(一)性能指標

　　為了協助比較不同實驗方法之效能，此單元將介紹以下幾種性能指標作為比較的標準，而在介紹性能指標前將定義多個公式以利此單元後半部性能指標公式之簡潔化。

(A) t_i 和 y_i 間的誤差

$$e_i = t_i - y_i, \tag{a}$$
$$i = 1, 2, \dots, n$$

where

n = 誤差值的數量
t_i = 第 i 對資料的真實數據
y_i = 第 i 對資料對真實數據的預測值

(B) 真實數據 t_i 的標準差, $\sigma_t = \text{std}(t_i)$ ⟨b⟩

(C) 預測值 y_i 的平均值, $\bar{y} = \text{mean}(y_i)$ ⟨c⟩

(D) 真實數據 t_i 的平均值, $\bar{t} = \text{mean}(t_i)$ ⟨d⟩

(E) 誤差 e_i 與 t_i 的百分比, $p_i = 100\left(\frac{e_i}{t_i}\right)$ ⟨e⟩

(F) 誤差 e_i 與 t_i 的百分比的共軛, $p_i^* = \text{conjugate}(p_i)$ ⟨f⟩

(G) 誤差 e_i 的絕對值, $|e_i| = \text{absolute}(e_i)$ ⟨g⟩

　　以下列出此篇報告比較基礎的性能指標之定義：

(A) 平均絕對偏差(Average absolute deviation, AAD)[1]，其公式為

$$\text{AAD} = mean(|e_i - mean(e_i)|) \tag{h}$$

(B) 絕對偏差中位數 (Median absolute deviation, MAD) [2]，其公式為

$$\text{MAD} = median(|e_i - median(e_i)|) \tag{i}$$

(C) 平均絕對誤差 (Mean absolute error, MAE)[3]，其公式為

$$\text{MAE} = mean(|e_i|) \tag{j}$$

(D) 平均絕對誤差百分比(Mean absolute percentage error, MAPE)[4]，其公式為

$$\text{MAPE} = mean(|p_i|) \tag{k}$$

(E) 平均絕對誤差比例(Mean absolute scaled error, MASE)[3]，其公式為

$$\text{MASE} = mean(|q_i|) \tag{l}$$

(F) 絕對誤差中位數 (Median absolute error, MdAE) [3]，其公式為

$$\text{MdAE} = median(|e_i|) \tag{m}$$

(G) 均方誤差絕對值 (Mean square absolute error, MSAE)，其公式為

$$\text{MSAE} = mean(|e_i|^2) \tag{n}$$

(H) 均方差(Mean squared error, MSE)，其公式為

$$\text{MSE} = mean(e_i^2)[5] \tag{o}$$

(I) 標準均方根誤差(Normalized Mean Square Error, NMSE) [6]，其公式為

$$NMSE = \frac{MSE}{\sigma_t^2} \tag{p}$$

（J）無因次誤差指數(Non-dimensional error index, NDEI)[8]，其公式為

$$NDEI = \sqrt{NMSE} = \frac{RMSE}{\sigma_t} \tag{q}$$

（K）平均絕對偏差百分比(Percentage mean absolute deviation , PMAD)[7]，其公式為

$$PMAD = \frac{MAE}{mean(|t_i|)} \tag{r}$$

（L）均方根差(Root mean square error, RMSE)[3]，其公式為

$$RMSE = \sqrt{MSE} \tag{s}$$

（M）均方根誤差百分比（Root mean square percentage error , RMSPE)[9]，其公式為

$$RMSPE = \sqrt{mean(p_i^2)} \ \ (or \ \sqrt{mean(p_i p_i^*)} \ if \ p_i \ is \ complex-valued) \tag{t}$$

（N）$RR = 1 - \frac{\sum_{i=1}^{n}(y_i - \bar{t})^2}{\sum_{i=1}^{n}(t_i - \bar{t})^2}$ \tag{u}

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