Bipartite Entanglement Entropy

Aditya Chincholi

June 28, 2021

Quasiperiodic Kicked Rotor

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- This has the drawback of increasing computational complexity of each individual step and the memory used at any given time is large.

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- This is better as the memory used is less but computation increases. Since fourier transforms are computationally cheap anyway, so it's fine.
- Peak memory required scales the same way but we have reduced it by a constant factor and it is not used in all calculations.

We use $\hbar = 2.85, \omega_2 = 2\pi\sqrt{5}, \omega_3 = 2\pi\sqrt{13}$, the momentum ranges from -10 to 10

$$H = \frac{p_1^2}{2} + p_2\omega_2 + p_3\omega_3 + Kcos(\theta_1)(1 + \alpha cos(\theta_2)cos(\theta_3))\sum_n \delta(t - n)$$

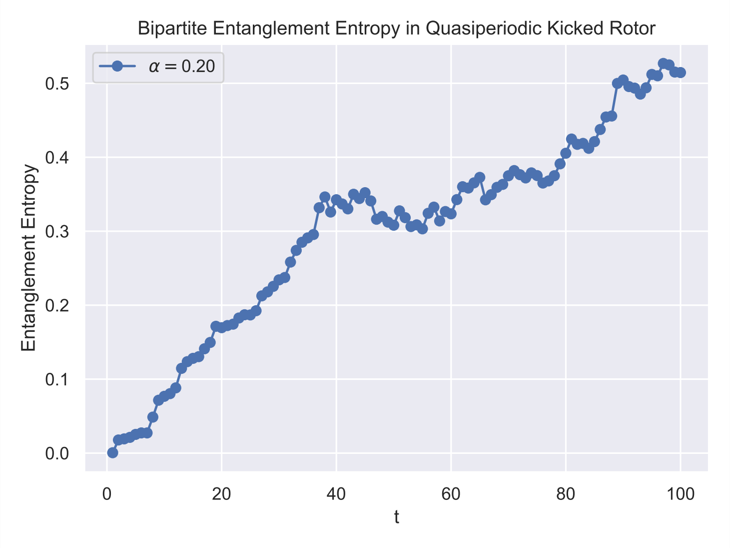

Figure 1: Precritical (Insulator): $K = 4, \alpha = 0.2$

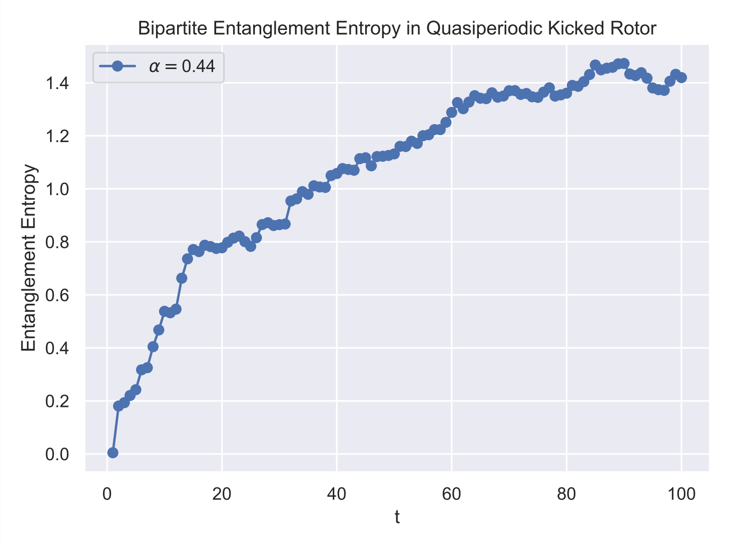

Figure 2: Critical: $K = 6.36, \alpha = 0.4375$

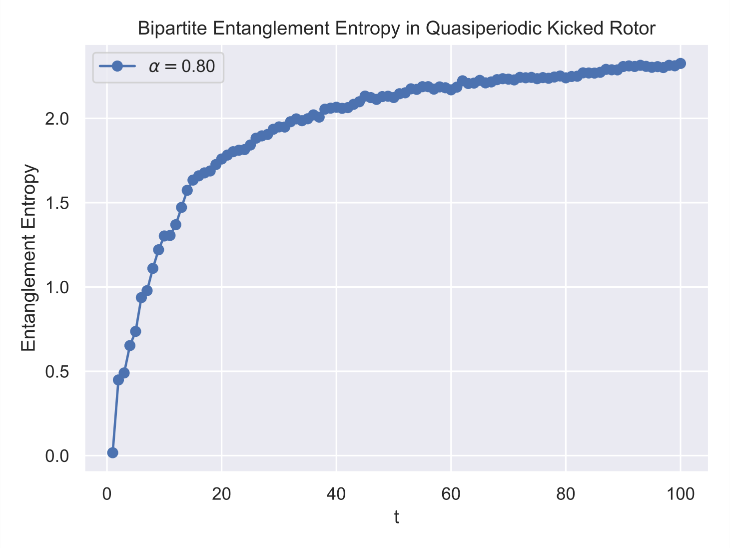

Figure 3: Post-critical (Metal): $K = 8, \alpha = 0.8$

- I don't see much of a trend here. The entanglement grows faster and higher with higher K values i.e. more diffusive the regime higher the entanglement for the same number of time steps but other than that, I don't see anything here.

Bypassing The Density Matrix

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- We plot the following quantities:
    - $P(p_1 = m\hbar)$
    - $E = p_1^2/2 + p_2\omega_2 + p_3\omega_3$
    - $S = -\rho_1 ln(\rho_1)$

