Bipartite Entanglement Entropy

Aditya Chincholi

June 28, 2021

Quasiperiodic Kicked Rotor

What I did earlier

- Constructing the full floquet operator in a single basis.

What I did earlier

- Constructing the full floquet operator in a single basis.
- Using a density matrix the whole time for calculations.

What I did earlier

- Constructing the full floquet operator in a single basis.
- Using a density matrix the whole time for calculations.
- This has the drawback of increasing computational complexity of each individual step and the memory used at any given time is large.

What I have done now

- Separate the floquet operator into momentum space and position space parts.

What I have done now

- Separate the floquet operator into momentum space and position space parts.
- Use a single pure state for calculations.

What I have done now

- Separate the floquet operator into momentum space and position space parts.
- Use a single pure state for calculations.
- Fourier transform the state at each time step and apply both parts of the floquet operator in their resp. basis.

What I have done now

- Separate the floquet operator into momentum space and position space parts.
- Use a single pure state for calculations.
- Fourier transform the state at each time step and apply both parts of the floquet operator in their resp. basis.
- This is better as the memory used is less but computation increases. Since fourier transforms are computationally cheap anyway, so it's fine.

What I have done now

- Separate the floquet operator into momentum space and position space parts.
- Use a single pure state for calculations.
- Fourier transform the state at each time step and apply both parts of the floquet operator in their resp. basis.
- This is better as the memory used is less but computation increases. Since fourier transforms are computationally cheap anyway, so it's fine.
- Peak memory required scales the same way but we have reduced it by a constant factor and it is not used in all calculations.

Results

We use $\hbar = 2.85, \omega_2 = 2\pi\sqrt{5}, \omega_3 = 2\pi\sqrt{13}$, the momentum ranges from -10 to 10

$$H = \frac{p_1^2}{2} + p_2\omega_2 + p_3\omega_3 + Kcos(\theta_1)(1 + \alpha cos(\theta_2)cos(\theta_3))\sum_n \delta(t - n)$$

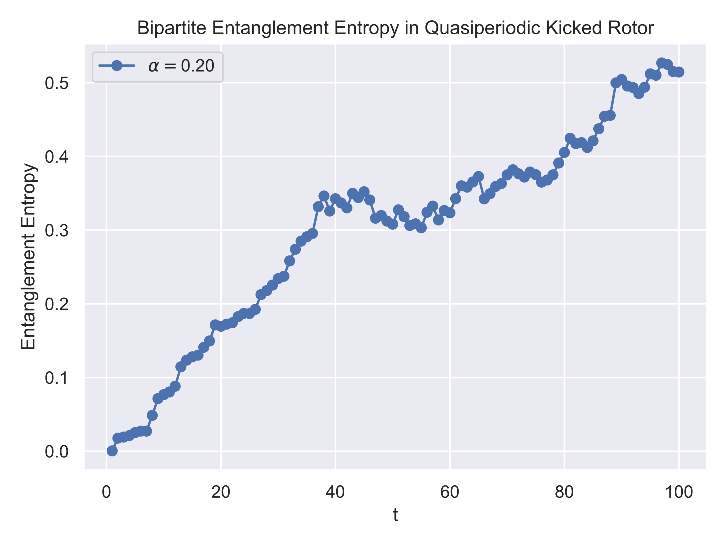


Figure 1: Precritical (Insulator): $K = 4, \alpha = 0.2$

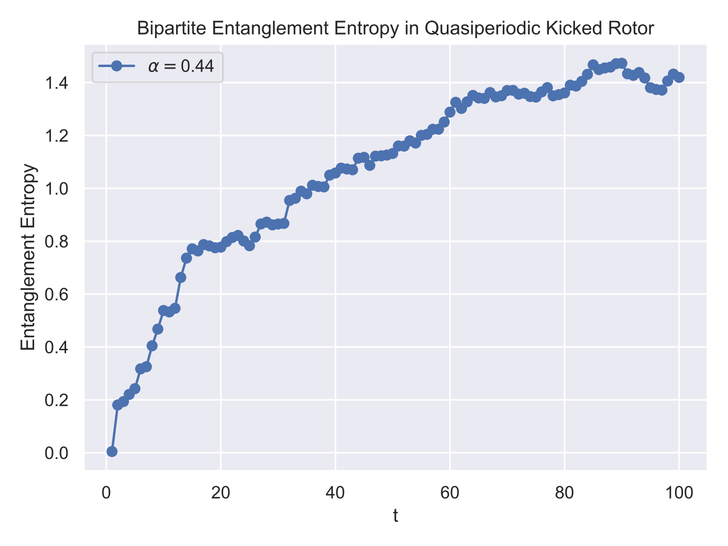


Figure 2: Critical: $K = 6.36, \alpha = 0.4375$

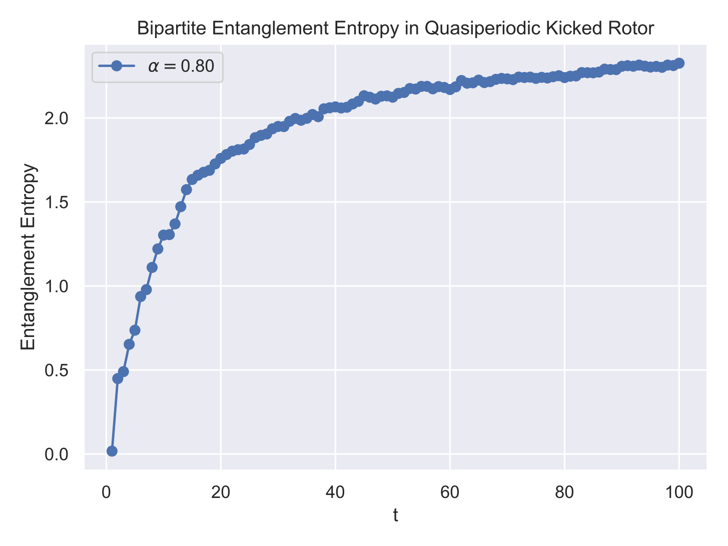


Figure 3: Post-critical (Metal): $K = 8, \alpha = 0.8$

- I don't see much of a trend here. The entanglement grows faster and higher with higher K values i.e. more diffusive the regime higher the entanglement for the same number of time steps but other than that, I don't see anything here.

