Report on Bipartite Entanglement Entropy

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June 28, 2021

Introduction

System

$$H = \frac{p_1^2}{2} + p_2\omega_2 + p_3\omega_3 + K cos(\theta_1)(1 + \alpha cos(\theta_2) cos(\theta_3)) \sum_n \delta(t-n)$$

- We are calculating the entanglement entropy between $\mathcal{H}_1$ and $\mathcal{H}_{23}$.

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    1. Entanglement entropy: $S = -Tr(\rho_1 ln(\rho_1))$

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  2. Energy: $E = p_1^2/2 + p_2\omega_2 + p_3\omega_3$

System

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4. Entropy Change: $S_{K_{n+1},\alpha_{n+1}} - S_{K_n,\alpha_n}$
5. Energy Change: $E_{K_{n+1},\alpha_{n+1}} - E_{K_n,\alpha_n}$

Parameters

- We use $\hbar = 1$, $\omega_2 = 2\pi\sqrt{5}$ and $\omega_3 = 2\pi\sqrt{13}$. The critical point of the metal insulator transition is known to occur at $K_c = 6.36 \pm 0.02$.

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- The value of α_c, however is not determined accurately, nor written in any literature as it doesn't form a part of the scaling function in a nice way, though it most certainly does matter as it generates the anisotropy. We determine the value of α_c roughly by using the fact that the authors of the cited papers traversed the $K - \alpha$ space along a straight line perpendicular to the transition parabola.[1] [2]

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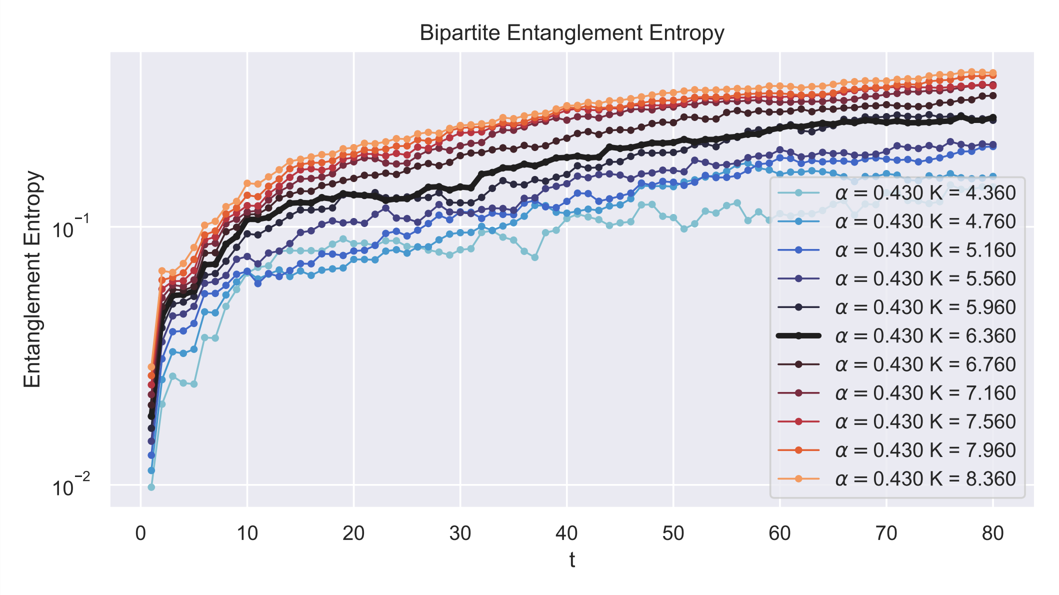
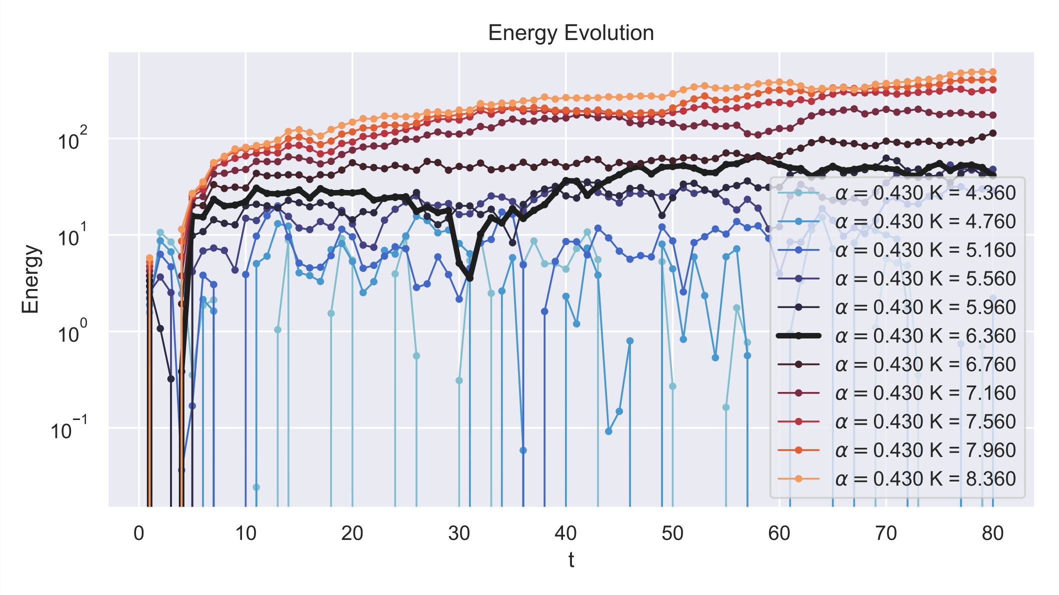
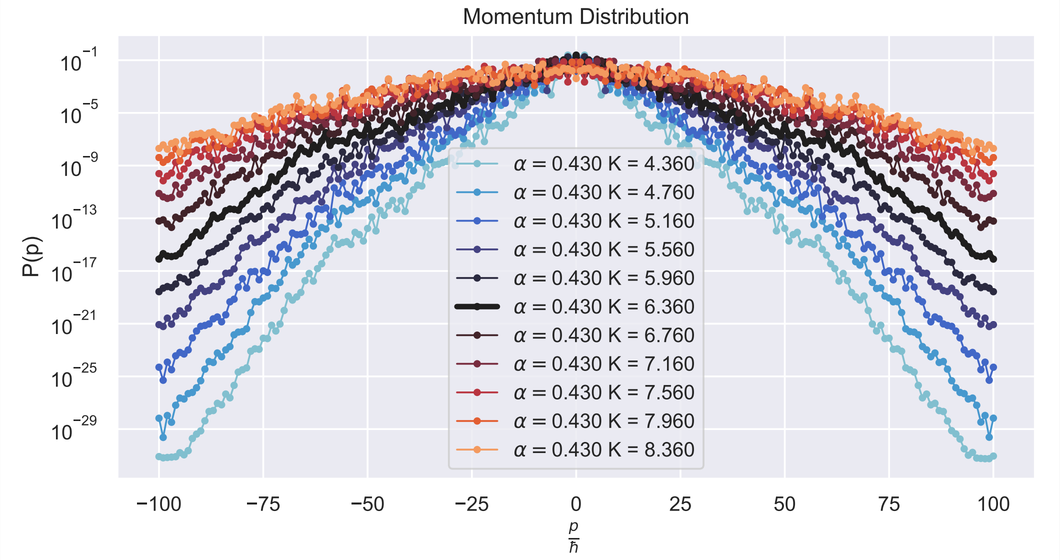
- TL;DR we used $K_c = 6.36$ and $\alpha_c = 0.4303$.
- We used a basis size of 201 (-100 to 100) for each of the 3 coordinates and the simulations were done for 80 timesteps.

