Localization in Spin-orbit Coupled Disordered Systems

Aditya Chincholi

Indian Institute of Science Education and Research, Pune

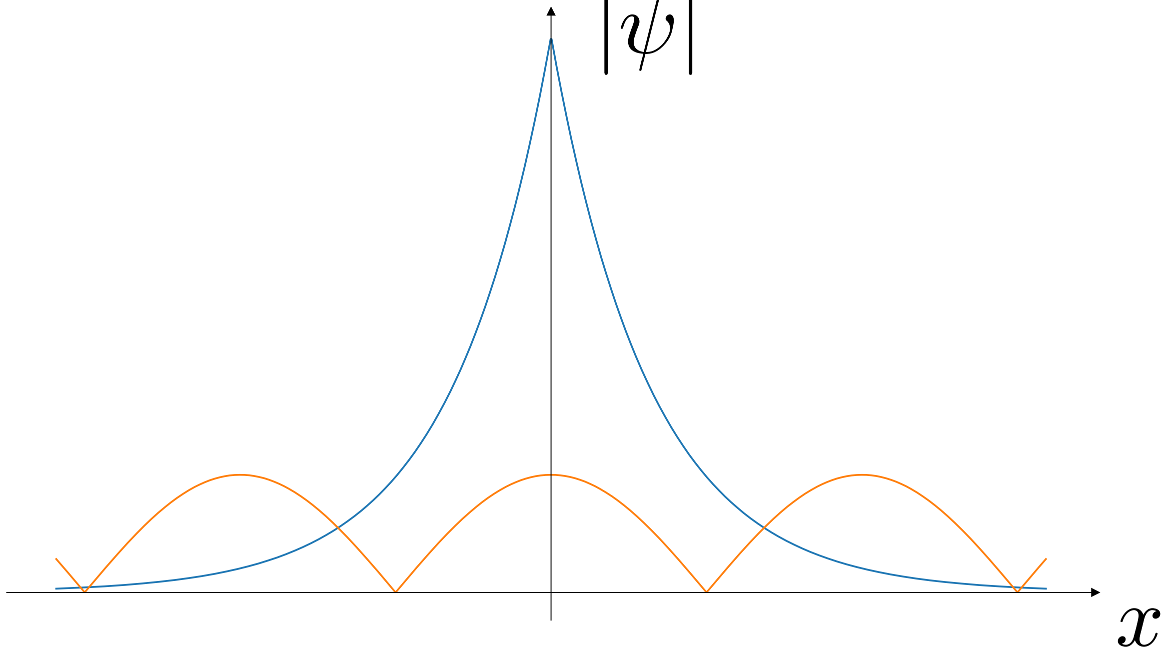
Supervised by: Dr Rajdeep Sensarma

Tata Institute of Fundamental Research, Mumbai

What Is Localization?

- Put a particle at r, then even after a long time it stays at r.
- More rigorously,

$$\psi(\mathbf{r}) \sim e^{\frac{|\mathbf{r}-\mathbf{r}_{\mathbf{0}}|}{2\xi}}$$



What is Localization?

- Inverse Participation Ratio (IPR) is defined as $\sum_{sites\ i} |\psi(i)|^4$
- For a localized wavefunction, IPR remains constant even if the system size is increased.
- For delocalized wavefunction, IPR scales as 1/N with system size.

What Is Disorder?

- Impurities or scatterers in the sample
- Usually random / quasirandom
- All materials created in a lab have it.
- Actually leads to some very interesting phenomena.

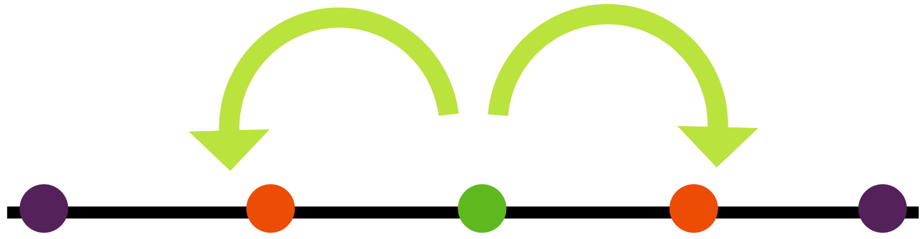
Localization Due To Disorder

- Anderson localization (*Anderson, P. W. "Absence of Diffusion in Certain Random Lattices." Physical Review 109, no. 5, 1958*)
- Quantum interference due to scattering from disorder leads to suppression of diffusion

In 1D

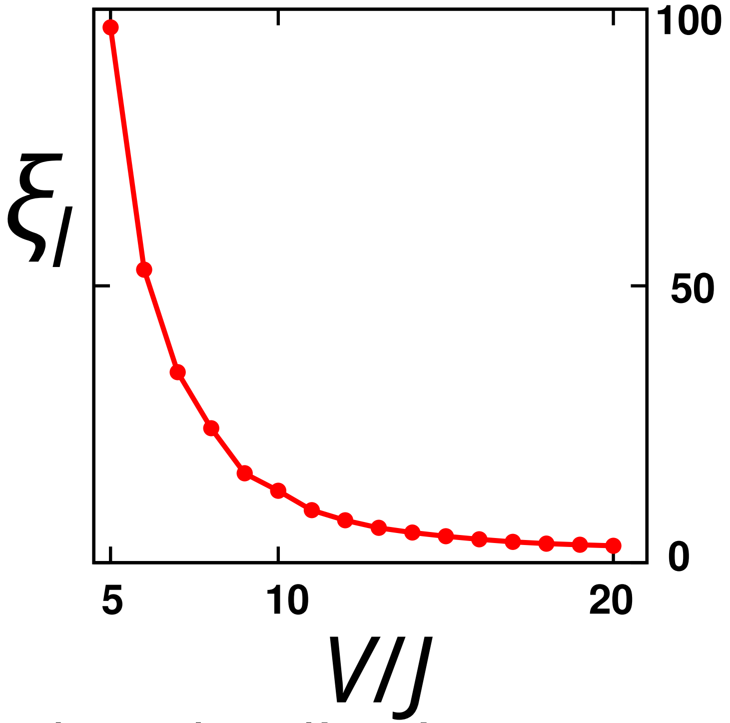
- Wavefunctions are always localized for non-zero disorder
- Localization length is of the order of the transport mean free path l: $\xi_{loc} \approx 2l$

$$H = \sum_i \epsilon_i c_i^\dagger c_i - t \sum_{\langle ij \rangle} c_i^\dagger c_j$$



Ref: Anderson, P. W. Physical Review 109, no. 5, 1958: 1492–1505

In 2D

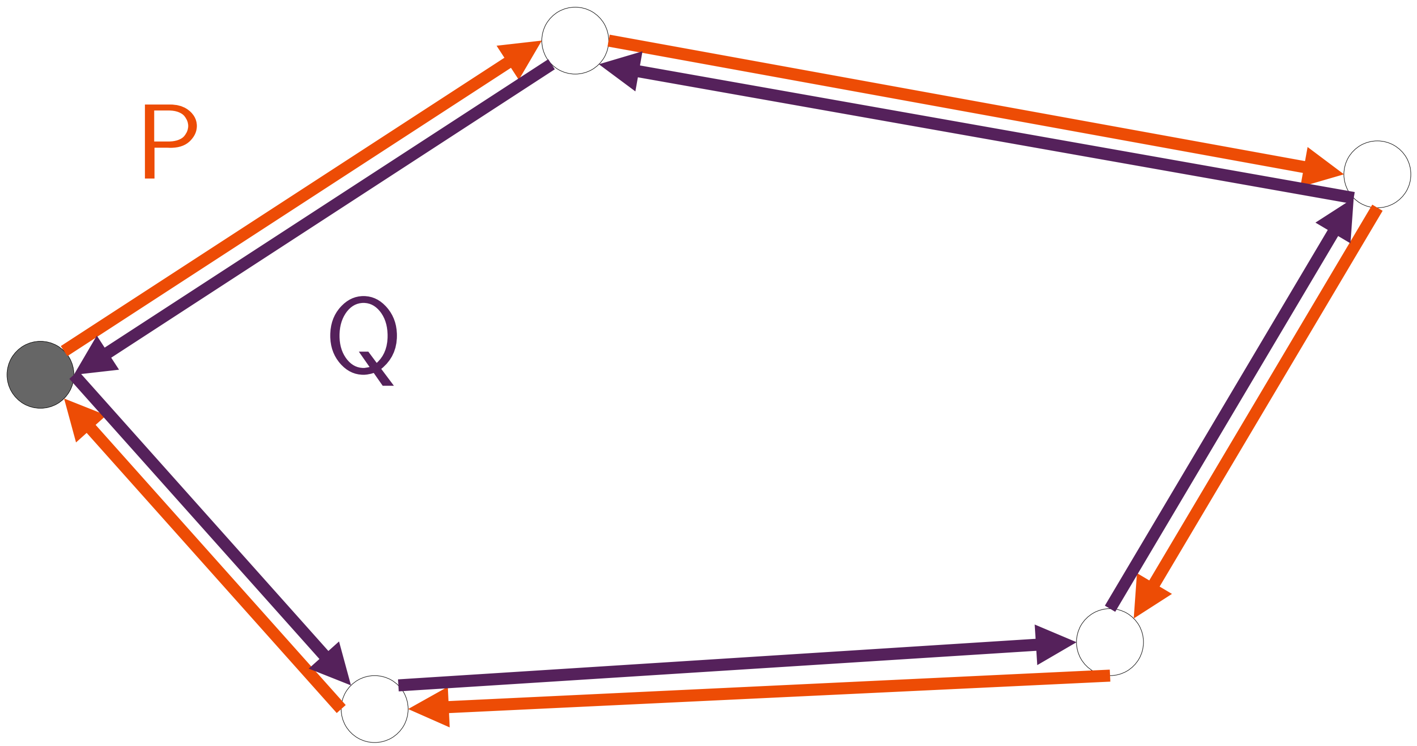


- Localization should occur for non-zero disorder
- But localization lengths can be very large
- $\xi_{loc} \sim e^{\frac{\pi}{2}kl}$
- Upper bound for the range in which non-zero disorder leads to localization
- Weak Localization correction terms are needed

