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A combined forecasting approach based on fuzzy soft sets

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ABSTRACT

Forecasting the export and import volume in international trade is the prerequisite of a government's policy-making and guidance for a healthier international trade development. However, an individual forecast may not always perform satisfactorily, while combination of forecasts may result in a better forecast than component forecasts. We believe the component forecasts employed in combined forecasts are a description of the actual time series, which is fuzzy. This paper attempts to use forecasting accuracy as the criterion of fuzzy membership function, and proposes a combined forecasting approach based on fuzzy soft sets. This paper also examines the method with data of international trade from 1993 to 2006 in the Chongqing Municipality of China and compares it with a combined forecasting approach based on rough sets and each individual forecast. The experimental results show that the combined approach provided in this paper improves the forecasting performance of each individual forecast and is free from a rough sets approach's restrictions as well. It is a promising forecasting approach and a new application of soft sets theory.

1. Introduction

Forecasting the export and import volume in international trade is the prerequisite for a government to make a relevant policy and guide the international trade industry to develop healthier. However, the time series of export and import volume is always fuzzy and nonlinear. The government concerns various categories of products which have their own developing characteristics and trends and are related to local economic policies. For example, agricultural products have a very strong seasonal feature, and for products that the government gives vigorous support for have a characteristic of rising exponentially, yet for those relatively mature types of products have very strong growth curve characteristic. We believe the time series of each type of product is uncertain or fuzzy. Therefore, using an individual specific method, such as the Box–Jenkins model, the Holt–Winters model, exponential smoothing, artificial neural networks, regression over time, etc., to forecast export and import volume cannot always meet the government's demand of high forecasting accuracy. In this case, there should be an expert system or algorithm to solve the difficulty, which can identify and combine one or several kinds of individual forecasting methods that are able to fit the time series better among the above. The process is to select and combine individual forecasting methods among several ones by a set of linear weights, and the better the forecasting methods fit the data, the greater is the weight.

The concept of combining forecasts started with the seminal work of Bates et al. in [1]. They demonstrated that a suitable linear combination of two forecasts may result in a better forecast than the two original ones. Dickinson [2] examined some of the theoretical implications of combining forecasts using a minimum variance criterion and proved that the error variance of the combining forecasts is no greater than that of any of the individual one. Makridakis and Andersen [3] introduced the well-known M-competition, in which combination of forecasts from more than one model often lead to improved forecasting

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performance. Throughout the years, many others have developed models to find the "optimal" combination of forecasts and it has been applied to many fields, such as business management, meteorology, economy, engineering, etc.

Regarding combining forecasts, there are two ways to construct the weight of each individual forecast method. First, much effort has been done to find the optimal fixed weights (FW) of individual forecasts to minimize the within-sample sum of squared forecast errors. Second, some researchers prefer changing the weights from time to time. Deutsch and Granger et al. [4] used changing weights derived from switching regression models or from smooth transition regression models. In their empirical examples method, changed weight is not always well-performed compared with FW in terms of root mean squared errors (RMSE). Chan and Kingsman et al. [5] assessed the value of the improvement in forecasting performance among Simple Average, Fixed Weighting, Rolling Window Weighting and Highest Weighting, and presented an application of combining forecasts in inventory forecast of a leading bank in Hong Kong. Zhong and Xiao [6] proposed a method of determining the weighting coefficient based on rough sets theory, in which they used the significance of attributes in rough sets as the weighting criterion. Zhong and Xiao [7] are on the basis of [6]. They revised their work by introducing knowledge entropy to compute the significance of attributes in rough sets. Prudêncio and Ludermir [8] employed a machine learning method to define the weight of each individual forecast and they implemented a multi-layer perceptron neural network (MLP) to combine two forecasts which have been brought into wide use. Zhang [9] have used a generalized autoregressive conditional heteroskedasticity (GARCH) model to determine the optimal weights of component forecasts, and used the model to forecast an actual time series of electronic products. The results shows that this weight-varying combinational method performs better than other commonly used forecasting approaches which are based on single model selection criteria or fixed weights.

We believe the individual methods in combined forecasts are a description of the actual time series, which are fuzzy. Meanwhile, determining the optimal weights is a decision-making problem to find a vector, by which the combined forecasts can minimize the within-sample sum of squared forecasts errors. Therefore, we use the relative error of each individual forecast as the criterion to construct the membership function of fuzzy sets and make it reflect in a tabular form of fuzzy soft sets. This paper attempts to introduce the concept of fuzzy soft sets to construct the computing combined forecast algorithm. Since the sample of export and import volume dataset is small and the application has to process millions of kinds of time series, we choose fixed weight as the means of combination.