Varying K


Bipartite Entanglement Entropy
Entanglement Entropy
10^{-1}
10^{-2}
0
10
20
30
40
50
60
70
80
t
α = 0.430 K = 4.360
α = 0.430 K = 4.760
α = 0.430 K = 5.160
α = 0.430 K = 5.560
α = 0.430 K = 5.960
α = 0.430 K = 6.360
α = 0.430 K = 6.760
α = 0.430 K = 7.160
α = 0.430 K = 7.560
α = 0.430 K = 7.960
α = 0.430 K = 8.360


Energy Evolution
Energy
t
$\alpha = 0.430$ K = 4.360
$\alpha = 0.430$ K = 4.760
$\alpha = 0.430$ K = 5.160
$\alpha = 0.430$ K = 5.560
$\alpha = 0.430$ K = 5.960
$\alpha = 0.430$ K = 6.360
$\alpha = 0.430$ K = 6.760
$\alpha = 0.430$ K = 7.160
$\alpha = 0.430$ K = 7.560
$\alpha = 0.430$ K = 7.960
$\alpha = 0.430$ K = 8.360


Momentum Distribution
10^{-1}
10^{-5}
10^{-9}
10^{-13}
10^{-17}
10^{-21}
10^{-25}
10^{-29}
P(p)
−100
−75
−50
−25
0
25
50
75
100
$\frac{p}{\hbar}$
$\alpha = 0.430$ K = 4.360
$\alpha = 0.430$ K = 4.760
$\alpha = 0.430$ K = 5.160
$\alpha = 0.430$ K = 5.560
$\alpha = 0.430$ K = 5.960
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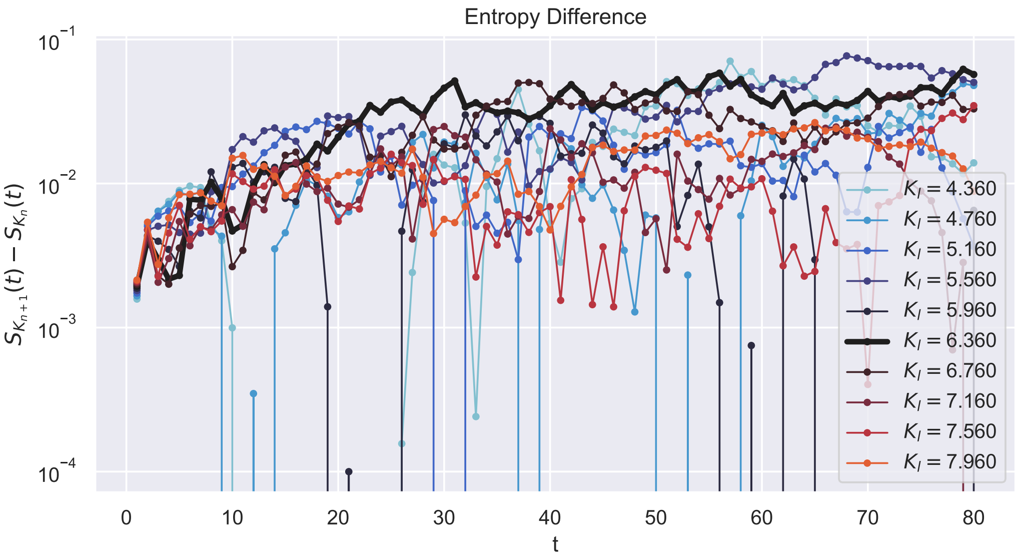
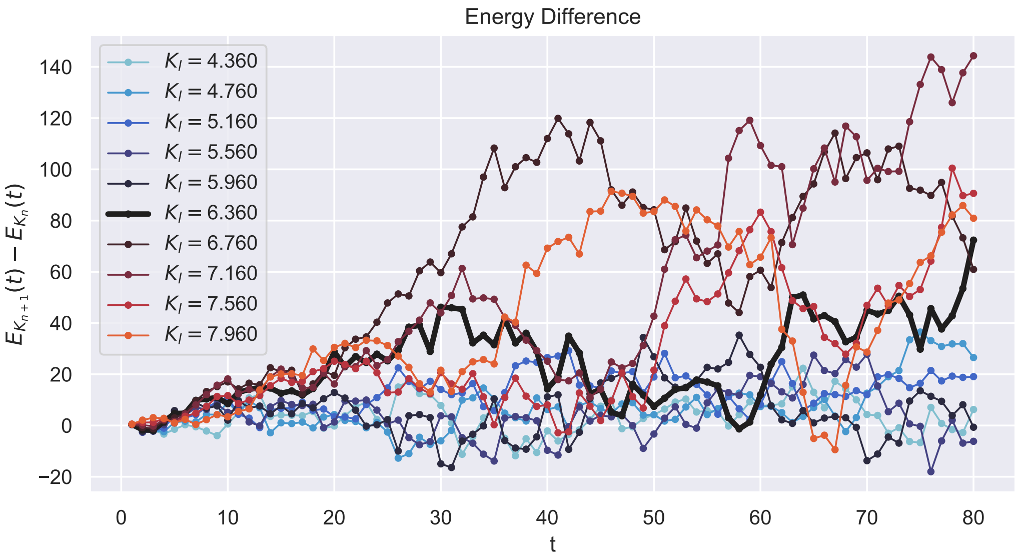
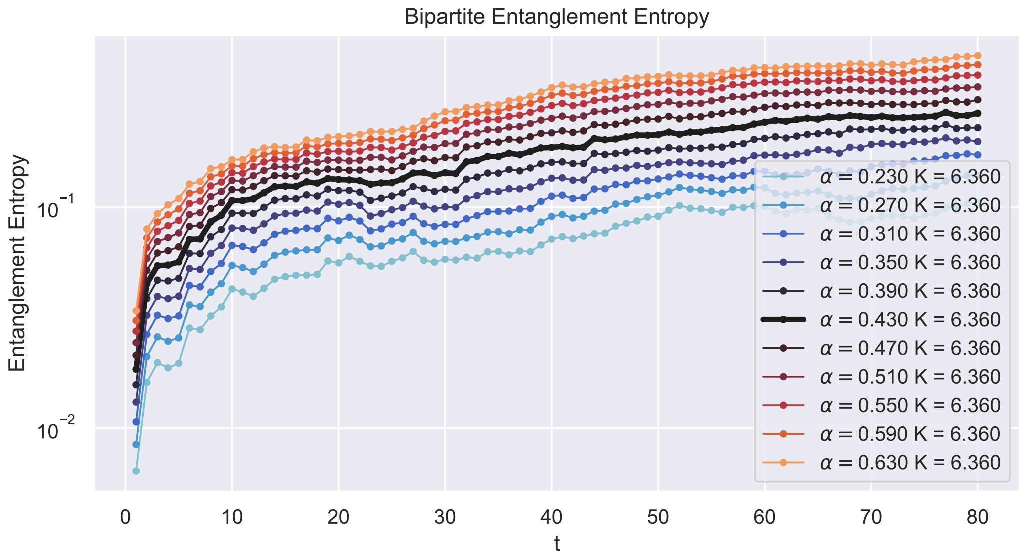
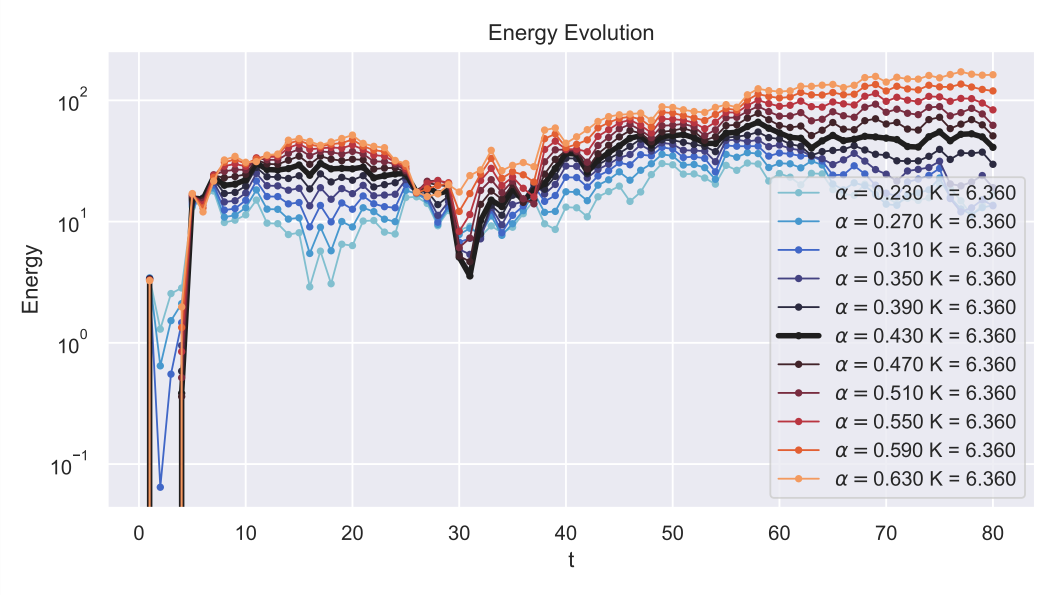
$\alpha = 0.430$ K = 7.160
$\alpha = 0.430$ K = 7.560
$\alpha = 0.430$ K = 7.960
$\alpha = 0.430$ K = 8.360


