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Refined composite multiscale weighted-permutation entropy of financial time series

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Highlights

- We propose the refined composite multiscale weighted-permutation entropy (RCMWPE) as a modification of multiscale weighted-permutation entropy (MWPE).
- We detect the temporal structures and investigate complexity of financial time series.
- RCMWPE can distinguish the differences between the Asian stock markets and European stock markets clearly.

Refined composite multiscale weighted-permutation entropy of financial time series

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Abstract

For quantifying the complexity of nonlinear systems, multiscale weighted-permutation entropy (MWPE) has recently been proposed. MWPE has incorporated amplitude information and been applied to account for the multiple inherent dynamics of time series. However, MWPE may be unreliable, because its estimated values show large fluctuation for slight variation of the data locations, and a significant distinction only for the different length of time series. Therefore, we propose the refined composite multiscale weighted-permutation entropy (RCMWPE). By comparing the RCMWPE results with other methods' results on both synthetic data and financial time series, RCMWPE method shows not only the advantages inherited from MWPE but also lower sensitivity to the data locations, more stable and much less dependent on the length of time series. Moreover, we present and discuss the results of RCMWPE method on the daily price return series from Asian and European stock markets. There are significant differences between Asian markets and European markets, and the entropy values of Hang Seng Index (HSI) are close to but higher than those of European markets. The reliability of the proposed RCMWPE method has been supported by simulations on generated and real data. It could be applied to a variety of fields to quantify the complexity of the systems over multiple scales more accurately.

Keywords: Refined composite multiscale weighted-permutation entropy (RCMWPE), Complexity, Financial time series

1. Introduction

Quantifying the complexity of nonlinear dynamical systems has recently gained increasing attention. Various information-theoretic methods for measuring the complexity have been proposed, such as various entropies [1–9], fractal dimensions [10], detrended fluctuation analysis [11–13], etc. One of the most reliable and conceptually simple tools is permutation entropy (PE) [2], which has been widely applied in many fields [14–21]. However, the main defect in the PE is in the fact that no other information besides the relative order structure is extracted. Then, a modication called weighted-permutation entropy (WPE) [22] has been proposed, which contains amplitude information from the relative order structure. More narrowly, WPE is able to distinguish between distinct fluctuations and small fluctuations of time series. In addition, WPE has better robustness and stability in the presence of higher levels of noise [22].

Time series derived from financial systems and physiology systems show structure on multiple temporal scales [4, 23, 24]. Nevertheless, WPE is based on single scale and does not take

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into account the complex temporal fluctuations inherent in these systems. Therefore, so called multiscale weighted-permutation entropy (MWPE) has been proposed [23, 25], which is based on multiscale analysis for exploring the multiple inherent dynamics of time series. However, some drawbacks of MWPE have been revealed. The estimated entropy values show large fluctuation for slight variation of the data locations, and a significant distinction only for the different length of time series, which may result in inaccurate estimates. The drawbacks may lead to MWPE not being applied widely. In this paper, we propose the refined composite multiscale weighted-permutation entropy (RCMWPE) to quantify the complexity of nonlinear time series, which can effectively overcome the shortcomings of MWPE and obtain more reliable estimates.

The aim of this study is twofold. The first and foremost aim is to introduce RCMWPE method for improving MWPE, and shows its power in complexity analysis. The second aim is to detect the inherent properties of financial time series. For the purposes, we extend MWPE method to construct RCMWPE algorithm by information-theoretic methods. We apply the method to analyze Gaussian white noise, binomial multifractal series and financial time series. In addition, we show its advantages by comparing with several current methods. The results of RCMWPE method on the daily price returns of stock also are discussed to reveal the inherent dynamics and reflect business behavior, systems mechanism and the influence of the external environment factors in financial markets.

The remainder of this paper is organized as follows. In the following section, we briefly introduce the methodologies of PE, WPE, MWPE and RCMWPE. For comparison, composite multiscale weighted-permutation entropy (CMWPE) and refined composite multiscale permutation entropy (RCMPE) [26] are also described briefly. In Sections 3 and 4, we present the empirical results obtained for synthetic data and stock markets indexes under MWPE, CMWPE, RCMWPE and RCMPE. The conclusions are shown at Section 5.

2. Methodologies

2.1. Multiscale weighted-permutation entropy

Permutation entropy (PE) was originally introduced by Bandt and Pompe [2] for measuring complexity of nonlinear time series. Considering a time series $\{x_i\}_{i=1}^N$, we focus on its subseries $X_j^m = \{x_j, x_{j+1}, \ldots, x_{j+m-1}\}$ with embedding dimension m. Each vector X_j^m is sorted in ascending order with permutation patterns π_i^m, and it is clear that π_i^m have $m!$ possible values. The symbol procedure is using patterns π_i^m to replace vector X_j^m for determining the relative frequency. More precisely as follows ($\sharp$ represent number)

$$p(\pi_i^m) = \frac{\sharp\{j|j = 1, ..., N-m+1;\ X_j^m\ has\ type\ \pi_i^m\}}{N-m+1} \tag{1}$$

PE is then defined as the Shannon entropy of the $m!$ distinguishable symbols $\{\pi_i^m\}_{i=1}^{m!}$:

$$H(x,m) = -\sum_{i:\pi_i^m \in \Pi} p(\pi_i^m) lnp(\pi_i^m) \tag{2}$$

where Π denotes $\{\pi_i^m\}_{i=1}^{m!}$. The constraint $N \gg m!$ must be satisfied in order to gain reliable statistics. For practical purpose, Bandt and Pompe recommend $m = 3, ..., 7$. PE assumes entropy value in the range [0, $ln(m!)$], and a completely increasing or decreasing series has a lowest value of 0. It can be proved that PE is invariant under nonlinear monotonic transformations. According to the definition, PE only considers the ordinal pattern and neglects the amplitude difference between the ordinal patterns. As a modification of PE, the weighted-permutation entropy (WPE) was been proposed [22]. In WPE method, each vector X_j^m is weighted with weight value w_j. Here, each weight value is the variance of each vector X_j^m. More precisely as follows:

$$w_j = \frac{1}{m}\sum_{t=1}^{m}[x_{j+t-1} - \bar{X}_j^m]^2 \tag{3}$$

where $\bar{X}_j^m$ denotes the arithmetic mean of X_j^m,

$$\bar{X}_j^m = \frac{1}{m}\sum_{l=1}^{m} x_{j+l-1} \tag{4}$$

Then, the weighted relative frequencies are calculated as follows:

$$p_w(\pi_i^m) = \frac{\sum_{j:X_j^m \ has\ type\ \pi_i^m} w_j}{\sum_{j \leq N-m+1} w_j} \tag{5}$$