Ref: Abrahams, E., P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan. Physical Review Letters 42, no. 10, 1979: 673–76

Img Ref: Chakraborty, Ahana, Pranay Gorantla, and Rajdeep Sensarma. Physical Review B 102, no. 22 (December 21, 2020): 224306

Weak Localization



Path P and it's time reversed counterpart Q interfere in a constructive manner leading to "coherent backscattering"

In 3D

- We see a phase transition: localized phase to delocalized phase (aka metal-insulator transition)
- Appearance of a mobility edge

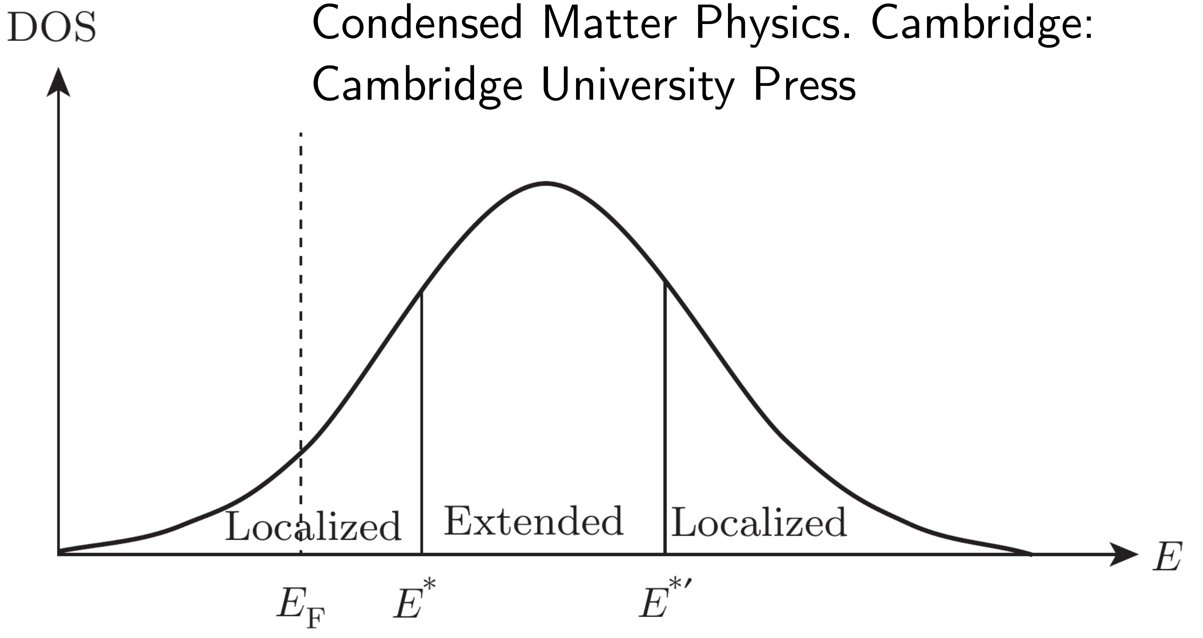
$W < W_c$ $\qquad$ W_c $\qquad$ $W > W_c$

Delocalized Phase (Metal) $\qquad$ Localized Phase (Insulator)

In 3D

- Mobility Edge is an energy level with the wavefunctions on one side being localized and the other side being delocalized.
- For high disorder, the middle region narrows and vanishes past the critical disorder.

Ref: Girvin, S., & Yang, K. (2019). Modern Condensed Matter Physics. Cambridge: Cambridge University Press

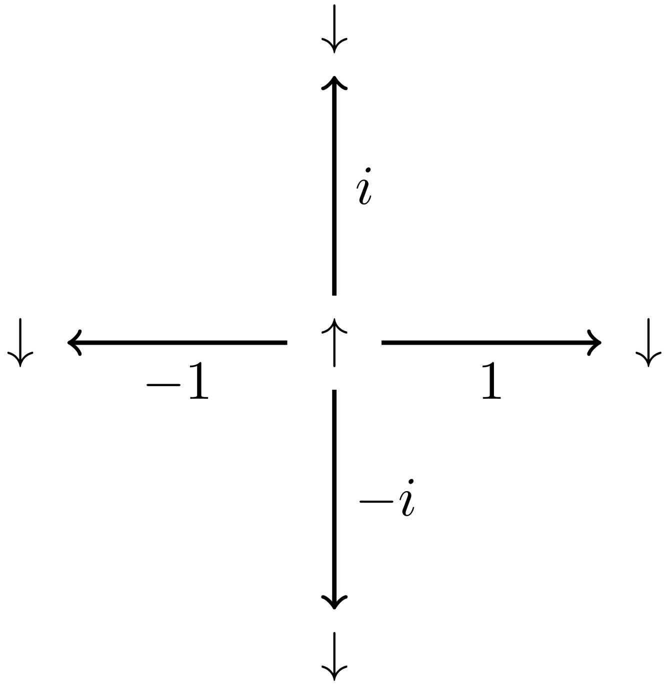


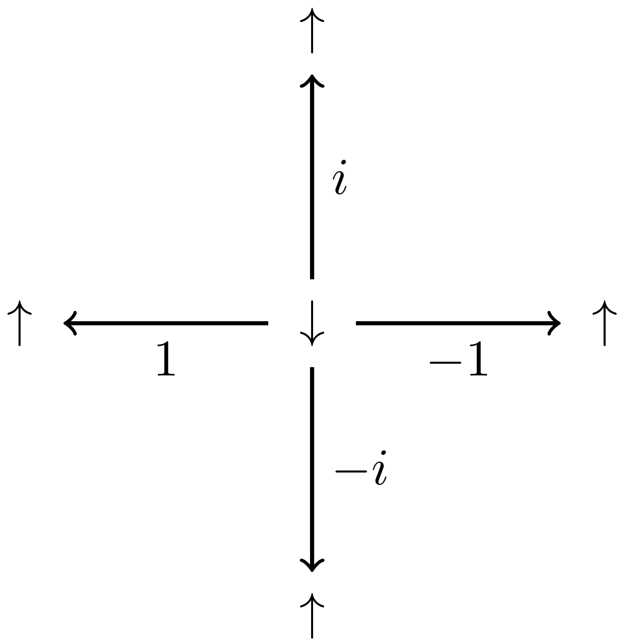
What is Spin-Orbit Coupling?

- The spin magnetic moment couples with the magnetic moment of the orbital motion
- Rashba coupling: $\alpha(\mathbf{k} \times \sigma) \cdot \hat{z}$

$$H_{so} = \sum_i (-\alpha c^\dagger_{i_{x+1}\uparrow} c_{i\downarrow} + \alpha c^\dagger_{i_{x+1}\downarrow} c_{i\uparrow} + \alpha c^\dagger_{i_{x-1}\uparrow} c_{i\downarrow} - \alpha c^\dagger_{i_{x-1}\downarrow} c_{i\uparrow} + i\alpha c^\dagger_{i_{y+1}\uparrow} c_{i\downarrow} + i\alpha c^\dagger_{i_{y+1}\downarrow} c_{i\uparrow} - i\alpha c^\dagger_{i_{y-1}\uparrow} c_{i\downarrow} - i\alpha c^\dagger_{i_{y-1}\downarrow} c_{i\uparrow})$$

What Is Spin-orbit Coupling?

