Accepted Manuscript

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PII: S0378-4371(17)30922-6
DOI: https://doi.org/10.1016/j.physa.2017.09.030
Reference: PHYSA 18635

To appear in: *Physica A*

Received date : 16 May 2017
Revised date : 21 August 2017

Please cite this article as: Z. Pan, L. Liu, Forecasting stock return volatility: A comparison between the roles of short-term and long-term leverage effects, *Physica A* (2017), https://doi.org/10.1016/j.physa.2017.09.030

Forecasting stock return volatility: A comparison between the roles of short-term and long-term leverage effects[☆]

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Abstract

In this paper, we extend the GARCH-MIDAS model proposed by [1] to account for the leverage effect in short-term and long-term volatility components. Our in-sample evidence suggests that both short-term and long-term negative returns can cause higher future volatility than positive returns. Out-of-sample results show that the predictive ability of GARCH-MIDAS is significantly improved after taking the leverage effect into account. The leverage effect for short-term volatility component plays more important role than the leverage effect for long-term volatility component in affecting out-of-sample forecasting performance.

Keywords: Volatility, GARCH-MIDAS, Leverage effect, Forecasting

JEL Classification: C58, C22, C53

1. Introduction

Modeling and forecasting financial market volatility is an important issue in the area of both financial economics and econophysics [2, 3, 4, 5, 6]. In particular, the relationship between return and volatility is greatly interested by academics [7, 8, 9]. In particular, the

[☆]We would like to show our sincere gratitude to three anonymous reviewers whose comments and suggestions greatly improved the quality of the manuscript. This work is supported by the National Science Foundation of China under the grant Nos. 71601161 (Zhiyuan Pan), 71401077 (Li Liu) and 71771124 (Li Liu). Li Liu's work is also sponsored by Qing Lan Project in Jiangsu province, and the financial support from the Top-notch Academic Programs Project of Jiangsu Higher Education Institutions (PPZY2015B104).

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"leverage effect" that negative stock returns lead to higher future volatility than positive returns is a well-established fact in the literature. The models used to detect the leverage effect include ARCH-type models (see, e.g., [10, 11, 12]) for daily or lower frequency data and the realized volatility models for intraday high frequency data (see, e.g., [13, 14, 15, 16, 17]). Aforementioned papers focus on the impact of the sign of returns on the total volatility. In this paper, we build upon these studies by disentangling the short-term and long-term leverage effects.

The plausible explanation about the asymmetric relationship between the sign of return and volatility is based on financial leverage. The decrease in the value of stock (i.e., negative return) increases financial leverage, which makes the stock riskier and therefore leads to increase in volatility [18, 19]. This hypothesis is so popular that leverage effect has been synonymous with the statistical relation itself, rather than the hypothesized explanation[1]. Actually, the financial leverage hypothesis can only explain the asymmetric volatility in the short-term. Since the financial leverage is always defined by the ratio of firm debt (noncommon equity liabilities) to equity value (see, e.g., [21, 22]), the decrease in lower firm value (i.e., negative return) indeed cause short-term higher financial leverage because the debt is less likely to change at daily or weekly frequency. However, the empirical evidence in the literature has shown that the expected stock returns are positively related to debt/equity ratio in the long-term [21]. In this sense, negative returns are always linked to lower debt/equity ratio, in contrast with the fact that the financial effect hypothesis advocates. Is the asymmetric relation between stock return and volatility in the short-term consistent with that in the long-term? This issue has been investigated in few studies except some notable papers such as [23].

The distinction between short-term and long-term volatility leverage effect is also of importance for market participants. According to the Heterogeneous Market Hypothesis of [24] , a financial market is composed by participants having different trading frequencies.

[1]Nowadays, leverage effect is not exactly what it sounds. For example, [20] find that the leverage effect is just as strong if not stronger, implying that the inverse relationship between price and volatility is not based on leverage.

Some investors such as dealers, market makers and daily speculators, with daily or even higher dealing frequency, pay attention to both short-term and long-term dynamics of stock prices. The other some investors such as commercial organization and pension fund investors, with monthly or lower dealing frequency, concentrate on stock market activity over the long horizon only. The behaviors of investors with various time horizons cause different types of volatility components. Therefore, the various types of investors are concerned about whether the different short-term or long-term component of stock return volatility is more strongly related to past negative return.