Bypassing The Density Matrix

- What I was doing: $|\psi\rangle \to \rho \to \rho_1 = Tr_{2,3}[\rho]$. This is a memory bottleneck as the full density matrix ρ still requires the $O(N^6)$ memory.

Bypassing The Density Matrix

- What I was doing: $|\psi\rangle \to \rho \to \rho_1 = Tr_{2,3}[\rho]$. This is a memory bottleneck as the full density matrix ρ still requires the $O(N^6)$ memory.
- What I realized: $|\psi\rangle \to \rho_1 = Tr_{2,3}[\rho]$ This gives a really high speedup and allows me to check if localisation is happening properly or not.

Bypassing The Density Matrix

- What I was doing: $|\psi\rangle \to \rho \to \rho_1 = Tr_{2,3}[\rho]$. This is a memory bottleneck as the full density matrix ρ still requires the $O(N^6)$ memory.
- What I realized: $|\psi\rangle \to \rho_1 = Tr_{2,3}[\rho]$ This gives a really high speedup and allows me to check if localisation is happening properly or not.
- I used these params to generate the following results: p-basis = [-100, 100], 80 timesteps, $\omega_2 = 2\pi\sqrt{5}$, $\omega_3 = 2\pi\sqrt{13}$, $\hbar = 2.85$.

Bypassing The Density Matrix

- What I was doing: $|\psi\rangle \to \rho \to \rho_1 = Tr_{2,3}[\rho]$. This is a memory bottleneck as the full density matrix ρ still requires the $O(N^6)$ memory.
- What I realized: $|\psi\rangle \to \rho_1 = Tr_{2,3}[\rho]$ This gives a really high speedup and allows me to check if localisation is happening properly or not.
- I used these params to generate the following results: p-basis = [-100, 100], 80 timesteps, $\omega_2 = 2\pi\sqrt{5}$, $\omega_3 = 2\pi\sqrt{13}$, $\hbar = 2.85$.
- We plot the following quantities:

Bypassing The Density Matrix

- What I was doing: $|\psi\rangle \to \rho \to \rho_1 = Tr_{2,3}[\rho]$. This is a memory bottleneck as the full density matrix ρ still requires the $O(N^6)$ memory.
- What I realized: $|\psi\rangle \to \rho_1 = Tr_{2,3}[\rho]$ This gives a really high speedup and allows me to check if localisation is happening properly or not.
- I used these params to generate the following results: p-basis = [-100, 100], 80 timesteps, $\omega_2 = 2\pi\sqrt{5}$, $\omega_3 = 2\pi\sqrt{13}$, $\hbar = 2.85$.
- We plot the following quantities:
    - $P(p_1 = m\hbar)$

Bypassing The Density Matrix

- What I was doing: $|\psi\rangle \to \rho \to \rho_1 = Tr_{2,3}[\rho]$. This is a memory bottleneck as the full density matrix ρ still requires the $O(N^6)$ memory.
- What I realized: $|\psi\rangle \to \rho_1 = Tr_{2,3}[\rho]$ This gives a really high speedup and allows me to check if localisation is happening properly or not.
- I used these params to generate the following results: p-basis = [-100, 100], 80 timesteps, $\omega_2 = 2\pi\sqrt{5}$, $\omega_3 = 2\pi\sqrt{13}$, $\hbar = 2.85$.
- We plot the following quantities:
  - $P(p_1 = m\hbar)$
  - $E = p_1^2/2 + p_2\omega_2 + p_3\omega_3$

Bypassing The Density Matrix

- What I was doing: $|\psi\rangle \to \rho \to \rho_1 = Tr_{2,3}[\rho]$. This is a memory bottleneck as the full density matrix ρ still requires the $O(N^6)$ memory.
- What I realized: $|\psi\rangle \to \rho_1 = Tr_{2,3}[\rho]$ This gives a really high speedup and allows me to check if localisation is happening properly or not.
- I used these params to generate the following results: p-basis = [-100, 100], 80 timesteps, $\omega_2 = 2\pi\sqrt{5}$, $\omega_3 = 2\pi\sqrt{13}$, $\hbar = 2.85$.
- We plot the following quantities:
  - $P(p_1 = m\hbar)$
  - $E = p_1^2/2 + p_2\omega_2 + p_3\omega_3$
  - $S = -\rho_1 ln(\rho_1)$

