

# A new hybrid method for time series forecasting: AR–ANFIS

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Received: 8 January 2016 / Accepted: 6 July 2016  
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**Abstract** In this study, a new hybrid forecasting method is proposed. The proposed method is called autoregressive adaptive network fuzzy inference system (AR–ANFIS). AR–ANFIS can be shown in a network structure. The architecture of the network has two parts. The first part is an ANFIS structure and the second part is a linear AR model structure. In the literature, AR models and ANFIS are widely used in time series forecasting. Linear AR models are used according to model-based strategy. A nonlinear model is employed by using ANFIS. Moreover, ANFIS is a kind of data-based modeling system like artificial neural network. In this study, a linear and nonlinear forecasting model is proposed by creating a hybrid method of AR and ANFIS. The new method has advantages of data-based and model-based approaches. AR–ANFIS is trained by using particle swarm optimization, and fuzzification is done by using fuzzy C-Means method. AR–ANFIS method is examined on some real-life time series data, and it is compared with the other time series forecasting methods. As a consequence of applications, it is shown that the proposed method can produce accurate forecasts.

**Keywords** Adaptive network fuzzy inference system · Autoregressive model · Fuzzy inference system · Time series · Particle swarm optimization · Fuzzy C-Means

## 1 Introduction

Classical methods used in time series analysis work with probabilistic approaches or some special stochastic processes. In recent years, alternative forecasting methods which are based on computational or fuzzy set theory are exposed in the literature. Normally, the estimated values of parameters used in these alternative methods change according to the random sample. Therefore, these methods can be examined same as probabilistic approaches. In the literature, adaptive network fuzzy inference system (ANFIS) is preferred as a forecasting method which doesn't need probabilistic approach. It is based on fuzzy set theory. ANFIS, first proposed by Jang [24], is a fuzzy inference system which was shown in network structure by suggestion of Takagi and Sugeno [42]. In the literature, ANFIS has been used in numerous studies for time series forecasting. Chen and Zhang [13] proposed time series prediction based on ensemble ANFIS method. Mohammadi et al. [30] used ANFIS approach for demand forecasting. Li et al. [27] proposed a hybrid model of wavelet transform (WT) and ANFIS. Mellit et al. [29] used ANFIS approach for forecasting solar radiation data. Yunos et al. [46] compared ANFIS and artificial neural network (ANN) for modeling composite index data. Support vector machine (SVM) and ANFIS on the short-term electric load prediction were compared by Escobar and Perez [20]. Yun et al. [45] proposed radial basis function (RBF) neural network and ANFIS-based short-term load forecasting approach. In this approach, forecasts were obtained by using RBF neural

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network and daily amount of change was used as the input of ANFIS. Tamura et al. [43] proposed a recurrent type ANFIS for time series forecasting. Chang [7] proposed a method about resolving the overshoot and volatility clustering effects in time series forecasting. Khadangi et al. [25] compared ANFIS and RBF models in daily stream flow forecasting. Cheng et al. [16] used an integrated ANFIS model in order to forecast electronic industry EPS. In another study, Cheng et al. [14] proposed a new ANFIS approach in order to forecast stock exchange data. Moreno [31] compared the multiple linear regression and autoregressive integrated moving average (ARIMA) models with ANFIS. Chen and Ma [12] used ANFIS for short-term traffic flow prediction. Chabaa et al. [6] used ANFIS to forecast Internet traffic time series. Shoorehdeli et al. [39], Pousinho et al. [34], Catalao et al. [5, 3, 4] and Pousinho et al. [35] used particle swarm optimization (PSO) for training ANFIS. Ho and Tsai [23] compared ANFIS and structural equation modeling (SEM) in linear and nonlinear forecasting of new product development performance. Li et al. [26] proposed an ANFIS approach based on genetic algorithm. Suhartono et al. [40] presented an ensemble method based on ANFIS-ARIMA. Lohani et al. [28] compared autoregressive techniques, neural network and ANFIS for hydrological time series modeling. Suhartono et al. [41] proposed two-level seasonal model based on hybrid ARIMA-ANFIS. Cheng et al. [15] presented the ordered weighted averaging (OWA) based on ANFIS model. Eğrioğlu et al. [21] proposed modified adaptive network based fuzzy inference system (MANFIS) for time series forecasting. Gocic et al. [22] used soft computing methods for reference evapotranspiration estimation. Chou et al. [17] used Taguchi method to gather data, applied hybrid Taguchi genetic evolutionary algorithm (HTGEA)-based ANFIS to model chemical mechanical polishing process of color filter (CMPPCF) and optimized CMPPCF input parameters by the HTGEA. Shojafar et al. [38] proposed a hybrid approach called FUGE that is based on fuzzy theory and a genetic algorithm. Protic [36] developed a model that is based on support vector machines (SVM) coupled with a discrete wavelet transform. Cordeschi et al. [18] proposed the optimal minimum energy scheduler. Wei [44] proposed a hybrid time series ANFIS for forecasting stock time series.

In the literature, ANFIS is one of the most commonly used methods for time series forecasting. One of the reasons is that inputs and outputs are generated by only one variable in time series analysis and its lags. Some of improvements about fuzzification and training ANFIS are made by Eğrioğlu et al. [21]. Also in the literature, the ANFIS method seems to reach the successful forecast

results in studies that are hybridized with other forecasting methods.

In this study, MANFIS is improved and a new hybrid forecasting method (AR-ANFIS) is proposed. The motivation of this study is listed below:

- Proposing a new hybrid method is based on data-based approach (ANFIS) and model-based approach (AR).
- Proposing hybrid method is based on linear (AR) and nonlinear (ANFIS) modeling approaches.
- Proposing a new hybrid fuzzy inference system that uses advantage of time series data like MANFIS.