We modify the GARCH-MIDAS model proposed by [1] by adding the asymmetry terms to capture the short-term and long-term leverage effect. The outstanding advantage of GARCH-MIDAS over the traditional ARCH-type models is that it can decompose the total volatility into two unobserved components: a short-term volatility component with higher frequency captured by a GARCH process [25] and a long-term volatility component with lower frequency modeled by a mixed-frequency data sampling (MIDAS) regression [26]. Certainly, our specification improves upon the leverage effect model of [23] in the way that it can deal with the mixed-frequency data. To account for short-term leverage effect, we extend the GARCH to GJR specification of [10] to describe the dynamics of short-term volatility component. For the purpose of detecting the long-term leverage effect, we extend the standard MIDAS regression for long-term volatility component in several ways[2]. First, we use an estimator proposed by [14] called "realized semivariance", which decompose monthly realized variance into a component that relates only to positive daily returns and a component that relates only to negative daily returns. A standard Wald method is employed to examine whether long-term leverage effect exists by testing the equal impacts of positive and negative semivariances on future long-term volatility. Second, we borrow the methodology of [17] which uses monthly realized semivariances to construct signed jumps and then test

[2]Indeed, we can also use the specification of HAR model of [27] to account for the long-term leverage effect. An outstanding advantage of MIDAS over HAR is that it can be used to model the mixed-frequency data and therefore can capture the effect of low-frequency macroeconomic variables on stock volatility in the futures work.

for the null hypothesis of equal impacts of positive and negative signed jumps.

We put forward three classes of asymmetric GARCH-MIDAS models: 2 models with only short-term leverage effect, 3 models with only long-term leverage effect and 3 models with dual leverage effects. The daily Standard and Poor 500 (S&P 500) index covering the period from January 2, 1991 to December 31,2015 is collected to do empirical analysis. Our in-sample evidence reveals the significant leverage effect for both short-term and long-term volatility components. In detail, the daily negative returns can cause higher short-term volatility than positive returns. Negative monthly semivariance (or negative signed jumps) provides significantly stronger predictive content for long-term volatility than positive semivariance (or positive signed jumps).

Furthermore, we investigate whether the differentiation between short-term and long-term leverage effects can improve the out-of-sample predictive ability of volatility models. We consider the forecasting horizons of 1, 5 and 22 days. The rolling window technique is applied to generate volatility forecasts from the year 2006 to 2015. We use six popular criteria of loss function to evaluate the accuracy of volatility forecasts, rather than make a single choice. The [28] method is used to test for the significance of the differences of loss functions between the benchmark of the standard GARCH-MIDAS and each of the asymmetric extensions. Our results suggest that accounting for leverage effects can lead to significantly more accurate volatility forecasts for all three horizons. The short-term leverage effect contributes more heavily than the long-term leverage effect to the improvement of predictive ability.

Because risk managers are more concerned about the predictive accuracy of high volatility, we further evaluate the forecasting performances of 8 asymmetric GARCH-MIDAS models when large fluctuations occur. The financial crisis in mid-2008-2009 caused a large crash in stock price. The unconditional variance of S&P 500 return during the financial crisis is almost as five times high as the variance during the whole sample period. Of the ten largest one-day movements in the S&P 500 index, eight appeared during the financial crisis period. Therefore, the financial crisis provides us a good example to evaluate the forecasting accuracy for large volatility. We find the consistent evidence that the asymmetric GARCH-

MIDAS with leverage effect can significantly beat the benchmark model without leverage effect. The reduction of forecasting loss of the asymmetric models over the benchmark model can achieve 16% for some cases. The predictive ability of the models accounting for long-term leverage effect even becomes stronger during the fluctuated time.

The loss functions are somewhat sensitive to the extreme value of forecasting error. Motivated by this fact, we calculate the success ratio, the number of the days when asymmetric GARCH-MIDAS forecasts are closer to the actual volatility than the benchmark forecasts divided by the total number of out-of-sample days. A success ratio higher than 0.5 suggests that the asymmetric model tested generates more accurate forecasts than the benchmark model most of time. Our results show that the success ratios GARCH-MIDAS with leverage effects are as high as 0.6 and are significantly higher than 0.5. The incorporation of short-term leverage effect in GARCH-MIDAS is more helpful to obtain higher success ratios than the incorporation of long-term leverage effect. The dependence test results also indicate that the event that each asymmetric model leads to more accurate forecasts occurs persistently. Therefore, it is possible to predict whether the models with leverage effects perform better out-of-sample.

The remainder of this paper is organized as follows: In Section 2, we give the methodology of extending GARCH-MIDAS to take into account short-term and long-term leverage effect. Section 3 shows the data description. Section 4 reports the main empirical results. The last section concludes the paper.

2. Econometric models

We investigate the effects of short-term and/or long-term "leverage effects" on the predictability of stock return volatility. For this purpose, we modify the GARCH-MIDAS models by incorporating asymmetry terms as additional explanatory variables. Prior to give the specifications of asymmetric GARCH-MIDAS models, we first describe the standard GARCH-MIDAS briefly.