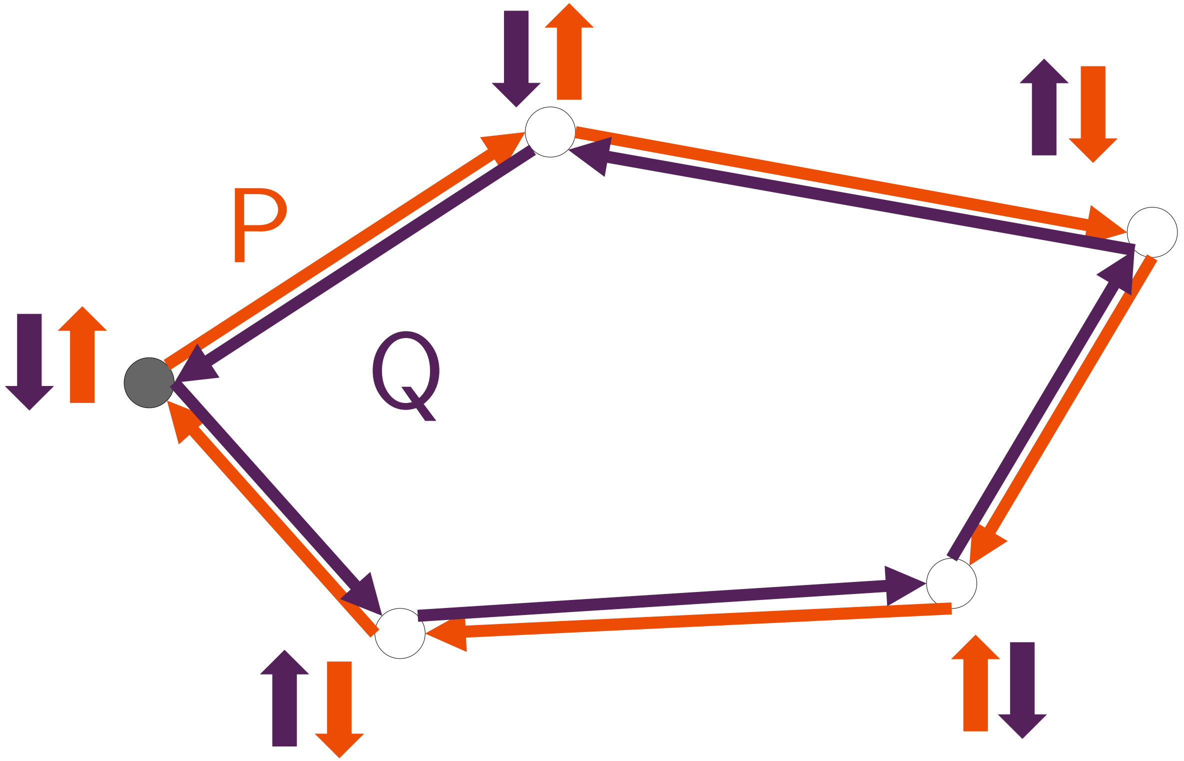




Why Spin-orbit Coupling?

- Destroys the weak localization effect as the time reversed path now has opposite spins.
- We see a phase transition

  in 2D now



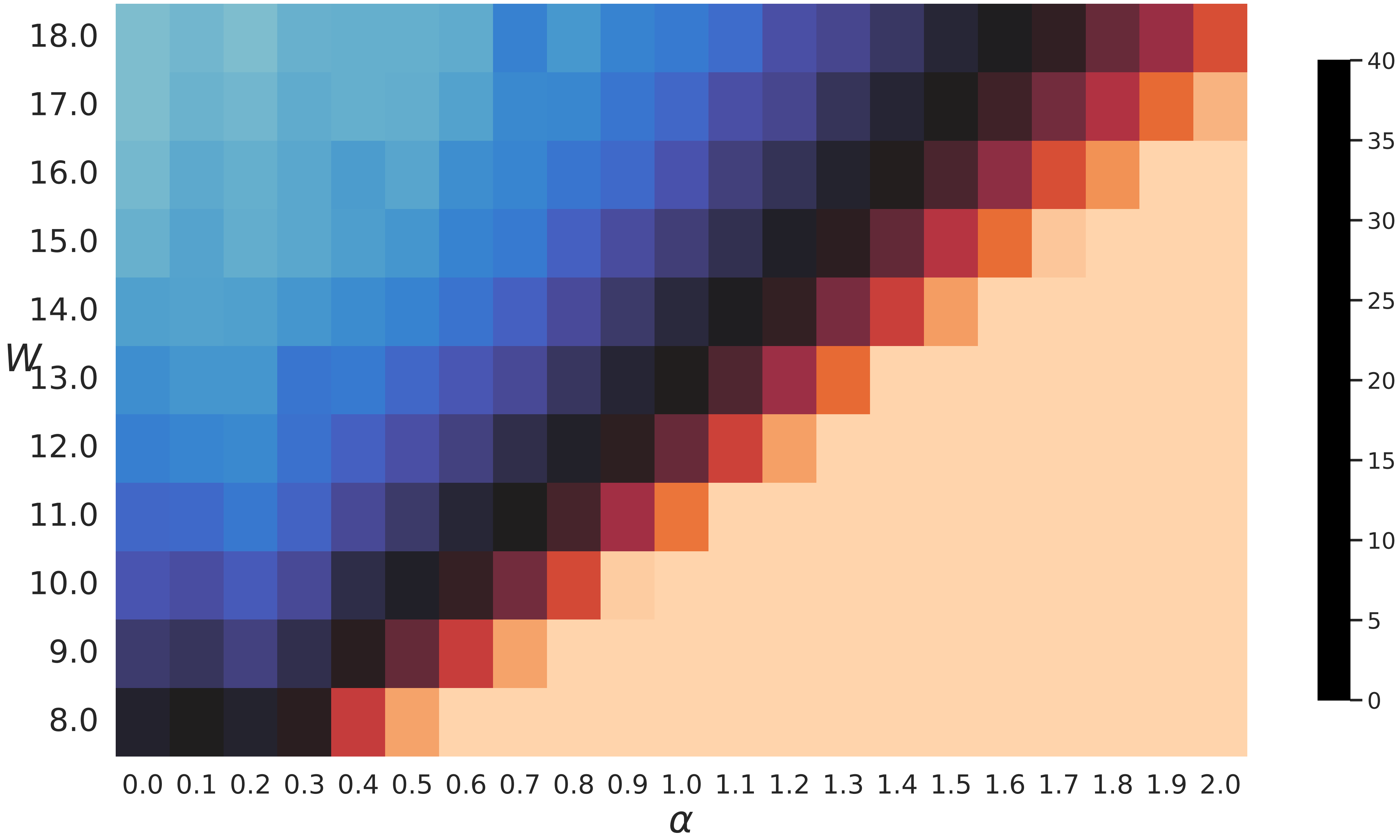
What Is In The Literature?

- (Evangelou, S. N., and T. Ziman, Journal of Physics C: Solid State Physics 20, no. 13 (May 1987): L235–40) showed that the transition exists in 2D if we add spin-orbit coupling.
- The exponent for the phase transition has been calculated to high degrees of accuracy (Asada, Yoichi, Keith Slevin, and Tomi Ohtsuki. Physical Review Letters 89, no. 25, 2002: 256601)
- Calculations are done for some parameters or for some modifications, but full phase space not documented anywhere.

What Did We Want?

- We wanted to ask a question about the information about the initial state that is retained by our system over long times. (More on that later)
- But we need to know what the Phase Space looks like.
- So we calculated that first

What Did We Find?