Momentum (p_1) distributions

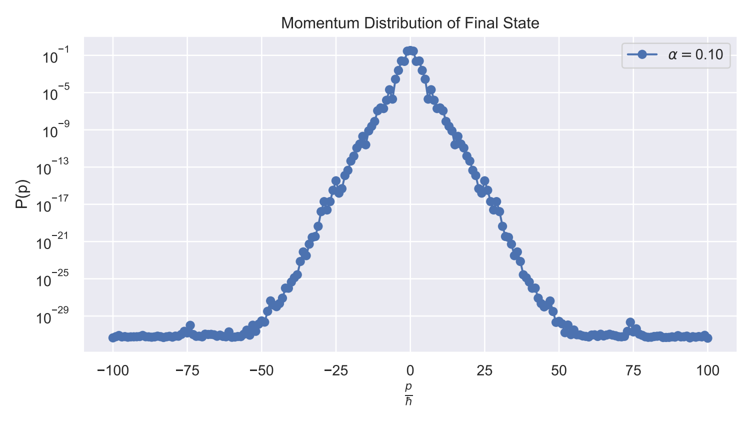


Figure 4: K $= 3$, $\alpha = 0.1$

Momentum (p_1) distributions

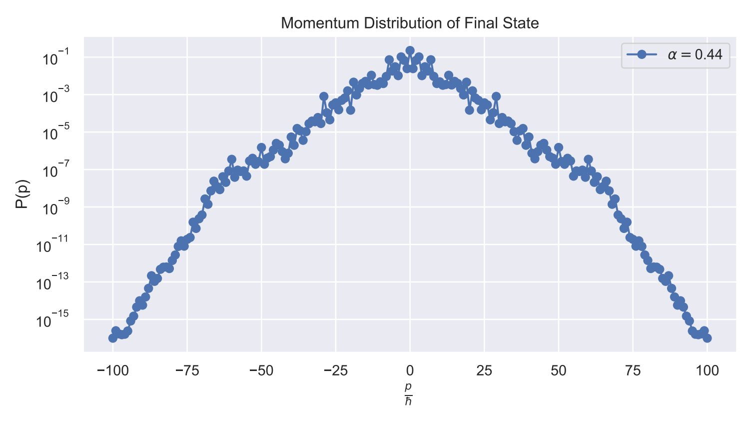


Figure 5: K $= 6.36$, $\alpha = 0.4375$

Momentum (p_1) distributions

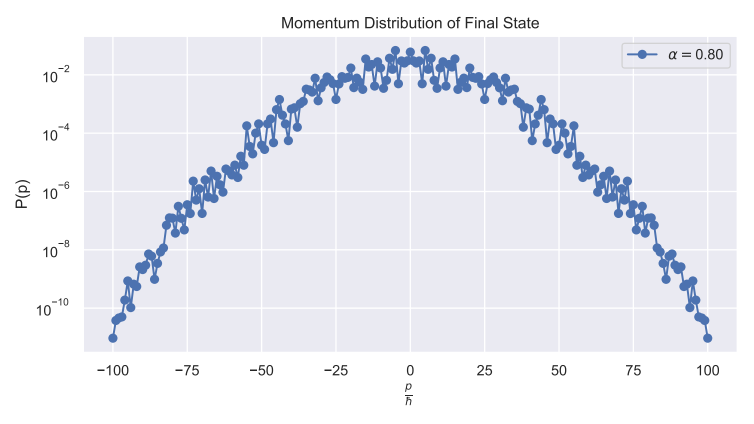


Figure 6: K $= 7$, $\alpha = 0.8$

Energy

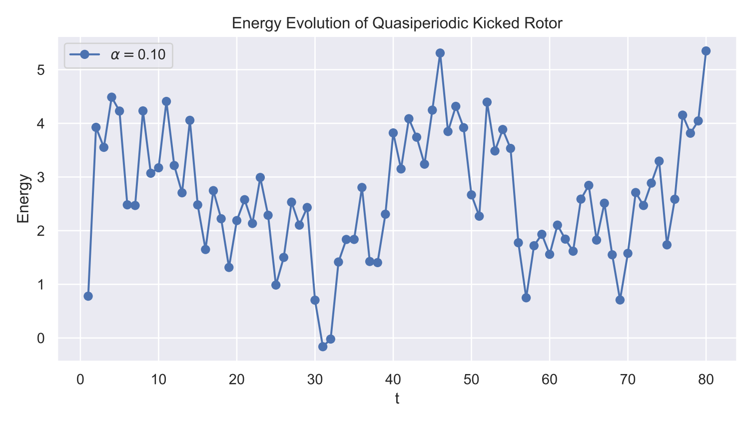


Figure 7: K $=$ 3, $\alpha = 0.1$

Energy

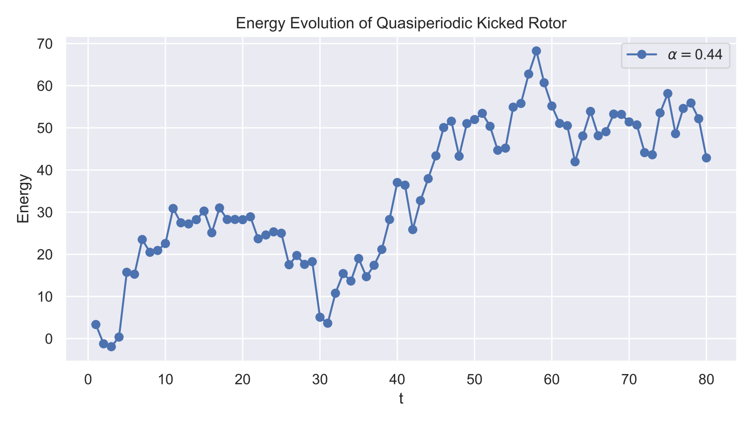


Figure 8: K = 6.36, $\alpha = 0.4375$

Energy

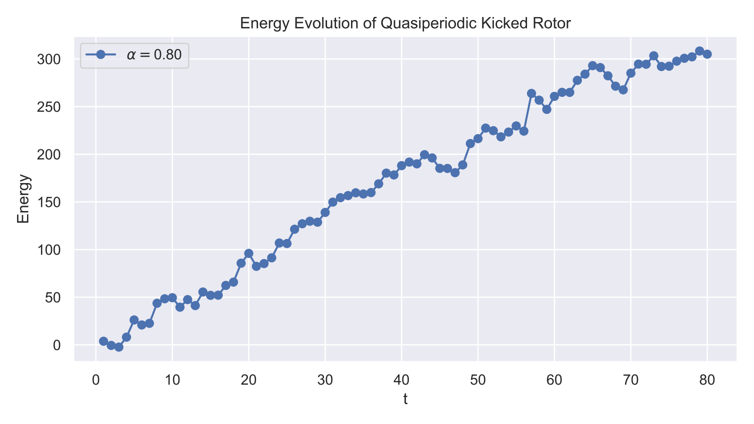


Figure 9: K $= 7$, $\alpha = 0.8$

Entropy

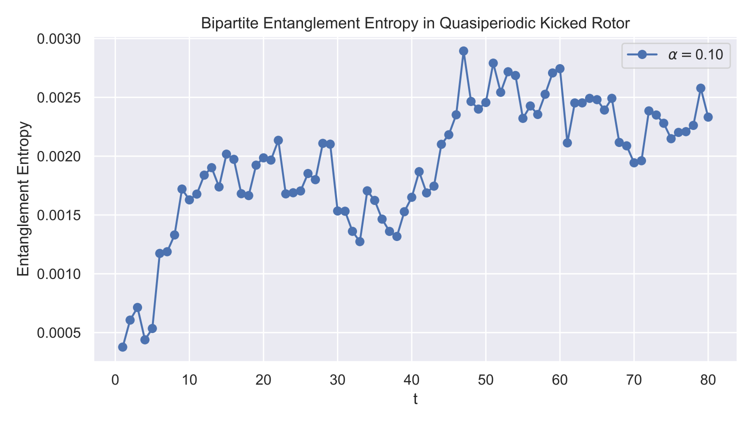