Many practical problems in engineering, social, management, economics etc. are fuzzy, imprecise, and uncertain. Soft sets theory is a newly-emerging tool to deal with uncertain problems. Molodtsov [10] initiated the concept of soft sets theory, in which the theory of probability, theory of fuzzy sets [11], and the interval mathematics were considered as mathematical tools for dealing with uncertainty, but all these theories have their own difficulties due to the inadequacy of the parameterization of the theory. He proposed the soft sets theory which may be free from such difficulties. On the basis of Molodtsov, Maji and Biswas [12] defined equality of two soft sets, subset and super set of soft set, complement of a soft set, null soft set, and absolute soft set with examples. Also they defined soft binary operation such as AND, OR and the operation of union, intersection and De Morgan's law. Hacı Aktaş and Naim Çağman [13] introduced the basic properties of soft sets to the related concept of fuzzy sets as well as rough sets, and then they gave a definition of soft group and derive basic properties using Molodtsov's definition of the soft sets. Jun and Park. [14] proposed the notion of soft ideals and idealistic soft BCK/BCI-algebras, and gave several examples. They also provided the relations between soft BCK/BCI-algebras and idealistic soft BCK/BCI-algebras and established intersection, union, AND operation, and OR operation of soft ideals and idealistic soft BCK/BCI-algebras.

With the establishment of soft sets theory, its application has boomed in recent years. Maji and Roy [15] addressed an application of soft sets theory in decision-making problems. Under Maji and Roy's inspiration, Xiao and Li et al. [16] introduced soft sets theory into the research of business competitive capacity evaluation. Mushrif and Sengupta et al. [17] proposed a new algorithm for classification of the natural textures. Chen and Tsang et al. [18] presented a revised definition of soft sets parameterization reduction, and compared with the related concept of attributes reduction in rough sets theory. Roy and Maji [19] use the notion of multi-observer and proposed a decision-making application of fuzzy soft sets. Moreover, an application example and its algorithm are also given. Although the algorithm is proved incorrect by Kong and Gao et al. [20], the fuzzy soft sets and multi-observer concept are valuable to successive researchers. Zou and Xiao [21] presented a data analysis approach of soft sets under incomplete information and gave an application example in quality evaluation of information systems.

The purpose of this paper is to introduce the soft sets theory for forecasting the export and import volume in international trade. We propose a combined forecasting approach based on the fuzzy soft sets (CFFSS) and an algorithm is also addressed in this paper. To prove the performance of the algorithm, we use the export dataset from 1993 to 2006 in Chongqing China and compare them with the combined forecasting approach based on the rough sets theory (CFRS) that was proposed in [7], and also with the individual forecasts as well. The results shows the fuzzy soft sets approach outperforms the individual forecast in most situations and is almost equal to the rough sets approach but has a low time-consuming operation compared with the rough sets approach. This is another new application of soft sets, which has enriched the soft sets theory application in real life. The initial work can be found in [19], which proposed a fuzzy soft sets conception and multi-observer data. In fact, in future work, we may consider other factors that affect export and import. Moreover, the powerful parameterization and Boolean operations may help us to construct a more practical and complicated model.

The rest of the paper is organized as follows. Section 2 introduces the basic principles of soft sets and fuzzy soft sets and gives a simple example; for more details one can refer to [19]. Section 3 gives a description of the approach taken in the study and the algorithm. Section 4 exams the performance of the approach with the export datasets from 1993 to 2006 in Chongqing China and compares the approach we provide with the rough sets approach proposed in [7], each individual forecast and some discussions. Finally Section 5 presents some conclusions from the research.

2. Preliminary

2.1. Soft sets

Definition 2.1 (*See [10]*). A pair (F, E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U.

In other words, the soft set is a parameterized family of subsets of the set U. Every set $F(\varepsilon)(\varepsilon \in E)$, from this family may be considered as the set of ε-elements of the soft sets (F, E), or as the set of ε-approximate elements of the soft set.

To illustrate this idea, let us consider the following example.

