Quantum Kicked Rotor Systems

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Abstract

Chapter 1

Introduction

The quantum kicked rotor is defined by the Hamiltonian:

$$H = \frac{p^2}{2} + kcos(\theta)\sum_{n\in\mathbb{Z}}\delta(t - n\tau) \tag{1.1}$$

This classical analog of this system reduces to the Chirikov standard map [2]:

$$p_{n+1} = p_n + ksin(\theta_n) \tag{1.2}$$
$$\theta_{n+1} = \theta_n + p_{n+1} \tag{1.3}$$

This map is chaotic and shows a great deal of complexity, but we will only deal with the quantum version here. The key point for us, is that this classical map displays diffusion in the angular momentum space. This can be seen qualitatively by looking at 1.3. When p_n becomes larger than 2π, which will happen after a few kicks if k is large, then the successive θ_n will become uncorrelated. As the sign of $sin(\theta_n)$ becomes random, the sequence p_n describes a random walk. This can be backed by a quantitative calculation which shows that the the distribution $f_N(p)$ i.e. the distribution of momentum after N steps wrt different initial conditions (θ_0) is well approximated by a gaussian distribution with a diffusion constant [2]:

$$D = \frac{k^2}{2} \tag{1.4}$$

As a time dependent problem, the quantum kicked rotor has no energy eigenstates. However, since it is a periodic kicking potential, we can use floquet operators to simulate it.

$$F = \lim_{\epsilon\to 0}\int_0^{\tau+\epsilon} exp(-iHt/\hbar)dt \tag{1.5}$$
$$= exp\left(\frac{-i}{\hbar}cos\ \theta\right)exp\left(\frac{-i\tau}{2\hbar}p^2\right) \tag{1.6}$$

The floquet operator allows us to obtain the state of the system at $t = n\tau$. Working in the eigenbasis of the (angular) momentum operator $p\,|n\rangle = \hbar n\,|n\rangle\,; \langle\theta|n\rangle = e^{in\theta}$, we get:

$$\langle m|F|n\rangle = exp\left(-\frac{i\tau}{2\hbar}m^2\right)i^{n-m}J_{n-m}\left(\frac{k}{\hbar}\right) \tag{1.7}$$

We can use this expression for the floquet matrix to simulate the quantum kicked rotor system as $F^N\,|\psi(0)\rangle = |\psi(N\tau)\rangle$. Even though, the system doesn't have stationary states, we can obtain floquet eigenstates by diagonalising the floquet operator.

An important variant of the kicked rotor system is the quasiperiodic kicked rotor, which is discussed further in the report. It is given by the hamiltonian:

$$H = \frac{p^2}{2} + \mathcal{K}(t)cos(\theta)\sum_{n\in\mathbb{Z}}\delta(t - n\tau) \tag{1.8}$$

where $\mathcal{K}(t) = k(1 + cos(\omega_2 t + \phi_2)cos(\omega_3 t + \phi_3))$.

Chapter 2

Readings and Work Done

2.1 Localisation

The quantum kicked rotor also shows the same diffusion mechanism as the classical analog in the beginning. If we take the initial state as $|0\rangle$ which has a uniform distribution of $\theta's$, then the system shows a gaussian shape in the momentum space. But after a certain time, the diffusion is suppressed by quantum effects which lead to an exponential localisation in the momentum space distribution around $|0\rangle$. This phenomenon is called 'dynamical localisation'. It is analogous to the Anderson localisation found in tight-binding systems.

2.2 Anderson Localisation

We now take a short detour to explain the phenomenon of Anderson Localisation. The contents of this section are largely taken from [1].

Consider a non-relativistic particle or gaussian wave packet propogating through a channel. If the channel has a constant potential throughtout then the wave will propogate through it unimpeded performing ballistic motion. If the channel has a constant potential with a small noise term, then the potential landscape will look like a series of speckles on an otherwise flat surface. Let us assume these speckles are well separated and look more like spikes rather than shallow hills.

