Edit or run this notebook

Table of Contents

```
GRBackend()
```

Exponential from discrete p

Let us say that you lend P_0 to someone at time zero and you expect to receive payment in return at time 1.

You charge 100% for each complete time period (a high interest rate!, but which facilitates comparisons). If the loan is returned before the complete time period, you obtain a proportional interest. At a payment date, you expect to receive the value deposited plus the interest obtained during the period.

How to calculate the value to be received?

You can simply say that your loan of P_0 will return $(1+1)P_0 = 2P_0$ at $t=1$.

However, if the borrower wants to repay what is due in the middle of the period, at $t=0.5$, then the payment would be $(1+1/2)P_0 = 1.5P_0$. The interest is calculated proportional to the period that P_0 was held. The interest due is only $(1+1/2)$ because the borrower rented the principal for only half of the period.

You could then lend the money to someone else. This other person would receive $(1+1/2)P_0$ and repay $(1+1/2) \times (1+1/2)P_0 = (1+1/2)^2 P_0$ at $t=1$.

But then we have $(1+1/2)^2 P_0 \neq (1+1)P_0$ as $(1+1/2)^2 = 2.25 > 2 = (1+1)$. Lending to someone, having the payment back in the middle of the period, and making another loan is better than lending and having the total payment back only in the end of the period.

No one would like to lend expecting to receive payment only in the end of the period. Lenders would stimulate borrowers to repay earlier so that they could lend to someone else to another shorter period. As it is not good business to lend to long time periods, the borrowers would not find lenders willing to lend to long time periods. In practie, no one would like to lend to any time period.

In order to solve the problem, the calculation of interest has to be made to compensate lenders. The value to be paid in each period should be calculated for small intervals of time. In this way, the borrower would a make a payment equal to the value that the lender would receive by lending to someone else. The lender would be indifferent between waiting until the end of the period and receiving payment for a short time period and lending to someone else.

Let us say that the period is one year and that the interest is calculated for each business day. One year is divided into 252 business days. So, if you lend P_0 at time $t=0$, the debt of the borrower at the next time interval will be $P_0(1+\frac{1}{252})$.

We divided the time interval from 0 to 1 into separate time intervals with [illegible] have transformed the interval $[0, 1]$ into 252 discrete time intervals.

In general, we can divide the interval $[0, 1]$ into n segments. For $n = 1$, we have only one segment, given by $[0, 1]$ itself. For $n = 2$, we have two segments given by $[0, 1/2]$ and $[1/2, 1]$ and so on.

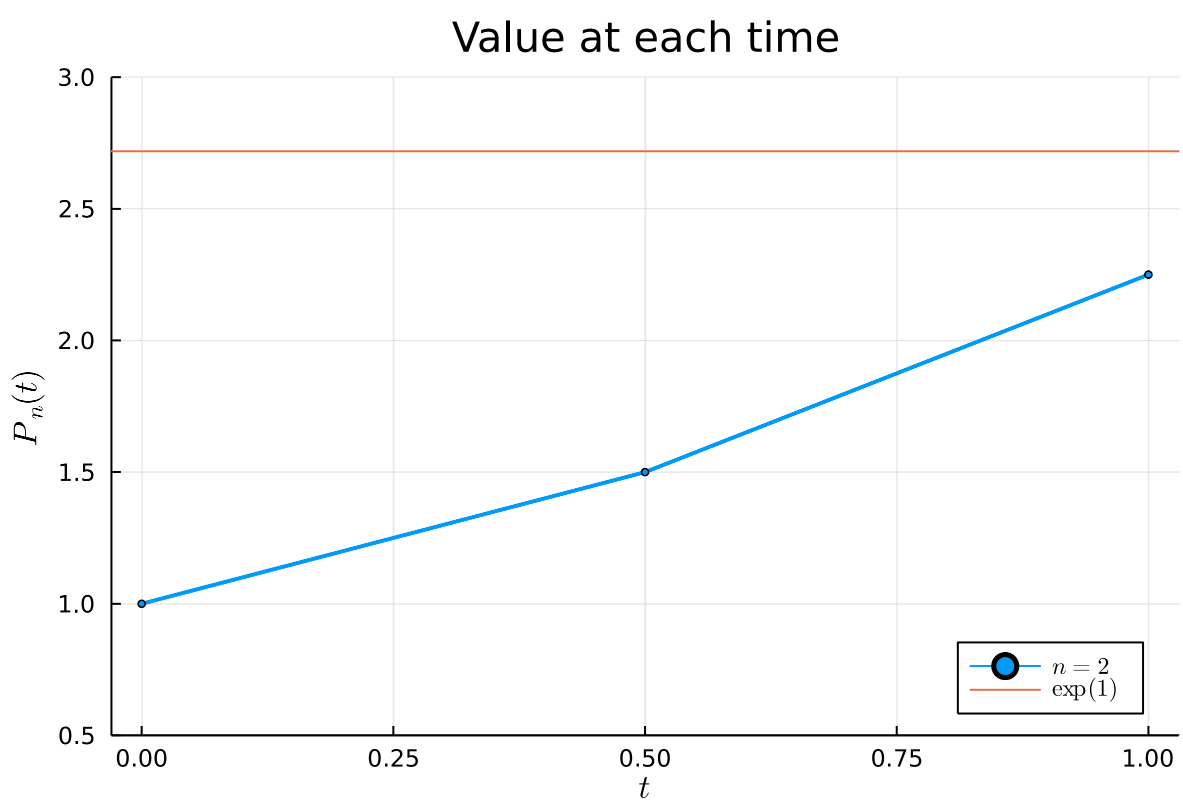
Let us calculate the value to be paid at the end of these discrete time intervals. Let us call $P_n(t)$ the amount owned at $t = 0, 1/n, 2/n, \ldots 1$ if the time $[0, 1]$ is divided into n segments. We have to calculate $P_n(0), P_n(1/n), P_n(2/n), \ldots, P_n(1)$.