Example 2.1. Let universe $U = \{h_1, h_2, h_3, h_4\}$ be a set of houses, a set of parameters $E = \{e_1, e_2, e_3, e_4\}$ be a set of status of houses which stand for the parameters "beautiful", "cheap", "in green surroundings", and "in good location" respectively. Consider the mapping F be a mapping of E into the set of all subsets of the set U. Now consider a soft set (F, E) that describes the "attractiveness of houses for purchase". According to the data collected, the soft set (F, E) is given by

$$\{F, E\} = \{(e_1, \{h_1, h_3, h_4\}), (e_2, \{h_1, h_2\}), (e_3, \{h_1, h_3\}), (e_4, \{h_2, h_3, h_4\})\}$$

where $F(e_1) = \{h_1, h_3, h_4\}$, $F(e_2) = \{h_1, h_2\}$, $F(e_3) = \{h_1, h_3\}$ and $F(e_4) = \{h_2, h_3, h_4\}$.In order to store a soft set in computer, a two-dimensional table is used to represent the soft set (F, E). Table 1 is the tabular form of the soft set (F, E). If $h_i \in F(e_j)$, then $h_{ij} = 1$, otherwise $h_{ij} = 0$, where h_{ij} are the entries (see Table 1).

2.2. Fuzzy soft sets

The real world is inherently uncertain, imprecise and vague. Traditional mathematical tools cannot deal with such problems. The fuzzy sets theory is widely employed in such kinds of problems [11]. Maji and Biswas et al. [22] proposed the notion of fuzzy soft sets and an example of decision-making was discussed in [19] as well.

Definition 2.2 (*See [19,22]*). Let $\mathscr{P}(U)$ denotes the set of all fuzzy sets of U. Let $A_i \subset E$.

A pair (F_i, A_i) is called fuzzy soft sets over U, where F_i is a mapping given by

$$F_i : A_i \to \mathscr{P}(U). \tag{2.1}$$

In view of the above discussions, we present an example below.

Example 2.2 (*See [19]*). Consider fuzzy soft sets (F, A) over the universal $U = \{h_1, h_2, h_3, h_4, h_5\}$, in which U represents the set of house $A = \{\text{blackish, reddish, green}\}$, and

$$F(\text{blackish}) = \{h_1/.4, h_2/.6, h_3/.5, h_4/.8, h_5/1\},$$
$$F(\text{reddish}) = \{h_1/1, h_2/.5, h_3/.5, h_4/1, h_5/.7\},$$
$$F(\text{green}) = \{h_1/.5, h_2/.6, h_3/.8, h_4/.8, h_5/.7\}.$$

Also, we can express it in a tabular form in Table 2.

3. Combined forecasts model based on fuzzy soft sets

3.1. Combined forecasts model

Definition 3.1 (*See [7]*). Consider the time series forecasting problem,assume that y is an actual time series vector, and y_t $(t = 1, 2, \ldots, n)$ is its time-point, $c_1, c_2, \ldots, c_m$ denote m kinds of individual forecasts and let c_{jt} $(j = 1, 2, \ldots, m; t = 1, 2, \ldots, n)$ denote the ith forecasting value of individual forecasts at time-point t, and λ_j $(j = 1, 2, \ldots, m)$ is the ith weight coefficient of individual forecasts, $\hat{y}_t$ denote the forecast value at time-point t, and then the combined forecasting model can be written as follows

$$\hat{y}_t = \sum_{j=1}^{m} \lambda_j c_{jt}. \tag{3.1}$$

Clearly, if we have a known each weight coefficient $\lambda_i = (i = 1, 2, \ldots, m)$ then we can forecast $\hat{y}_t$ by the formula (3.1).

Table 1
The tabular representation of (F, E).

U	e_1	e_2	e_3	e_4
h_1	1	1	1	0
h_2	0	1	0	1
h_3	1	0	1	1
h_4	1	0	0	1

Table 2
An example of fuzzy soft sets.

U	**'Blackish'**	**'Reddish'**	**'Green'**
h_1	0.4	1	0.5
h_2	0.6	0.5	0.6
h_3	0.5	0.5	0.8
h_4	0.8	1	0.8
h_5	1	0.7	0.8

Table 3
A tabular form of fuzzy soft sets for CFFSS.