In the second section of this paper, fuzzy C-Means method is described. Afterward, particle swarm optimization method is described in Sect. 3. A new hybrid AR-ANFIS approach is introduced in Sect. 4. The application results which are obtained from the real-life time series are summarized in Sect. 5. In Sect. 6, the results and recommendations for future are discussed.

## 2 Fuzzy C-Means clustering method

In this study, fuzzy C-Means (FCM) clustering algorithm is used for calculating membership values of inputs. FCM is one of the most widely used methods in the literature. This method was proposed by Bezdek in 1981. The main purpose of this method is to minimize the distance between observations and centers of cluster. FCM clustering algorithm needs to the number of fuzzy clusters ( $c$ ), the observation values ( $X = \{x_1, x_2, \dots, x_n\}$ ), the number of observations ( $n$ ), the membership value matrix ( $U$ ), the membership value ( $\mu_{ik}$ ), the norm matrix ( $A$ ), the matrix of cluster centers ( $V$ ), the cluster center vector ( $v_i$ ) and the fuzziness parameter ( $m \in (1, \infty)$ ). It tried to minimize the objective function as follows:

$$\min J(X; U; V) = \sum_{i=1}^c \sum_{k=1}^n (\mu_{ik})^m d^2(x_k, v_i)_A \quad (1)$$

Firstly,  $c$  and  $m$  are determined to minimize the objective function. For most data,  $1.5 \leq m \leq 3$  gives good results [2]. The equations given in (2), (3), (4) and (5) are used in FCM algorithm.

$$v_i^{(t)} = \frac{\left( \sum_{k=1}^n (\mu_{ik}^{(t)})^m x_k \right)}{\sum_{k=1}^n (\mu_{ik}^{(t)})^m}, \quad \forall i = 1, \dots, c \quad (2)$$

$$\mu_{ik}^{(t)} = \left[ \sum_{j=1}^c \left( \frac{d(x_k, v_i^{(t-1)})}{d(x_k, v_j^{(t-1)})} \right)^{\frac{2}{m-1}} \right]^{-1} \quad (3)$$

$$E_t = \sum_{i=1}^c \|v_{i,(t+1)} - v_{i,(t)}\| \quad (4)$$

Alternatively,  $E_t$  can be calculated according to membership values by using the equation given in (5).

$$E_t = \|U^{(t)} - U^{(t+1)}\| \quad (5)$$

FCM method is described step by step in the following algorithm:

**Algorithm 2.1. Fuzzy C-Means**

- Step 1 Initialize the membership matrix  $U$ , randomly.
- Step 2 Initialize  $c$ ,  $m$  and  $\varepsilon$ .
- Step 3 Calculate the vector of cluster center  $v_i$  by using Eq. (2).
- Step 4 Calculate the membership values by using Eq. (3).
- Step 5 Calculate  $E_t$  by using Eq. (4).
- Step 6 If  $E_t \leq \varepsilon$ , then stop. Otherwise, return to Step 3.

### 3 Particle swarm optimization

Particle swarm optimization (PSO) was proposed by Eberhart and Kennedy [19]. PSO is a population-based optimization method. This method is initialized with a population of random solutions and it searches on optimal solution by updating generations. In PSO, the potential solutions are called particles. The best individual performances of  $i$ th particle are stored in the vector called  $pbest$ , and  $gbest$  represents the best particle which has the best evaluation function value, found so far [19]. Positions and velocities of each particles are stored in  $X_i$ ,  $V_i$ .

$$X_i = (x_{i1}, x_{i2}, \dots, x_{iD}) \quad (6)$$

$$V_i = (v_{i1}, v_{i2}, \dots, v_{iD}) \quad (7)$$

$pbest_i$  and  $gbest$  particles are given in (8), (9), respectively:

$$pbest_i = (p_{i1}, p_{i2}, \dots, p_{iD}) \quad (8)$$

$$gbest = (p_1, p_2, \dots, p_D) \quad (9)$$

Velocities are updated by using the equation given in (10).

$$v_{id}^{k+1} = w \cdot v_{id}^k + c_1 \cdot \text{rand}_1^k \cdot (pbest_{id}^k - x_{id}^k) + c_2 \cdot \text{rand}_2^k \cdot (gbest^k - x_{id}^k) \quad (10)$$

In PSO,  $w$  is the inertia parameter,  $k$  is the number of iteration and  $\text{rand}_1^k$ ,  $\text{rand}_2^k$  are random values from the interval [0,1]. Cognitive ( $c_1$ ), social ( $c_2$ ) coefficients and  $w$  are updated by using the equations given in (11), (12) and (13), respectively [37].

$$c_1 = (c_{1f} - c_{1i}) \frac{t}{\text{maxt}} + c_{1i} \quad (11)$$

$$c_2 = (c_{2f} - c_{2i}) \frac{t}{\text{maxt}} + c_{2i} \quad (12)$$

$$w = (w_2 - w_1) \frac{\text{maxt} - t}{\text{maxt}} + w_1 \quad (13)$$

Here,  $\text{maxt}$  defines the number of iteration and  $t$  defines the valid iteration number, and  $(c_{1f}, c_{1i})$ ,  $(c_{2f}, c_{2i})$  and  $(w_1, w_2)$  are possible intervals for  $c_1$ ,  $c_2$  and  $w$ , respectively.

The new position of the particle is calculated by using (14).

$$x_i^{(k+1)} = x_i^k + v_i^{(k+1)} \quad (14)$$

All of this update process is continued until it reaches the specified value of an error or iteration.

Guaranteed convergence particle swarm optimization (GCP SO) was proposed by Peer et al. [33]. New velocity formula is proposed in this method because PSO velocity formula is insufficient for the calculation of velocity of the best particle. In this study, GCP SO is used for training ANFIS and estimating AR parameters.