Then, WPE is computed as

$$H_w(x, m) = -\sum_{i:\pi_i^m \in \Pi} p_w(\pi_i^m) ln p_w(\pi_i^m) \tag{6}$$

In real world, many complex systems show structure on multiple spatiotemporal scales, such as financial systems and physiology systems. If only considering on single scale analysis, WPE measures may be inaccurate estimates or insufficient descriptions of systems when analyzing these systems. Then, WPE has been extended to multiscale analysis called multiscale weighted-permutation entropy (MWPE) [25]. The algorithm consists of two procedures. Firstly, we construct coarse-gained series from original series $\{x_i\}_{i=1}^{N}$ with time series factor τ. More precisely, the k-th coarse-grained time series $y_k^{(\tau)} = \{y_{k,1}^{(\tau)}, y_{k,2}^{(\tau)}, ..., y_{k,p}^{(\tau)}\}$ in time scale τ is defined as follows:

$$y_{k,j}^{(\tau)} = \frac{1}{\tau}\sum_{i=(j-1)\tau+k}^{j\tau+k-1} x_i. \quad 1 \leqslant j \leqslant \frac{N}{\tau}, 1 \leqslant k \leqslant \tau \tag{7}$$

For each given τ, the original series is divided into non-overlapping segments of length τ, and data points in each segment are calculated to the arithmetic mean as representation of this segment. Secondly, we calculate WPE, which replaces original time series x with coarse-grained sequence y. For time scale τ, MWPE is defined as

$$H_w(y_k^{(\tau)}, m) = -\sum_{i:\pi_i^m \in \Pi} p_w^{(\tau,k)}(\pi_i^m) ln p_w^{(\tau,k)}(\pi_i^m) \tag{8}$$

In previous WPE method, the first coarse-grained series (that is $k = 1$) was used. For MWPE method, the constraint $N/\tau \gg m!$ must be satisfied in order to gain reliable statistics.

2.2. Refined composite multiscale weighted-permutation entropy

In the subsection, we perform MWPE analysis on Gaussian white noise. As shown in Table 1, the constrain $N/\tau \gg m!$ is satisfied. However, the estimated values of MWPE show larger standard deviations, which may lead to inaccurate measures in real data. In next section, we also will perform the MWPE method on another synthetic data to show serious sensitivity to the data locations.

Table 1 gives standard deviations of MWPE, CMWPE and RCMWPE for 100 realizations of Gaussian white noise at $\tau = 20$ and $m = 3$.

method	sample number				
	500	1000	3000	5000	10000
MWPE	0.0919	0.0558	0.0194	0.0107	0.0052
CMWPE	0.0357	0.0224	0.0074	0.0036	0.0020
RCMWPE	0.0327	0.0161	0.0052	0.0024	0.0015

Focusing on the larger fluctuation of estimated values, the most obvious solution is computing the mean of the τ entropy values of τ coarse-grained series (see Eq.(7)) at a certain scale factor τ. Based on this thinking and inspired by Wu et al. [1], composite multiscale weighted-permutation entropy (CMWPE) can be proposed. Then, we define CMWPE as follows:

$$CMWPE(x, m, \tau) = \frac{1}{\tau}\sum_{k=1}^{\tau} H_w(y_k^{(\tau)}, m) = -\frac{1}{\tau}\sum_{k=1}^{\tau}\sum_{i:\pi_i^m \in \Pi} p_w^{(\tau,k)}(\pi_i^m) ln p_w^{(\tau,k)}(\pi_i^m) \tag{9}$$

As shown in Table 1, estimated values of CMWPE are more stable. However, CMWPE and MWPE cannot resolve the decrease of estimated values as the length of time series decreases. As shown in Fig.1, CMWPE and MWPE show more dependent on the length of series. At large time scales especially when $N/\tau \gg m!$ does not been satisfied, the drawback is more obvious.

It is well know that the inherent property of Gaussian white noise is invariant, no matter long or short series. For any time scale, a reliable analysis method should not show significantly variant results with the changes only on the length of time series. For resolving the drawback, we propose RCMWPE method. Inspired by Wu et al. [6] and Humeau-Heurtier et al. [26], it is defined as

$$RCMWPE(x, m, \tau) = -\sum_{i:\pi_i^m \in \Pi} \bar{p}_w^{(\tau)}(\pi_i^m) ln \bar{p}_w^{(\tau)}(\pi_i^m) \tag{10}$$

where $\bar{p}_w^{(\tau)}(\pi_i^m)$ is the arithmetic mean of $\{p_w^{(\tau,k)}(\pi_i^m)\}_{k=1}^{\tau}$:

$$\bar{p}_w^{(\tau)}(\pi_i^m) = \frac{1}{\tau}\sum_{k=1}^{\tau} p_w^{(\tau,k)}(\pi_i^m) \tag{11}$$