Momentum (p_1) distributions

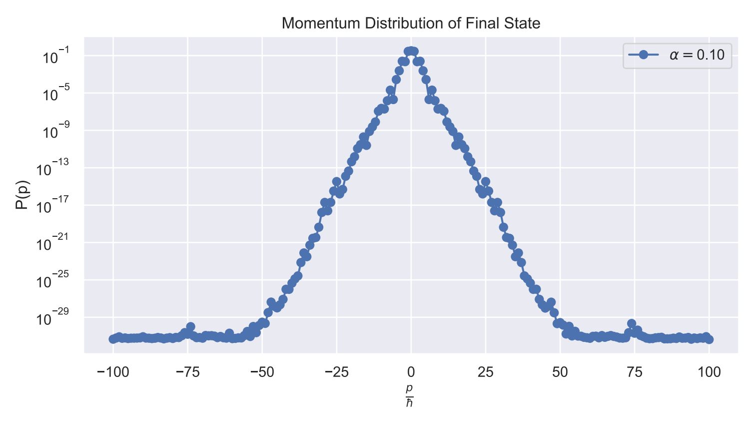


Figure 4: K = 3, α = 0.1

Momentum (p_1) distributions

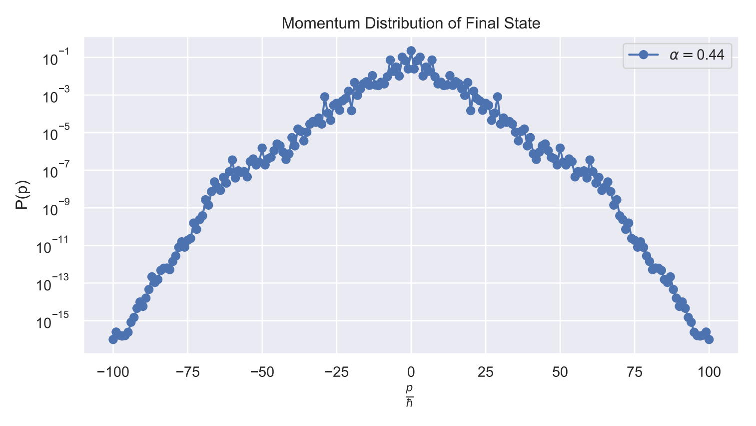


Figure 5: K = 6.36, α = 0.4375

Momentum (p_1) distributions

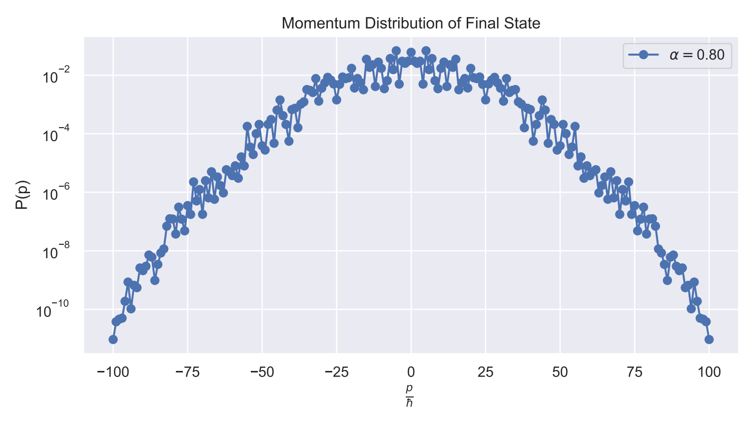


Figure 6: K = 7, α = 0.8

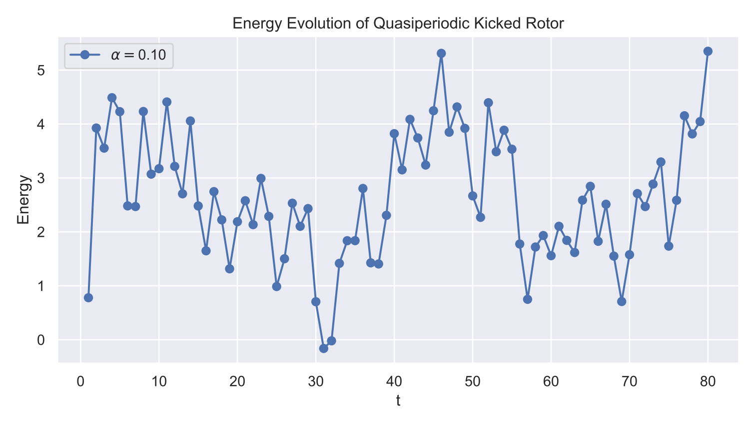


Figure 7: K = 3, α = 0.1

Energy

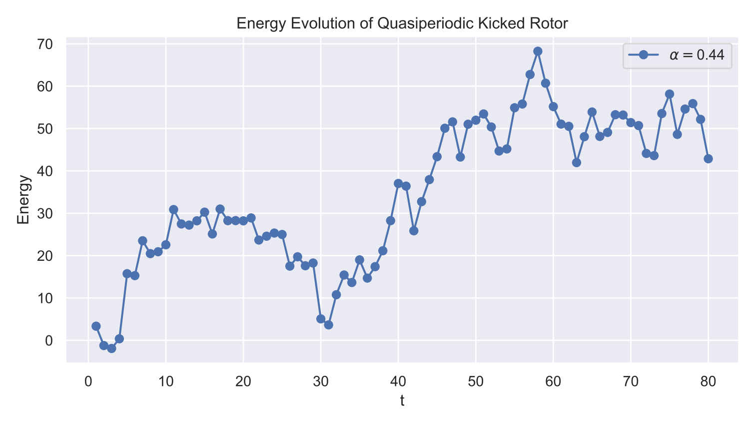


Figure 8: K = 6.36, α = 0.4375

Energy

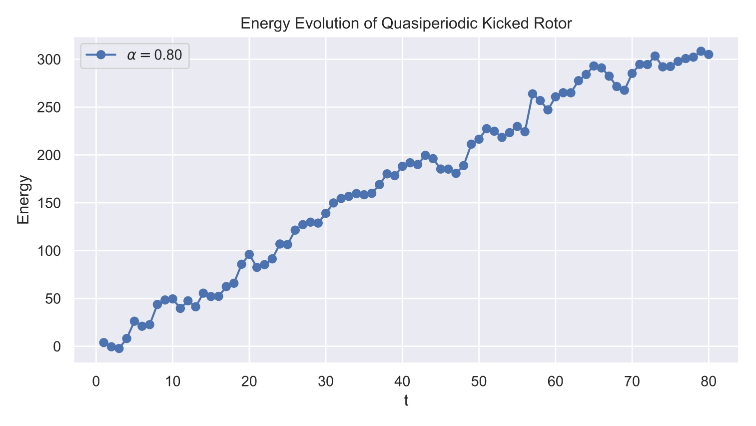


Figure 9: $K = 7$, $\alpha = 0.8$

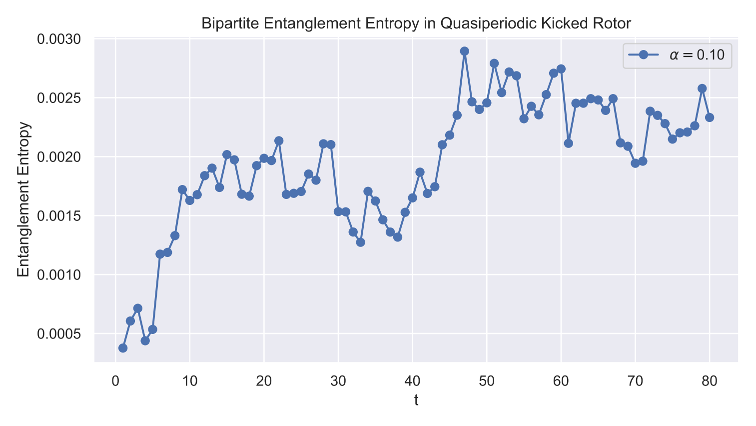


Figure 10: K = 3, α = 0.1

Entropy

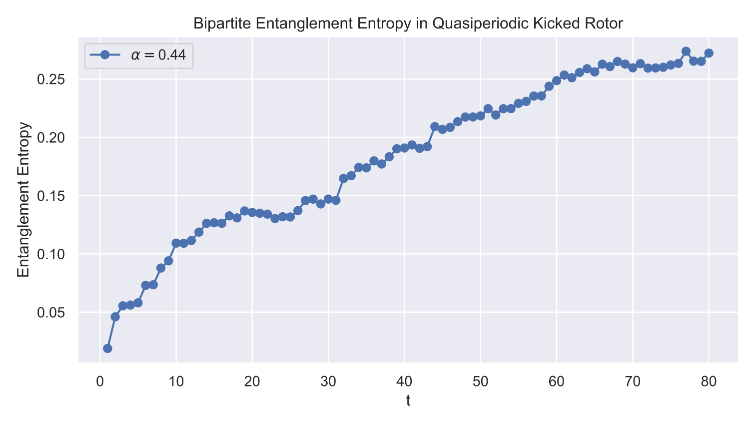


Figure 11: K = 6.36, α = 0.4375

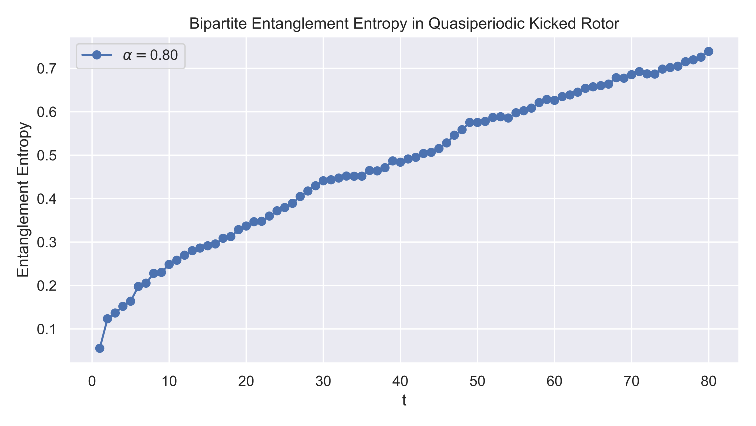


Figure 12: K = 7, α = 0.8

Multiple K Values

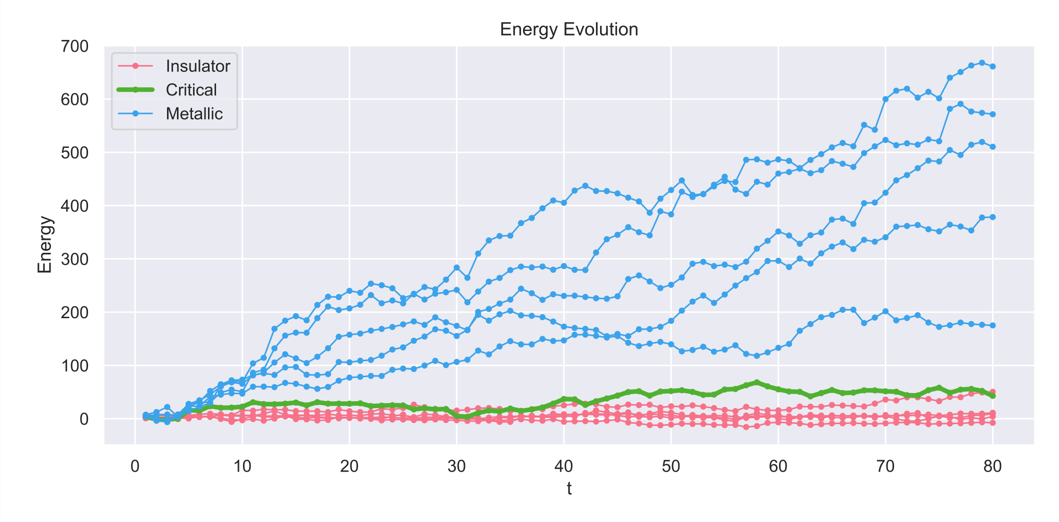
- We look at the following plots for the following at different K, α values from the insulator to the metallic regime:
  1. Energy Expectation Value
  2. Entanglement Entropy
  3. Momentum Distribution
- To study the changes in the energy and entropy values with K, we have plotted the following quantities:
  1. Entropy Difference: $S(K_{n+1}, \alpha_{n+1}) - S(K_n, \alpha_n)$ vs t
  2. Energy Difference: $E(K_{n+1}, \alpha_{n+1}) - E(K_n, \alpha_n)$ vs t

Multiple K Values

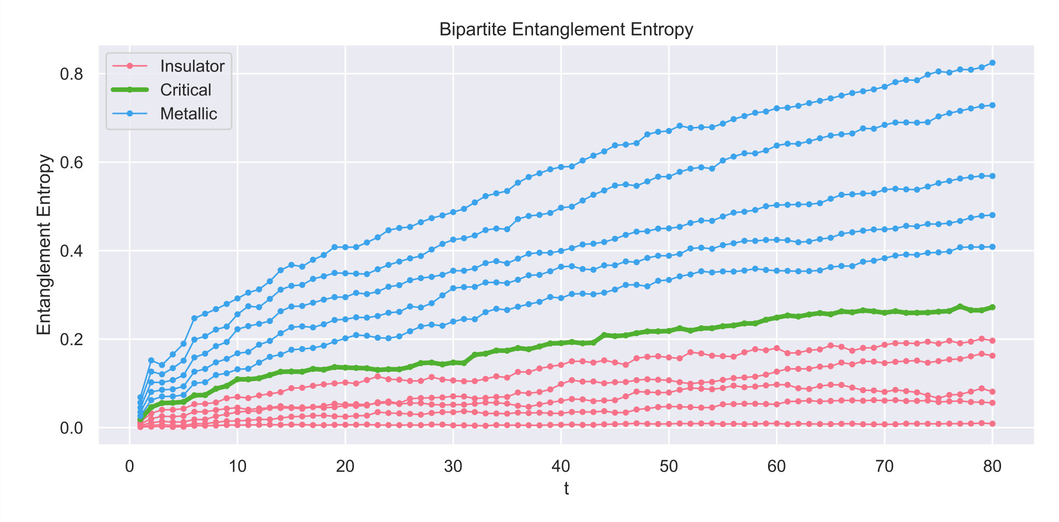
- First we take a big picture look with 11 values with $K \in [3.00, 9.72]$ and $\alpha \in [0.200, 0.6750]$. Both ranges are centred around the critical point[1]. Momentum range is -100 to 100 with 80 timesteps.
- Then we look very close to the critical point with 11 values with $K \in [6.30, 6.42]$ and $\alpha \in [0.4000, 0.4750]$. Both ranges are centred around the critical point. Momentum range is -100 to 100 with 80 timesteps.