GCP SO velocity formula is given in (15)

$$v_{id}^k = w * v_{id}^k - x_{id}^k + gbest_d^k + \rho(k) * \text{rand}_3 \quad (15)$$

In Eq. (15),  $\text{rand}_3$  is the random value from the interval [0,1] and  $\rho(0)$  is defined as 1.  $\rho(k+1)$  is calculated as follows:

$$\rho(k+1) = \begin{cases} 2\rho(k), & S_n > S_c \\ 0.5\rho(k), & f_n > f_c \\ \rho(k), & \text{otherwise} \end{cases} \quad (16)$$

where  $S_n$  represents the number of success, and  $f_n$  represents the number of failure. The threshold parameters  $S_c$  and  $f_c$  are defined empirically. For a high-dimensional search space, it is difficult to obtain a better value in only a few iterations. Therefore,  $S_c = 15$  and  $f_c = 5$  are suggested [32].

### 4 The proposed method: AR-ANFIS

ANFIS is a data-based system which describes the non-linear relationship between inputs and outputs. If ANFIS is used for time series forecasting, model inputs are selected as lagged variables. In this case, it is easier to define the rules because inputs and output are generated by one time series. Eğrioğlu et al. [21] proposed modified ANFIS (MANFIS) for time series forecasting, and they used the practical method for defining the fuzzy rules. In both ANFIS and MANFIS, there is a nonlinear connection between inputs and outputs. In the literature, linear models

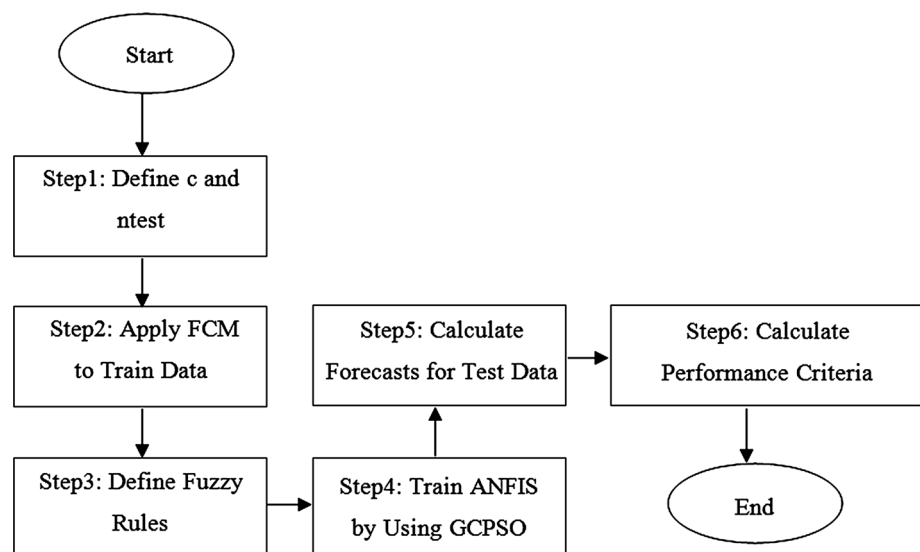
are widely used in time series forecasting. The linear models can give the simple relationship between inputs and outputs. However, in real-life time series, the relationship among lagged variables cannot be known linear or non-linear. Thus, there are numerous studies in recent years about hybridizing linear and nonlinear models. Zhang [47] proposed a time series forecasting method by using a hybrid ARIMA and neural network model. Suhartono et al. [41] proposed a hybrid ARIMA–ANFIS method by using Zhang’s [47] two-stage approach.

In this study, AR and ANFIS are hybridized in one stage and a new hybrid method is proposed. The proposed method is called autoregressive adaptive network fuzzy inference system (AR–ANFIS). The differences between AR–ANFIS and ANFIS are listed as follows:

- ANFIS approach is redesigned and inputs are directly connected to the last layer. The output of AR–ANFIS is the linear combination of AR and classical ANFIS.
- Minimal operator is used instead of multiplication in the combination of inputs.
- Rules are defined by using the rule defining method given in [21].
- Membership functions are not used for calculating the membership values. Alternatively, the rule fire strengths are calculated based on membership values which are obtained by FCM. Thus, it is possible to work with less parameters than classical ANFIS approach.
- Consequent and AR–ANFIS combination parameters are predicted by using one-stage GCPSO.

The flowchart and the network structure are shown in Figs. 1 and 2, respectively. The algorithm of generating fuzzy rules, calculating the output and training AR–ANFIS is explained step by step in the following algorithms:

**Fig. 1** Flowchart for AR–ANFIS



#### Algorithm 4.1. Generating Fuzzy Rules

**Step 1** The fuzzy set which has maximum membership value is determined by using the membership values obtained by FCM. Thus, fuzzy observations are obtained. This method is explained by using an example. Let  $c$  be three and a time series have four observations. The time series and membership values of its observations for fuzzy sets are given in Table 1.

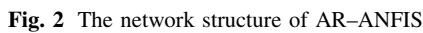
For example, first observation has maximum membership values for  $L_2$  and hereby the fuzzy value for the first observation is obtained as  $L_2$ . Similarly, other fuzzy observations are determined as  $L_3, L_1, L_1$ , respectively.

**Step 2** Fuzzy observations and the model degree are used for defining Mamdani-type rules. It is assumed that there are two inputs  $(x_{t-1}, x_{t-2})$  of the system. Mamdani-type rules can be obtained from data. Since  $x_1 = L_2$ ,  $x_2 = L_3$  and  $x_3 = L_1$ , Mamdani-type rule is obtained as ‘If  $x_{t-1} : L_3$  and  $x_{t-2} : L_2$ , then  $x_t : L_1$ .’ In Table 2, this rule is presented by  $L_2, L_3 \rightarrow L_1$ . Mamdani-type rules can be easily converted to Sugeno-type rules like in Table 2.

**Step 3** Mamdani-type rules which are obtained from the sample time series are converted to Sugeno-type rules in Table 2.