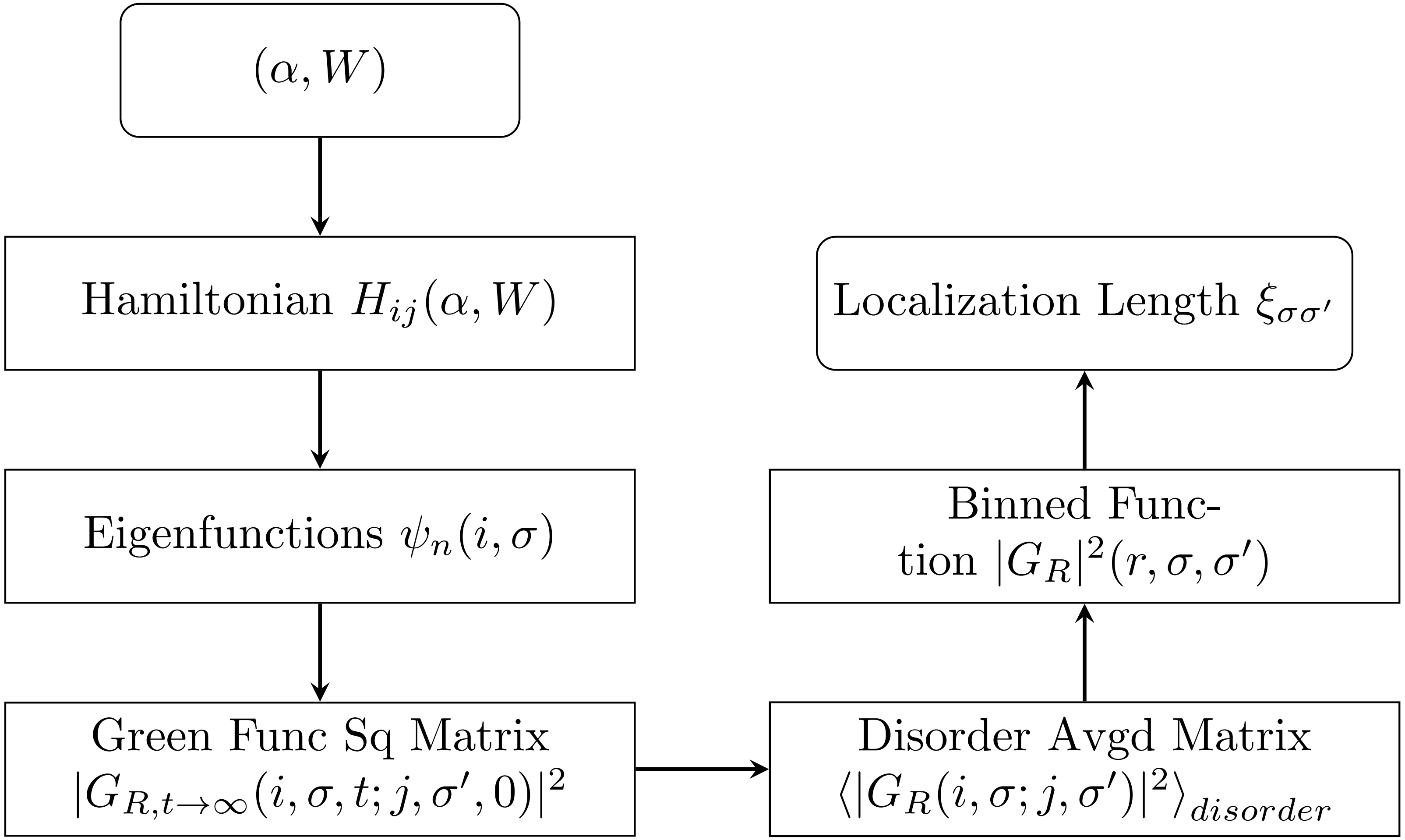


How Did We Find It?

Localization Length can be calculated in several ways:

1. Scaling of IPR ($\sum_{sites\ i} |\psi(i)|^4$) with the system size
2. $\boxed{\text{Green's Function goes as } e^{\frac{-r}{\xi}}\text{. The square of the green's function also has a nice expression for the long time limit.}}$
3. For 1D, it can be calculated using the energy spectrum.

Overview of Method



System

$$H = \sum_i \epsilon_i c_i^\dagger c_i - t \sum_{<ij>} c_i^\dagger c_j + H_{so}$$

$$\begin{aligned} H_{so} = \sum_i (& -\alpha c^\dagger_{i_{x+1}\uparrow} c_{i\downarrow} + \alpha c^\dagger_{i_{x+1}\downarrow} c_{i\uparrow} + \alpha c^\dagger_{i_{x-1}\uparrow} c_{i\downarrow} - \alpha c^\dagger_{i_{x-1}\downarrow} c_{i\uparrow} \\ & + i\alpha c^\dagger_{i_{y+1}\uparrow} c_{i\downarrow} + i\alpha c^\dagger_{i_{y+1}\downarrow} c_{i\uparrow} - i\alpha c^\dagger_{i_{y-1}\uparrow} c_{i\downarrow} - i\alpha c^\dagger_{i_{y-1}\downarrow} c_{i\uparrow}) \end{aligned}$$

Here $\epsilon_i \in [-W/2, W/2]$ are drawn from a uniform distribution. $t = 1$ is the hopping strength and is usually set to unity. α is the spin-orbit coupling strength.

Green's Function

For the long time limit, we get the square of the green's function to be

$$
\begin{aligned}
|G_R|^2(i,\sigma;j,\sigma') = &\sum_n |\psi_n(i,\sigma)^*\psi_n(j,\sigma')|^2 \\
&+ \sum_{E_m=E_n} \psi_n(i,\sigma)^*\psi_n(j,\sigma')\psi_m(j,\sigma')^*\psi_m(i,\sigma)
\end{aligned}
$$

This quantity is not translationaly invariant, we need to average over many disorder realizations in order to get a translationally invariant quantity.