2.1. GARCH-MIDAS models

The GARCH-MIDAS model proposed by [1] decomposes the volatility into two components, a short-term component driven by a GARCH process for high-frequency data and a long-term volatility component captured by a MIDAS process for low-frequency data[3]. This model can flexibly deal with mixed frequency data and is increasingly popular in analyzing the macroeconomic determinants of financial return volatility. In the GARCH-MIDAS specification, the return $r_{i,t}$ on high-frequency i (say day) in low-frequency t (say month) is modeled as,

$$\begin{aligned} r_{i,t} &= \mu + \sqrt{\tau_t \times g_{i,t}}\epsilon_{i,t}, \qquad \forall i = 1, 2, \cdots, N_t, \\ \epsilon_{i,t} | I_{i-1,t} &\sim \mathcal{N}(0, 1), \end{aligned} \tag{1}$$

where $I_{i-1,t}$ denotes the information set available at day $i-1$ of period t and N_t is the number of days in month t. [1] use the GARCH process of [25] to model the short-term volatility component, $g_{i,t}$, which can be given as,

$$g_{i,t} = (1 - \alpha - \beta) + \alpha \frac{\epsilon_{t-1}^2}{\tau_t} + \beta g_{i-1,t}, \tag{2}$$

The process of long-term component τ_t is captured by a MIDAS regression,

$$\tau_t = m + \theta \sum_{k=1}^{K} \varphi_k(\omega) RV_{t-k}, \tag{3}$$

where RV_t is the realized volatility in month t, defined as,

$$RV = \sum_{i=1}^{N_t} r_{i,t}^2. \tag{4}$$

To incorporate more updated information about long-term volatility, we alternatively consider a rolling window specification for the MIDAS filter following [1]. In detail, we

[3]Note that the short-term and long-term components are not observable in real-time but the model-implied series.

remove the restriction that τ_t is fixed for month t and let τ and g both change at daily frequency. This rolling window RV can be defined as follows:

$$RV_i^{(rw)} = \sum_{j=1}^{N} r_{i-j}^2, \tag{5}$$

where r_{i-j} denotes the jth daily return before the day i across various period t. To be consistent, we consider the monthly realized volatility by setting N=22. In this way, equation (3) becomes,

$$\tau_i^{(rw)} = m^{(rw)} + \theta^{(rw)} \sum_{k=1}^{K} \varphi_k(\omega) RV_{i-k}^{(rw)}, \tag{6}$$

for simplicity, we drop the superscript "(rw)" in the following equations. Following [29], the flexible and popular one-parameter Beta polynomial is employed as the weighting scheme,

$$\varphi_k(\omega) = \frac{(1-k/K)^{\omega-1}}{\sum_{j=1}^{K}(1-j/K)^{\omega-1}}. \tag{7}$$

2.2. GARCH-MIDAS with short-term leverage effect

To account for the short-term leverage effect, we use the GJR(1,1) specification [10] to model the process of short-term component, given by,

$$g_{i,t} = (1-\alpha-\gamma/2-\beta) + (\alpha+\gamma I(r_{i-1,t}<0))\frac{\epsilon_{i-1,t}^2}{\tau_t} + \beta g_{i-1,t}, \tag{8}$$

where $I(\cdot)$ is an indicator function which takes the value of 1 when the condition in the parenthesis is satisfied and 0 otherwise. The parameter γ captures the "leverage effect".

We use two different specifications to model the long-term component. The first is the MIDAS regression, which is exactly the same to equation (3). The second model based on the decomposition of realized volatility is in the spirit of [17], which is given by,

$$\tau_i = m + \theta^J \sum_{k=1}^{K} \varphi_k(\omega)\Delta J_{i-k}^2 + \theta^{BV} \sum_{k=1}^{K} \varphi_k(\omega) BV_{i-k}, \tag{9}$$

where $\Delta J_i^2 = RS_i^+ - RS_i^-$ is the signed jump variation, a simple statistic to isolate the information from signed jumps; and $RS_i^+ \sum_{j=1}^{N} r_{i-j}^2 I(r_{i-j}>0)$ and $RS_i^- \sum_{j=1}^{N} r_{i-j}^2 I(r_{i-j}<0)$ are the realized semivariance measures which can capture the variation only due to positive and negative returns, respectively [14]. The bipower variation (BV) is defined as $BV_i = \sum_{j=1}^{N} |r_{i-j} r_{i-j-1}|$.

2.3. GARCH-MIDAS with long-term leverage effect

To take into account the leverage effect in the long-term volatility component, we modify the GARCH-MIDAS by adding the asymmetry variables in MIDAS regressions. This is inspired by [17] who investigate the role of "leverage effect" in realized volatility models. Specifically, these modified GARCH-MIDAS models employ the same GARCH to capture short-term volatility component (equation (2)), but use several different MIDAS regressions for long-term component. The first model takes the semivariances as the explanatory variables for realized volatility, given by,

$$\tau_i = m + \theta^+ \sum_{k=1}^{K} \varphi_k(\omega) RS_{i-k}^+ + \theta^- \sum_{k=1}^{K} \varphi_k(\omega) RS_{i-k}^-. \tag{10}$$

The standard Wald test for the null hypothesis $\theta^+ = \theta^-$ is employed to examine the significance of leverage effect. The second specification additionally uses a lagged realized volatility interacted with an indicator for negative returns, which is consistent with the method for detecting the classic leverage effect. This specification can be written as follows:

$$\tau_i = m + \theta^+ \sum_{k=1}^{K} \varphi_k(\omega) RS_{i-k}^+ + \theta^- \sum_{k=1}^{K} \varphi_k(\omega) RS_{i-k}^- + \delta \sum_{k=1}^{K} \varphi_k(\omega) RV_{i-k} \cdot I(R_{i-k} < 0), \tag{11}$$

where R_i is the accumulated return over past N days. We can test for the null hypothesis $\theta^+ = \theta^-$ and $\delta = 0$ to find whether the leverage effect is significant based on this specification.