[1]The (G. Lemarié, Grémaud, and Delande 2009) paper gives the value of K at critical point, but not of α.

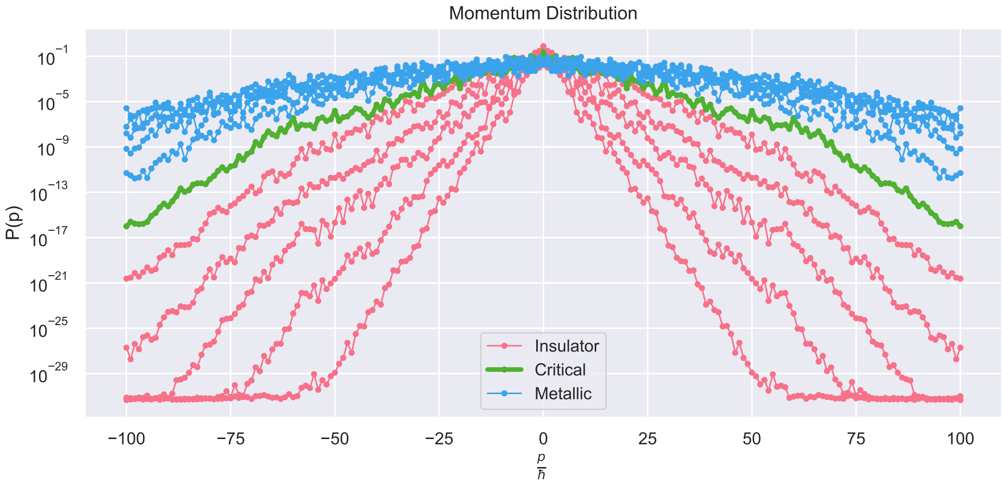
The Big Picture



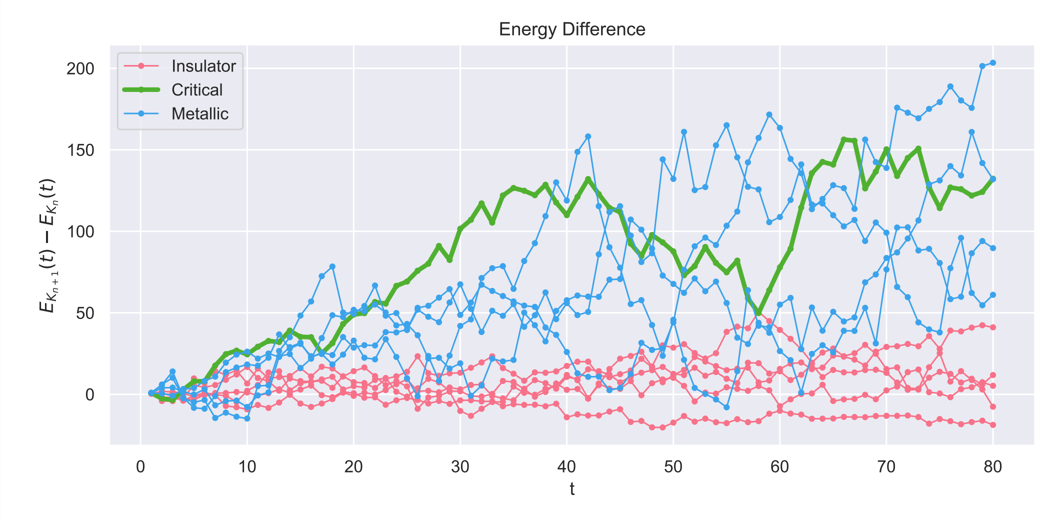
Energy Evolution
Insulator
Critical
Metallic
Energy
t
0
100
200
300
400
500
600
700
0
10
20
30
40
50
60
70
80



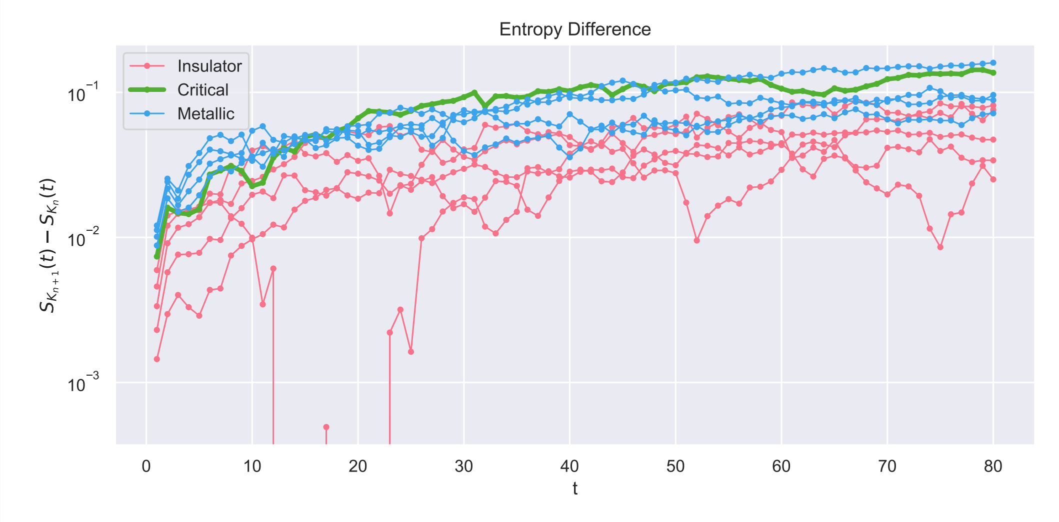
Bipartite Entanglement Entropy
Insulator
Critical
Metallic
Entanglement Entropy
0.0
0.2
0.4
0.6
0.8
0
10
20
30
40
50
60
70
80
t



Momentum Distribution
P(p)
10^{-1}
10^{-5}
10^{-9}
10^{-13}
10^{-17}
10^{-21}
10^{-25}
10^{-29}
−100
−75
−50
−25
0
25
50
75
100
$\frac{p}{\hbar}$
Insulator
Critical
Metallic

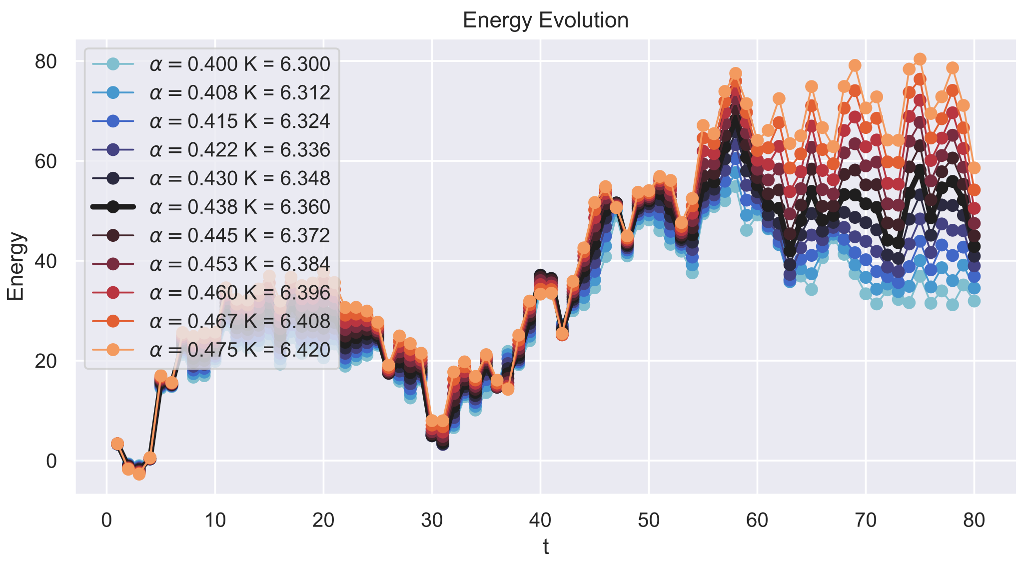


Energy Difference
Insulator
Critical
Metallic
200
150
100
50
0
0
10
20
30
40
50
60
70
80
t
$E_{K_{n+1}}(t) - E_{K_n}(t)$