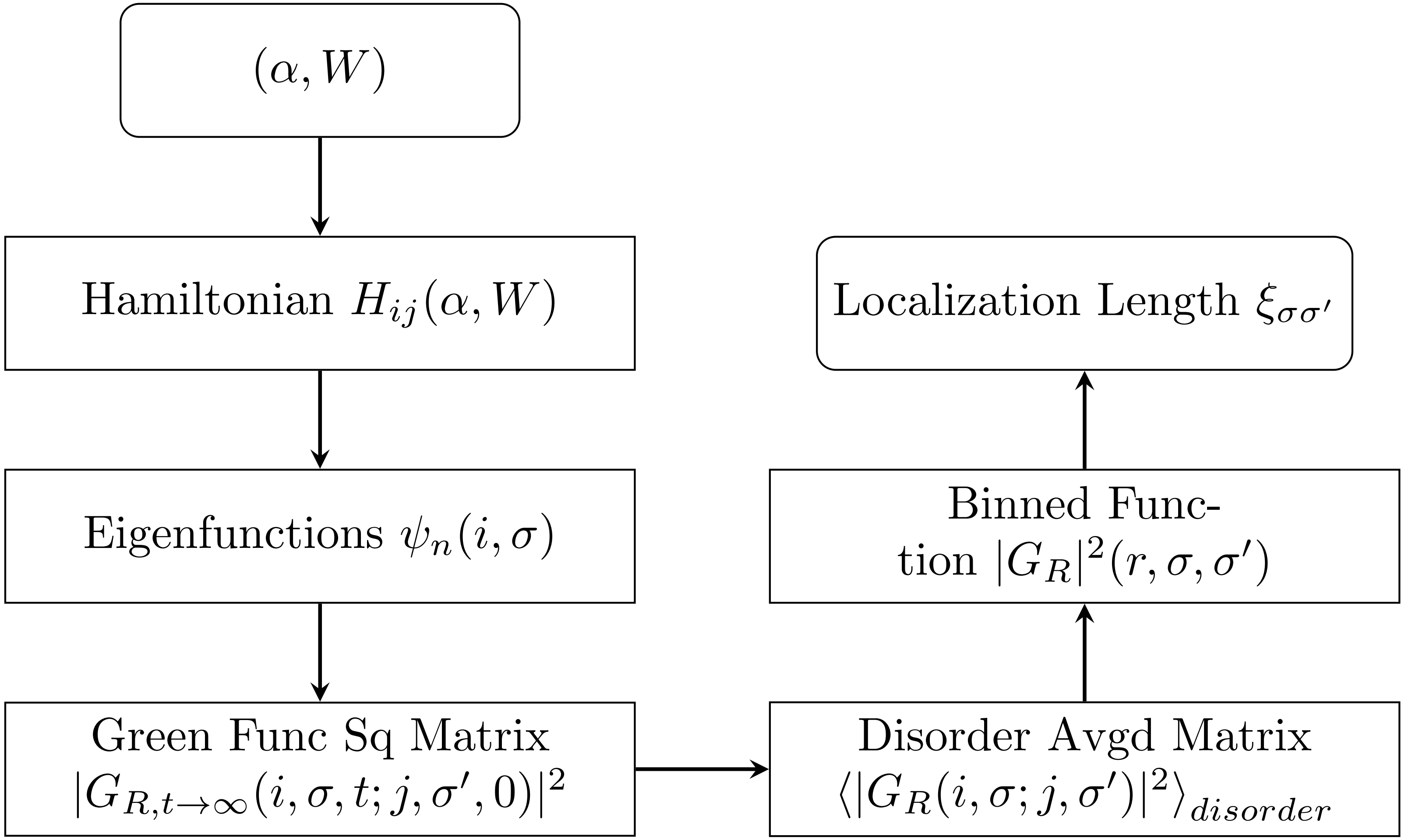
Binning and Fitting

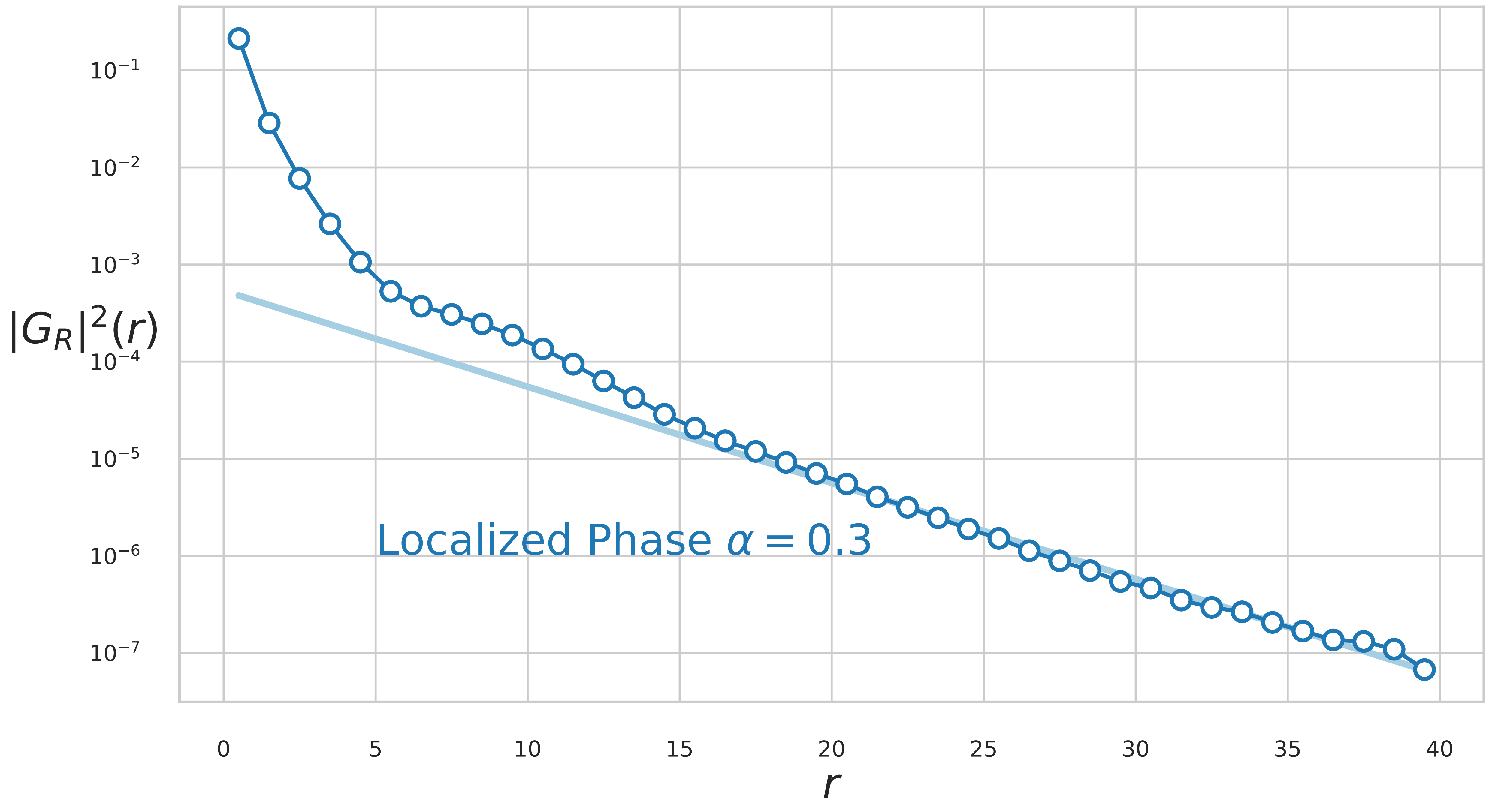
- We construct a curve from the matrix by binning it:

$$|G_R|^2(r,\sigma,\sigma') = \frac{1}{N_r} \sum_{|\mathbf{r_i}-\mathbf{r_j}|\in[r,r+\delta r]} |G_R|^2(i,\sigma;j,\sigma')$$

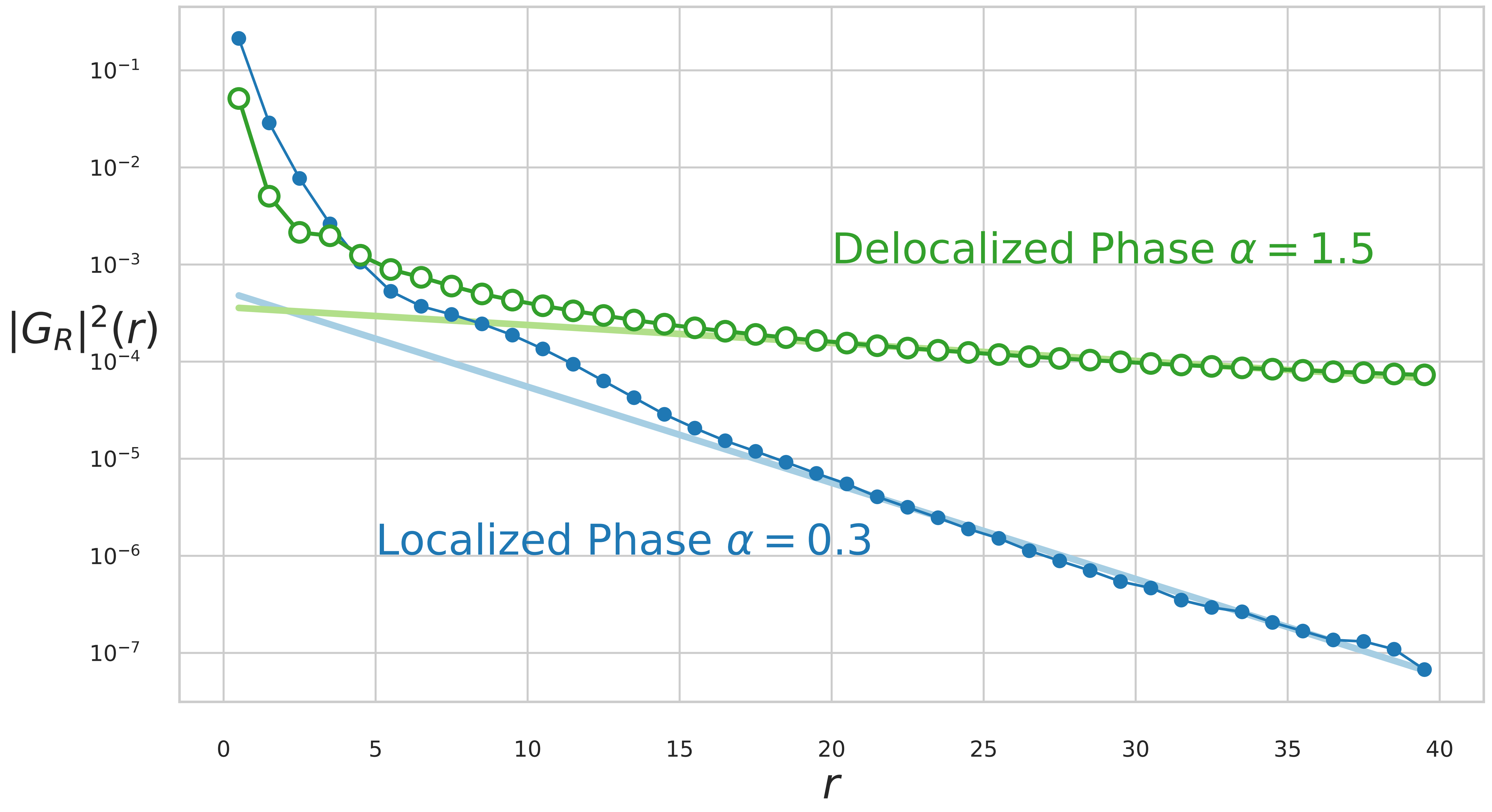
- The curve shows exponential decay of the form $\exp\left(\frac{-2r}{\xi}\right)$ So we fit an exponential to it to get the value of ξ_{loc}

Overview of Method

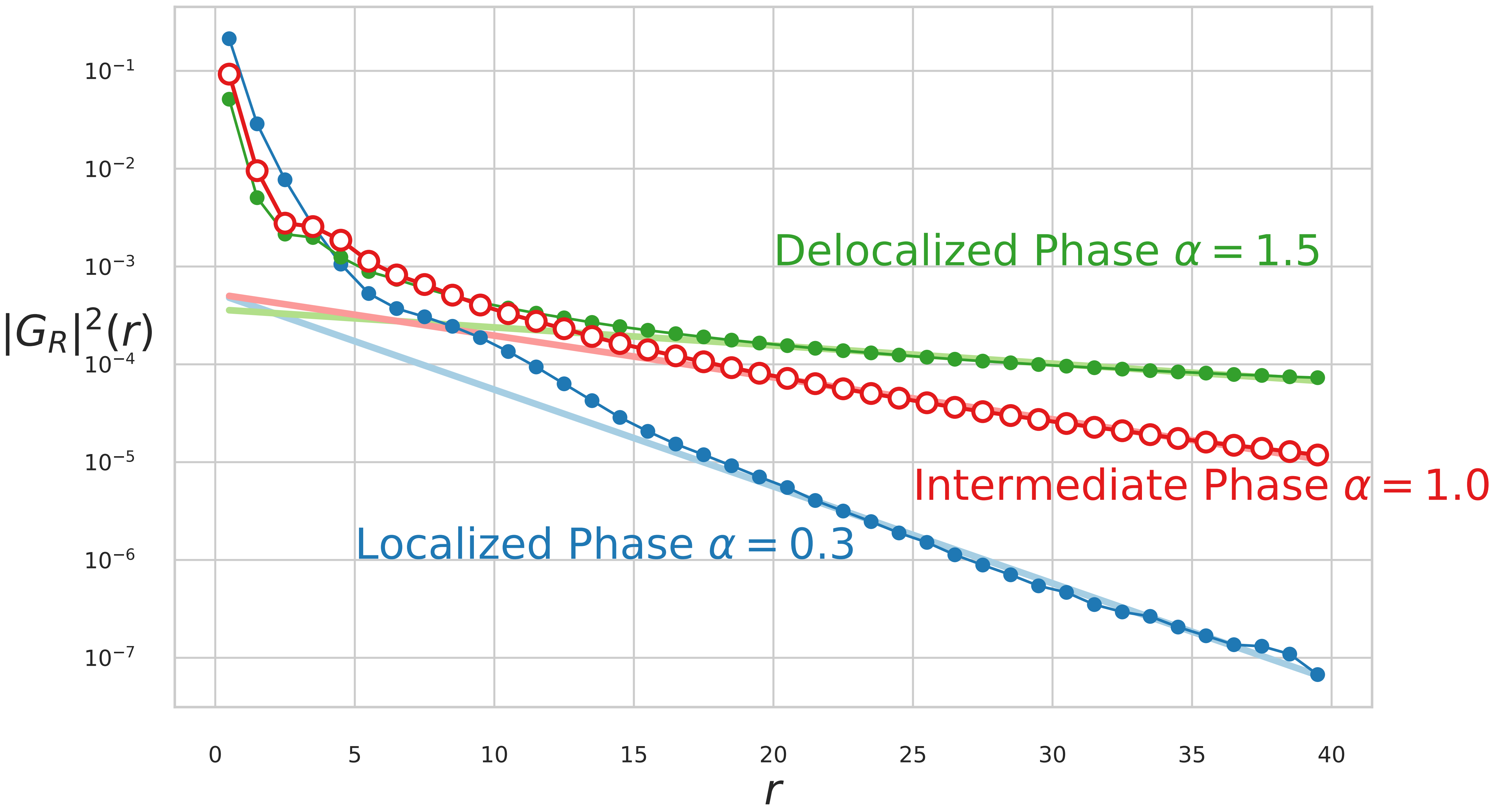




$|G_R|^2(r)$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
Localized Phase $\alpha = 0.3$
0
5
10
15
20
25
30
35
40
r