The third model considers the asymmetric predictive information of signed jumps, given by,

$$\tau_i = m + \theta^{J+} \sum_{k=1}^{K} \varphi_k(\omega) \Delta J_{i-k}^{2+} + \theta^{J-} \sum_{k=1}^{K} \varphi_k(\omega) \Delta J_{i-k}^{2-} \theta^{BV} \sum_{k=1}^{K} \varphi_k(\omega) BV_{i-k}, \tag{12}$$

where the positive and negative jumps are defined as $\Delta J_i^{2+} = (RS_i^+ - RS_i^-) I(RS_i^+ - RS_i^- > 0)$ and $\Delta J_i^{2-} = (RS_i^+ - RS_i^-) I(RS_i^+ - RS_i^- < 0)$, respectively. We can test for the null hypothesis $\theta^{J+} = \theta^{J-}$ to examine the significance of leverage effect using the standard Wald statistic.

2.4. GARCH-MIDAS with dual leverage effects

We have separately considered the short-term or long-term leverage effects using the asymmetric extensions of GARCH-MIDAS models. This subsection develops a set of models with leverage effect in processes of both short-term and long-term volatility components. Similarly, we use GJR to model short-term volatility component and three GARCH-MIDAS specifications developed in Section 2.3 to model long-term component. In this way, we can obtain 3 GARCH-MIDAS models with dual leverage effects.

In summary, we have a total of 8 extensions of GARCH-MIDAS. 2 of them take into account leverage effect in the short-term, 3 models consider leverage effect in the long-term and the remaining 3 models account for both short-term and long-term one. A typology of these models is given in Table 1.

Table 1 about here

3. Data

We collect daily data of Standard and Poor 500 (S&P 500) index[4]. Our data sample covers the period from January 2, 1991 to December 31, 2015, resulting in 6300 observations. We obtain the daily returns by computing the first order differences of logarithmic prices. Following the majority of the literature, the squared daily return is taken as the actual value of ex post volatility. Figure 1 represents the graphical illustrations of daily price, return and volatility of S&P 500 index.

Figure 1 about here

4. Empirical results

In this section, we give the in-sample results about the significance of leverage effect using the asymmetric extensions of GARCH-MIDAS. We also show the out-of-sample performances of asymmetric GARCH-MIDAS models relative to the original symmetric one to

[4]Data source: Federal Reserve Bank of Saint Louis (http://www.stlouisfed.org/)

find whether introducing the leverage effect can improve the forecast ability. The predictive ability is evaluated under different criteria.

4.1. In-sample results

Figure 2 plots the in-sample total conditional volatility and its secular component obtained from each of 8 modified GARCH-MIDAS models. We can see that the secular component dominates the dynamics of total volatility. Table 2 reports the estimation results of the GARCH-MIDAS models with "leverage effects". First, looking at the parameter estimates in the equation for short-term volatility component, the asymmetric parameter γ is significantly positive. This evidence indicates the existence of leverage effect in the dynamics of short-term volatility component. The values of $\alpha+\beta+\gamma/2$ which are rather close to 1 reveal the stylized fact of strong short-term volatility persistence. Second, turning to the MIDAS regression for long-term component, the sign of the coefficient of positive return semivariance θ^+ is mixed, depending on the model specification. The significance of θ^+ is not consistent across various models. Differently, the coefficient of negative semivariance θ^- is significantly positive for each of 4 models (Models 3, 4, 6, and 7). This result suggests that higher negative semivariance can lead to higher long-term volatility in the future, while the impact of positive semivariance is minor. The Wald statistic testing for the null hypothesis $\theta^+ = \theta^-$ is 6.127 and shows rejection at 5% significance level, implying the significant leverage effect in the process of long-term component of realized volatility. The long-term leverage effect is also revealed by the significantly positive coefficient δ in Model 4 and Model 7, which tells us that higher volatility is more strongly related to negative returns in the long-term. The absolute value of coefficient of negative return jump θ^{J-} is greater than θ^{J+} in Model 5 and Model 8. The Wald statistic which is 8.509, rejects the null hypothesis $\theta^{J-} = \theta^{J+}$, also suggesting that negative (positive) jumps lead to significantly higher (lower) long-term volatility.