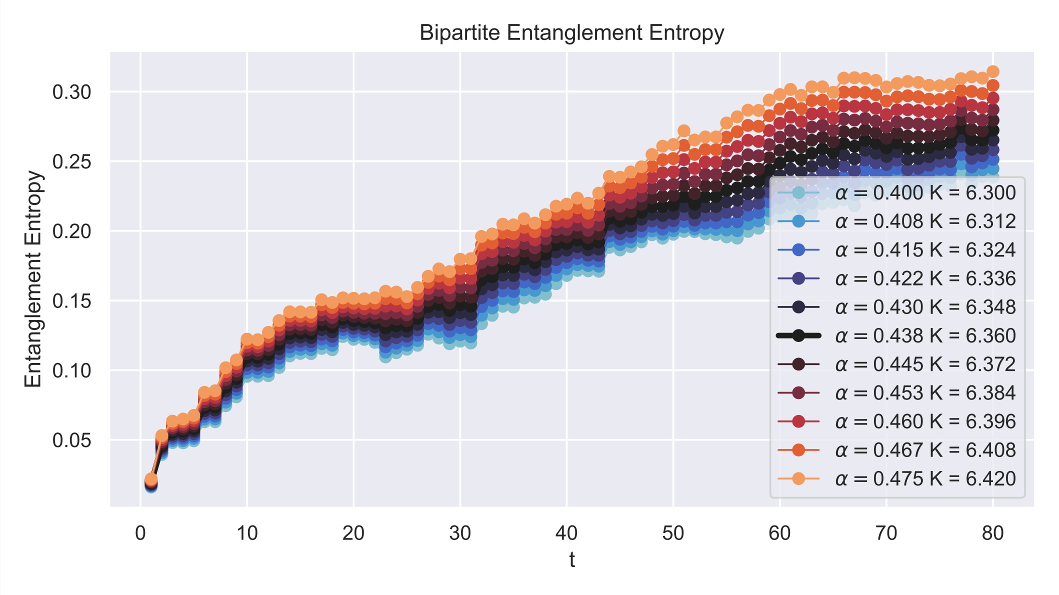


Entropy Difference
Insulator
Critical
Metallic
10^{-1}
10^{-2}
10^{-3}
$S_{K_{n+1}}(t) - S_{K_n}(t)$
0
10
20
30
40
50
60
70
80
t

The Microscopic Picture



Energy Evolution
α = 0.400 K = 6.300
α = 0.408 K = 6.312
α = 0.415 K = 6.324
α = 0.422 K = 6.336
α = 0.430 K = 6.348
α = 0.438 K = 6.360
α = 0.445 K = 6.372
α = 0.453 K = 6.384
α = 0.460 K = 6.396
α = 0.467 K = 6.408
α = 0.475 K = 6.420
Energy
t
0
20
40
60
80
0
10
20
30
40
50
60
70
80



Bipartite Entanglement Entropy
Entanglement Entropy
0.30
0.25
0.20
0.15
0.10
0.05
0
10
20
30
40
50
60
70
80
t
α = 0.400 K = 6.300
α = 0.408 K = 6.312
α = 0.415 K = 6.324
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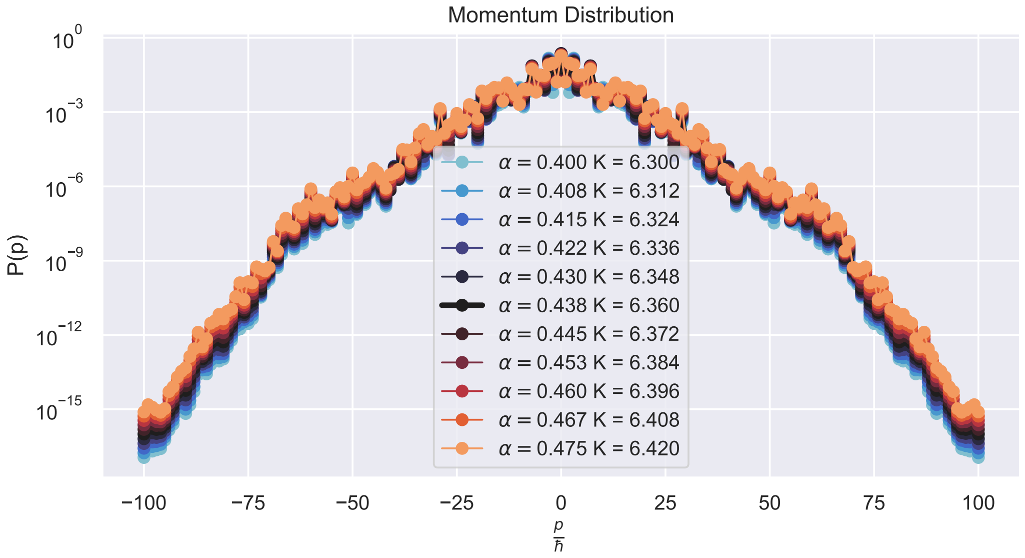
Momentum Distribution
10^{0}
10^{-3}
10^{-6}
10^{-9}
10^{-12}
10^{-15}
P(p)
−100
−75
−50
−25
0
25
50
75
100
$\frac{p}{\hbar}$
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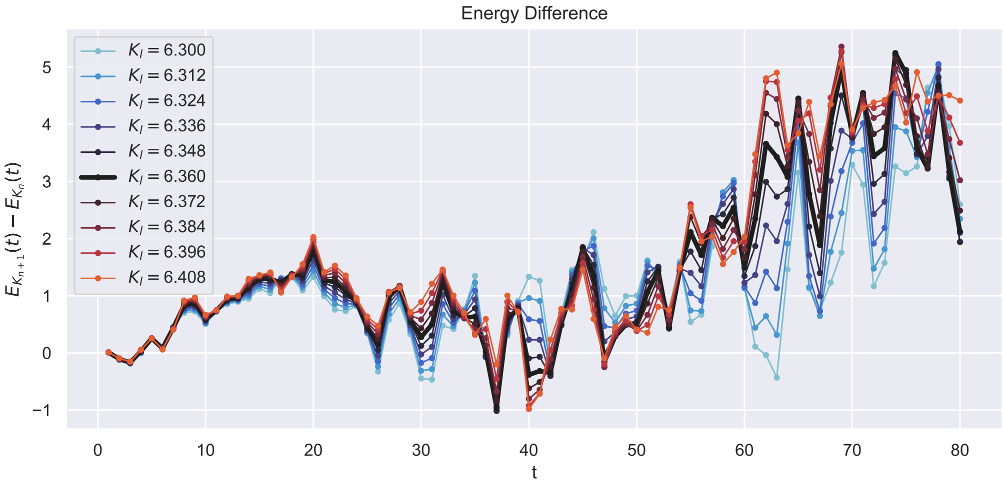
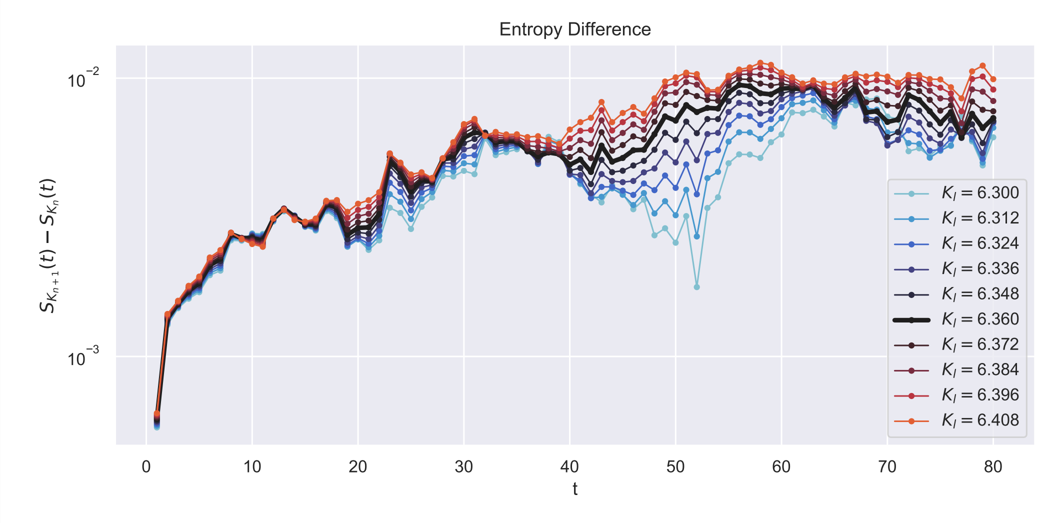


Energy Difference
$K_l = 6.300$
$K_l = 6.312$
$K_l = 6.324$
$K_l = 6.336$
$K_l = 6.348$
$K_l = 6.360$
$K_l = 6.372$
$K_l = 6.384$
$K_l = 6.396$
$K_l = 6.408$
$E_{K_{n+1}}(t) - E_{K_n}(t)$
t
0
10
20
30
40
50
60
70
80
−1
0
1
2
3
4
5


Entropy Difference
$S_{K_{n+1}}(t) - S_{K_n}(t)$
10^{-2}
10^{-3}
0
10
20
30
40
50
60
70
80
t
$K_l = 6.300$
$K_l = 6.312$
$K_l = 6.324$
$K_l = 6.336$
$K_l = 6.348$
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Parameters

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- The value of α_c, however is not determined accurately, nor written in any literature as it doesn't form a part of the scaling function in a nice way, though it most certainly does matter as it generates the anisotropy. We determine the value of α_c roughly by using the fact that the authors of the cited papers traversed the $K - \alpha$ space along a straight line perpendicular to the transition parabola. (G. Lemarié, Grémaud, and Delande 2009) (Gabriel Lemarié et al. 2009)

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- TL;DR we used $K_c = 6.36$ and $\alpha_c = 0.4303$.