U	c_1	c_2	c_3	$\dots$	c_m
o_1	$f(\xi_{1,1})$	$f(\xi_{1,2})$	$f(\xi_{1,3})$	$\dots$	$f(\xi_{1,m})$
o_2	$f(\xi_{2,1})$	$f(\xi_{2,2})$	$f(\xi_{2,3})$	$\dots$	$f(\xi_{2,m})$
o_3	$f(\xi_{3,1})$	$f(\xi_{3,2})$	$f(\xi_{3,3})$	$\dots$	$f(\xi_{3,m})$
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
o_n	$f(\xi_{n,1})$	$f(\xi_{n,2})$	$f(\xi_{n,3})$	$\dots$	$f(\xi_{n,m})$

3.2. Weight determination based on fuzzy soft sets

Definition 3.2. Let (F, A) denote the fuzzy soft sets over a common universal $U = \{o_1, o_2, o_3, \dots, o_n\}$, where U represents the universal set of time-points in time series, and $A = \{c_1, c_2, c_3, \dots, c_m\}$ denote the set of individual forecasts, in which c_j $(j = 1, 2, 3, \dots, m)$ is an individual forecasts, and then F is a mapping given by

$$F : A \to \mathscr{P}(U). \tag{3.2}$$

Definition 3.3. Let ξ_{ij} $(i = 1, 2, \dots, n; j = 1, 2, \dots, m)$ be a fuzzy variable defined on the fuzzy soft sets (F, A), and then its membership function is defined as follows

$$f(\xi_{ij}) = (1 - |\hat{y}_{ij} - y_i|/y_i) \vee 0. \tag{3.3}$$

The definition states that the membership function of each fuzzy variable is the forecast accuracy of an individual forecasts at a time-point, in which $\hat{y}_{ij}$ denote the forecasted value of the individual forecasts c_j at ith time-point and y_i denote the actual value at ith time-point. In some extreme situation, $(1 - |\hat{y}_{ij} - y_i|/y_i)$ may be negative, and we interpret accuracy as a number between 0% and 100%, thus rigorously we add a maximum operation with 0 to the formula. Obviously, $f(\xi_{ij}) \in [0, 1]$ and $f(\xi_{ij})$ is a measure of how close the actual are to the forecast quantity. Furthermore, we can represent the fuzzy soft sets (F, A) as a tabular form, see Table 3.

Definition 3.4. Suppose that a vector $W = [w_1, w_2, w_3, \dots, w_m]$, in which w_j $(l = 1, 2, 3, \dots, m)$ denote the sum of the individual forecast c_j's forecasted value, and then

$$w_j = \sum_{i=1}^{n} f(\xi_{ij}). \tag{3.4}$$

Also, we define the weight coefficient λ_j of the individual forecasts c_j as follows

$$\lambda_j = \frac{w_j}{\sum\limits_{i=1}^{m} w_i}. \tag{3.5}$$

Table 4
Monthly data from 1993 to 2006 in Chongqing China.

	Actual	C1	C2	C3
1	11,208,336.00	11,208,336.00	11,208,336.00	11,208,336.00
2	31,864,836.00	31,864,836.00	31,864,836.00	31,864,836.00
3	21,888,531.00	36,173,506.00	21,888,531.00	21,888,531.00
4	20,294,793.00	14,775,098.00	20,294,793.00	21,653,901.00
5	22,199,802.00	13,157,754.00	19,676,569.00	24,682,720.00
6	30,792,280.00	27,395,528.00	21,538,307.00	21,461,042.00
7	36,140,781.00	38,421,041.00	26,163,354.00	24,428,958.00
…	…	…	…	…
164	319,501,346.00	284,076,498.00	314,944,453.00	294,243,826.00
165	289,978,690.00	303,831,328.00	333,131,987.00	302,194,979.00
166	277,099,968.00	278,763,400.00	334,627,298.00	298,234,445.00
167	.	261,866,840.00	325,734,319.00	295,526,668.00

Table 5
The tabular representation of the fuzzy soft sets tabular form.

