Quantum Kicked Rotor

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1 Introduction

2 Questions

1. What does kicking the rotor periodically have anything to do with a random walk?

2. What is the analogy with Anderson localization? After all, Anderson localisation is about a diffusing wavefunction which encounters disorder in the form of passive scatterers and gets reflected/transmitted with certain probability. This transmission amplitude goes down exponentially with length of the sample. In contrast, in the kicked rotor we have an initial condition of uniform distribution in the position space. We have active "kicks" which pump energy into the system and these kicks strengths are pseudo-random apparently.

3. If we take a quantum rotor and kick it with a truly random kick strength like Ka_j at $t = j$ where $a_j = 1$ with probability p and $a_j = -1$ with probability $1 - p$, would it also show initial diffusion and subsequent localisation?

4. In the Anderson localisation (at least for d=1 case), we see the "transport" being matter transport i.e. we comment on the chances that the matter particle is transmitted across the sample. One could also say that the energy of the particle (which is constant) gets transported to position states far from the origin. But in the case of the kicked rotor, what exactly is being transported? Energy is being pumped into and taken out of this system at all levels, so what is being transported?

5. In the simulation, given a particular dimension of the fourier space of $|\psi\rangle$, how do we get a bound on the maximum timestep?

3 Partial Answers

1. Let us try and answer the question 3. First consider a hamiltonian given by:

$$H = \frac{L^2}{2} + \hbar K \sum_{j=0}^{\infty} a_j \delta(t-j) \tag{1}$$

where $P(a_j = 1) = p, P(a_j = -1) = 1 - p$. We can take $p = \frac{1}{2}$ for simplicity. We then have the unitary operator U_j to evolve the state from after the (j-1)th kick to after the jth kick.

$$U_j = exp(-iKa_j)exp(-i\frac{L^2}{2\hbar}) \tag{2}$$

$$U_j |m\rangle = exp(-i(Ka_j + \frac{\hbar m^2}{2})) |m\rangle \tag{3}$$

where $|m\rangle$ is the eigenstate of angular momentum operator L. We can see clearly here that this random "kick" actually does nothing. It doesn't project our system from one angular momentum eigenstate to another. So this is just a phase shift of each existing eigenstate. No new $|m\rangle$ states can be occupied which weren't occupied before. Clearly, the disorder $\leftrightarrow$ random kick strength analogy fails in this respect. Lets try the following general hamiltonian and find the problem.

$$H = \frac{L^2}{2} + \hbar K V(\theta) \sum_{j=0}^{\infty} a_j \delta(t-j) \tag{4}$$

Then we get

$$U_j = exp(-iKa_jV(\theta))exp(-i\frac{L^2}{2\hbar}) \tag{5}$$

$$U_j |m\rangle = \frac{e^{-i\hbar m^2/2}}{\sqrt{2\pi}} \int exp(-iKa_jV(\theta))exp(-im\theta) |\theta\rangle \, d\theta \tag{6}$$

$$= \frac{e^{-i\hbar m^2/2}}{\sqrt{2\pi}} \int exp(-i(Ka_j\frac{V(\theta)}{\theta} + m)\theta) |\theta\rangle \, d\theta \tag{7}$$

$$[\text{And if we take } V(\theta) = \theta] = e^{-i\hbar m^2/2} |m + Ka_j\rangle \tag{8}$$

So clearly, $V(\theta)$ needs at least a θ term in order to kick the system into other states. The problem is that without theta dependence, the initial m term will never break into pieces which hop to other states. ~~The issue is one cannot think of the state hopping from $|m\rangle$ to $|n\rangle$, rather one must look at it from the θ space perspective.~~

2. A naive answer to question 5 might be to do the following energy calculation:

$$Tk = \frac{1}{2}\hbar^2 L_{max}^2 \tag{9}$$

$$T \leq \frac{\hbar^2 L_{max}^2}{2k} \tag{10}$$

$$[\text{If we take } \hbar = 1, L_{max} = 1000, k = 5]$$

$$T \leq 10^5 \tag{11}$$

which undoubtedly seems like an overestimate.

4 Week 7

Deadline: Next Saturday (10 April 2021)

4.1 Objectives

1. Work out the a_j model analytically and computationally both.
2. Add noise to kick period in the kicked rotor: Kick the rotor at $\tau \pm \delta\tau$ where $\delta\tau$ is drawn from a uniform distribution. Loss of localization is expected.

3. Add noise to kick strength in the kicked rotor: Kick the rotor with strength $k \pm \delta k$ where δk is from uniform distribution. Loss of localization is expected.

4. Read up on the quasi-periodic kicked rotor and the metal-insulator transition in it.

References

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