Figure 10: K $= 3$, $\alpha = 0.1$

Entropy

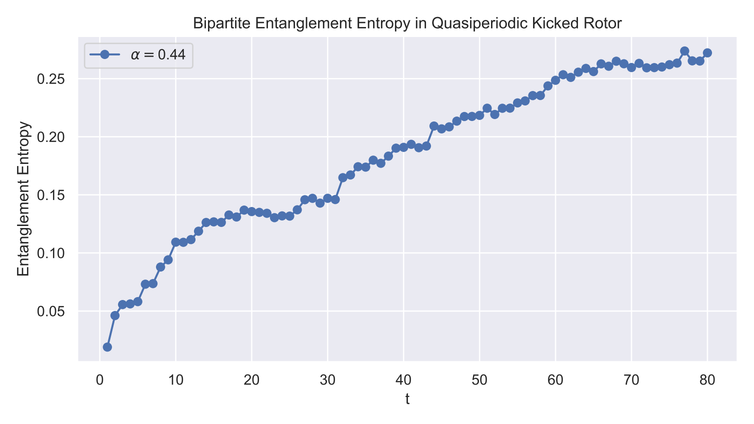


Figure 11: K $= 6.36$, $\alpha = 0.4375$

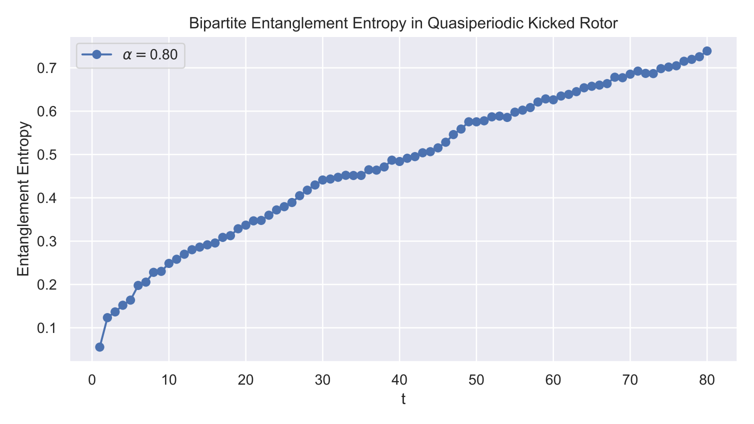


Figure 12: K = 7, $\alpha = 0.8$

Multiple K Values

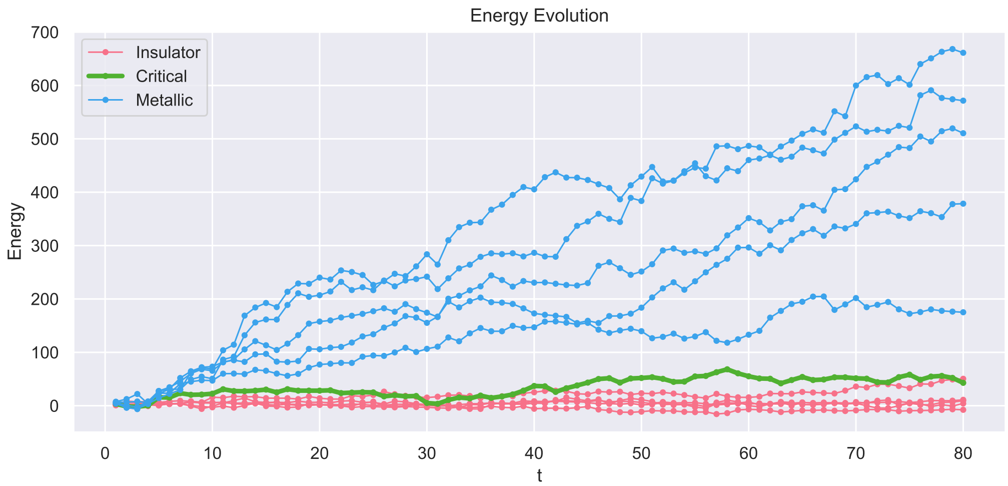
- We look at the following plots for the following at different K, α values from the insulator to the metallic regime:
  1. Energy Expectation Value
  2. Entanglement Entropy
  3. Momentum Distribution
- To study the changes in the energy and entropy values with K, we have plotted the following quantities:
  1. Entropy Difference: $S(K_{n+1}, \alpha_{n+1}) - S(K_n, \alpha_n)$ vs t