The value $P_n(1)$ is especially interesting. This is the value that the borrower has to repay if the debt is paid at once at time 1.

Making the calculations

Move the slider to change the number of segments in $[0, 1]$.

n = 2



As the number of segments increases, the value of $P_n(1)$ approaches $e =$

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$P_n(1) \to e$ as $n \to \infty$. When $n \to \infty$, we are in reality in continuou divided into infinitely many points. We say that interest rates are compounded continuously in this case.

```
Ptn (generic function with 1 method)
```

```julia
function Ptn(n)

    xx = 0:1/n:1

    P0 = 1

    nxx = 0:(length(xx)-1)

    yy = P0*(1+1/n).^nxx

    p = plot(xx,yy,title="Value at each time",marker = 2,lw=2,ylim=(0.5,3),xlabel=L"t",ylabel=L"P_n(t)",legend=:bottomright,label=L"\textit{n = %$n}")

    plot!([exp(1)], seriestype = :hline,label=L"\mathrm{exp(1)}")

end
```

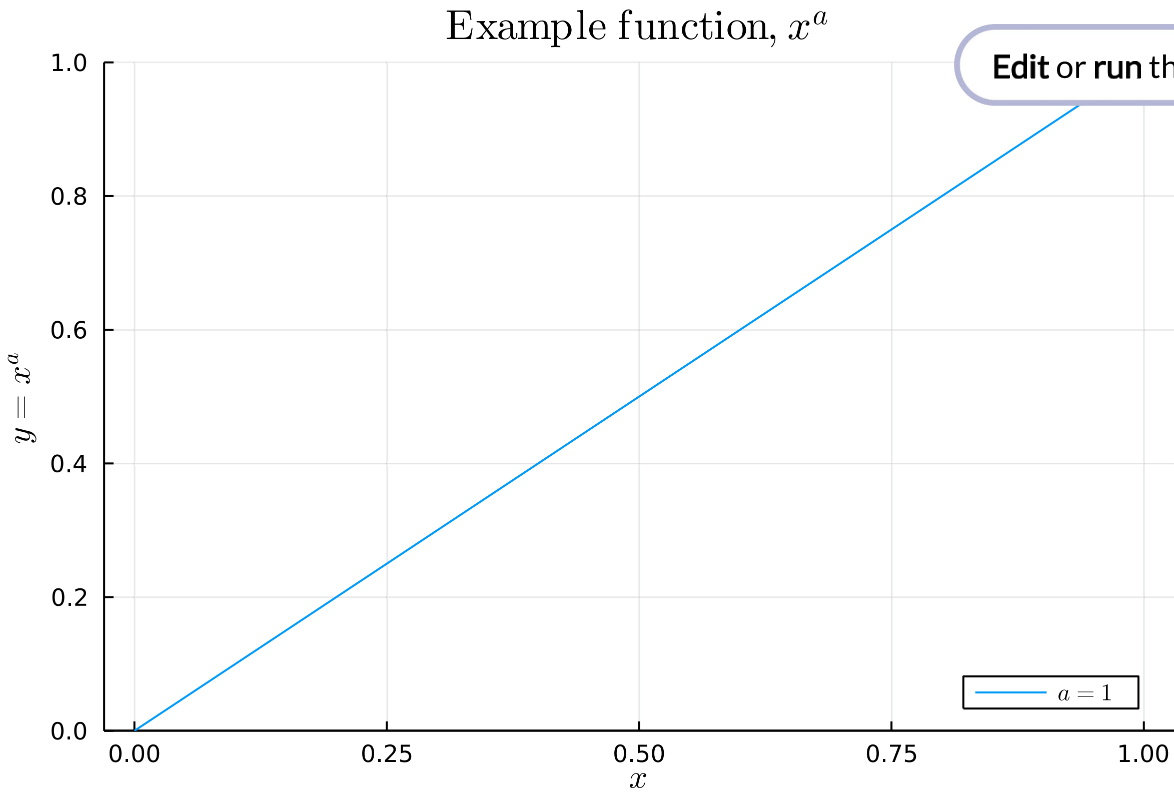
Example function

This is just another example to practice the use of functions and plots.

a = 1

```julia
md"""
a = $(@bind power_a Slider(0:0:0.1:3, show_value=true, default=1))
"""
```

Edit or run this notebook




```
begin
    a = power_a
    explot1(a)
end
```

explot1 (generic function with 1 method)

```
function explot1(a)

    xx = 0:0.01:1
    yy = zeros(1,length(xx))

    yy = xx.^a

    p = plot(xx,yy,ylim=(0,1),title=L"\textrm{Example\ function},
    x^a",label=L"\textit{a = %$a}",legend=:bottomright,xlabel=L"x",ylabel=L"y=x^a")

end
```