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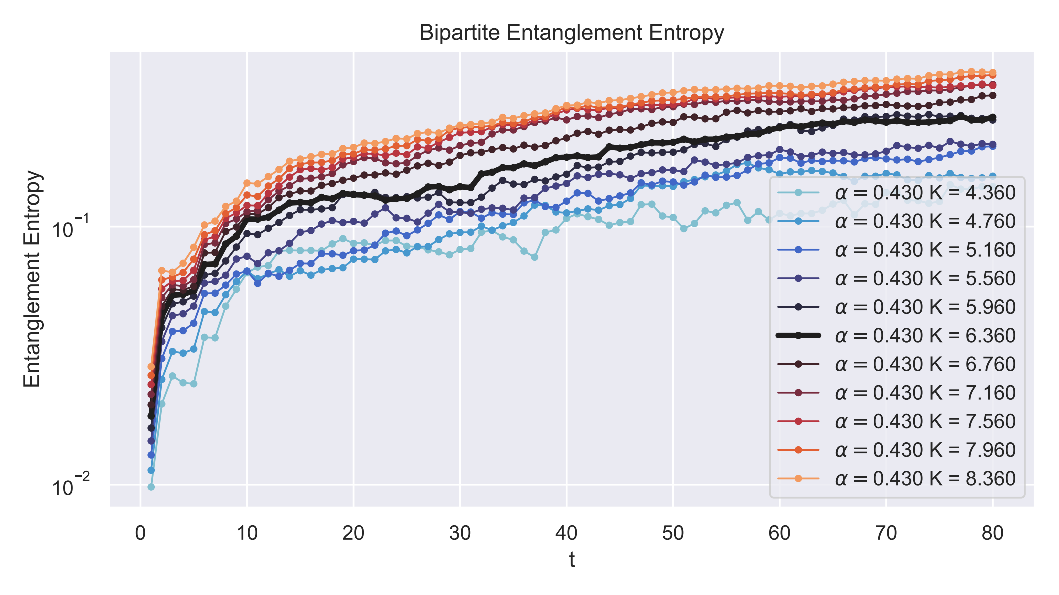
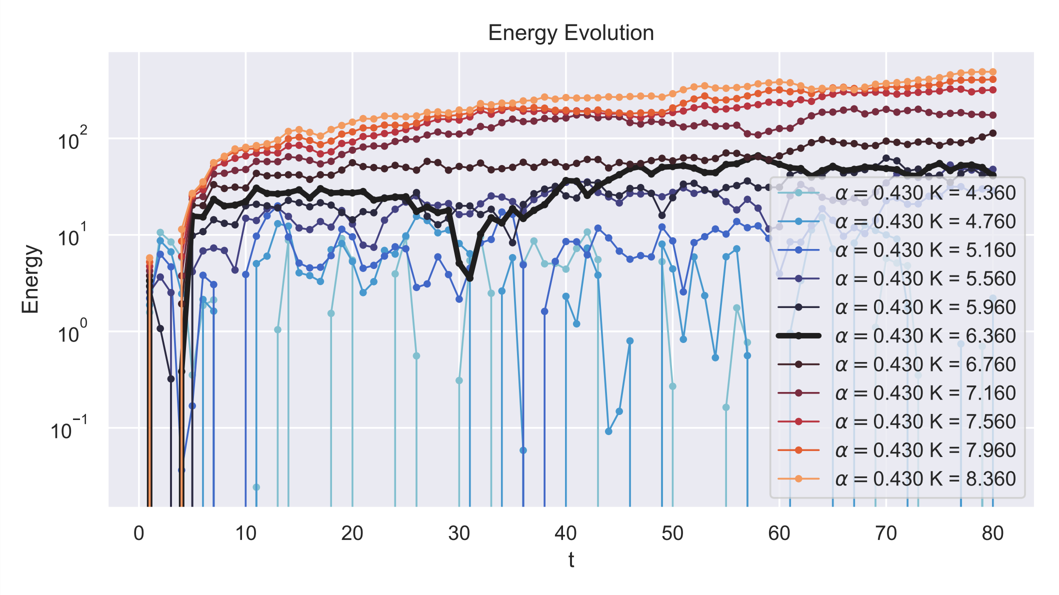
- TL;DR we used $K_c = 6.36$ and $\alpha_c = 0.4303$.
- We used a basis size of 201 (-100 to 100) for each of the 3 coordinates and the simulations were done for 80 timesteps.

Varying K


Bipartite Entanglement Entropy
Entanglement Entropy
10^{-1}
10^{-2}
0
10
20
30
40
50
60
70
80
t
α = 0.430 K = 4.360
α = 0.430 K = 4.760
α = 0.430 K = 5.160
α = 0.430 K = 5.560
α = 0.430 K = 5.960
α = 0.430 K = 6.360
α = 0.430 K = 6.760
α = 0.430 K = 7.160
α = 0.430 K = 7.560
α = 0.430 K = 7.960
α = 0.430 K = 8.360


Energy Evolution
Energy
t
10^{2}
10^{1}
10^{0}
10^{-1}
0
10
20
30
40
50
60
70
80
α = 0.430 K = 4.360
α = 0.430 K = 4.760
α = 0.430 K = 5.160
α = 0.430 K = 5.560
α = 0.430 K = 5.960
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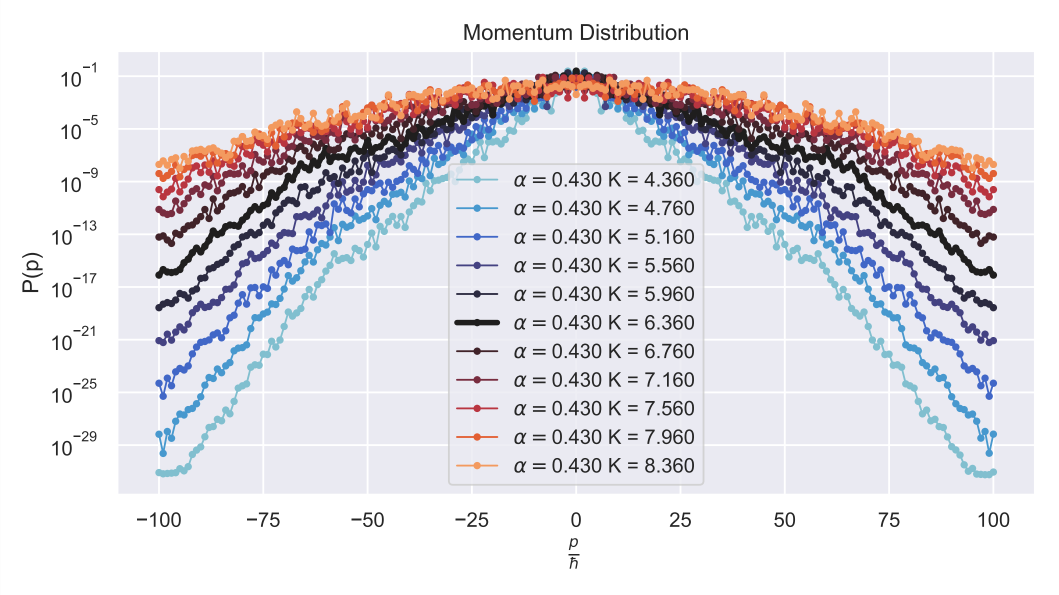
α = 0.430 K = 6.760
α = 0.430 K = 7.160
α = 0.430 K = 7.560
α = 0.430 K = 7.960
α = 0.430 K = 8.360