  2. Energy Difference: $E(K_{n+1}, \alpha_{n+1}) - E(K_n, \alpha_n)$ vs t

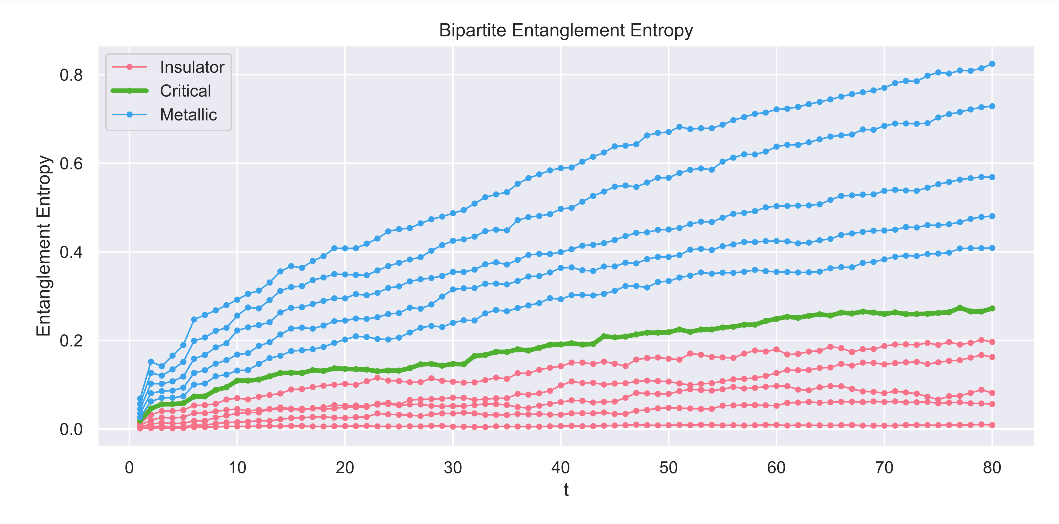
Multiple K Values

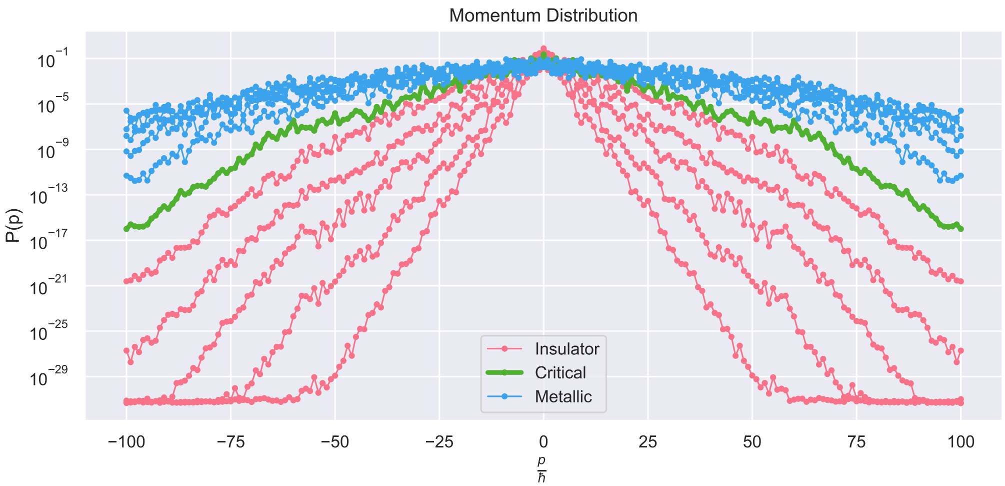
- First we take a big picture look with 11 values with $K \in [3.00, 9.72]$ and $\alpha \in [0.200, 0.6750]$. Both ranges are centred around the critical point[1]. Momentum range is -100 to 100 with 80 timesteps.
- Then we look very close to the critical point with 11 values with $K \in [6.30, 6.42]$ and $\alpha \in [0.4000, 0.4750]$. Both ranges are centred around the critical point. Momentum range is -100 to 100 with 80 timesteps.