Entropy Difference
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$S_{K_{n+1}}(t) - S_{K_n}(t)$
0
10
20
30
40
50
60
70
80
t
$K_l = 4.360$
$K_l = 4.760$
$K_l = 5.160$
$K_l = 5.560$
$K_l = 5.960$
$K_l = 6.360$
$K_l = 6.760$
$K_l = 7.160$
$K_l = 7.560$
$K_l = 7.960$


Energy Difference
$K_l = 4.360$
$K_l = 4.760$
$K_l = 5.160$
$K_l = 5.560$
$K_l = 5.960$
$K_l = 6.360$
$K_l = 6.760$
$K_l = 7.160$
$K_l = 7.560$
$K_l = 7.960$
$E_{K_{n+1}}(t) - E_{K_n}(t)$
140
120
100
80
60
40
20
0
−20
0
10
20
30
40
50
60
70
80
t

Varying α


Bipartite Entanglement Entropy
Entanglement Entropy
10^{-1}
10^{-2}
0
10
20
30
40
50
60
70
80
t
α = 0.230 K = 6.360
α = 0.270 K = 6.360
α = 0.310 K = 6.360
α = 0.350 K = 6.360
α = 0.390 K = 6.360
α = 0.430 K = 6.360
α = 0.470 K = 6.360
α = 0.510 K = 6.360
α = 0.550 K = 6.360
α = 0.590 K = 6.360
α = 0.630 K = 6.360