Momentum Distribution
P(p)
10^{-1}
10^{-5}
10^{-9}
10^{-13}
10^{-17}
10^{-21}
10^{-25}
10^{-29}
−100
−75
−50
−25
0
25
50
75
100
$\frac{p}{\hbar}$
α = 0.430 K = 4.360
α = 0.430 K = 4.760
α = 0.430 K = 5.160
α = 0.430 K = 5.560
α = 0.430 K = 5.960
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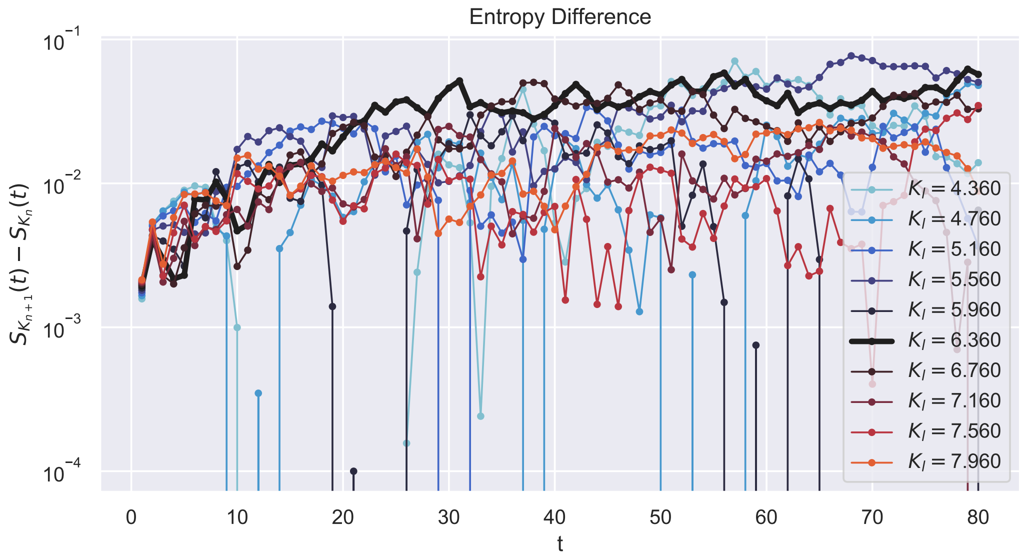
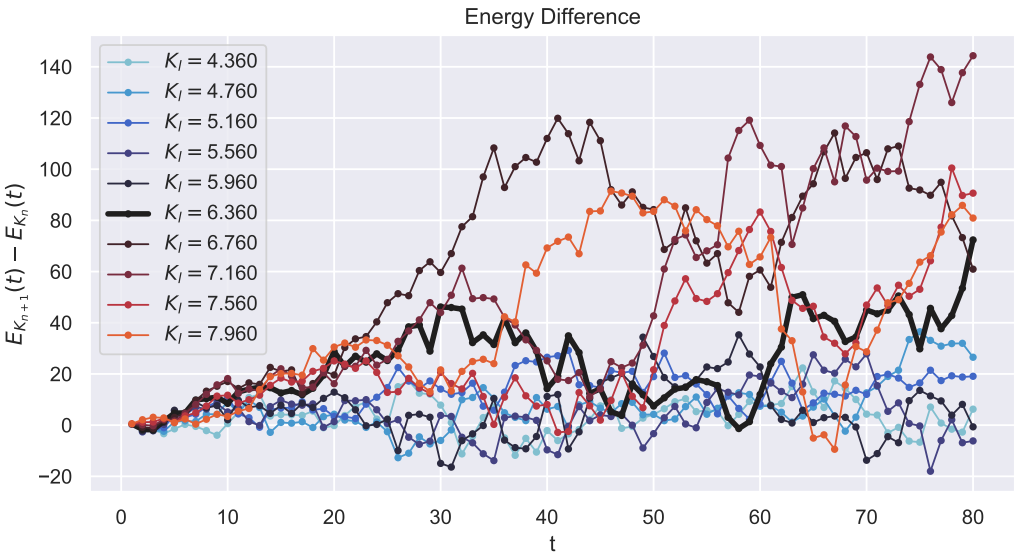
α = 0.430 K = 6.760
α = 0.430 K = 7.160
α = 0.430 K = 7.560
α = 0.430 K = 7.960
α = 0.430 K = 8.360

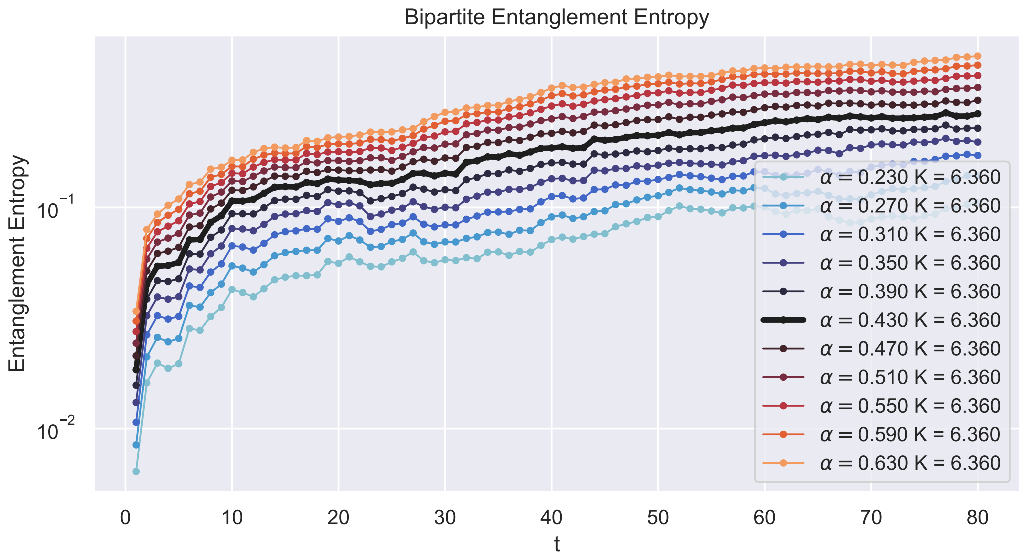


Entropy Difference
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$S_{K_{n+1}}(t) - S_{K_n}(t)$
0
10
20
30
40
50
60
70
80
t
$K_l = 4.360$
$K_l = 4.760$
$K_l = 5.160$
$K_l = 5.560$
$K_l = 5.960$
$K_l = 6.360$
$K_l = 6.760$
$K_l = 7.160$
$K_l = 7.560$
$K_l = 7.960$

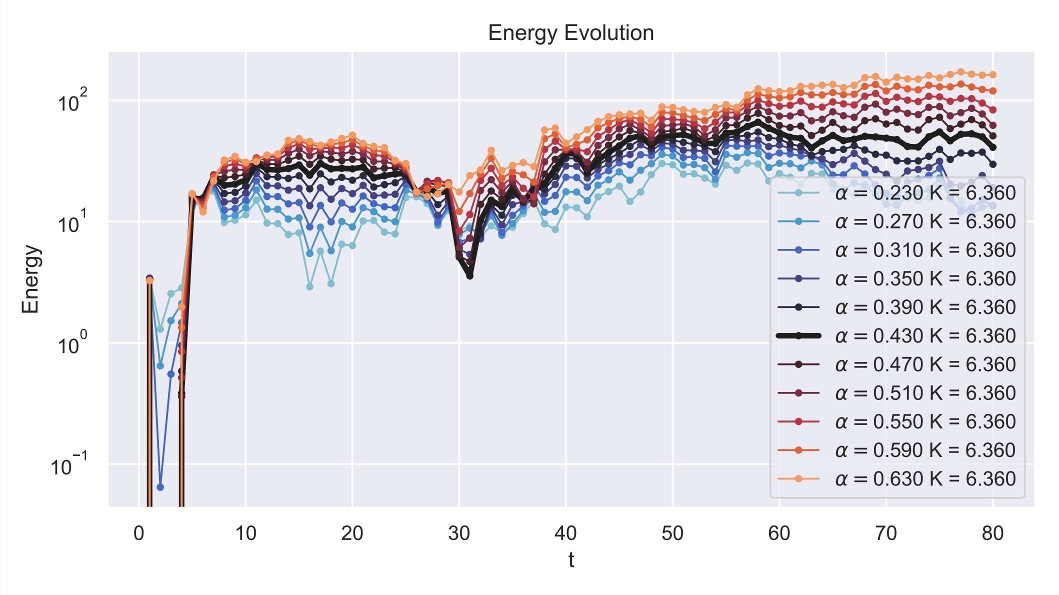


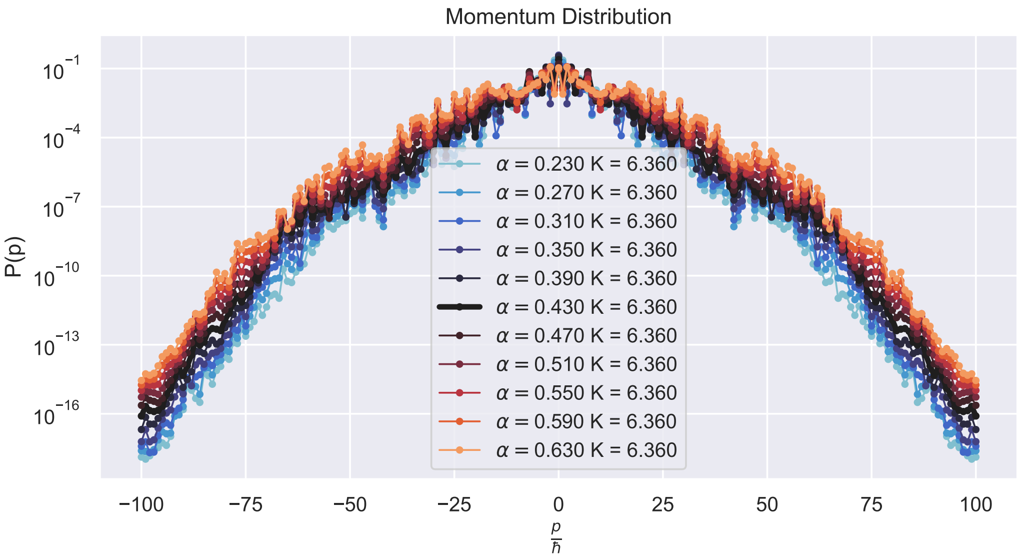
Energy Difference
$K_l = 4.360$
$K_l = 4.760$
$K_l = 5.160$
$K_l = 5.560$
$K_l = 5.960$
$K_l = 6.360$
$K_l = 6.760$
$K_l = 7.160$
$K_l = 7.560$
$K_l = 7.960$
$E_{K_{n+1}}(t) - E_{K_n}(t)$
t
140
120
100
80
60
40
20
0
−20
0
10
20
30
40
50
60
70
80