Table 2 and Figure 2 about here

4.2. Out-of-sample results

Our in-sample evidence shows the significant leverage effect in the processes of both short-term and long-term components of daily volatility. To investigate implication of leverage effect for the predictive ability of GARCH-MIDAS models, we evaluate their out-of-sample forecasting performances. We employ the rolling window technique to generate out-of-sample volatility forecasts. In detail, the initial subsample for parameter estimation covers the period of 15 years from January 1991 to December 2005. The actual volatility proxied by squared daily return during the remaining 10 years from January 2006 to December 2015 is used to evaluate the forecasting accuracy. When adding a new observation to the estimation sample, we discard the earliest one. In this way, the length of estimation sample is fixed. Figures 3-5 plot the volatility forecasts of GARCH-MIDAS models with leverage effect, as well as the true volatility, for the forecasting horizons of 1 day, 5 days (1 week) and 22 days (1 month), respectively.

Figures 3-5 about here

To evaluate the accuracy of volatility forecasts, we follow the literature in using loss functions. Because the evaluation results rely on the selection of loss function, we consider six popular criteria rather than make a single choice. These loss criteria are given as follows:

$$MSE = \frac{1}{M}\sum_{i=1}^{M}(\sigma_i^2 - \hat{\sigma}_i^2)^2, \quad (13)$$

$$MAE = \frac{1}{M}\sum_{i=1}^{M}|\sigma_i^2 - \hat{\sigma}_i^2|, \quad (14)$$

$$HMSE = \frac{1}{M}\sum_{i=1}^{M}(1 - \sigma_i^2/\hat{\sigma}_i^2)^2, \quad (15)$$

$$HMAE = \frac{1}{M}\sum_{i=1}^{M}|1 - \sigma_i^2/\hat{\sigma}_i^2|, \quad (16)$$

$$R^2LOG = \frac{1}{M}\sum_{i=1}^{M}(\ln(\sigma_i^2/\hat{\sigma}_i^2))^2 \quad (17)$$

$$QLIKE = \frac{1}{M}\sum_{i=1}^{M}(\ln(\hat{\sigma}_t^2) + \sigma_t^2/\hat{\sigma}_t^2), \quad (18)$$

where M is the number of out-of-sample forecasts; σ_i^2 and $\hat{\sigma}_i^2$ are the actual value and forecast of realized volatility, respectively. MSE and MAE are the mean squared error and mean absolute error, respectively. $HMSE$ and $HMAE$ are the heteroscedasticity adjusted version of MSE and MAE, respectively. R^2LOG is similar to the R^2 of the Mincer-Zarnowitz regressions. [30] finds that MSE and $QLIKE$ are robust to the imperfect proxy of actual volatility. This is important because even if the squared daily return may not be a good proxy of actual volatility compared with some measures for intraday high-frequency data such as realized volatility [31, 32, 33], we can still guarantee that the evaluation results based on these two loss functions are correct.

Table 3 shows the forecasting performances for the horizon of 1 day evaluated by six loss functions. We report the ratios of loss functions of the proposed asymmetric GARCH-MIDAS relative to the benchmark of the traditional symmetric model of [1]. Therefore, a loss function ratio smaller than 1 suggests that the corresponding asymmetric GARCH-MIDAS with leverage effect can result in more accurate volatility forecasts than the GARCH-MIDAS without leverage effect. We use the [28] method to test for the null hypothesis that under a specific criterion the loss function of the related asymmetric GARCH-MIDAS is higher than or equal to the loss function of the benchmark model, against the alternative hypothesis that the asymmetric model has lower forecasting loss.

The results in Table 3 suggest that under each of six loss criteria the loss function ratios of most asymmetric models are lower than 1. Therefore, the incorporation of the leverage effect can improve the 1-day-ahead forecasting ability of GARCH-MIDAS models. The DM statistics show that Model 1 and Model 2 accounting for short-term leverage effect can significantly outperform the benchmark model under all six loss criteria. Models 3-5 with long-term leverage effect perform as well as the benchmark GARCH-MIDAS for most cases. In particular, Model 4 can significantly beat the benchmark of traditional symmetric GARCH-MIDAS under only three out of 6 loss functions. This evidence indicates that the long-term leverage effect plays less important role than the short-term one in affecting the predictive ability of volatility models. Nevertheless, we can still conclude that accounting for the long-term leverage is helpful to improve the predictability. The meaningful evidence is that GARCH-MIDAS with both two types of leverage effect (Model 7 and Model 8) displays lower loss ratios than the GARCH-MIDAS with only short-term leverage effect (Model 1 and Model 2). Certainly, they can significantly outperform the benchmark model under all criteria. The percent reductions of forecasting losses of models with dual leverage effects change from 3% to 10%, depending on which loss function is used.