#### Algorithm 4.2. Calculating the output of AR–ANFIS

**Step 1** The membership values for the fuzzy sets are calculated by using centers of cluster in FCM. The membership values are calculated by using the equation given in (3). For example, it is assumed that there are two inputs  $(x_{t-1}, x_{t-2})$ ,



$x_t$	$L_1$	$L_2$	$L_3$	Fuzzy observations
30	0.2	0.6	0.2	$L_2$
40	0.1	0.1	0.8	$L_3$
10	0.6	0.3	0.1	$L_1$
20	0.5	0.3	0.2	$L_1$

$x_t$	Fuzzy observations	Mamdani-type rules	Sugeno-type rules
30	$L_2$	–	–
40	$L_3$	–	–
10	$L_1$	$L_2, L_3 \rightarrow L_1$	If $x_{t-1} : L_3$ and $x_{t-2} : L_2$ , then $f_1 = p_1 x_{t-1} + q_1 x_{t-2} + r_1$
20	$L_1$	$L_3, L_1 \rightarrow L_1$	If $x_{t-1} : L_1$ . and $x_{t-2} : L_3$ , then $f_2 = p_2 x_{t-1} + q_2 x_{t-2} + r_2$

$$O_i^1 = \mu_{L_i}(x), \quad i = 1, 2 \quad (17)$$

$$O_i^1 = \mu_{L_{i-2}}(x), \quad i = 3, 4 \quad (18)$$

$$O_1^2 = w_1 = \min(O_1^1, O_4^1) \quad (19)$$

$$O_2^2 = w_2 = \min(O_2^1, O_3^1) \quad (20)$$

Step 3 The rule fire strengths are normalized. The normalized rule fire strengths for the network structure given in Fig. 2 are calculated as follows:

$$O_i^3 = \bar{w}_i = \frac{w_i}{w_1 + w_2}, \quad i = 1, 2 \quad (21)$$

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structure in Fig. 2 has two rules and output values of the linear functions are given as follows:

$$\hat{x}_t^1 = p_1 x_{t-1} + q_1 x_{t-2} + r_1 \quad (22)$$

$$\hat{x}_t^2 = p_2 x_{t-1} + q_2 x_{t-2} + r_2 \quad (23)$$

Step 5

The parameter set  $\{p_1, p_2, q_1, q_2, r_1, r_2\}$  in (22) and (23) will be denoted as consequent parameters.

The output of ANFIS is calculated by using the equation given in (24).

$$\hat{x}_{t,ANFIS} = \bar{w}_1 \hat{x}_t^1 + \bar{w}_2 \hat{x}_t^2 \quad (24)$$

Step 6

The output of AR is calculated as follows:

$$\hat{x}_{t,AR} = \varphi_1 x_{t-1} + \varphi_2 x_{t-2} \quad (25)$$

Step 7

The output of AR–ANFIS is calculated as follows:

**Table 3** The positions of a particle

Pos.	$x_{k1}$	$x_{k2}$	$x_{k3}$	$x_{k4}$	$x_{k5}$	$x_{k6}$	$x_{k7}$	$x_{k8}$	$x_{k9}$	$x_{k10}$	$x_{k11}$
Par.	$p_1$	$q_1$	$r_1$	$p_2$	$q_2$	$r_2$	$v_1$	$v_2$	$v_3$	$\varphi_1$	$\varphi_2$

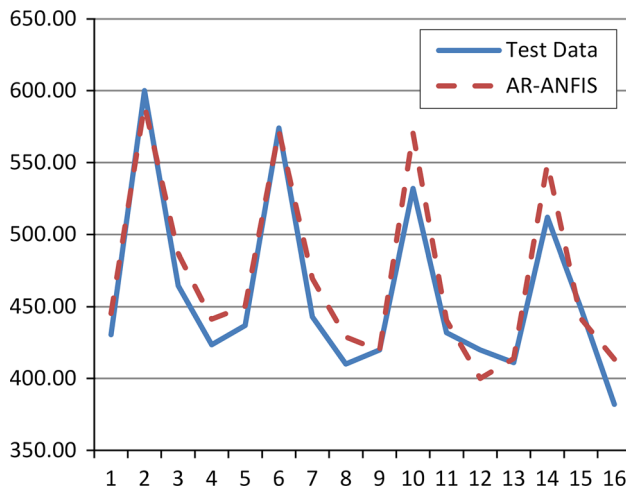
**Table 4** The names and features of time series

The number of series	Series/year	Number of observation	Number of lag	Number of cluster	n test
1	ABC	148	4,8	3:10	16
2	BIST100/2009	103	1:5	3:7	7, 15
3	BIST100/2010	104	1:5	3:7	7, 15
4	BIST100/2011	106	1:5	3:7	7, 15
5	BIST100/2012	106	1:5	3:7	7, 15
6	BIST100/2013	106	1:5	3:7	7, 15
7	TAIEX1999	266	1:5	3:7	45
8	TAIEX2000	271	1:5	3:7	47
9	TAIEX2001	244	1:5	3:7	43
10	TAIEX2002	248	1:5	3:7	43
11	TAIEX2003	249	1:5	3:7	43
12	TAIEX2004	250	1:5	3:7	45

**Table 5** Results obtained for ABC data

Test data	WMES	SARIMA	FFANN	ANFIS	MANFIS	AR–ANFIS
430.50	453.91	452.72	453.8838	446.7141	451.456	445.231
600.00	575.22	578.29	557.8151	553.7286	575.632	589.867
464.50	502.32	487.7	497.5159	482.0749	494.066	486.806
423.60	444.73	446.28	437.393	434.1992	436.565	441.308
437.00	459.66	456.77	449.0035	438.5516	444.696	450.353
574.00	582.48	583.51	569.0025	559.006	575.423	571.947
443.00	508.64	492.13	471.0758	472.5198	481.273	469.621
410.00	450.31	450.36	424.3307	427.5736	414.437	428.886
420.00	465.4	461.01	448.8667	445.0127	430.307	419.428
532.00	589.74	588.96	560.0436	562.9364	565.178	570.378
432.00	514.96	496.77	447.0135	459.1439	452.052	440.589
420.00	455.89	454.64	408.6362	416.1582	392.138	400.060
411.00	471.15	465.46	428.1073	431.7013	419.331	413.925
512.00	597.00	594.71	537.6988	544.9831	536.878	549.318
449.00	521.28	501.67	438.433	444.3101	446.324	441.676
382.00	461.46	459.17	420.5827	426.01	406.637	413.590
RMSE	53.3295	47.0367	24.1052	25.0500	21.3728	20.5791
MAPE	0.1072	0.0949	0.0476	0.0467	0.0400	0.0376