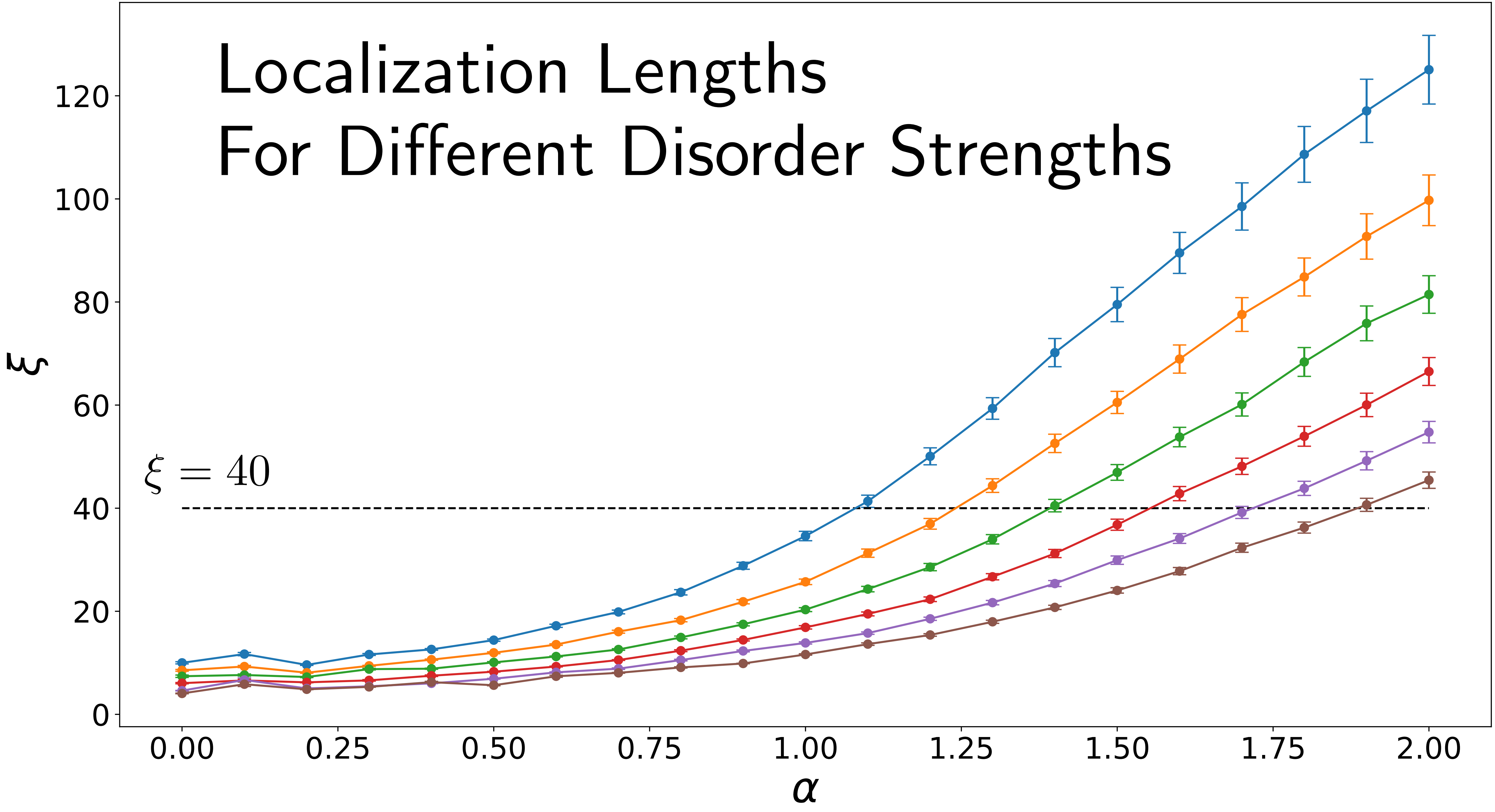
Delocalized Phase $\alpha = 1.5$
Localized Phase $\alpha = 0.3$
$|G_R|^2(r)$
r
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0 5 10 15 20 25 30 35 40



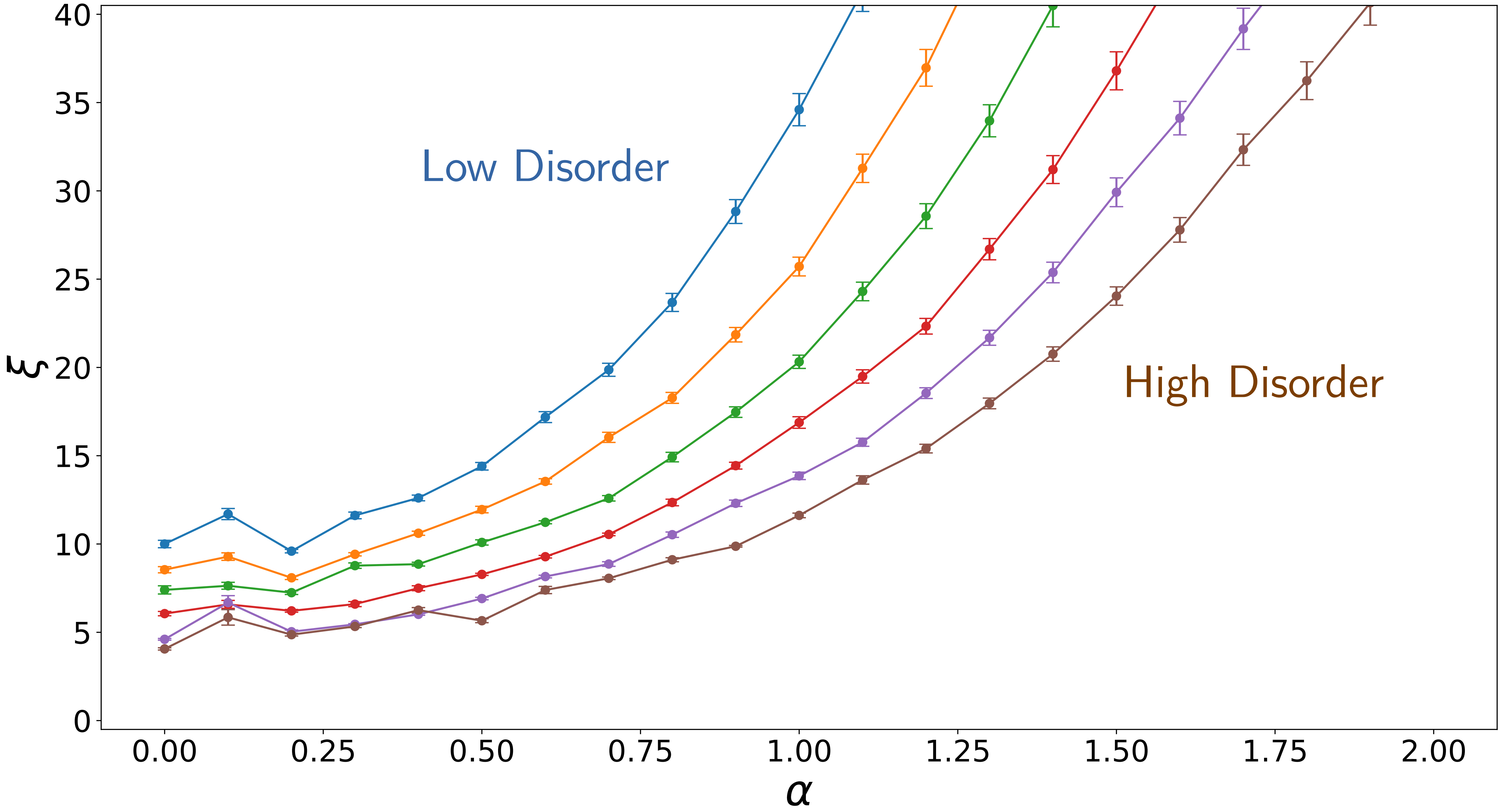
Delocalized Phase $\alpha = 1.5$
Intermediate Phase $\alpha = 1.0$
Localized Phase $\alpha = 0.3$
$|G_R|^2(r)$
r
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
5
10
15
20
25
30
35
40