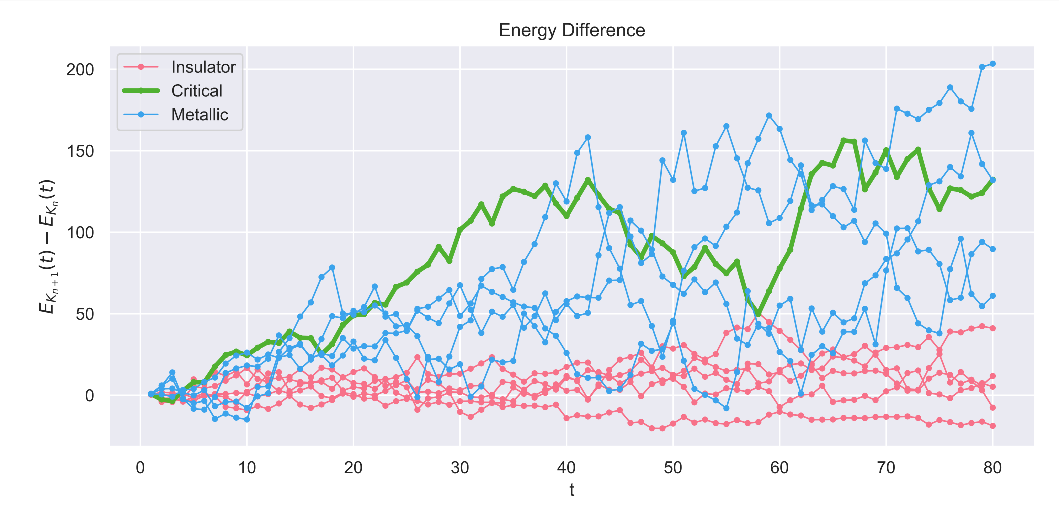
[1] The (Lemarié, Grémaud, and Delande 2009) paper gives the value of K at critical point, but not of α.

The Big Picture

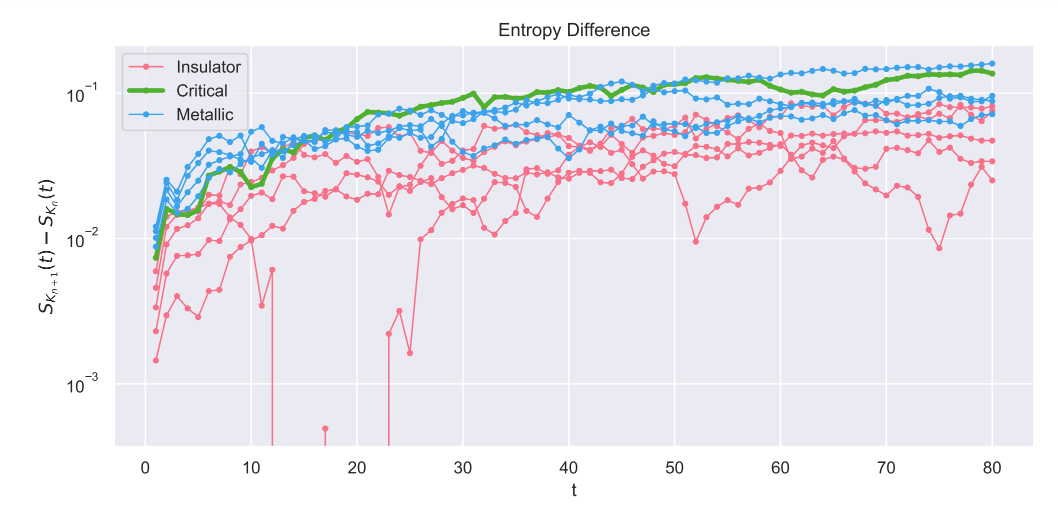

Energy Evolution
Insulator
Critical
Metallic
Energy
t
700
600
500
400
300
200
100
0
0
10
20
30
40
50
60
70
80


Bipartite Entanglement Entropy
Insulator
Critical
Metallic
0.8
0.6
0.4
0.2
0.0
Entanglement Entropy
0
10
20
30
40
50
60
70
80
t

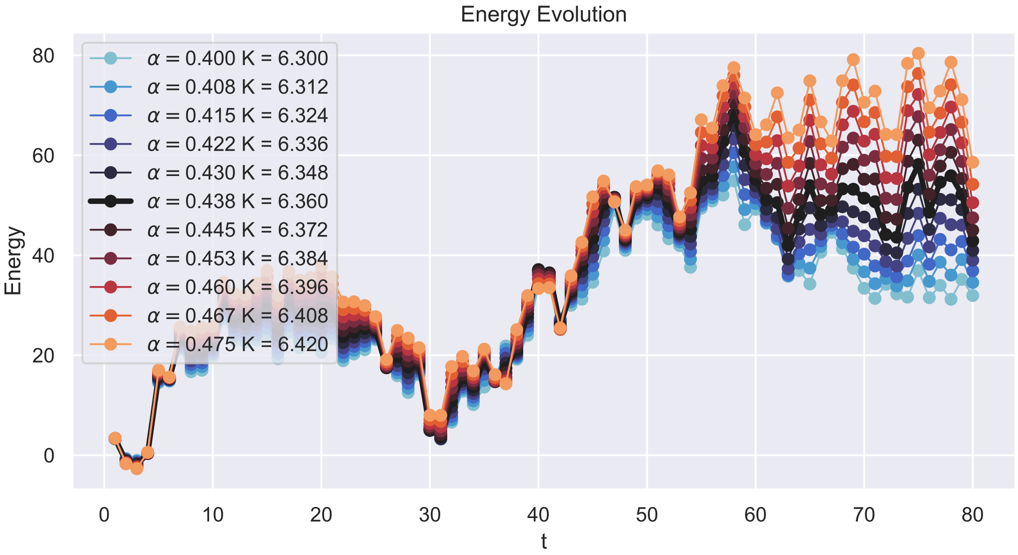

Momentum Distribution
P(p)
10^{-1}
10^{-5}
10^{-9}
10^{-13}
10^{-17}
10^{-21}
10^{-25}
10^{-29}
−100
−75
−50
−25
0
25
50
75
100
$\frac{p}{\hbar}$
Insulator
Critical
Metallic