**Fig. 3** Test data and AR-ANFIS forecasts for ABC series

$$\hat{x}_t = v_1 \hat{x}_{t,ANFIS} + v_2 \hat{x}_{t,AR} + v_3 \quad (26)$$

The mathematical structure of hybrid method is given in closed form in Eq. (26). It is clear that hybrid AR-ANFIS is a linear combination of AR and ANFIS models, but the combination weights are estimated with other model parameters in the single optimization processes by employing GCP SO.

#### Algorithm 4.3. Training AR-ANFIS with PSO

- Step 1** The initial parameters of PSO method are determined. These parameters are denoted as cognitive, social and inertia parameters ( $c_{1i}, c_{1f}, c_{2i}, c_{2f}, w_1, w_2$ ), the number of particles and number of maximum iteration.
- Step 2** Particle's initial positions and the velocity values are generated randomly. The positions of the particles in PSO are equal to the parameters in

AR-ANFIS. A particle in PSO is given Table 3. All particle positions are generated by using the uniform distribution from the interval  $(-1, 1)$ , and the velocity values are generated by using the uniform distribution from the interval  $(0, 1)$ .

$$x_{ij} \sim \text{Uniform}(-1, 1) \quad (27)$$

$$v_{ij} \sim \text{Uniform}(0, 1) \quad (28)$$

- Step 3** The evaluation function is selected as the mean square error (MSE), and its value is calculated for each particle. MSE can be calculated by using the equation given in (29)

$$\text{MSE} = \frac{1}{n} \sum_{i=1}^n (x_t - \hat{x}_t)^2 \quad (29)$$

In this equation,  $\hat{x}_t$ ,  $t = 1, 2, \dots, n$  is calculated by using Algorithm 4.2.

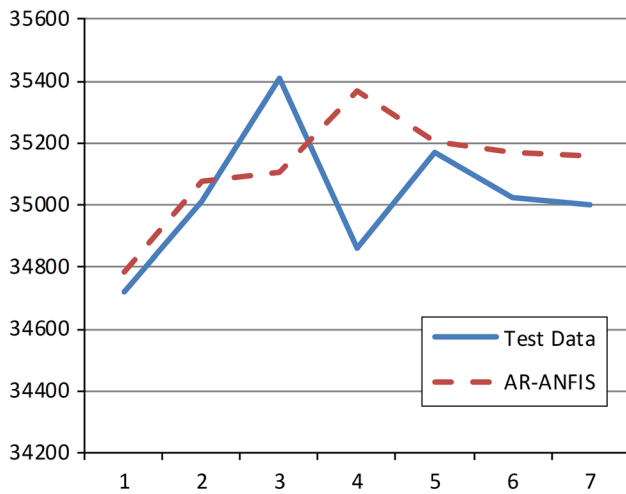
- Step 4** Initially, pbest and gbest are determined. Then, pbest and gbest are updated for each iteration. Termination criteria are controlled.
- Step 5** Cognitive, social and inertia parameters are calculated by using Eqs. (11), (12) and (13), respectively. Indexes of failure and success are updated.
- Step 6** The new velocity and position values of particles are calculated by using Eqs. (14) and (15). The evaluation values are calculated for new particles in Step 3 and the process continues with Step 4.

## 5 Applications

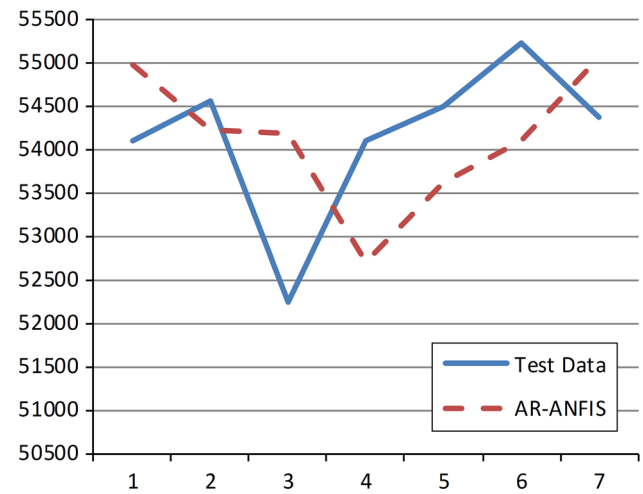
In this study, 12 time series are analyzed by using MATLAB (11.B) for the evaluation of the forecasting performance of AR-ANFIS. The first time series comprises quarterly data