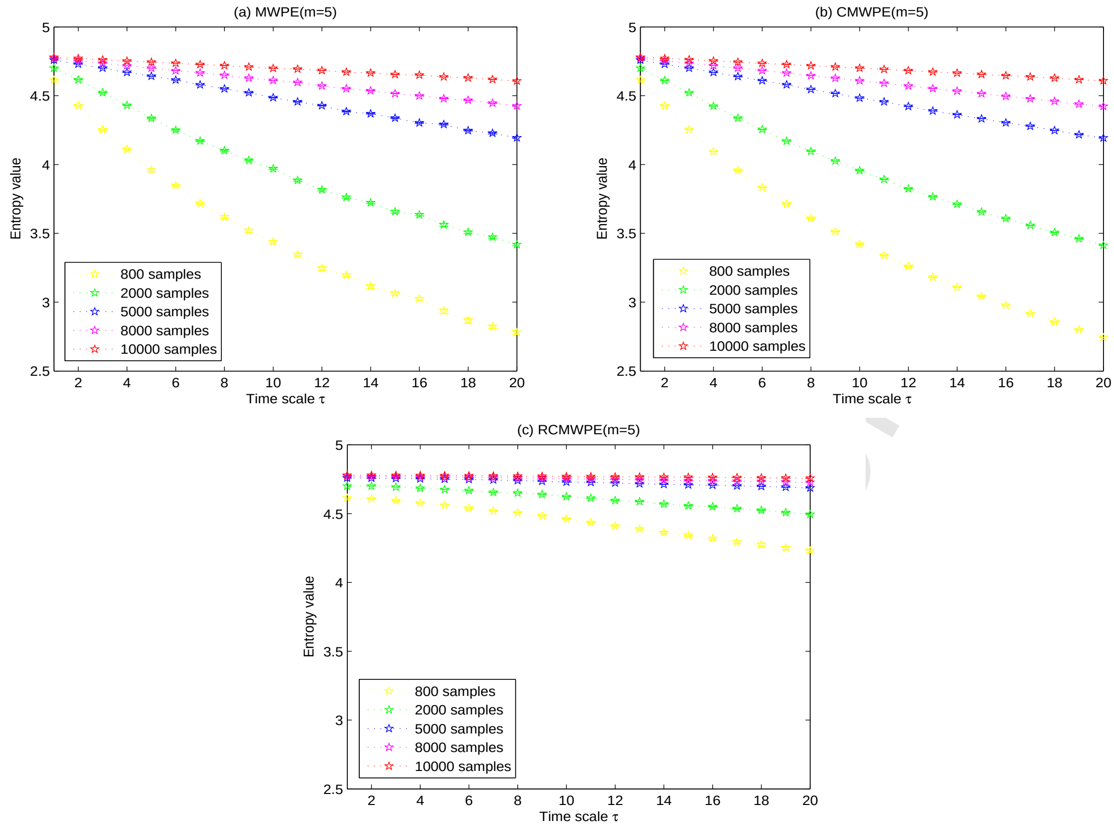

Fig.1. MWPE, CMWPE and RCMWPE values of Gaussian white noise for different lengths are shown, where embedding dimension m=5, and each dot is an average over 100 realizations. RCMWPE shows much less dependent on the length of series compared with MWPE and CMWPE.

In Table 1, the standard deviations of RCMWPE are lowest, and showing significantly lower standard deviations compared with MWPE. In Fig.1, RCMWPE shows much less dependent on the length of series compared with MWPE and CMWPE. To sum up in conclusion, RCMWPE method effectively overcomes the shortcomings of MWPE, which is more stable and much less dependent on the length of series.

For comparison, we review refined composite multiscale permutation entropy (RCMPE) [26] as follows

$$RCMPE(x, m, \tau) = -\sum_{i:\pi_i^m \in \Pi} \bar{p}^{(\tau)}(\pi_i^m) ln\bar{p}^{(\tau)}(\pi_i^m) \tag{12}$$

where $\bar{p}^{(\tau)}(\pi_i^m)$ is the arithmetic mean of $\{p^{(\tau,k)}(\pi_i^m)\}_{k=1}^{\tau}$, which does not consider amplitude information.

3. Analysis on synthetic data

For showing the drawback of MWPE and the advantage of RCMWPE, we review the binomial multifractal series [27–30]. The series is popular and suitable for studying complex models. The series of $N = 2^{n_{max}}$ points is defined as

$$x_i = a^{n(i-1)}(1-a)^{n_{max}-n(i-1)} \tag{13}$$

where $0.5 < a < 1$ and $n(i)$ is the number of digits equal to 1 in the binary representation of the index i, e.g., $n(12)$=2, since 12 corresponds to binary 1100. In this section, we generate binomial multifractal series with $a = 0.75$ and $n_{max} = 16$ (that is generating 2^{16} samples) shown in Fig.2.

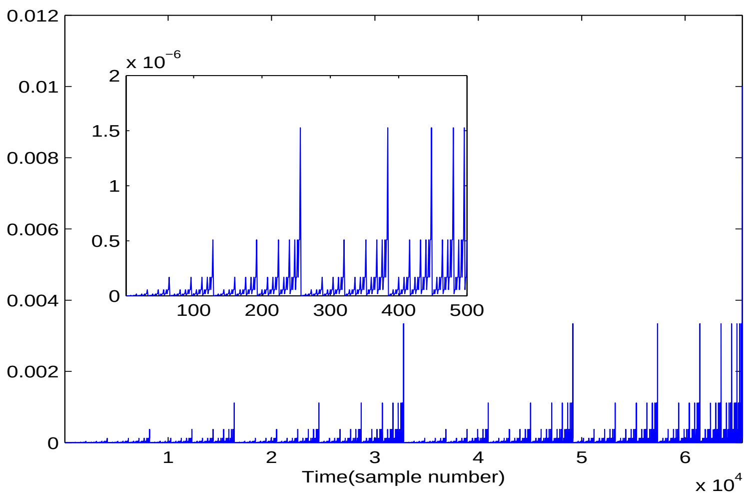

Fig.2. The sample of binomial multifractal series with $a = 0.75$ and $n_{max} = 16$

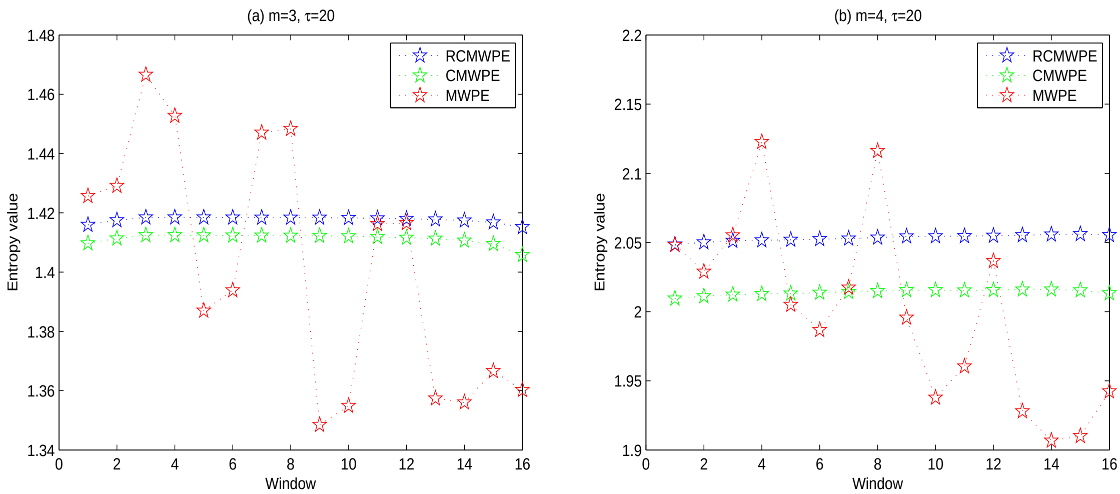

Fig.3. The values of MWPE, CMWPE and RCMWPE for binomial multifractal series in Fig.2, and the calculation with windows of (2^{16}-16) points slide by one point under m=3, 4 and τ = 20, where $N/\tau \gg m!$ has been satisfied. Each window covers more than 99.97 percent of the generated series.