Table 3 about here

Table 4 and Table 5 report the forecasting performances for the 5-day and 22-day horizons, respectively. In general, we can find the consistent evidence that short-term leverage effect can affect model predictive ability more greatly than the long-term one. But the GARCH-MIDAS models incorporating leverage effects in both short-term and long-term volatility equation result in most accurate volatility forecasts. Both types of leverage effects are meaningful for improving predictive ability.

Table 4 and Table 5 about here

The large fluctuations have more important implications than small fluctuations in risk management [34, 35]. Motivated by this fact, we investigate the forecasting performances of our asymmetric GARCH-MIDAS during the period when stock prices are more volatile. The financial crisis in mid-2008-2009 provides a good example for us to examine the predictability during the volatile period. Table 6 gives the loss ratios of 8 asymmetric GARCH-MIDAS models over the benchmark model, as well as their DM statistics for the horizon of 1 day[5]. We can find the consistent evidence that Model 1 and Model 2 with short-term leverage effect can perform significantly better than the benchmark model. Differently, the GARCH-MIDAS with long-term leverage effect becomes a bit stronger during the financial crisis period. In particular, the forecasts of Model 4 and Model 5 are significantly more accurate than the benchmark forecasts under 4 out of 6 loss criteria. The models which take into account dual

[5]To save space, we do not report the results for 5-day and 22-day horizons but they are available upon request.

leverage perform better than the models with single leverage effect. The reduction of loss function during financial crisis can achieve as high as 16% (see Model 8).

Table 6 about here

It may be argued that the loss function is sensitive to the extreme values of forecast errors. Due to this consideration, we employ an alternative evaluation criterion, success ratio (SR), which measures how often a tested model results in better volatility forecasts than benchmark one. The success ratio of Model j is given by,

$$SR_j = \frac{1}{M}\sum_{i=1}^{M} l_{i,j}, \quad l_{i,j} = I\big((\sigma_i^2 - \hat{\sigma}_{i,j}^2)^2 < (\sigma_i^2 - \hat{\sigma}_{i,benchmark}^2)^2\big), \tag{19}$$

where $\hat{\sigma}_{i,j}^2$ is the volatility forecast of model j and $\hat{\sigma}_{i,benchmark}^2$ is the forecast of the benchmark model (i.e., the original GARCH-MIDAS of [1] (Model 0).). SR quantifies the probability that the tested model produces more accurate forecasts. An SR higher than 0.5 implies that the model of interest can result in more accurate forecasts than the benchmark model most of time. Obviously, the value of SR is less affected by outliers. We use the [36] test to examine the significance of the difference between SR and 0.5.

Table 7 gives the success ratios of 8 asymmetric GARCH-MIDAS models for three forecasting horizons. Model 1 and Model 2, which consider short-term leverage effect, display the success ratios significantly higher than 0.5. The predictive ability of the models with long-term leverage effect (Models 3-5) evaluated by SR is somewhat weaker, but their forecasts also significant more accurate than the benchmark model for the horizons of 1 day and 22 days. Models 6-8 with both types of asymmetry can also significantly outperform the benchmark model for all three horizons, suggested by the SRs. The probability that the proposed asymmetric models result in more accurate volatility forecasts can reach 60%.

Insert Table 7 here

5. Conclusions

The leverage effect (or asymmetric effect) in the return-volatility relation is a central issue in finance. In this paper, we introduce the asymmetry terms to GARCH-MIDAS model of

[1] to disentangle short-term and long-term leverage effects. We develop 8 extensions of GARCH-MIDAS to account for leverage effects in short-term and/or long-term volatility components. Our in-sample evidence reveals that future volatility of S&P 500 index return is more strongly related to past negative return both in the short-term and long-term. The out-of-sample findings based on six criteria of loss functions suggest that the asymmetric GARCH-MIDAS can significantly beat the benchmark of the existing model without leverage effect. The models with leverage effect can generate more accurate volatility forecasts at the probability of 60%. Moreover, the improvement of predictive ability of volatility models is more attributed to the introduction of short-term leverage effect than to the long-term one.

We would like to conclude the paper by outlining some issues which deserve future work. First, one can make the model specification more parsimonious. The model specification of our asymmetric GARCH-MIDAS may be too parameterized. This is also the reason why we do not consider more sophisticated process for short-term component such as EGARCH and FIGARCH. Second, we can investigate the effect of low-frequency macro variables on stock volatility during bear and bull periods to highlight the advantage of GARCH-MIDAS. Finally, more accurate volatility measures such as realized kernal can be considered to replace the realized volatility in the equation for long-term volatility component.