**Table 6** Results obtained for BIST100 data

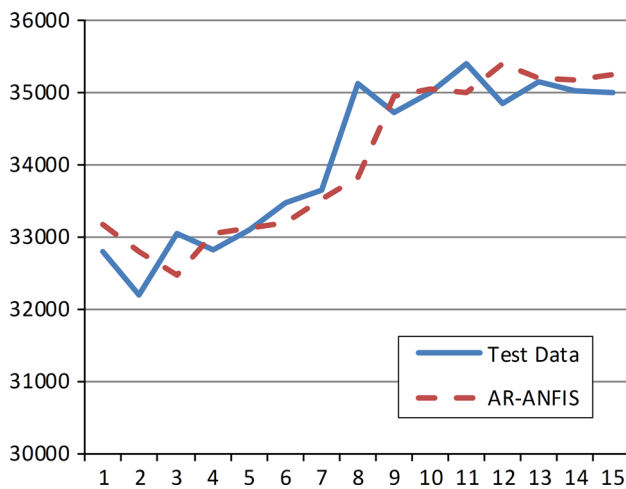
Time series	Length of test set	ARIMA		ANFIS		MANFIS		AR-ANFIS	
		RMSE	MAPE	RMSE	MAPE	RMSE	MAPE	RMSE	MAPE
BIST100/2009	7	344.91	0.0087	405.15	0.0097	261	0.0059	240.39	0.0052
BIST100/2009	15	540.21	0.0120	647.28	0.0157	503	0.0112	467.99	0.0102
BIST100/2010	7	1221	0.0183	1141.30	0.0153	1144	0.0187	1136.70	0.0191
BIST100/2010	15	1612	0.0219	2033	0.0277	1303	0.0193	1451.80	0.023
BIST100/2011	7	1057.60	0.0144	1007	0.0141	960	0.0141	987.042	0.0134
BIST100/2011	15	1129.60	0.0150	1134	0.0153	1009	0.0136	999.532	0.013
BIST100/2012	7	651	0.0084	634	0.0092	634	0.0082	631.94	0.0087
BIST100/2012	15	621	0.0088	938	0.0139	629	0.0094	619.58	0.0086
BIST100/2013	7	1361.60	0.0116	1477	0.0131	1418	0.0134	1362.3	0.0118
BIST100/2013	15	1268.7	0.0109	1413	0.0125	1264	0.0109	1256.46	0.0108



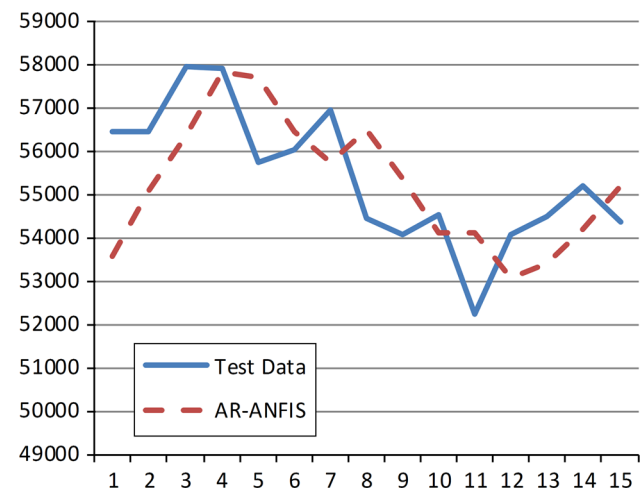
**Fig. 4** Time series graph of test data (ntest = 7) and AR-ANFIS forecasts for Series 2



**Fig. 6** Time series graph of test data (ntest = 7) and AR-ANFIS forecasts for Series 3



**Fig. 5** Time series graph of test data (ntest = 15) and AR-ANFIS forecasts for Series 2



**Fig. 7** Time series graph of test data (ntest = 15) and AR-ANFIS forecasts for Series 3

between 1956 and 1994 related to Australian Beer Consumption (ABC). The following five time series comprise daily data between 2009 and 2013 from Istanbul Stock Exchange Market (BIST100) Index. The last six time series comprise daily data between 1999 and 2004 from the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX). These time series and their features are presented in Table 4. The methods are compared by using root-mean-square error (RMSE) and mean absolute percentage error (MAPE) criteria. RMSE and MAPE are calculated by using the equations given in (30) and (31), respectively.

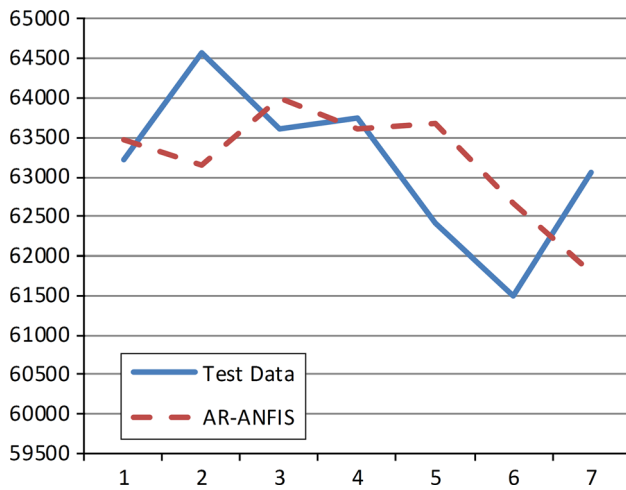
$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{t=1}^n (x_t - \hat{x}_t)^2} \quad (30)$$

$$\text{MAPE} = \frac{1}{n} \sum_{t=1}^n \left| \frac{x_t - \hat{x}_t}{x_t} \right| \quad (31)$$

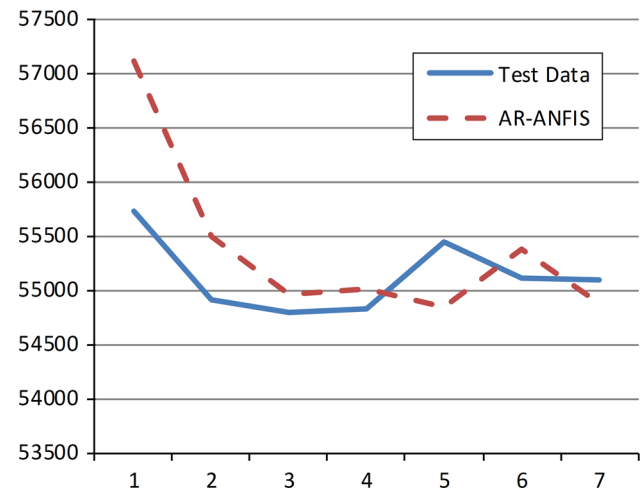
The forecasting results for ABC data are given in Table 5. The results obtained from AR-ANFIS are compared with Winter's Multiplicative Exponential Smoothing (WMES), Seasonal Autoregressive Integrated Moving Average (SARIMA), Feed-Forward Artificial Neural Network (FFANN), ANFIS and MANFIS methods, and these application results are quoted from [21].