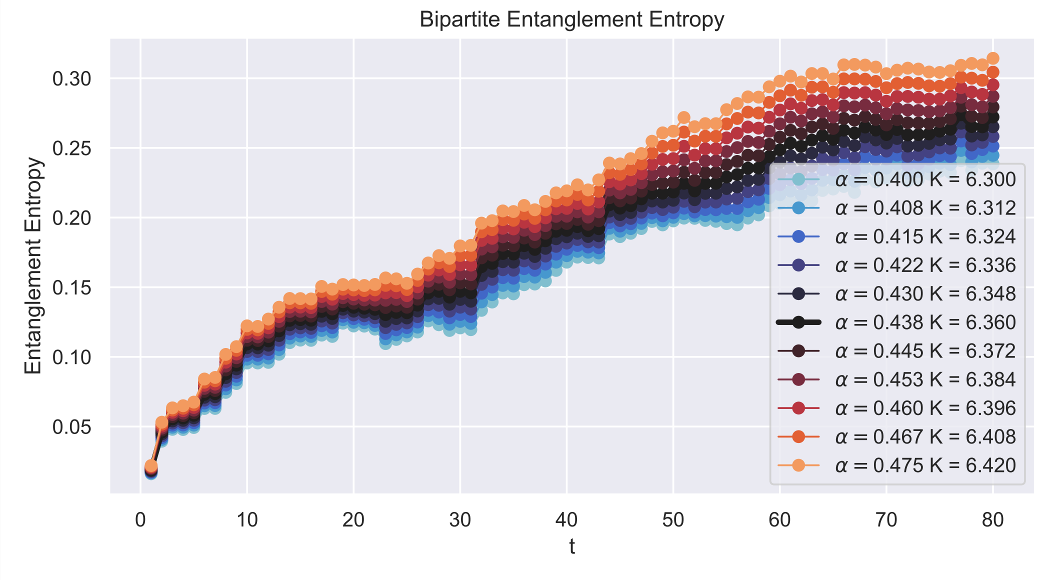


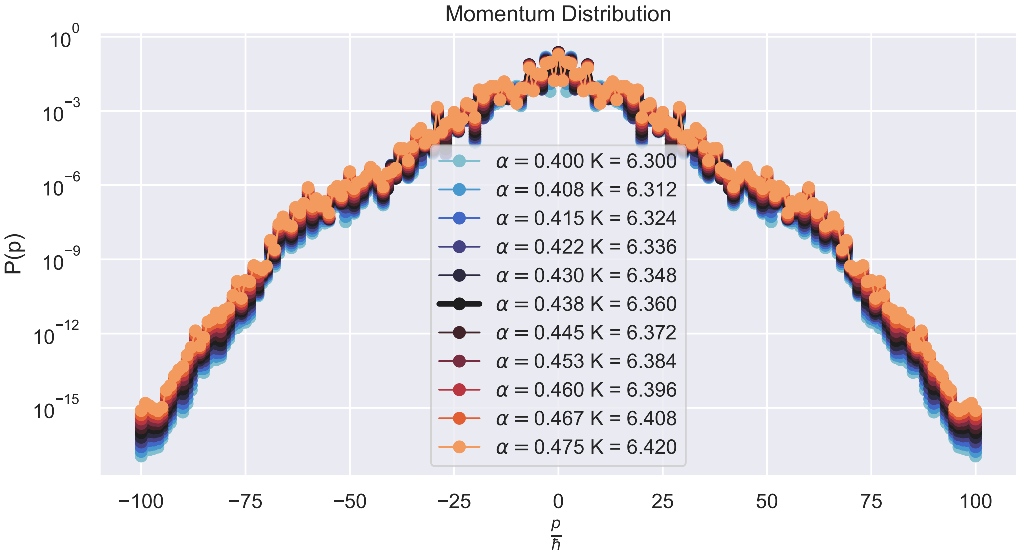
Energy Difference
Insulator
Critical
Metallic
200
150
100
50
0
0
10
20
30
40
50
60
70
80
t
$E_{K_{n+1}}(t) - E_{K_n}(t)$


Entropy Difference
Insulator
Critical
Metallic
10^{-1}
10^{-2}
10^{-3}
$S_{K_{n+1}}(t) - S_{K_n}(t)$
0
10
20
30
40
50
60
70
80
t