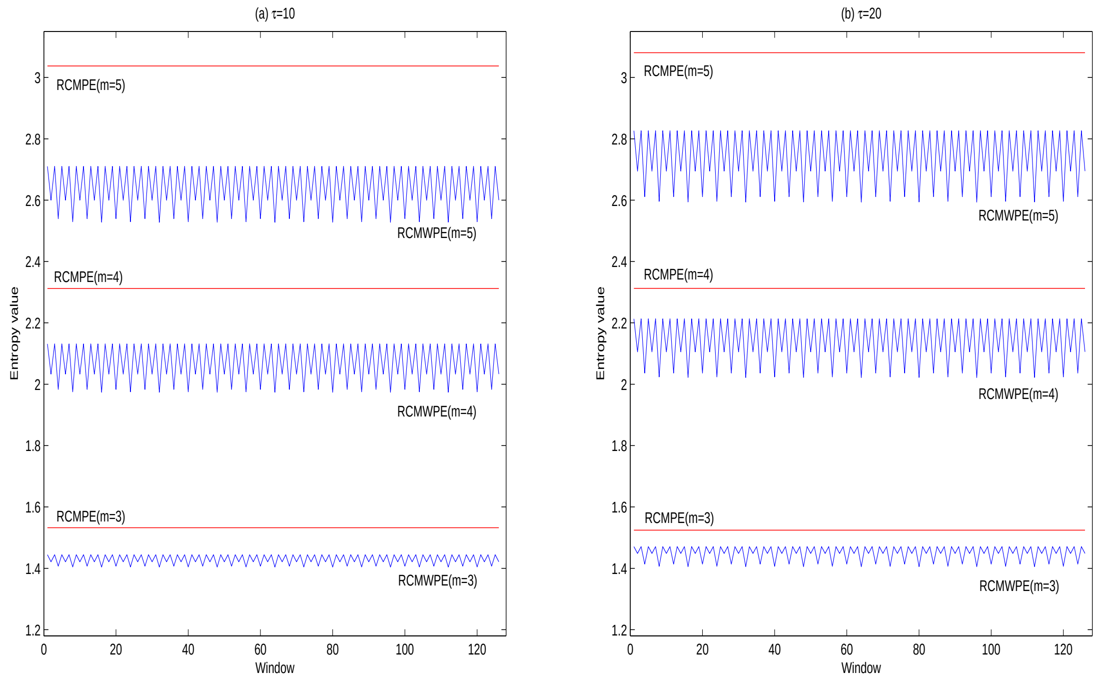

Fig.4. The values of RCMWPE (blue lines) and RCMPE (red lines) for binomial multifractal series in Fig.1, and the calculation with windows of 2^{10} points slide by 2^9 points under m=3, 4, 5 and $\tau = 10, 20$.

We calculate MWPE, CMWPE and RCMWPE for each sliding window of binomial multifractal series (see Fig.3). Windows of (2^{16}-16) points sliding by one point are used, which each window nearly covers entire binomial multifractal series and the difference between these windows on contained data points is extremely small and even negligible. However, for time scale $\tau = 20$, it is obvious that significant changes in the results of MWPE with different windows. At m=3, MWPEs fluctuate in the range [1.348, 1.467]. At m=4, MWPEs fluctuate in the range [1.907, 2.123]. For different parameters, similar results can be obtained, showing the unreliability of MWPE. The extremely small difference on data points lead to significant difference on estimating values, which may be serious drawback in practical application. For example, it is difficult that the data of European stock markets and Asian stock markets could be synchronized without changing their structures, because these markets have the different opening dates and time zones, then these data always have the difference on the data locations, which may lead to wrong analysis results. Moreover, MWPE measures may also be inaccurate estimates when time series processed has missing values. Here, we can see that the estimated values of RCMWPE show more stable. In addition, the values of RCMWPE are always higher than those of CMWPE. According to Eq.(11), the probability distribution of the weighted relative frequencies of RCMWPE are more homogeneous and stable, which leads to higher entropy values.

For binomial multifractal series, windows of 2^{10} points slide by 2^9 points are also used. As shown in Fig.2, these windows are similar on data ordinal patterns, but their amplitudes have large differences. The values of RCMPE and RCMWPE are shown in Fig.4. It is obvious that there is no significant variation in the results of RCMPE, but RCMWPE drops consistently for the region with fluctuations. For a fixed embedded dimension m, RCMWPE is lower than RCMPE, because RCMWPE is applying the variance distribution to increasing the weight to regular spiky patterns corresponding to a higher amount of information. It is evidence that RCMWPE inherits the advantage (reflecting amplitude information) of MWPE.

4. Analysis on financial time series

In the section, we investigate the daily price returns of stock-indexes during 1997-2016. The daily price return $x(n)$, is calculated by $x(n) = ln(s(n)) - ln(s(n-1))$, where $s(n)$ denotes the daily closing price at time n. We first select Hong Kong's Hang Seng Index (HSI). Hong Kong is one of the biggest financial centers in the world, and its stock market is a special market which is an apart of Asia markets but maintains British management mechanism due to implementing China's policy of "one country, two systems". Therefore, besides HSI, we choose the three important stock indexes from each side (i.e. Europe and Asia) in order to better demonstrate the application of the introduced method in practical situation. The 7 indexes are from three European markets: SMI (Switzerland), DAX (Germany), CAC 40 (France), and four Asian markets: N225 (Japan), TWII (Taiwan), HSI (Hong Kong), SSEC (China). The data was collected from the Yahoo Financial web site (https://finance.yahoo.com/).