Energy Evolution
Energy
t
10^{2}
10^{1}
10^{0}
10^{-1}
0
10
20
30
40
50
60
70
80
α = 0.230 K = 6.360
α = 0.270 K = 6.360
α = 0.310 K = 6.360
α = 0.350 K = 6.360
α = 0.390 K = 6.360
α = 0.430 K = 6.360
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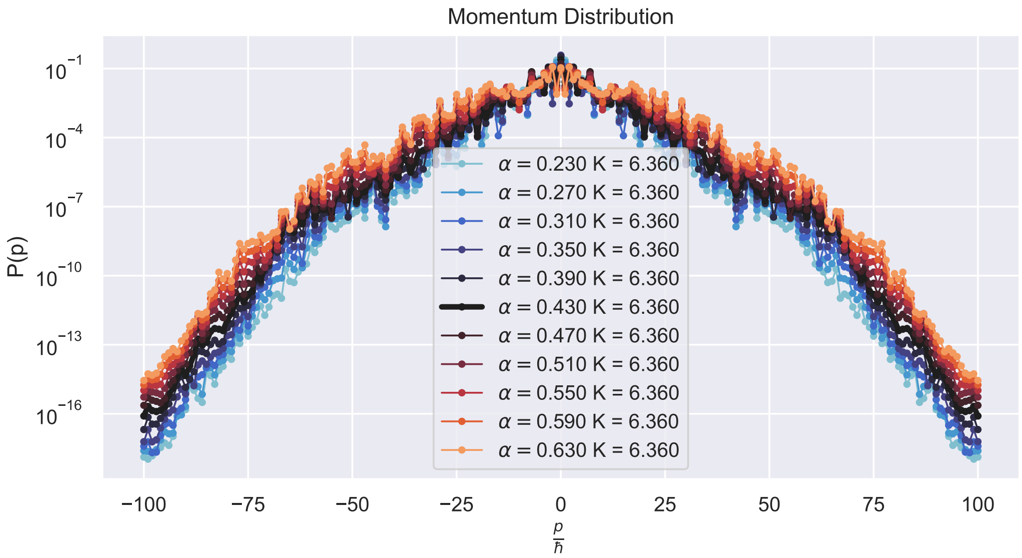
α = 0.550 K = 6.360
α = 0.590 K = 6.360
α = 0.630 K = 6.360


Momentum Distribution
P(p)
10^{-1}
10^{-4}
10^{-7}
10^{-10}
10^{-13}
10^{-16}
−100
−75
−50
−25
0
25
50
75
100
$\frac{p}{\hbar}$
$\alpha = 0.230$ K = 6.360
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$\alpha = 0.550$ K = 6.360
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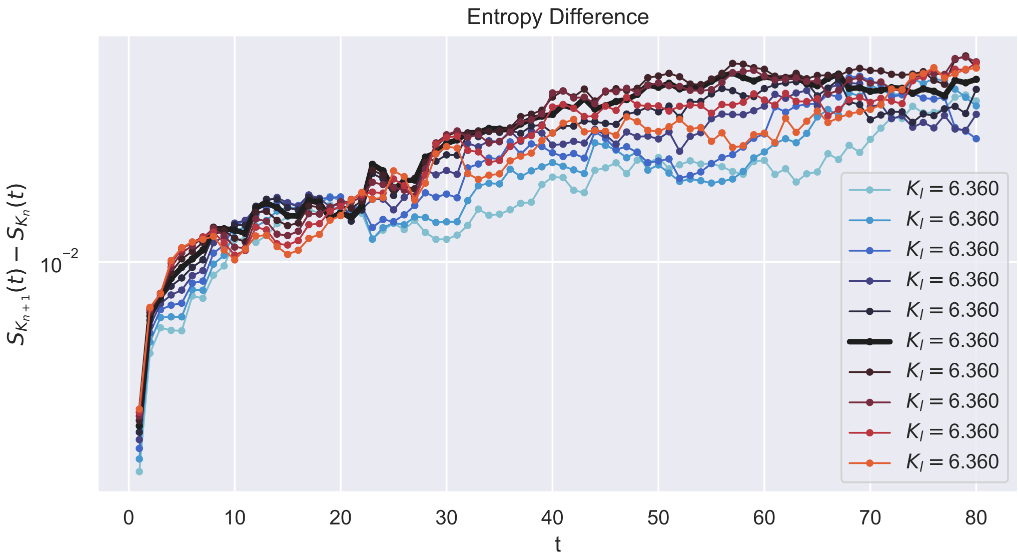