ABC series has 148 observations, the number of lag 4 and 8, the number of fuzzy sets varied between 3 and 10 and ntest=16, and it is seen from Table 4. Best case results are obtained and given in Table 5. Moreover, the number of cluster and the number of lag for the optimal solution are given in Table 8.

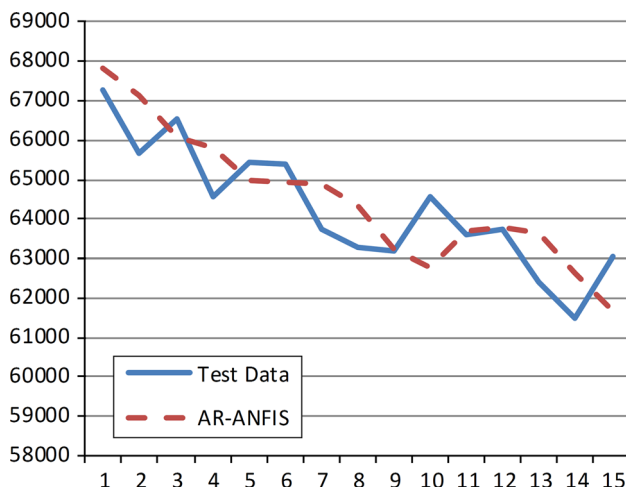




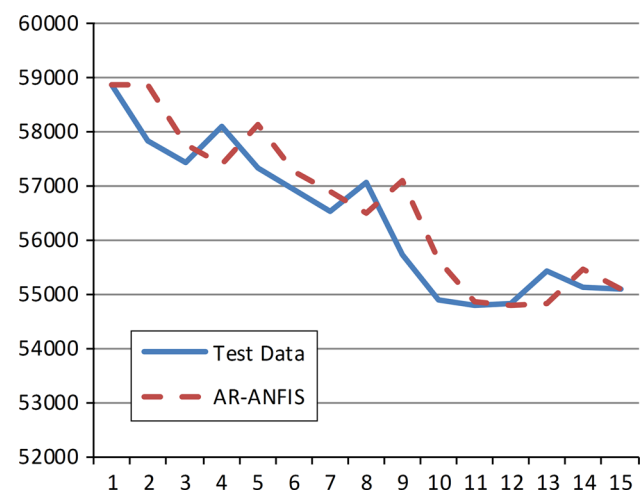
**Fig. 8** Time series graph of test data ( $n_{test} = 7$ ) and AR-ANFIS forecasts for Series 4



**Fig. 10** Time series graph of test data ( $n_{test} = 7$ ) and AR-ANFIS forecasts for Series 5



**Fig. 9** Time series graph of test data ( $n_{test} = 15$ ) and AR-ANFIS forecasts for Series 4



**Fig. 11** Time series graph of test data ( $n_{test} = 15$ ) and AR-ANFIS forecasts for Series 5

It is seen that AR-ANFIS can produce lower RMSE and MAPE values in comparison with other methods. The AR-ANFIS improved 3 and 17 % the forecast result of MANFIS and ANFIS results, respectively. Test data and AR-ANFIS application results are given as a graph in Fig. 3

The performance of the proposed method is examined on five time series from BIST100. The proposed method is compared with ARIMA, ANFIS and MANFIS methods. ARIMA method application results are quoted from Baş et al. [1]. The application results are given in Table 6.

It can be seen that the RMSE values for AR-ANFIS are smaller than other methods in seven cases in Table 6 and the success rate is 0.70. Similarly, the MAPE values for AR-ANFIS are smaller than other methods in 6 cases in Table 6, and the success rate is 0.60. It is clear that the proposed method can produce better forecast results for small test set. When the

test set is seven, the smaller RMSE and MAPE values can be obtained according to 15 test data. The forecast graphs for test data and AR-ANFIS are obtained by using series 2–6 and it is given in Figs. 4, 5, 6, 7, 8, 9, 10, 11, 12, 13.

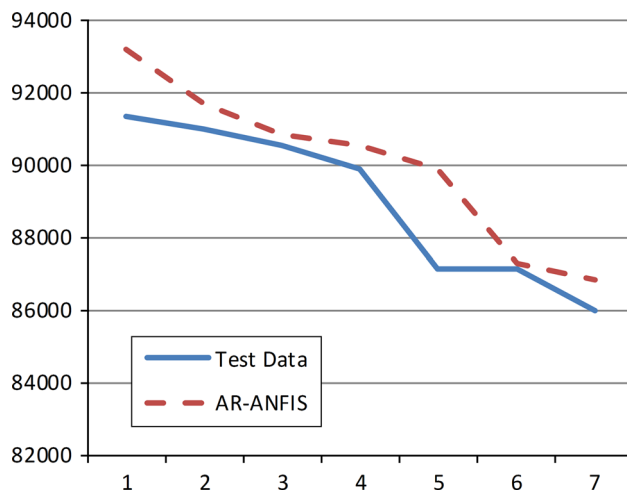
Finally, TAIEX is used in the last application. The result of the proposed method is compared with Chen [8], Chen and Chang [9], Chen and Chen [10] and Chen et al. [11]. Application results of mentioned methods were quoted from Baş et al. [1].

The results obtained for TAIEX data are given in Table 7. The RMSE values for AR-ANFIS are smaller than other methods three cases in Table 7 and the success rate is 0.50. In terms of the average value of RMSE, AR-ANFIS method gives the best result Fig. 14.