Varying α

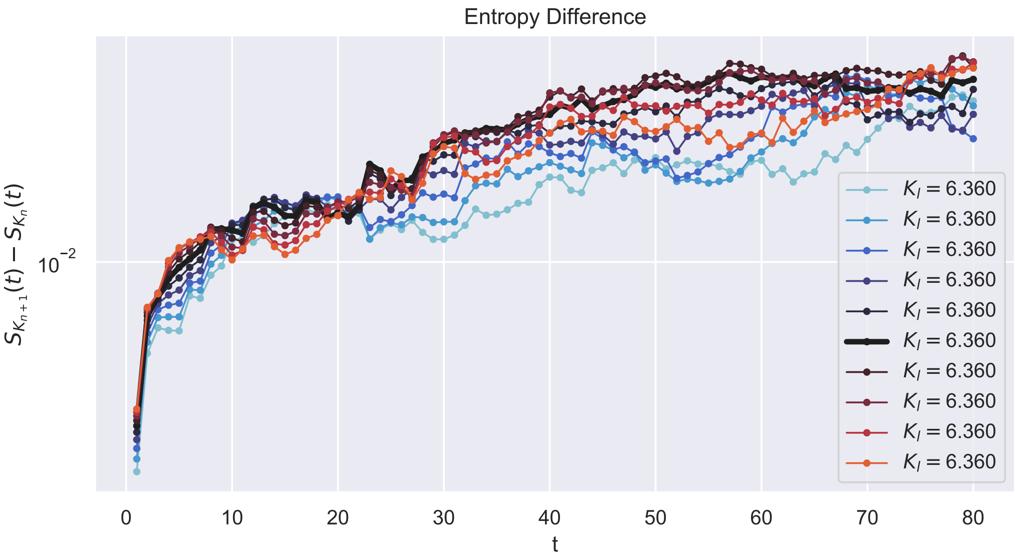


Bipartite Entanglement Entropy
Entanglement Entropy
10^{-1}
10^{-2}
0
10
20
30
40
50
60
70
80
t
α = 0.230 K = 6.360
α = 0.270 K = 6.360
α = 0.310 K = 6.360
α = 0.350 K = 6.360
α = 0.390 K = 6.360
α = 0.430 K = 6.360
α = 0.470 K = 6.360
α = 0.510 K = 6.360
α = 0.550 K = 6.360
α = 0.590 K = 6.360
α = 0.630 K = 6.360

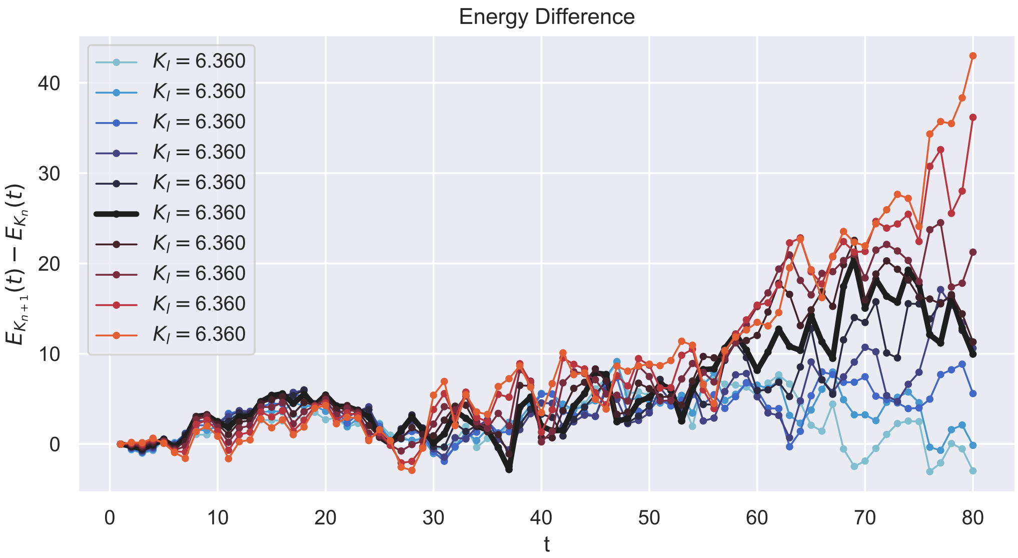

Energy Evolution
Energy
t
10^2
10^1
10^0
10^{-1}
0
10
20
30
40
50
60
70
80
α = 0.230 K = 6.360
α = 0.270 K = 6.360
α = 0.310 K = 6.360
α = 0.350 K = 6.360
α = 0.390 K = 6.360
α = 0.430 K = 6.360
α = 0.470 K = 6.360
α = 0.510 K = 6.360
α = 0.550 K = 6.360
α = 0.590 K = 6.360
α = 0.630 K = 6.360



Momentum Distribution
P(p)
10^{-1}
10^{-4}
10^{-7}
10^{-10}
10^{-13}
10^{-16}
−100
−75
−50
−25
0
25
50
75
100
$\frac{p}{\hbar}$
$\alpha = 0.230$ K = 6.360
$\alpha = 0.270$ K = 6.360
$\alpha = 0.310$ K = 6.360
$\alpha = 0.350$ K = 6.360
$\alpha = 0.390$ K = 6.360
$\alpha = 0.430$ K = 6.360
$\alpha = 0.470$ K = 6.360
$\alpha = 0.510$ K = 6.360
$\alpha = 0.550$ K = 6.360
$\alpha = 0.590$ K = 6.360
$\alpha = 0.630$ K = 6.360

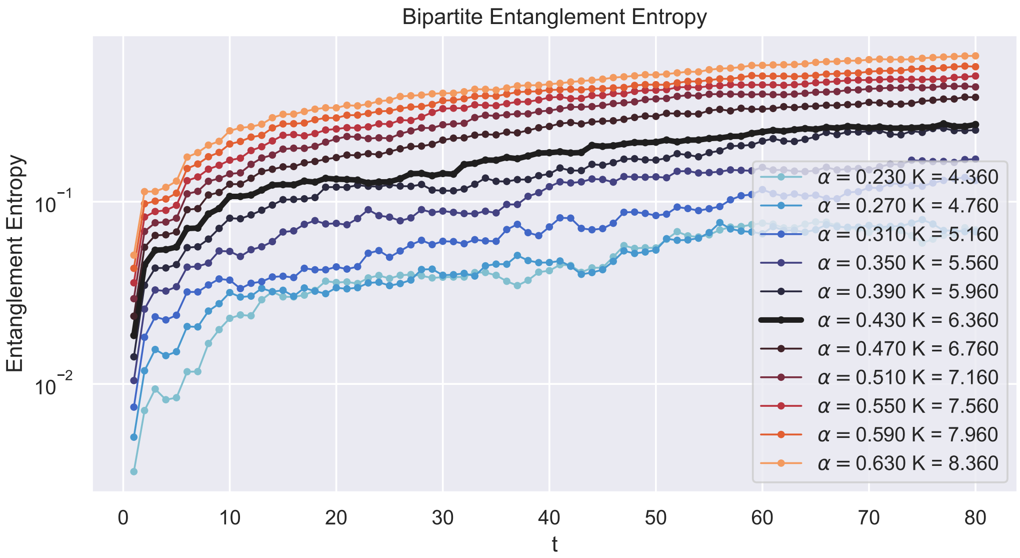


Entropy Difference
$S_{K_{n+1}}(t) - S_{K_n}(t)$
10^{-2}
0
10
20
30
40
50
60
70
80
t
$K_l = 6.360$
$K_l = 6.360$
$K_l = 6.360$
$K_l = 6.360$
$K_l = 6.360$
$K_l = 6.360$
$K_l = 6.360$
$K_l = 6.360$
$K_l = 6.360$
$K_l = 6.360$

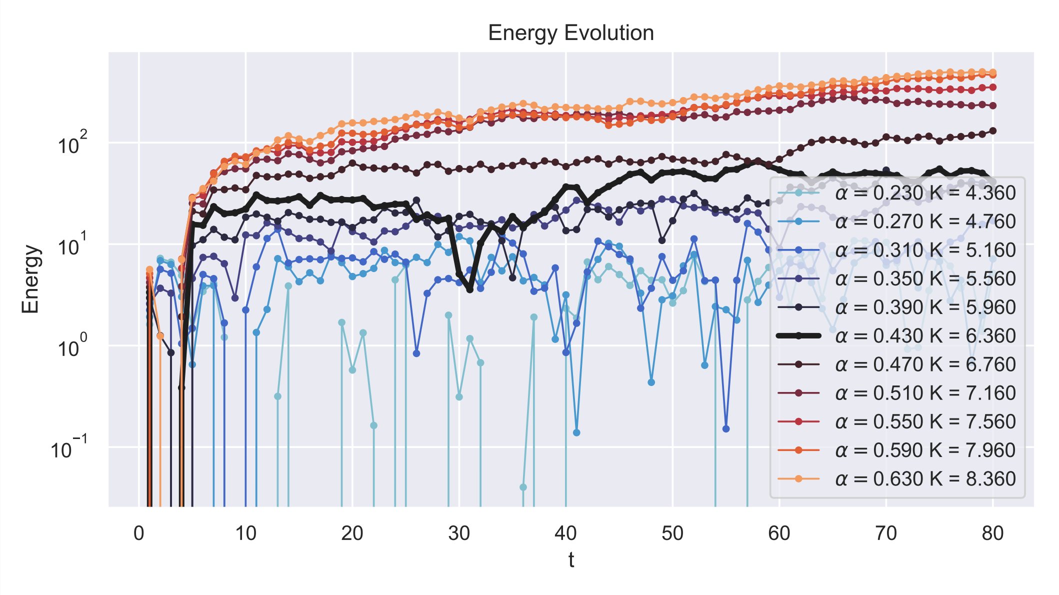


Energy Difference
$K_l = 6.360$
$K_l = 6.360$
$K_l = 6.360$
$K_l = 6.360$
$K_l = 6.360$
$K_l = 6.360$
$K_l = 6.360$
$K_l = 6.360$
$K_l = 6.360$
$K_l = 6.360$
$E_{K_{n+1}}(t) - E_{K_n}(t)$
t
0
10
20
30
40
50
60
70
80