In the following, we apply RCMWPE, RCMPE, CMWPE and MWPE methods to analyze the complexities of European indexes and Asian indexes. Moreover, by comparing with RCMPE, CMWPE and MWPE methods, we demonstrate the reliability of RCMWPE method. In addition, we present and discuss the results of RCMWPE method on these time series.

4.1. Comparison of RCMWPE with current methods

In order to obtain reliable statistics, the length of coarse-gained series N/τ and embedding dimension m must satisfy $N/\tau \gg m!$. Here, each index is used in around 5000 data points. If we determine that time scale τ is up to 20, so making the shortest coarse-grained series is about 250 points. Saying $250 \gg 5! = 120$ seems inappropriate, but it haven't too much effect to RCMWPE analysis as discussed above. For fully showing the advantage of RCMWPE method, $m = 6$ also is considered. Then, we set $\tau \leq 20$ and $m = 3, 4, 5, 6$ in this research. We calculate the estimated values of RCMWPE, CMWPE, MWPE and RCMPE at $m = 3, 4, 5, 6$ shown in Fig.5, Fig.6, Fig.7 and Fig.8, respectively. The same area markets should be closer because the business behavior of the stock markets in same area are influenced by similar rules and interactions between them. As mentioned before, financial systems show structure on multiple temporal scales. By multiscale analysis, we can detect the inherent dynamics of financial time series to distinguish between Asian markets and European markets.

At $m = 3$, RCMWPE, CMWPE, MWPE and RCMPE results of stock indexes are shown in Fig.5. See Figs.5(a) and 5(b), the results of RCMWPE and CMWPE are very similar, and both of them can distinguish between Asian markets and European markets well. Fig.5(c) shows the results of MWPE analysis, which fails to classify these markets, and the fluctuations of estimated values are larger. It is because MWPE method shows the sensitivity to the data locations, which may lead to wrong economic decisions when applied in the economic analysis. Fig.5(d) shows the results of RCMPE analysis, which shows the ability to distinguish between different markets but RCMWPE is better than it. More precisely, when these markets can be classified, the distance between Asia market and Europe market in RCMWPE analysis is about 0.02, and that in RCMPE analysis only is around 0.005. RCMWPE method is more effective than RCMPE method, because RCMWPE can extract more information from financial time series by incorporating amplitude information, but RCMPE does not it. At $m = 4, 5$, both of them show similar results that RCMWPE is most effective method in analyzing financial time series. The difference is that the ability of CMWPE in distinguish different markets are decreased when $m = 4, 5$.

At $m = 6$, the constrain of permutation entropy $N/\tau \gg m!$ isn't satisfied especially when scale factors $\tau > 2$. The results of four methods are shown in Fig.8. See Fig.8(a), Asian markets (expect HSI series) and European markets gradually show significant difference with the increase of time scales τ, and the entropy values of Asian markets are higher than those of European markets at large time scales. However, CMWPE, MWPE and RCMPE fail to distinguish the two kinds of markets, and Asian markets (except N225 series) are always similar to with European markets.

In conclusion, RCMWPE is most effective and reliable compared with CMWPE, MWPE and RCMPE in analyzing the complexity of time series. RCMPE and RCMWPE are both helpful to characterize the multiscale properties of each financial time series and distinguish the two different continents. However, the RCMPE method will be influenced by the embedded dimension m, which means the possible permutations $m!$ increase may lead to important vector weight reduction. When the amount of data that can be obtained is small, or the subject shows structure on more multiple spatiotemporal scales, the advantage of RCMWPE method is more obvious.

4.2. RCMWPE analysis in financial time series

In the subsection, we discuss the results of RCMWPE analysis on 7 stock indexes. According to the above discussion, RCMWPE analysis shows a similar result at the different embedded dimensions m. Then, we set $m = 4$ as representation of RCMWPE analysis in financial time series, and its results is shown in Fig.6(a).

For scale one, by definition, RCMWPE reduces to WPE. Asian markets and European markets are intertwined, which each series is relatively concentrated, then we cannot obtain too much information from the analysis results. WPE is based on single scale and fails to account for the multiple temporal inherent dynamics of the financial systems.

For larger scales, we find that the SSEC series are assigned the highest values of entropy and the SMI series are assigned the lowest, while other series are assigned between them. We think that SSEC is assigned by relatively higher entropy values because Chinese markets as emerging markets have full of vitality. For relatively lower entropy values of SMI, we think that Swiss economy shows a relatively certain profile at larger scales (as megatrends) because Switzerland is one of the most stable economies in the world.

As clearly shown in these figures, the selection for τ has a significant influence on the performance of the RCMWPE analysis for distinguishing between different indexes. When scale factor τ is up to 13, the separation between the Asia market and Europe market has started. At larger scales, the differences of the two kinds of markets are very significant, and the entropy values of Asian markets are higher than those of Europe. The result is roughly consistent with earlier research of multiscale sample entropy (MSE) analysis [31], but the difference of the two kinds of markets is more significant in RCMWPE analysis. It means that RCMWPE is capable of describing the multiscale structure of complexity of time series. Meanwhile, we also can find that HSI is close to European markets (seeing Figs.7(a) and 8(a) are more obvious), possibly because the Hong Kong financial market remains the system mechanism and business practices of the UK financial market. However, the entropy values of HSI series are higher than European markets, because the business behavior of the Hong Kong market is influenced by other Asian markets and some policies. For example, Hong Kong's stock market has found itself beholden to shifts in mainland Chinese policy, which leads to HSI similar to other Asian markets. More remarkable, we cannot see the relativity between HSI and European markets when m=3 (see Fig.5(a)), but HSI gradually close to European markets with the increase of m (see Figs.6(a), 7(a) and 8(a)). In this light, through altering the embedded dimension m within the constraint, we may get more abundant information and better detect the inherent dynamics of the financial time series.