Entropy Difference
$S_{K_{n+1}}(t) - S_{K_n}(t)$
10^{-2}
0
10
20
30
40
50
60
70
80
t
$K_l = 6.360$
$K_l = 6.360$
$K_l = 6.360$
$K_l = 6.360$
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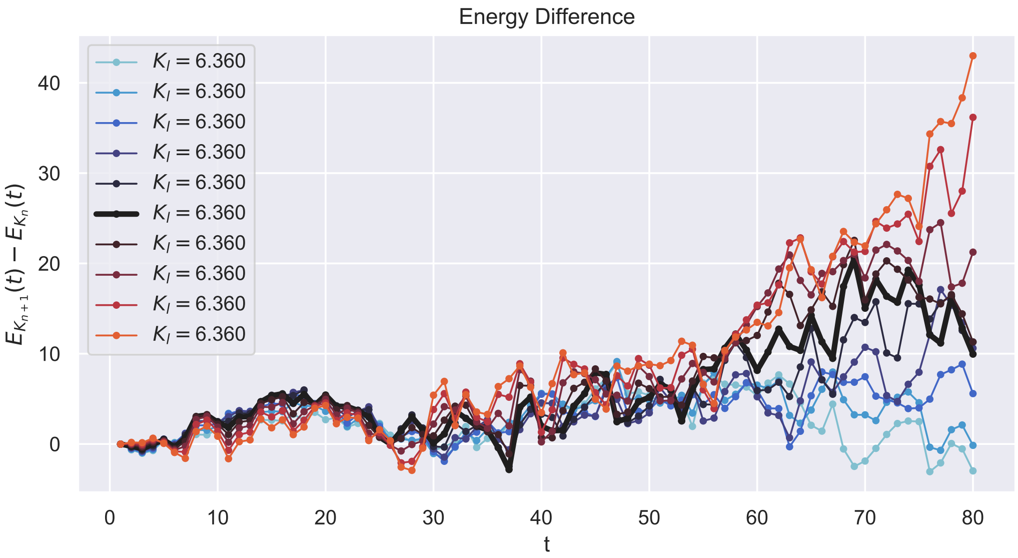
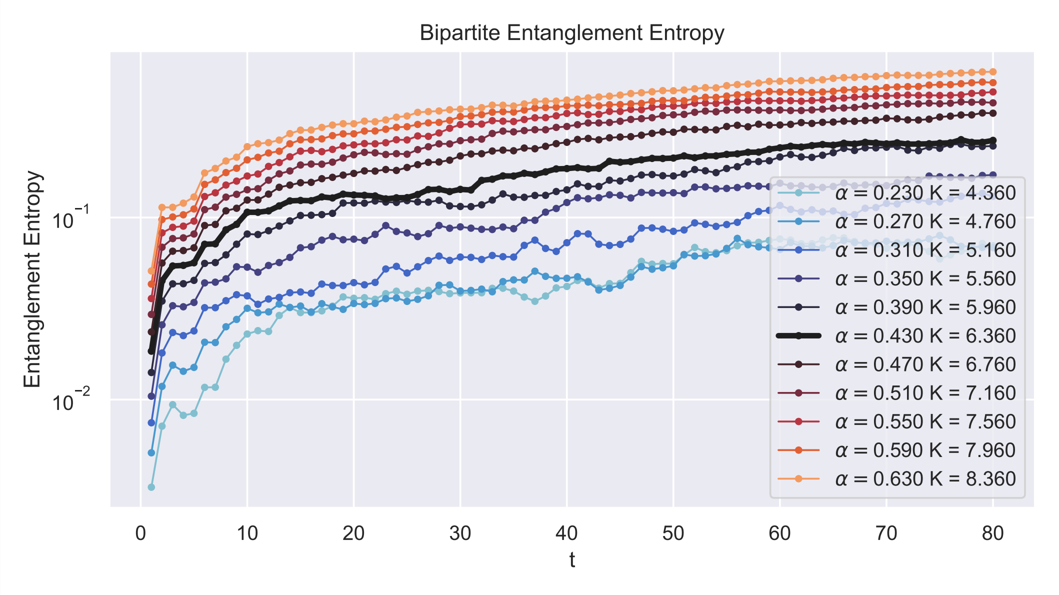


Energy Difference
$K_l = 6.360$
$K_l = 6.360$
$K_l = 6.360$
$K_l = 6.360$
$K_l = 6.360$
$K_l = 6.360$
$K_l = 6.360$
$K_l = 6.360$
$K_l = 6.360$
$K_l = 6.360$
$E_{K_{n+1}}(t) - E_{K_n}(t)$
t
0
10
20
30
40
50
60
70
80