Finite Size

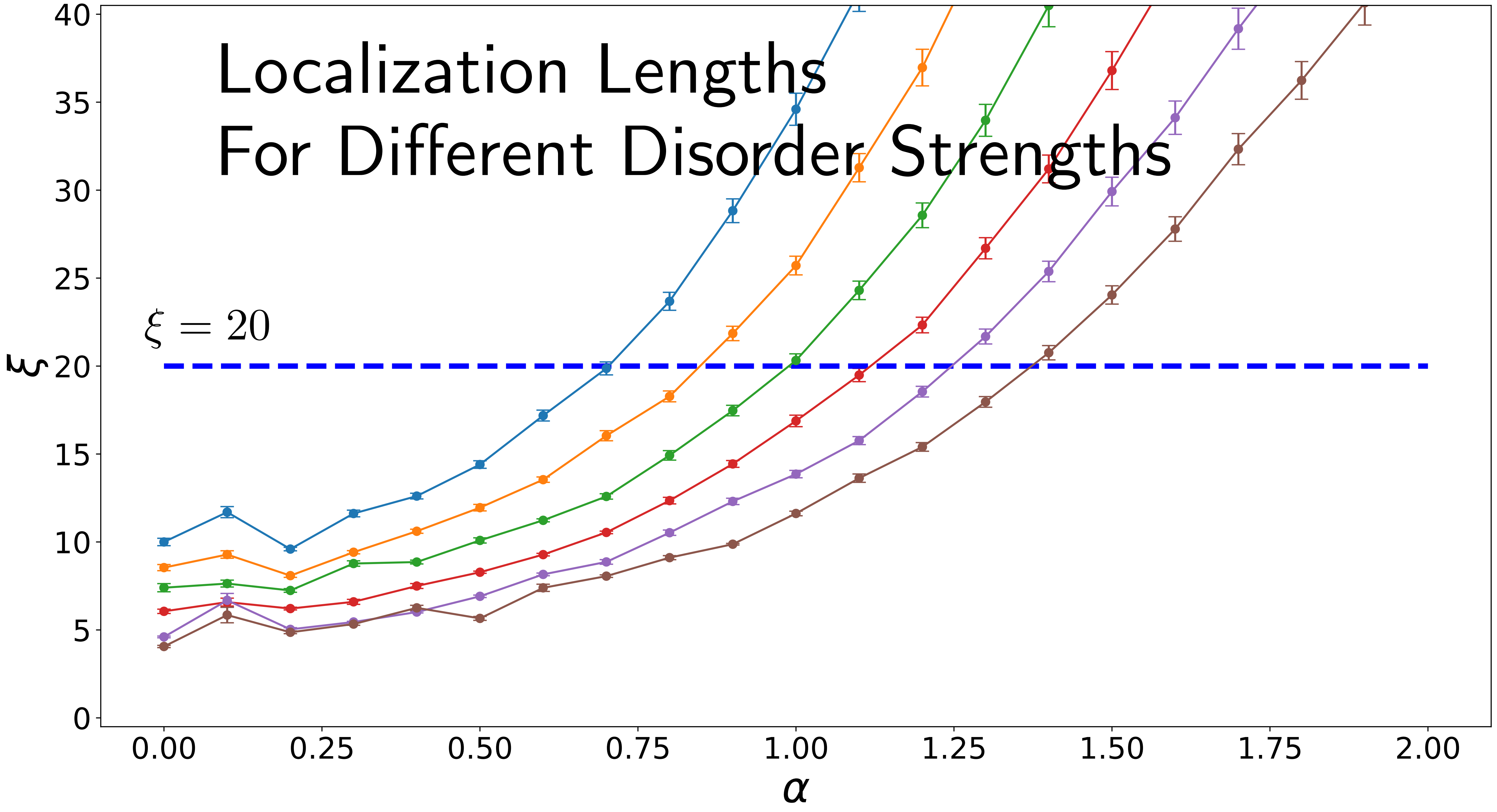
- $\xi_{loc} > L$ makes no physical sense. Such measurements are unreliable.
- We need a good threshold that allows us to draw a line as to what localization lengths we can reliably calculate in our finite system.

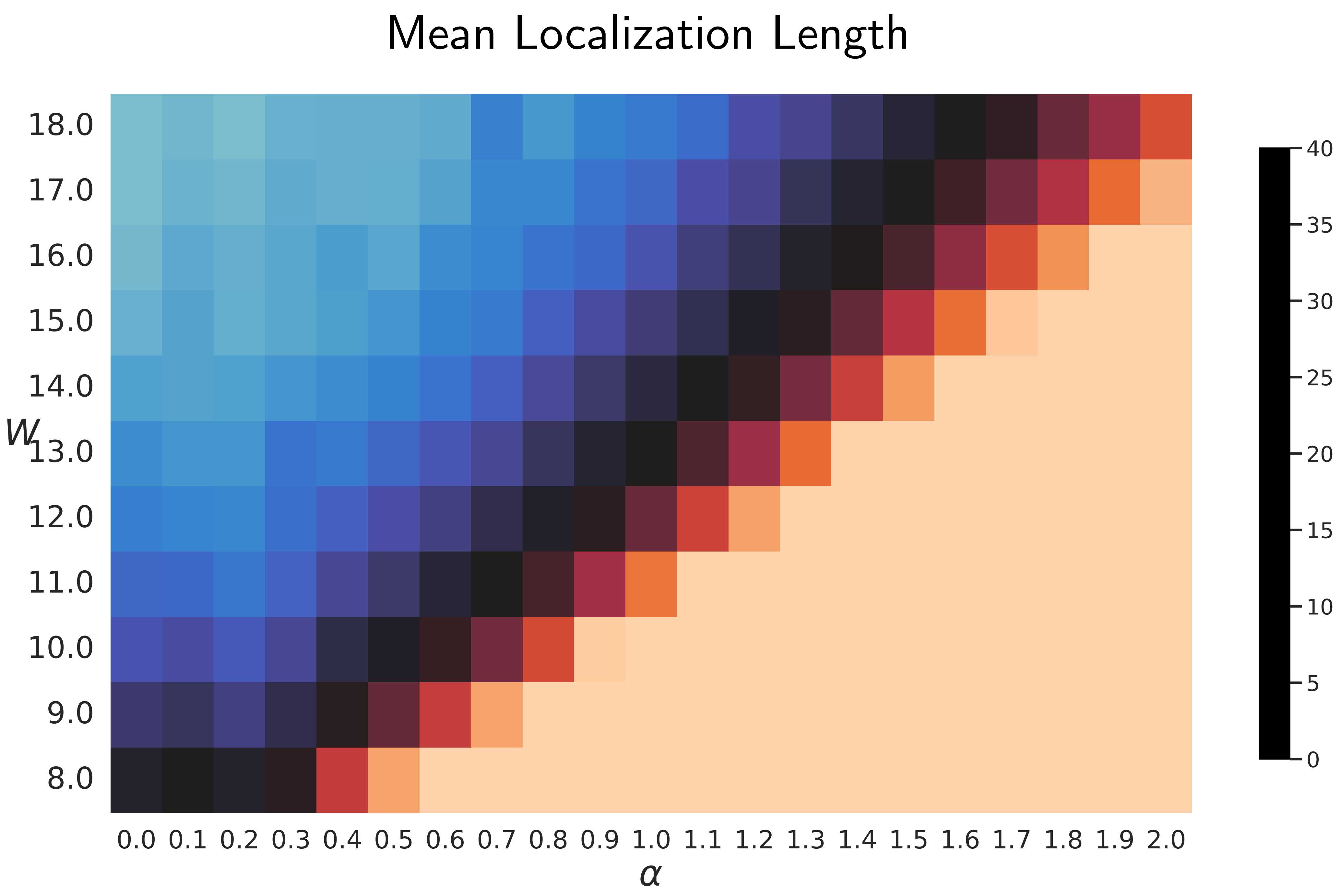


Localization Lengths
For Different Disorder Strengths
$\xi = 40$
ξ
α
0
20
40
60
80
100
120
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