Table 1: The typology of GARCH-MIDAS and its extensions

Model number	Short-term leverage effect?	Long-term leverage effect?	Model for short-term component	Explanatory variables for long-term component	Eq. number for short-term component	Eq. number for long-term component
0.	$\times$	$\times$	GARCH	RV	Eq.(2)	Eq.(6)
1.	$\surd$	$\times$	GJR	RV	Eq.(8)	Eq.(6)
2.	$\surd$	$\times$	GJR	ΔJ_i^2 and BV	Eq.(8)	Eq.(9)
3.	$\times$	$\surd$	GARCH	RS^+ and RS^-	Eq.(2)	Eq.(10)
4.	$\times$	$\surd$	GARCH	RS^+, RS^- and $RV \cdot I(R_{i-k} < 0)$	Eq.(2)	Eq.(11)
5.	$\times$	$\surd$	GARCH	$\Delta J^{2+}, \Delta J^{2-}$ and BV	Eq.(2)	Eq.(12)
6.	$\surd$	$\surd$	GJR	RS^+ and RS^-	Eq.(8)	Eq.(10)
7.	$\surd$	$\surd$	GJR	RS^+, RS^- and $RV \cdot I(R_{i-k} < 0)$	Eq.(8)	Eq.(11)
8.	$\surd$	$\surd$	GJR	$\Delta J^{2+}, \Delta J^{2-}$ and BV	Eq.(8)	Eq.(12)

Table 2: Estimation results of GARCH-MIDAS and its extensions

	Model 0	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
μ	0.059***	0.027***	0.027	0.057***	0.050***	0.057***	0.068***	0.025**	0.063***
	(5.798)	(2.622)	(1.127)	(5.641)	(4.314)	(5.870)	(4.590)	(2.515)	(5.844)
α	0.098***	0.001	0.001	0.097***	0.085***	0.093***	0.068	0.001	0.108***
	(8.564)	(0.010)	(0.002)	(8.665)	(5.559)	(8.579)	(1.543)	(0.004)	(2.875)
γ		0.198***	0.195***				0.124***	0.175***	0.193***
		(8.635)	(6.759)				(3.737)	(5.213)	(3.598)
β	0.860***	0.844***	0.853***	0.855***	0.854***	0.863***	0.823***	0.850***	0.755***
	(43.86)	(47.60)	(42.34)	(42.46)	(51.54)	(52.67)	(18.52)	(26.71)	(24.63)
m	0.365***	0.331***	0.336***	0.362***	0.272***	2.764***	0.762***	0.324***	0.019
	(3.096)	(4.392)	(2.617)	(3.558)	(4.646)	(2.889)	(8.144)	(5.648)	(0.246)
ω	2.819**	3.560***	3.114***	3.224**	8.359**	0.331***	6.081***	8.871*	9.695***
	(2.495)	(3.298)	(2.980)	(2.053)	(2.056)	(3.305)	(4.135)	(1.934)	(3.425)
θ	0.028***	0.028***							
	(4.772)	(6.962)							
θ^J			0.028						
			(0.225)						
θ^+				0.002	-0.011		-0.048**	-0.016	
				(0.259)	(-0.277)		(-2.394)	(-0.406)	
θ^-				0.056***	0.066**		0.078**	0.054***	
				(5.181)	(2.517)		(2.502)	(2.627)	
δ					0.149***			0.139***	
					(6.060)			(3.743)	
θ^{J+}						0.058**			0.011
						(1.725)			(0.267)
θ^{J-}						-0.127***			-0.121***
						(-3.409)			(-3.058)
θ^{BV}			0.061***			0.011			0.097***
			(6.410)			(0.870)			(4.976)
Wald Stat							6.127**	30.47***	8.509***

Notes: This table provides the parameter estimates of GARCH-MIDAS and its various extensions. The typology of these models is given in Table 1. The last row reports the Wald statistics for the null hypotheses $\theta^+ = \theta^-$ (Model 6), $\theta^+ = \theta^-, \delta = 0$ (Model 7) and $\theta^{J+} = \theta^{J-}$ (Model 8). The numbers in parentheses are the t-statistics. The asterisks *, ** and *** denote rejection of null hypothesis at 10%, 5% and 1% significance levels, respectively.

Table 3: 1-day-ahead forecasting performances of GARCH-MIDAS models with leverage effect

	MSE	MAE	HMSE	HMAE	R^2LOG	QLIKE
Model 1	0.940*	0.977***	0.938**	0.967***	0.949***	0.976***
	(-1.396)	(-2.418)	(-1.703)	(-4.340)	(-8.398)	(-6.896)
Model 2	0.942*	0.973***	0.937***	0.973***	0.969***	0.978***
	(-1.538)	(-3.157)	(-2.072)	(-3.689)	(-11.05)	(-8.072)
Model 3	0.993	1.010	0.995	0.991***	1.026	1.002
	(-0.882)	(3.928)	(-0.328)	(-3.724)	(8.867)	(2.591)
Model 4	0.942	0.973***	0.914	0.998	0.920***	0.987***
	(-1.253)	(-2.679)	(-0.652)	(-0.171)	(-7.816)	(-2.983)
Model 5	0.977	1.003	1.017	0.995	0.945	0.982
	(-0.971)	(0.489)	(0.413)	(-1.102)	(0.247)	(-0.656)
Model 6	0.939*	0.978***	0.949	0.967***	0.967***	0.977***
	(-1.473)	(-2.442)	(-1.143)	(-4.549)	(-8.060)	(-6.973)
Model 7	0.906*	0.961***	0.882**	0.962***	0.932***	0.975***
	(-1.456)	(-3.315)	(-1.807)	(-4.159)	(-10.82)	(-7.927)
Model 8	0.928*	0.975***	0.899***	0.963***	0.966***	0.974***
	(-1.514)	(-2.602)	(-2.805)	(-4.910)	(-8.974)	(-7.811)