Varying K and α


Bipartite Entanglement Entropy
Entanglement Entropy
10^{-1}
10^{-2}
0
10
20
30
40
50
60
70
80
t
α = 0.230 K = 4.360
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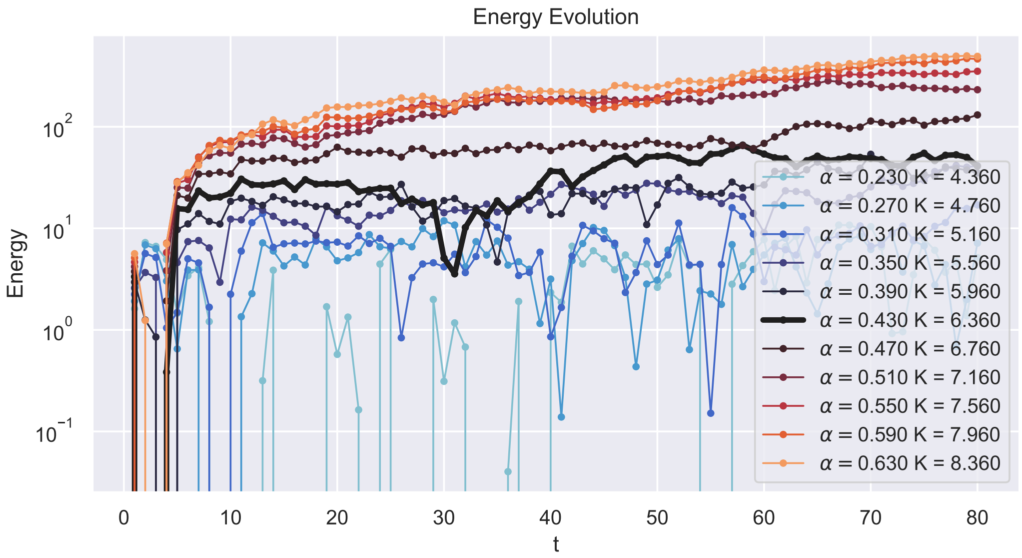
α = 0.470 K = 6.760
α = 0.510 K = 7.160
α = 0.550 K = 7.560
α = 0.590 K = 7.960
α = 0.630 K = 8.360


Energy Evolution
Energy
t
10^2
10^1
10^0
10^{-1}
0
10
20
30
40
50
60
70
80
α = 0.230 K = 4.360
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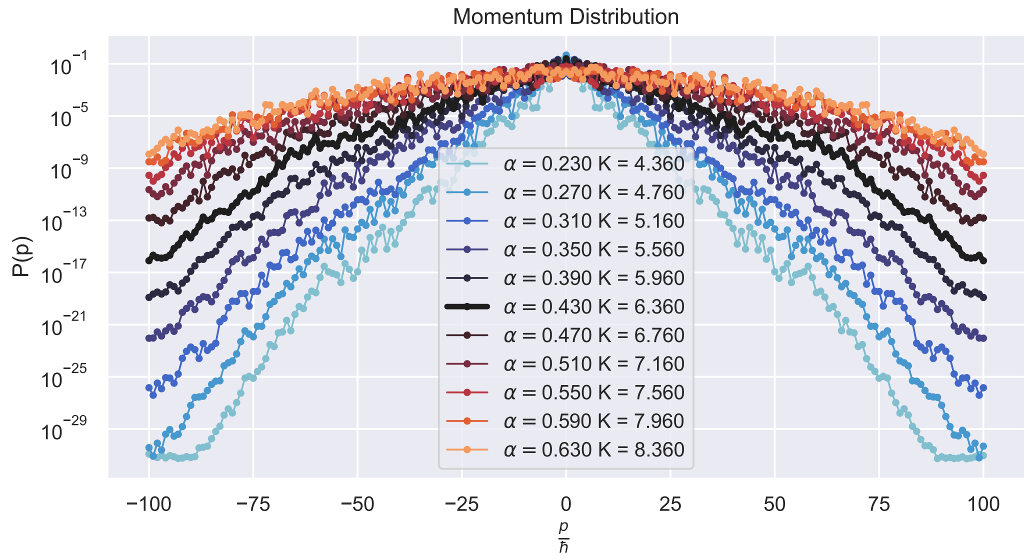
α = 0.590 K = 7.960
α = 0.630 K = 8.360