U	C1	C2	C3
1	1	1	1
2	1	1	1
3	0.3473	1	1
4	0.7280	1	0.9330
5	0.5926	0.8863	0.8881
6	0.8896	0.6994	0.6969
7	0.9369	0.7239	0.6759
…	…	…	…
164	0.8891	0.9857	0.9209
165	0.9522	0.8511	0.9578
166	0.9939	0.7923	0.9237

3.3. Algorithm

Step1: Input the actual time series y_t $(t = 1, 2, \ldots, n)$, and forecast it with each individual forecast y_{tj} $(t = 1, 2, 3, \ldots, n; j = 1, 2, 3, \ldots, m)$
Step2: Use formula $f(\xi_{ij}) = (1 - |\hat{y}_{ij} - y_i|/y_i) \vee 0$ to construct fuzzy soft sets (F, A)
Step3: Initial vector $W = [0, 0, 0, \ldots, 0]$ and then calculate each w_j with formula $w_j = \sum_{i=1}^{n} f(\xi_{ij})$
Step4: Compute each weight λ_j with $\lambda_j = w_j / \sum_{i=1}^{m} w_i$
Step5: forecast with formula $\hat{y}_t = \sum_{j=1}^{m} \lambda_j c_{jt}$.

4. Empirical results and discussions

4.1. Empirical analysis of export trade forecasting for Chongqing China

We implement the algorithm with Microsoft Visual Studio 2005 in C# language. Table 4 gives the monthly data of export from 1993 to 2006 in Chongqing Municipality of China, in which Column C1, C2 and C3 represent the forecasting time series by Autoregressive Integrated Moving Average Model (ARIMA) that is proposed in [23], Holt–Winters seasonal model (HW) and Moving Average Model (MA) respectively. And these actual data are all derive from the statistical yearbook of Chongqing Foreign Trade and Economic Relations Commission.

The tabular representation of the fuzzy soft sets can be seen in Table 5. And the membership function $f(\xi_{ij})$ is defined in formula (3.3).

By means of formula (3.5), we can get the combined forecasting weight coefficient vector which is [0.4502, 0.2865, 0.2631]. The mean absolute percentage error (MAPE) and the root mean squared error (RMSE), both of which are the most commonly used error measures in business, are used to evaluate the forecast models in this paper [24].

The results of these analyses are summarized in Table 6. The data plotted in Fig. 1 consists of 165 (more than 13 years) of monthly time-points and 166 CFFSS time-points, began in January 1996, and the horizontal coordinate refers to time-point and the vertical the export volume. Vividly, Fig. 1 reveals the combined forecasting approach fit the actual time series well. Furthermore, as shown in Table 6 CFFSS reduces the RMSE and MAPE of the three individual forecasts, thus it improves the forecasting accuracy of each individual forecast.

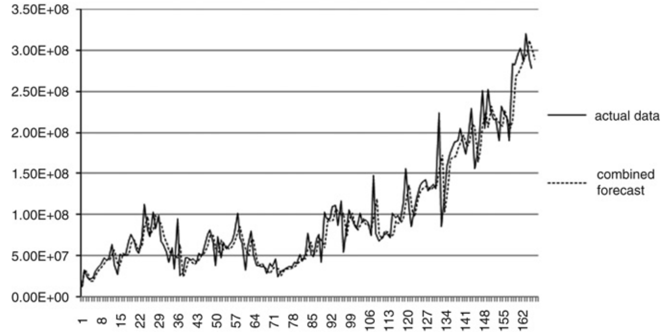

Fig. 1. The comparison between actual data and CFFSS.

Table 6
Forecasting error.

Forecasts	Sum of RMSE	MAPE
ARIMA	21704695.54	0.20
HW	24977400.63	0.22
MA	23694402.95	0.22
CFFSS	18990026.07	0.16

Table 7
The definition of dataset.

Dataset	Code	Dataset	Code
Electromechanical equipment	d1	Europe	d6
Chemical products	d2	USA	d7
Light industrial products	d3	Japan	d8
Asia	d4	Germany	d9
North America	d5	Gross volume of chongqing	d10

Table 8
The comparison with CFRS and each individual forecast.

Dataset	MAPE				
	CFRS	CFFSS	ARIMA	HW	MA
d1	0.2	0.2	0.24	0.27	0.24
d2	0.19	0.18	0.21	0.18	0.18
d3	0.23	0.24	0.34	0.33	0.29
d4	0.24	0.24	0.27	0.31	0.25
d5	0.2	0.19	0.24	0.24	0.23
d6	0.15	0.16	0.18	0.21	0.21
d7	0.21	0.2	0.24	0.24	0.23
d8	0.27	0.26	0.33	0.26	0.25
d9	0.24	0.25	0.29	0.33	0.29
d10	0.15	0.16	0.28	0.22	0.21

4.2. Comparison with CFRS

In order to compare CFFSS algorithm with CFRS of the forecast accuracy, and the combined forecasting approach based on the rough sets theory [6,7,25,26] is implemented with Microsoft Visual Studio 2005 in C# language as well. We employ the monthly export data from 1993 to 2006 in Chongqing China to compare CFFSS with CFRS and each individual forecast. Besides the gross volume of Chongqing we also include other different datasets, see Table 7.