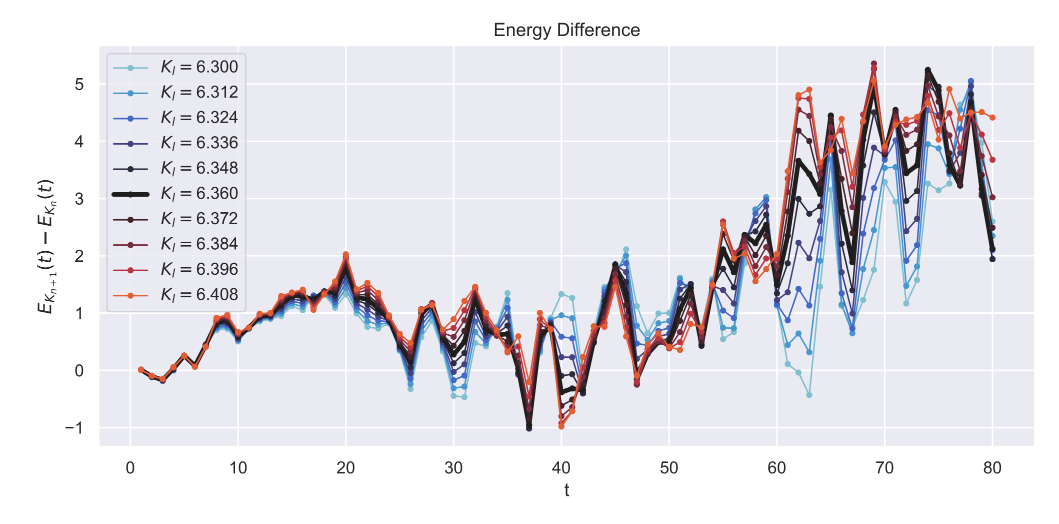
The Microscopic Picture


Energy Evolution
α = 0.400 K = 6.300
α = 0.408 K = 6.312
α = 0.415 K = 6.324
α = 0.422 K = 6.336
α = 0.430 K = 6.348
α = 0.438 K = 6.360
α = 0.445 K = 6.372
α = 0.453 K = 6.384
α = 0.460 K = 6.396
α = 0.467 K = 6.408
α = 0.475 K = 6.420
Energy
0
20
40
60
80
t
0
10
20
30
40
50
60
70
80

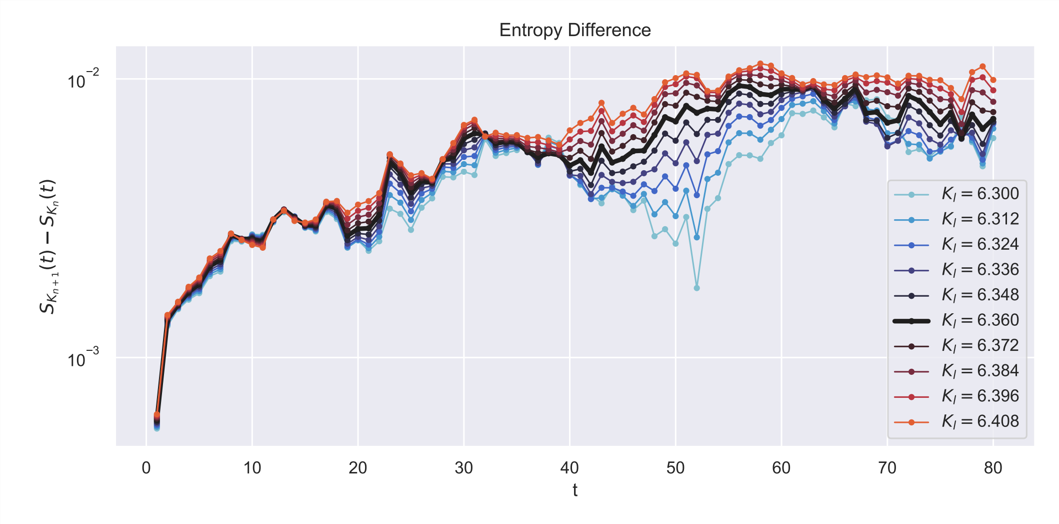

Bipartite Entanglement Entropy
Entanglement Entropy
0.30
0.25
0.20
0.15
0.10
0.05
0
10
20
30
40
50
60
70
80
t
α = 0.400 K = 6.300
α = 0.408 K = 6.312
α = 0.415 K = 6.324
α = 0.422 K = 6.336
α = 0.430 K = 6.348
α = 0.438 K = 6.360
α = 0.445 K = 6.372
α = 0.453 K = 6.384
α = 0.460 K = 6.396
α = 0.467 K = 6.408
α = 0.475 K = 6.420



Momentum Distribution
P(p)
10^{0}
10^{-3}
10^{-6}
10^{-9}
10^{-12}
10^{-15}
−100
−75
−50
−25
0
25
50
75
100
$\frac{p}{\hbar}$
α = 0.400 K = 6.300
α = 0.408 K = 6.312
α = 0.415 K = 6.324
α = 0.422 K = 6.336
α = 0.430 K = 6.348
α = 0.438 K = 6.360
α = 0.445 K = 6.372
α = 0.453 K = 6.384
α = 0.460 K = 6.396
α = 0.467 K = 6.408
α = 0.475 K = 6.420



Energy Difference
$K_l = 6.300$
$K_l = 6.312$
$K_l = 6.324$
$K_l = 6.336$
$K_l = 6.348$
$K_l = 6.360$
$K_l = 6.372$
$K_l = 6.384$
$K_l = 6.396$
$K_l = 6.408$
$E_{K_{n+1}}(t) - E_{K_n}(t)$
t
0
10
20
30
40
50
60
70
80
−1
1
2
3
4
5


Entropy Difference
$S_{K_{n+1}}(t) - S_{K_n}(t)$
10^{-2}
10^{-3}
0
10
20
30
40
50
60
70
80
t
$K_l = 6.300$
$K_l = 6.312$
$K_l = 6.324$
$K_l = 6.336$
$K_l = 6.348$
$K_l = 6.360$
$K_l = 6.372$
$K_l = 6.384$
$K_l = 6.396$
$K_l = 6.408$

References

Lemarié, G., B. Grémaud, and D. Delande. 2009. "Universality of the Anderson Transition with the Quasiperiodic Kicked Rotor." *Europhys. Lett.* 87 (3): 37007. https://doi.org/10.1209/0295-5075/87/37007.