Spacing Ratio Distribution

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- Or is it $t \to -t$, $\theta \to -\theta$, $p \to p$?
- The issue is the second definition is the one used by Lemarie et al in the "Universality of the Anderson Transition" paper (Lemarié, Grémaud, and Delande 2009).

The Problem

$$H = \frac{p^2}{2} + Kcos(\theta)\sum_n \delta(t - n\tau)$$

This hamiltonian is symmetric wrt $t \to -t$, $\theta \to -\theta$ i.e time reversal and wrt $\theta \to -\theta$, $p \to -p$ i.e. parity. The time reversal only holds if we consider only $\Delta t = N\tau$ but that is fine I guess.

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- ~~But even then, the distribution should be e^{-s} for the spacing. which is not the case.~~
- The Izrailev and Atas papers I was talking about last time were indeed on kicked rotors on a torus as you suspected.

Parity is a headache (cont.)

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- ~~All in all, I have no clue what is going wrong.~~

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- I found a few places where it was detailed and I managed to implement GUE, GOE, GSE, CUE, COE, CSE generators. (Mezzadri 2007; Edelman and Rao 2005; Sirca and Horvat 2012)
- It turned out that my program worked correctly for the unitary and orthogonal ensembles but failed for the symplectic ensembles because they show degeneracy.

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- The symplectic ensembles show Kramer's degeneracy due to the half integral spin. Because of that, each energy level has at least a double degeneracy (up and down spin for spin $\frac{1}{2}$).

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- And this worked for the earlier case of the quantum kicked rotor as well. Using the parity eigenvalue would be more accurate in some sense because we don't remove other symmetries that may exist that we don't know of, but it is not good numerically.

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- And this worked for the earlier case of the quantum kicked rotor as well. Using the parity eigenvalue would be more accurate in some sense because we don't remove other symmetries that may exist that we don't know of, but it is not good numerically.
- This method removes the degeneracies arising from all the symmetries, but if we are sure that there is only one symmetry this will work. By taking the ratio of number of unique eigenvalues to the original number of eigenvalues, we can find the mean degeneracy.

Results

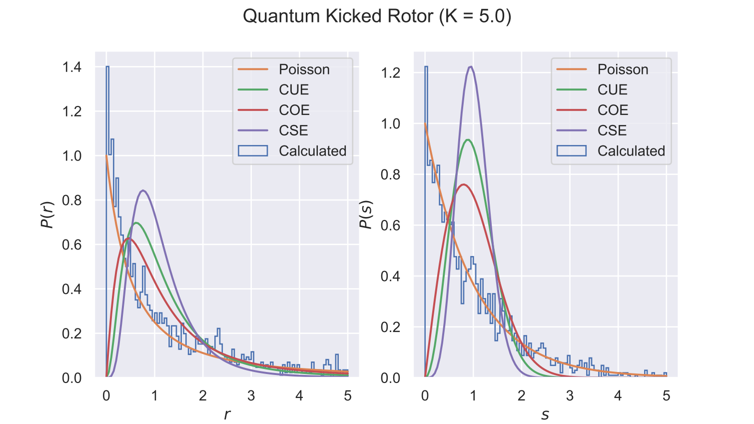

Figure 1: Standard kicked rotor with $K = 5$, p-basis $[-2000, 2000]$, 3991 eigenphases used and 10 discarded (tol $= 0.1$). Uniqueness ratio is 0.52 with tol $= 10^{-8}$

References

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