Table 8 offers a comparison between CFFSS and CFRS and each individual forecast. The results reveal that CFFSS and CFRS are more or less of equal performance in many situations. Moreover, the MAPE of CFFSS and CFRS are lower than each individual forecast.

Although the performance of CFFSS and CFRS are almost equal, the rough sets approach has some drawbacks. Firstly, the CFRS algorithm uses many highly time-consuming operations. Secondly, it has more steps than the CFFSS. Since the rough sets process discrete information system, the first step of CFRS is to construct a discrete information system. And the discretization of the continuous data method we employed is an equal distance algorithm, in which the time complexity is $O(N)$. Moreover, the discretization of CFRS is arbitrary and needs being specified manually. The second step of CFRS is to compute the equivalence classes of each attribute, in which the time complexity is $O(NM^2)$ [27].

Table 9
The comparison of characteristic between CFFSS and CFRS.

	Time-consuming	Time complexity	Restriction	Robustness
CFRS	High	$O(N) + O(NM^2)$	Restricted to "thin" data (large sample and the number of component forecasts is small in term of sample)	Low
CFFSS	Low	$O(NM)$	None	High

It is a dilemma for CFRS, for if on one hand it is discretized into too many intervals, there will be more time-points for computing the conditional entropy or it will throw an exception of dividing by zero when computing the significance of each attribute; on the other hand if discretized into too few intervals then it will be so rough for extracting each individual forecast's feature that we cannot get a weight vector for combined forecasting with high forecasting accuracy. In other words, the process of computing conditional entropy tends to throw a dividing by zero exception when there is a high ratio of the number of component forecasts time-points i.e. more component forecasts but fewer time-points. So it is not very robust in a sense. In our experiments, when the number of individual forecasts is 3 and the number of time-points is 166, discretizing into 4 intervals is suitable for a better result. Whereas when the number of component forecasts is very large and the number of time-point sets is small, a divide by zero exception for the computing of conditional entropy will always be thrown.

Comparing with the CFRS, CFFSS can achieve an almost equal performance to the CFRS. Obviously, the CFFSS has a much lower time-consumption and is free from such complex operations as discretization, computing equivalence class and conditional entropy. And its time complexity is only $O(NM)$, while the CFRS is $O(N) + O(NM^2)$. In practice, there are many kinds of data, thus such highly time-consuming operations cannot be afforded. More importantly, since CFFSS is free from such operations, there is no such restriction of the number of component forecasts and the number of time-points. Table 9 is the comparison of characteristics between CFFSS and CFRS.

5. Conclusion

This paper proposes a combined forecasting approach based on fuzzy soft sets by using an export dataset of Chongqing Municipality China from 1993 to 2006 and compares CFFSS with the combined forecasting approach based on the rough sets theory (CFRS) which was proposed by [7], and also with the individual forecasts as well. The empirical results show that CFFSS outperforms the individual forecast in most situations and is almost equal to CFRS but is a low time-consuming operation and is free from CFRS's restriction of the number of component forecasts and the length of observation data. It is a promising combined forecasting approach and is another new application of soft sets, which has enriched the wide application of soft sets theory in real life.

The core of this approach is to construct the fuzzy membership function and the tabular form of the fuzzy soft sets model. This paper uses the accuracy of each time-point as the criterion of fuzzy membership and employs the sum of fuzzy membership to design the determination of weight vector of combined forecasts.

Although CFFSS can reduce the error of individual forecasts, the export volume in international trade is affected by many factors such as supply and demand, and to a large extent, governmental intervention (trade barriers and subsidies), GDP, FDI, interest rate, tax reimbursement, price of labor power, etc. It is difficult to forecast the export volume very accurately with a single factor, which requires a multidimensional time series to analyze such a problem. Since the multidimensional time series are always fuzzy and nonlinear, the fuzzy soft sets are very suitable to resolve such a problem with its parameterization, multi-observer notion in the further study and binary operations. In fact, to it can be added multidimensional time series by the notion of multi-observers and the use of soft binary operations for more complex model construction to describe such kinds of problem.

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