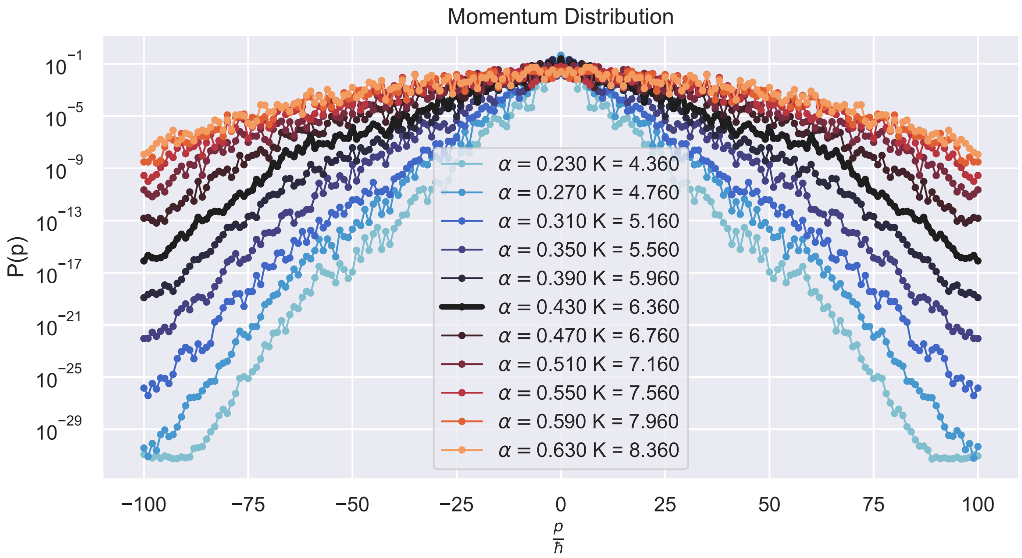
Varying K and α

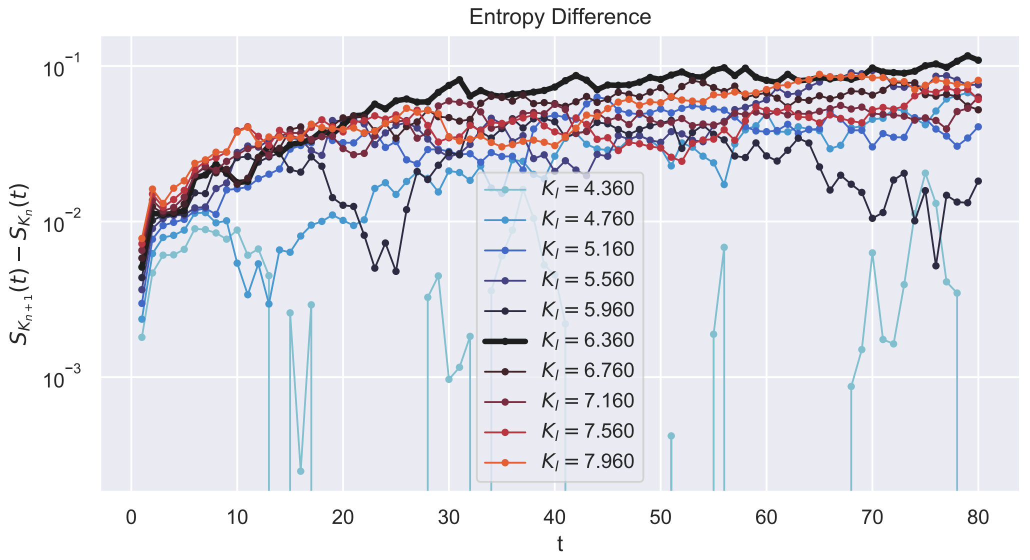


Bipartite Entanglement Entropy
Entanglement Entropy
10^{-1}
10^{-2}
0
10
20
30
40
50
60
70
80
t
α = 0.230 K = 4.360
α = 0.270 K = 4.760
α = 0.310 K = 5.160
α = 0.350 K = 5.560
α = 0.390 K = 5.960
α = 0.430 K = 6.360
α = 0.470 K = 6.760
α = 0.510 K = 7.160
α = 0.550 K = 7.560
α = 0.590 K = 7.960
α = 0.630 K = 8.360

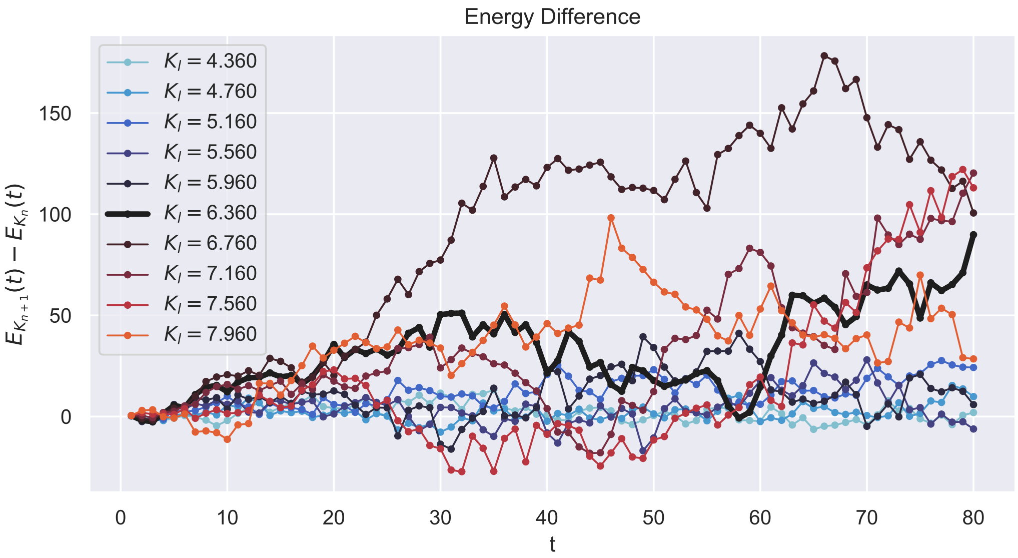


Energy Evolution
Energy
t
10^{2}
10^{1}
10^{0}
10^{-1}
0
10
20
30
40
50
60
70
80
α = 0.230 K = 4.360
α = 0.270 K = 4.760
α = 0.310 K = 5.160
α = 0.350 K = 5.560
α = 0.390 K = 5.960
α = 0.430 K = 6.360
α = 0.470 K = 6.760
α = 0.510 K = 7.160
α = 0.550 K = 7.560
α = 0.590 K = 7.960
α = 0.630 K = 8.360


Momentum Distribution
P(p)
10^{-1}
10^{-5}
10^{-9}
10^{-13}
10^{-17}
10^{-21}
10^{-25}
10^{-29}
−100
−75
−50
−25
0
25
50
75
100
$\frac{p}{\hbar}$
$\alpha = 0.230$ K = 4.360
$\alpha = 0.270$ K = 4.760
$\alpha = 0.310$ K = 5.160
$\alpha = 0.350$ K = 5.560
$\alpha = 0.390$ K = 5.960
$\alpha = 0.430$ K = 6.360
$\alpha = 0.470$ K = 6.760
$\alpha = 0.510$ K = 7.160
$\alpha = 0.550$ K = 7.560
$\alpha = 0.590$ K = 7.960
$\alpha = 0.630$ K = 8.360



Entropy Difference
$S_{K_{n+1}}(t) - S_{K_n}(t)$
10^{-1}
10^{-2}
10^{-3}
0
10
20
30
40
50
60
70
80
t
$K_l = 4.360$
$K_l = 4.760$
$K_l = 5.160$
$K_l = 5.560$
$K_l = 5.960$
$K_l = 6.360$
$K_l = 6.760$
$K_l = 7.160$
$K_l = 7.560$
$K_l = 7.960$



Energy Difference
$K_l = 4.360$
$K_l = 4.760$
$K_l = 5.160$
$K_l = 5.560$
$K_l = 5.960$
$K_l = 6.360$
$K_l = 6.760$
$K_l = 7.160$
$K_l = 7.560$
$K_l = 7.960$
$E_{K_{n+1}}(t) - E_{K_n}(t)$
150
100
50
0
0
10
20
30
40
50
60
70
80
t

References

Lemarié, Gabriel, Julien Chabé, Pascal Szriftgiser, Jean Claude Garreau, Benoît Grémaud, and Dominique Delande. 2009. "Observation of the Anderson Metal-Insulator Transition with Atomic Matter Waves: Theory and Experiment." *Phys. Rev. A* 80 (4): 043626. https://doi.org/10.1103/PhysRevA.80.043626.

Lemarié, G., B. Grémaud, and D. Delande. 2009. "Universality of the Anderson Transition with the Quasiperiodic Kicked Rotor." *Europhys. Lett.* 87 (3): 37007. https://doi.org/10.1209/0295-5075/87/37007.