat{\varepsilon}_3 \hat{\varepsilon}_4 \gamma^a \hat{P}_{14}] \cdot 8$

$$\Delta_{ab} = \left[ -g_{ab} \mathrm{Tr}[\hat{\varepsilon}_3 \hat{\varepsilon}_4 \hat{P}_{13} \hat{P}_{1134}] + \mathrm{Tr}[\hat{\varepsilon}_3 \gamma^a \hat{\varepsilon}_4 \hat{P}_{134} \gamma^b \hat{P}_1] \right] \cdot 8$$

$$\Delta_{ab\gamma} = 0$$

so that

$$d_1 = 8 \cdot c \cdot S_1 \cdot \frac{1}{(k^2-m^2)} \frac{1}{(k^2)^2} \frac{1}{d} \Big[ \varepsilon(\varepsilon_3, \varepsilon_4, P_1, P_4) \cdot 2 \cdot (4-d) + \varepsilon(\varepsilon_3, \varepsilon_4, P_3, P_4) \cdot (4-d) \Big]$$

$$\Rightarrow d_1 = \frac{8\, c\, S_1}{m^4} \frac{4-d}{(k^2-m^2)} \frac{1}{d} \Big[ 2\, \varepsilon(\varepsilon_3 \varepsilon_4 P_1 P_4) + \varepsilon(\varepsilon_3 \varepsilon_4 P_3 P_4) \Big]$$

For $d_2$ $\quad \Delta_a = -\mathrm{Tr}[\varepsilon_3 \varepsilon_4 \gamma^a P_{13}] \cdot 8$

$$\Delta_{ab} = 8 \cdot \left\{ g^{ab} \mathrm{Tr}[\varepsilon_3 P_{1134} \varepsilon_4 P_{14}] + \mathrm{Tr}[\varepsilon_3 P_{134} \gamma^a P_1 \varepsilon_4 \gamma^b] \right\}$$

$$\Delta_{ab\gamma} = 0$$

so that

$$d_2 = \frac{8\, c}{m^4} S_2 \frac{4-d}{k^2-m^2} \frac{1}{d} \Big[ 2\, \varepsilon(\varepsilon_3 \varepsilon_4 P_1 P_4) + \varepsilon(\varepsilon_3 \varepsilon_4 P_3 P_4) \Big]$$

Note: the Lorentz part of $d_1$ and $d_2$ are equal !! The Kreimer scheme clearly breaks the symmetry of $3 \leftrightarrow 4$ since we would naively expect, given $d_1$, that $d_2$ would be $\sim \varepsilon(\varepsilon_3 \varepsilon_4 P_1 \underline{\underline{P_3}})$