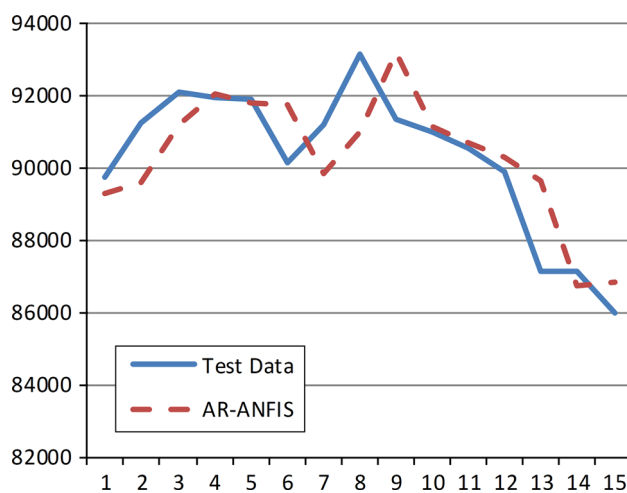
For ABC data set, ANFIS and MANFIS application results are quoted from Eğrioglu et al. [21] because the

number of cluster and the number of lag are not included in Table 8.

It is clear that AR–ANFIS can produce better forecast results for the examined real-world time series. The RMSE



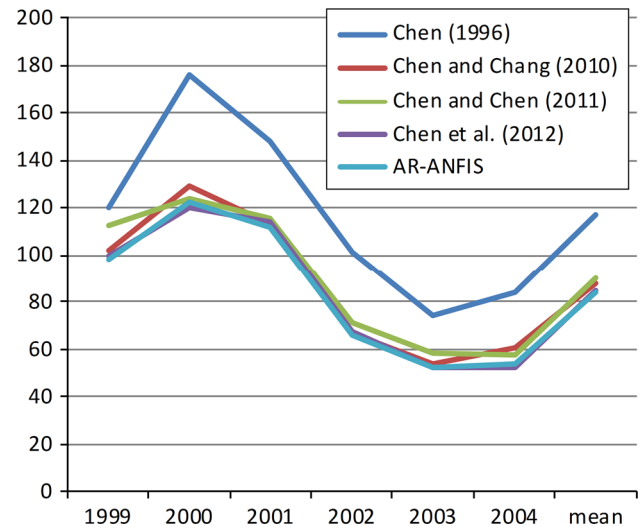
**Fig. 12** Time series graph of test data ( $n_{test} = 7$ ) and AR–ANFIS forecasts for Series 6



**Fig. 13** Time series graph of test data ( $n_{test} = 15$ ) and AR–ANFIS forecasts for Series 6

**Table 7** Results obtained for TAIEX data

Methods	RMSE						Mean
	1999	2000	2001	2002	2003	2004	
[8]	120	176.32	147.84	101.18	74.46	84.28	117.34
[9]	101.97	129.42	113.33	66.82	53.51	60.48	87.58
[10]	112.47	123.62	115.33	71.01	58.06	57.73	89.70
[11]	99.87	119.98	114.47	67.17	52.49	52.27	84.37
ANFIS	101.16	137.02	114.72	65.99	57.04	61.36	89.54
MANFIS	101.94	124.92	112.47	62.57	52.33	53.66	84.64
AR–ANFIS	98.37	122.81	111.49	65.86	51.83	53.63	84.00



**Fig. 14** The average RMSE values of Chen [8], Chen and Chang [9], Chen and Chen [10], Chen et al. [11] and AR–ANFIS methods

values for AR–ANFIS are smaller than MANFIS in eight cases in Table 6 and the success rate is 0.80. Similarly, the RMSE values for AR–ANFIS are smaller than MANFIS in five cases in Table 7 and the success rate is 0.83.

## 6 Conclusion

Fuzzy inference systems have been frequently used in time series forecasting problems. ANFIS is a data-based method based on fuzzy set theory. In this study, an alternative ANFIS method is proposed for time series forecasting.

AR–ANFIS integrates data-based and model-based approaches in one network. Moreover, AR–ANFIS can be used for both linear and nonlinear forecast approaches. Furthermore, it works with less parameters through the membership functions which are not used for defining membership values of AR–ANFIS inputs. The fuzzy rules are defined by using [21]. Thus, it can be used with less parameters and rules.

**Table 8** Conditions for the best results of ANFIS, MANFIS and AR–ANFIS

Time series	ANFIS		MANFIS		AR–ANFIS		ntest
	Number of cluster	Number of lag	Number of cluster	Number of lag	Number of cluster	Number of lag	
ABC	–	–	–	–	6	8	16
BIST100/2009	3	2	5	4	5	4	7
BIST100/2009	2	2	7	2	3	5	15
BIST100/2010	2	2	5	3	4	4	7
BIST100/2010	3	4	4	5	6	5	15
BIST100/2011	2	2	3	4	4	3	7
BIST100/2011	2	3	3	3	7	4	15
BIST100/2012	3	5	3	3	5	4	7
BIST100/2012	2	2	6	5	7	2	15
BIST100/2013	3	2	7	2	6	3	7
BIST100/2013	3	3	5	2	6	3	15
TAIEX/1999	3	3	3	1	6	3	45
TAIEX/2000	4	5	5	2	5	5	47
TAIEX/2001	3	2	5	4	4	5	43
TAIEX/2002	4	2	4	5	5	3	43
TAIEX/2003	4	2	3	3	6	3	43
TAIEX/2004	5	5	5	3	4	4	45

According to application results, it can be clearly seen that AR–ANFIS has successful forecasting performance and it can produce better forecasts for small length test sets. In the future studies, ARIMA and ANFIS can be hybridized in order to propose a new hybrid method.

#### Compliance with ethical standards

**Conflict of interest** The author declares that there is no conflict of interests regarding the publication of this paper.

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