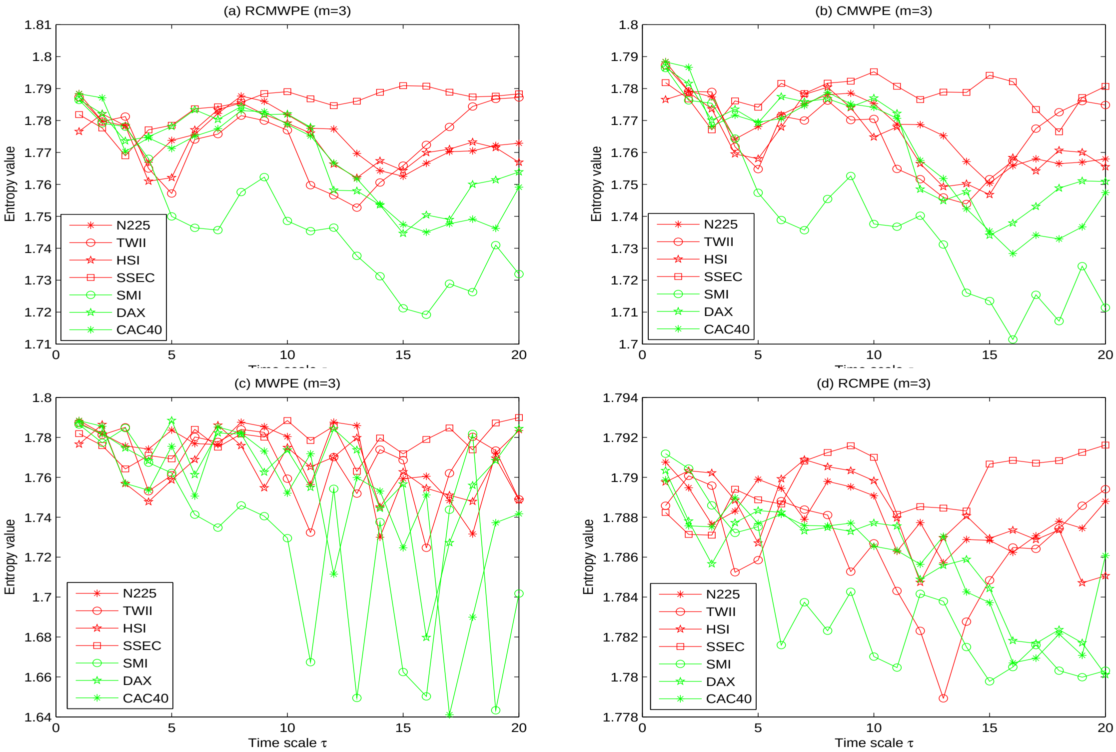

Fig.5. RCMWPE, CMWPE, MWPE and RCMPE results of stock indexes on various time scale τ when m=3. Red line stand for the Asian markets, and green line stand for the European markets.

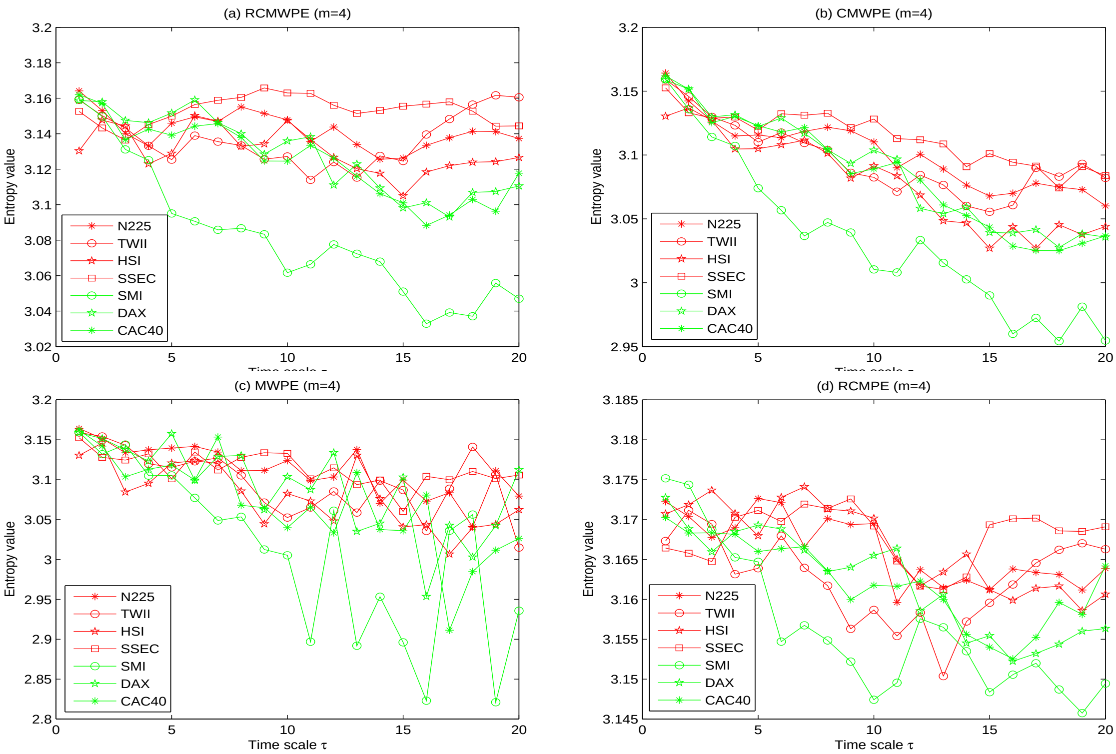

Fig.6. RCMWPE, CMWPE, MWPE and RCMPE results of stock indexes on various time scale τ when m=4. Red line stand for the Asian markets, and green line stand for the European markets.