Momentum Distribution
P(p)
10^{-1}
10^{-5}
10^{-9}
10^{-13}
10^{-17}
10^{-21}
10^{-25}
10^{-29}
−100
−75
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−25
0
25
50
75
100
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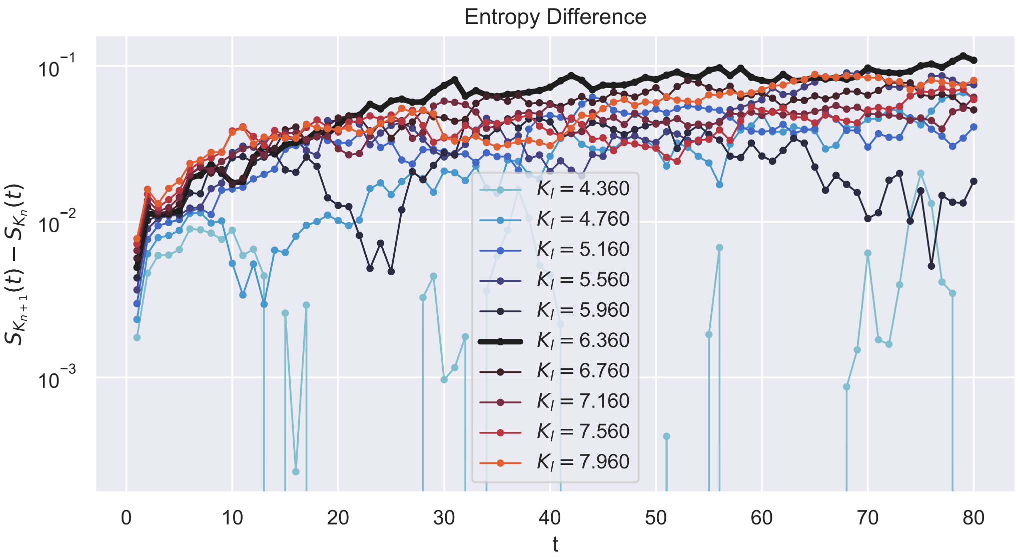
Entropy Difference
$S_{K_{n+1}}(t) - S_{K_n}(t)$
10^{-1}
10^{-2}
10^{-3}
0
10
20
30
40
50
60
70
80
t
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$K_l = 4.760$
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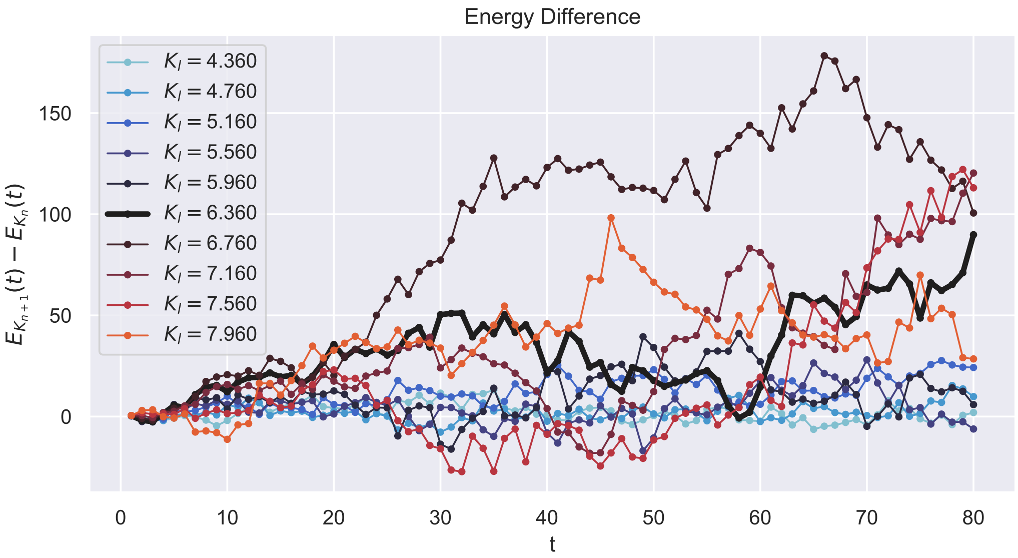
Energy Difference
$K_l = 4.360$
$K_l = 4.760$
$K_l = 5.160$
$K_l = 5.560$
$K_l = 5.960$
$K_l = 6.360$
$K_l = 6.760$
$K_l = 7.160$
$K_l = 7.560$
$K_l = 7.960$
150
100
50
0
$E_{K_{n+1}}(t) - E_{K_n}(t)$
0
10
20
30
40
50
60
70
80
t

Trial Math

- We use α from 0.1 to 0.8 and $\alpha_c = 0.5$.

1. Lemarié, G., Grémaud, B. & Delande, D. Universality of the Anderson transition with the quasiperiodic kicked rotor. *Europhys. Lett.* **87**, 37007 (2009).

2. Lemarié, G. *et al.* Observation of the Anderson metal-insulator transition with atomic matter waves: Theory and experiment. *Phys. Rev. A* **80**, 043626 (2009).