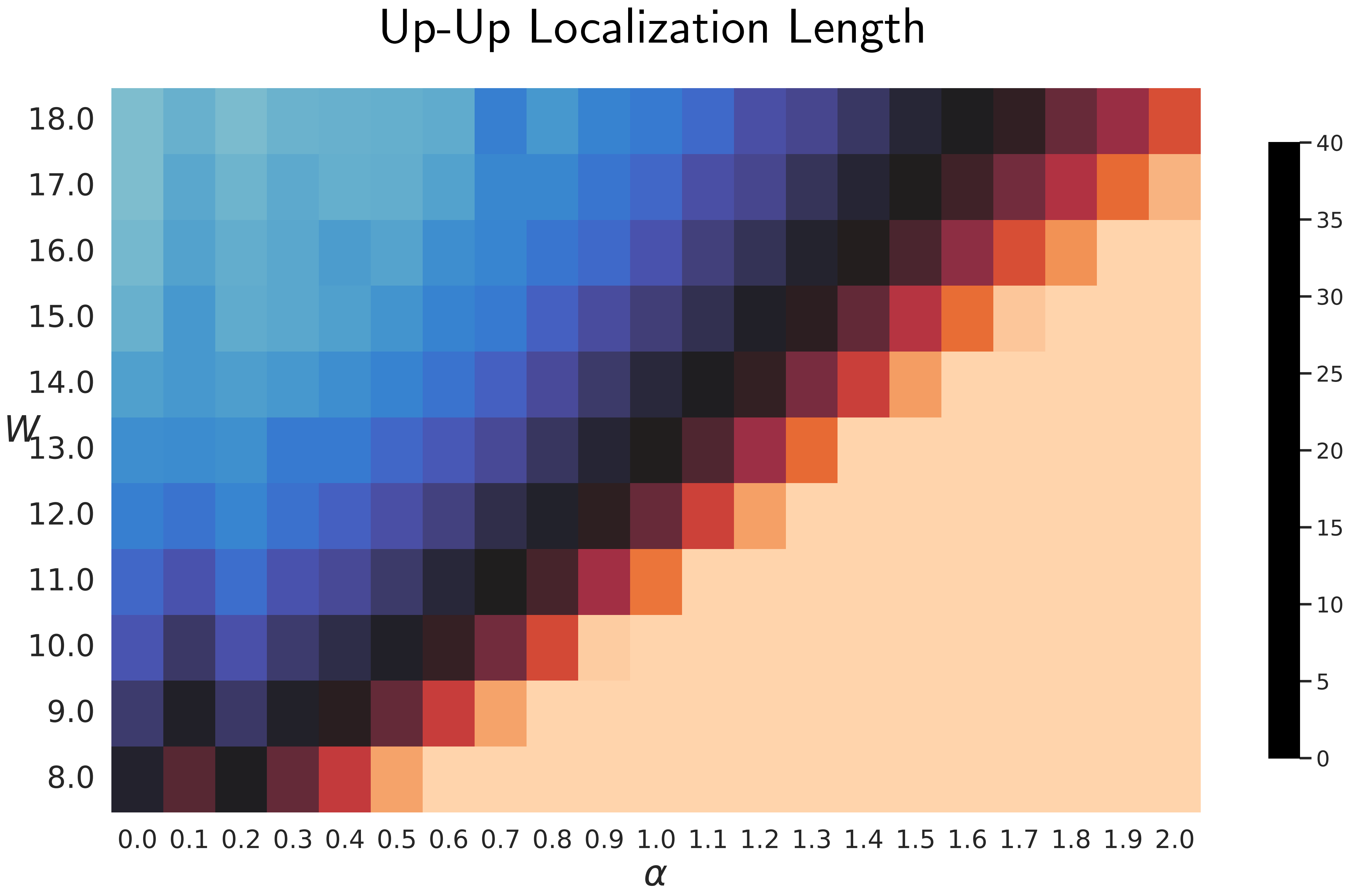


Low Disorder
High Disorder
ξ
α
0
5
10
15
20
25
30
35
40
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

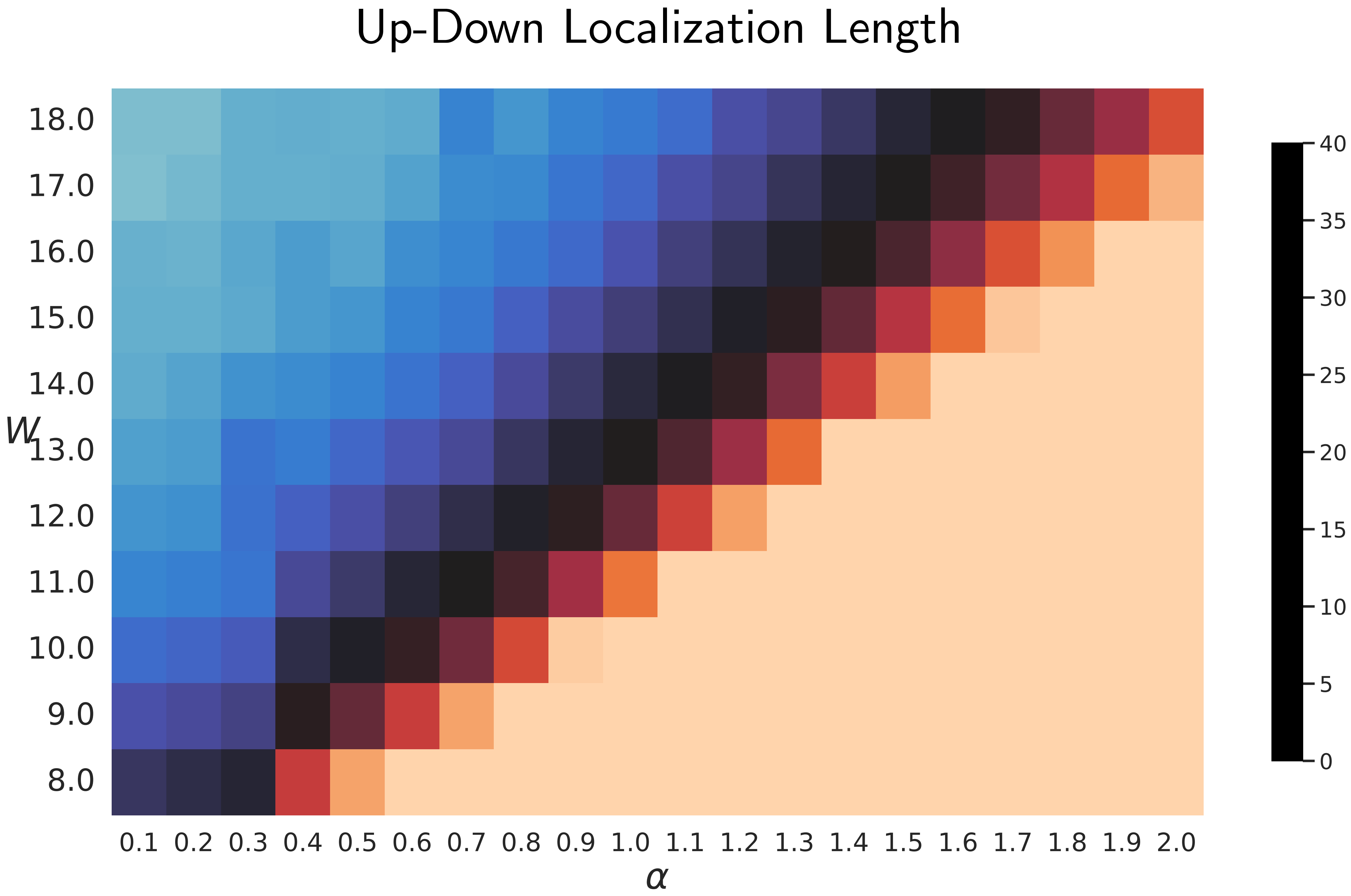

Localization Lengths
For Different Disorder Strengths
$\xi = 20$
ξ
α
0
5
10
15
20
25
30
35
40
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00



Mean Localization Length
W
18.0
17.0
16.0
15.0
14.0
13.0
12.0
11.0
10.0
9.0
8.0
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0
α
40
35
30
25
20
15
10
5
0



Up-Up Localization Length
W
α
18.0
17.0
16.0
15.0
14.0
13.0
12.0
11.0
10.0
9.0
8.0
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0
40
35
30
25
20
15
10
5
0


Up-Down Localization Length
W
18.0
17.0
16.0
15.0
14.0
13.0
12.0
11.0
10.0
9.0
8.0
α
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2.0
40
35
30
25
20
15
10
5
0

Why Did We Find This?

- In disordered systems without spin-orbit coupling, the system retains memory about its initial state.
- Initial state with the sites in set A occupied and set B are unoccupied
- After long times, is particle density on A different from that on B?

Memory of Initial States

- The 2D Anderson model does retain information about its initial states
- Imbalance: $I(t) := \dfrac{\sum_{i\in A} n_i - \sum_{i\in B} n_i}{\sum_{i\in A} n_i + \sum_{i\in B} n_i}$

$$\langle I(\infty)\rangle = \frac{2}{N_s}\sum_i \sum_{k\in A} o_i \exp(\frac{-2|r_i - r_j|}{\xi})$$

Ref: Chakraborty, Ahana, Pranay Gorantla, and Rajdeep Sensarma. Physical Review B 102, no. 22 (December 21, 2020): 224306.

Memory of Initial States

- To study imbalance in our system, we need to know how the localization length over the phase space since the imbalance depends on it strongly.
- Hence we calculated the phase space.

Imbalance in Spin Systems

- For spin systems, we have two densities involved: charge/particle density and spin density
- These are linked by the spin-orbit coupling
- Dynamics could transform imbalance in one to imbalance in the other

Next Steps?

- Imbalance for different initial states. Is information about initial states retained with spin?
- Can certain spin patterns/charge patterns show interesting behaviour in terms of localization and imbalance?
- Energy dependence and mobility edge

Appendix