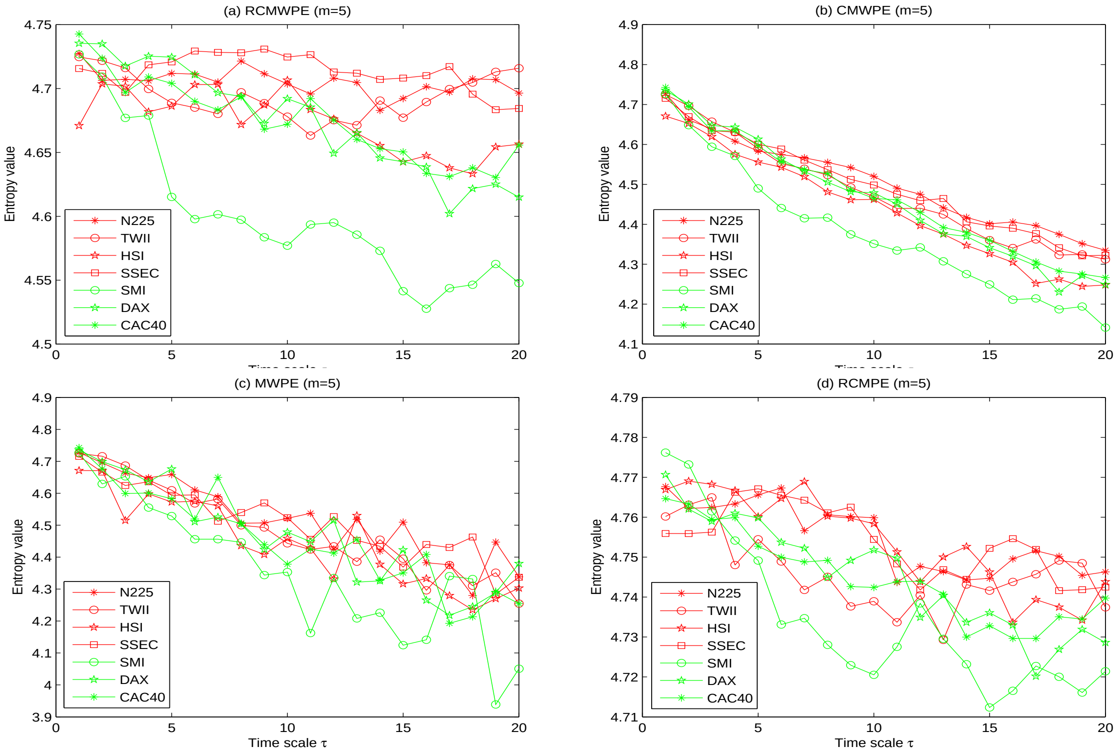

Fig.7. RCMWPE, CMWPE, MWPE and RCMPE results of stock indexes on various time scale τ when m=5. Red line stand for the Asian markets, and green line stand for the European markets.

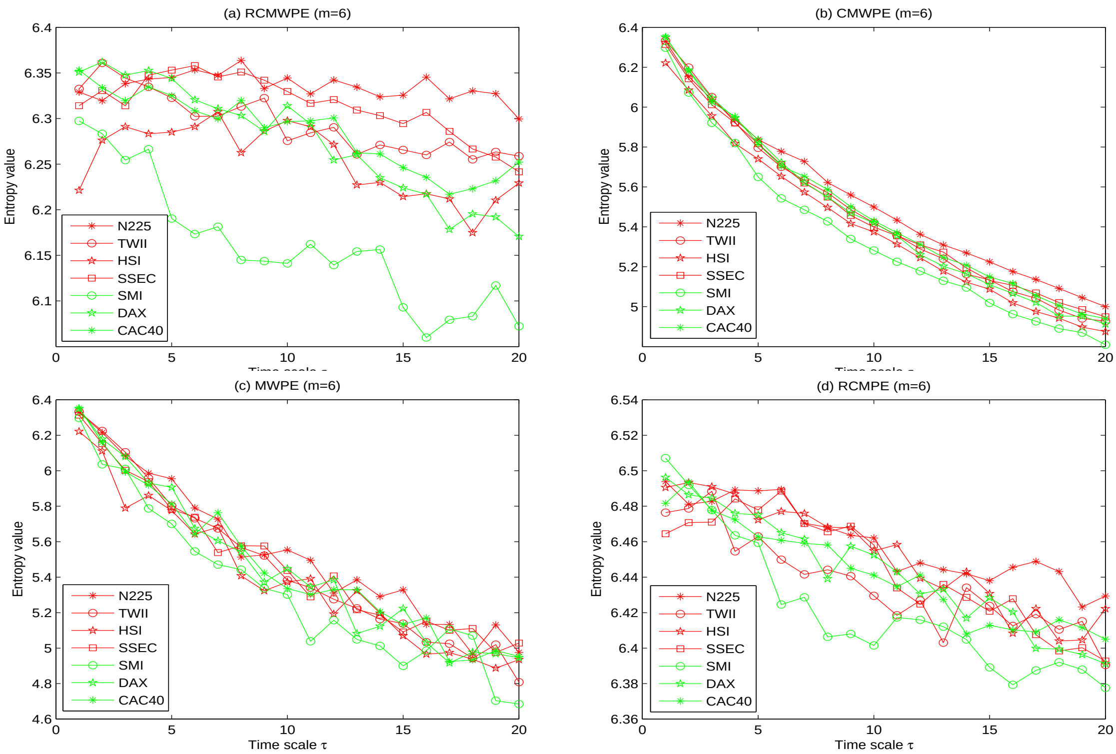

Fig.8. RCMWPE, CMWPE, MWPE and RCMPE results of stock indexes on various time scale τ when m=6. Red line stand for the Asian markets, and green line stand for the European markets.

5. Conclusions

As an efficient measure, PE has been widely applied to various fields. For more complex systems, MWPE based on PE has been proposed. However, in real data analysis, MWPE shows some serious shortcomings such as stronger sensitivity to the data locations and more dependent on the length of series. In the paper, we focus on the shortcoming of MWPE, proposing RCMWPE as an improved method. By comparing with MWPE method, RCMWPE has lower sensitivity to the data locations, is more stable and much less dependent on the length of time series. RCMWPE also inherits the advantages of MWPE, then it shows more reliable than RCMPE in exploring complexity of time series. Through the analyses of synthetic data and financial time series, the reliability of RCMWPE is shown.

Moreover, we discuss the results of RCMWPE analysis in the daily price returns of 7 stock markets from Asia and Europe. In scale one, Asian markets and European markets are relatively concentrated. The selection for τ has a significant influence on the performance of the RCMWPE analysis. At larger scales, the differences between the two kinds of markets are very significant, and the entropy values of Asian markets are higher than those of Europe. Meanwhile, we also can find that the entropy values of HSI are close to but higher than those of European markets.