We have three length scales here, l the spacing between these speckles, δ the width of the speckle and λ the de Broglie wavelength of the particle. We assume $\delta << \lambda << l$ and thus, the particle sees the speckles as well-separated δ-spikes.

Each of these speckles - wells and peaks - acts as a scatter for our particle. Quantum mechanically, each of them has a finite, non-zero probability of both reflecting and transmitting the particle.

Suppose then that our particle hits scatterer 1 and gets transmitted with some probability. It then performs ballistic motion and hits scatterer 2. Again there is some probability of transmission and reflection. The particle may get transmitted immediately, or it may undergo reflection twice (a complete internal reflection) and then get transmitted. It may undergo a complete internal reflection multiple times before crossing the scatterer. This leads to the net transmission probability being dependent on the phase difference accumulated over a complete internal reflection.

The net transmission probability can be found by multiplying the transfer matrices of the two scatterers and obtaining an overall transfer matrix. This yields the expression:

$$T_{12} = \frac{T_1 T_2}{1 + \sqrt{R_1 R_2} e^{i\theta}} \tag{2.1}$$

where T_i and R_i denote the transmission and reflection probabilities of the ith scatterer, θ is the phase accumulated in a complete internal reflection.

This phase is distributed randomly irrespective of the distribution of distances between the scatterers as $\lambda << l$ and thus, we may assume a uniform distribution for θ.

Decoherence can occur if during the ballistic part, the particle couples to an external degree of freedom. In such a scenario, we can assume a uniform distribution for θ and obtain an average transmission probability. In doing this, we are effectively killing any interference effects as we are considering the phase to be completely scrambled by the time it reaches scatterer 2. The calculations then yield $\langle T_{12} \rangle = T_1 T_2/(1 - R_1 R_2)$. For a channel of length L and nearly uniform scatterer density n, this gives us $R/T \propto nL$ which is Ohm's law.

But things are much more interesting if we have phase-coherent transmission through the channel. We must then find a quantity that is additive for such a transmission and average over it. $\kappa = -ln\ T$ is such a quantity. It exhibits the property $\langle ln\ T_{12} \rangle = ln\ T_1 + ln\ T_2$ and due to this, it displays self-averaging.

Therefore, for a channel of length L, $|\langle ln\ T \rangle| \propto nL$. Since $T < 1$, $ln\ T < 0$ and thus we have $exp(\langle ln\ T \rangle) \sim exp(-L/\xi_{loc})$ where ξ_{loc} is called the localisation length. This exponential localisation in the absence of absorption is a hallmark of strong localisation.

Here, $ln\ T$ exhibits a normal distribution and its peak corresponds to the most likely value $T_{typ} = exp(\langle ln\ T \rangle) = e^{-L/\xi_{loc}}$. This phenomenon is called Anderson localisation.

In 3D, the situation becomes more complicated and the system can either allow transmission or show localisation based on disorder strength. These two regimes are separated by a 2nd order phase transition known as the Anderson transition or the metal-insulator transition, metal referring to transmission and insulator referring to localisation of the particle.

Even though Anderson originally introduced it in the form of a tight-binding model with discrete spatial sites, our presented model also exhibits the same behaviour and thus has been used in this report.

2.3 Robustness

2.4 Anderson Transition

2.5 Bipartite Entanglement

2.6 Spectral Analysis

Chapter 3

Epilogue

3.1 Future Prospects

3.2 Conclusion

Bibliography

[1] Cord A. Müller and Dominique Delande. "Disorder and interference: localization phenomena". In: *arXiv:1005.0915 [cond-mat, physics:quant-ph]* (June 1, 2016). arXiv: 1005.0915. URL: http://arxiv.org/abs/1005.0915 (visited on 04/21/2021).

[2] Hans-Jürgen Stöckmann. *Quantum Chaos: An Introduction*. Cambridge University Press, 1999.