When the constrain $N/\tau \gg m!$ is satisfied, RCMWPE shows more stable and reliable because τ coarse-grained series can be used and it has incorporated amplitude information. When the amount of data that can be obtained is small, or the subject shows structure on more multiple spatiotemporal scales, the advantage of RCMWPE method is more obvious. Therefore, the proposed RCMWPE method could replace MWPE and RCMPE, and be directly applied to a variety of fields to quantify the complexity of time series over multiple scales more accurately. In the research of physiology systems, we can also use RCMWPE method to account for inherent in healthy physiologic control systems.

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References

[1] S.D. Wu, C.W. Wu, S.G. Lin, C.C. Wang, K.Y. Lee, Time series analysis using composite multiscale entropy, Entropy 15 (3) (2013) 1069-1084.

[2] C. Bandt, B. Pompe, Permutation entropy: a natural complexity measure for time series, Phys. Rev. Lett. 88 (17) (2002) 174102.

[3] Y. Yin, P. Shang, Multivariate multiscale sample entropy of traffic time series, Nonlinear Dyn. 86 (1) (2016) 1-10.

[4] M. Costa, A.L. Goldberger, C.K. Peng, Multiscale entropy analysis of complex physiologic time series, Phys. Revi. Lett. 89 (6) (2002) 068102.

[5] J.S Richman, J.R. Moorman, Physiological time-series analysis using approximate entropy and sample entropy, Am. J. Physiol. 278 (6) (2000) H2039-H3049.

[6] S.D. Wu, C.W. Wu, S.G. Lin, K.Y. Lee, C.K. Peng, Analysis of complex time series using refined composite multiscale entropy, Phys. Lett. A. 378 (20) (2014) 1369-1374.

[7] S.M. Pincus, Approximate entropy as a measure of system complexity, Proc. Nati. Acad. Sci. USA. 88 (6) (1991) 2297.

[8] S. Pincus, B.H. Singer, Randomness and degrees of irregularity, Proc. Nati. Acad. Sci. USA. 93 (5) (1996) 2083.

[9] T. Schreiber, Measuring information transfer, Phys. Rev. Lett. 85 (2) (2000) 461-464.

[10] P. Grassberger, I. Procaccia, Measuring the strangeness of strange attractors, Phys. D Nonl. Phen. 9 (1983) 189-208.

[11] C.K. Peng, S.V. Buldyrev, S. Havlin, M. Simons, H.E. Stanley, A.L. Goldberger, Mosaic organization of DNA nucleotides, Phys. Rev. E 49 (2) (1994) 1685-1689.

[12] C.K.Peng, S. Havlin, H.E. Stanley, A.L. Goldberger, Quantification of scaling exponents and crossover phenomena in nonstationary heartbeat time series, Chaos 5 (1) (1995) 82-87.

[13] N. Xu, P. Shang, S. Kamae, Modeling traffic flow correlation using DFA and DCCA, Nonlinear Dyn. 61 (1-2) (2010) 207-216.

[14] E. Olofsen, J.W. Sleigh, A. Dahan, Permutation entropy of the electroencephalogram: a measure of anaesthetic drug effect, British Journal of Anaesthesia 101 (6) (2008) 810-821.

[15] X. Li, S. Cui, L.J. Voss, Using permutation entropy to measure the electroencephalographic effects of sevoflurane, Anesthesiology 109 (3) (2008) 448-456.

[16] X. Li, G. Ouyang, D.A. Richards, Predictability analysis of absence seizures with permutation entropy, Epilepsy Res. 77 (1) (2007) 70-74.

[17] Y. Cao, W. Tung, J. Gao, V. Protopopescu, L. Hively, Detecting dynamical changes in time series using the permutation entropy, Phys. Rev. E 70 (2) (2004) 046217.

[18] L. Zunino, D.G. Pérez, M.T. Martn, M. Garavaglia, A. Plastino, O.A. Rosso, Permutation entropy of fractional brownian motion and fractional gaussian noise, Phys. Lett. A 372 (27-28) (2008) 4768-4774.

[19] R. Yan, Y. Liu, R.X. Gao, Permutation entropy: a nonlinear statistical measure for status characterization of rotary machines, Mech. Syst. Signal Process 29 (5) (2012) 474484.

[20] W. Shi, P. Shang, A. Lin, The coupling analysis of stock market indices based on cross-permutation entropy, Nonlinear Dyn. 79 (4) (2014) 2439-2447.

[21] L. Zunino, M. Zanin, B.M. Tabak, D.G. Pérez, O.A. Rosso, Forbidden patterns, permutation entropy and stock market inefficiency, Physica A 388 (14) (2009) 2854-2864.

[22] B. Fadlallah, B. Chen, A. Keil, J. Príncipe, Weighted-permutation entropy: a complexity

measure for time series incorporating amplitude information, Phys. Rev. E 87 (2) (2013) 022911.

[23] Y. Yin, P. Shang, Weighted multiscale permutation entropy of financial time series, Nonlinear Dyn. 78 (2014) 2921-2939.

[24] Y. Yin, P. Shang, Detection of multiscale properties of financial market dynamics based on an entropic segmentation method, Nonlinear Dyn. 83 (3) (2016) 1743-1756.

[25] J. Xia, P. Shang, J. Wang, W. Shi, Permutation and weighted-permutation entropy analysis for the complexity of nonlinear time series, Commun. Nonlinear Sci. Numer. Simul. 31 (1-3) (2016) 60-68.

[26] A. Humeau-Heurtier, C.W. Wu, S.D. Wu, Refined composite multiscale permutation entropy to overcome multiscale permutation entropy length dependence, IEEE Signal Process. Lett. 22 (12) (2015) 2364-2367.

[27] J. Feder, Fractals, New York: Plenum Press (1988).

[28] A.L. Barabási, T. Vicsek, Multifractality of self-affine fractals, Phys. Rev. A 44 (4) (1991) 2730-2733.

[29] J.W. Kantelhardt, S.A. Zschiegner, E. Koscielny-Bunde, S. Havlin, A. Bunde, H.E. Stanley, Multifractal detrended fluctuation analysis of nonstationary time series, Physica A 316 (1-4) (2002) 87-114.

[30] H. Xiong, P. Shang, Weighted multifractal analysis of financial time series, Nonlinear Dyn. 87 (2017) 2251-2266.

[31] J. Xia, P. Shang, J. Wang, W. Shi, Classifying of financial time series based on multiscale entropy and multiscale time irreversibility, Physica A 400 (2) (2014) 151-158.