Notes: This table reports the loss function ratio of the model of interest to the benchmark of standard GARCH-MIDAS model (Model 0). The typology of these models is given in Table 1. A loss function ratio lower than 1 implies that the model of interest results in more accurate volatility forecasts than the benchmark model. The significance of the difference of loss function between the tested model and the benchmark one is analyzed by the [28] (DM) statistic with the standard normal distribution. We use one-side test and report the DM statistics in the parentheses, in which a negative value indicates that the loss function of the corresponding model is lower than the benchmark model. The asterisks *, ** and *** denote rejection of null hypothesis at 10%, 5% and 1% significance levels, respectively.

Table 4: 5-day-ahead forecasting performances of GARCH-MIDAS models with leverage effect

	MSE	MAE	HMSE	HMAE	R^2LOG	QLIKE
Model 1	0.940*	0.971***	0.959	0.971***	0.967***	0.945***
	(-1.369)	(-2.930)	(-0.967)	(-3.722)	(-9.708)	(-7.366)
Model 2	0.939**	0.965***	0.945**	0.977***	0.960***	0.942***
	(-1.729)	(-4.106)	(-1.823)	(-3.054)	(-11.73)	(-8.075)
Model 3	1.001	1.004	1.009	1.000	1.002	1.003
	(0.267)	(3.957)	(1.170)	(-0.018)	(4.748)	(2.770)
Model 4	0.976	0.990*	1.013	1.013	0.990***	0.996
	(-0.776)	(-1.366)	(0.550)	(2.055)	(-3.979)	(-0.647)
Model 5	0.997	0.986***	1.054	1.006	1.000	1.005
	(-0.255)	(-2.743)	(1.451)	(2.303)	(0.475)	(2.021)
Model 6	0.941*	0.973***	0.974	0.972***	0.968***	0.946***
	(-1.431)	(-2.929)	(-0.484)	(-3.777)	(-9.792)	(-7.376)
Model 7	0.935*	0.970***	0.897***	0.964***	0.966***	0.940***
	(-1.609)	(-3.453)	(-2.555)	(-4.803)	(-10.10)	(-8.032)
Model 8	0.937**	0.957***	0.974	0.977***	0.964***	0.946***
	(-1.679)	(-4.550)	(-0.670)	(-3.137)	(-10.62)	(-7.573)

Notes: See Table 3.

Table 5: 22-day-ahead forecasting performances of GARCH-MIDAS models with leverage effect

	MSE	MAE	HMSE	HMAE	R^2LOG	QLIKE
Model 1	0.952*	0.973***	0.960	0.969***	0.966***	0.942***
	(-1.398)	(-2.967)	(-0.824)	(-4.063)	(-9.918)	(-7.727)
Model 2	0.945**	0.966***	0.926***	0.974***	0.962***	0.941***
	(-1.672)	(-4.159)	(-2.217)	(-3.531)	(-11.11)	(-8.125)
Model 3	1.026	1.009	1.000	1.005	0.998***	1.001
	(2.600)	(1.892)	(0.013)	(3.186)	(-3.025)	(0.895)
Model 4	1.030	0.999	1.054	1.011	0.995***	1.002
	(1.907)	(-0.103)	(1.411)	(2.581)	(-2.832)	(0.506)
Model 5	0.997	0.984***	1.002	1.009	0.995***	1.001
	(-0.309)	(-4.864)	(0.073)	(4.246)	(-6.380)	(0.341)
Model 6	0.956*	0.975***	0.949*	0.970***	0.967***	0.945***
	(-1.419)	(-2.982)	(-1.385)	(-4.112)	(-9.765)	(-7.800)
Model 7	0.959	0.979***	0.875***	0.959***	0.971***	0.941***
	(-1.197)	(-2.194)	(-2.867)	(-5.414)	(-8.550)	(-8.014)
Model 8	0.934***	0.955***	0.919***	0.972***	0.963***	0.941***
	(-2.042)	(-5.563)	(-2.347)	(-3.841)	(-10.93)	(-8.239)

Notes: See Table 3.

Table 6: Forecasting performances during the period of financial crisis (1-day-ahead)

	MSE	MAE	HMSE	HMAE	R^2LOG	QLIKE
Model 1	0.936*	0.965***	0.939	0.991	0.949***	0.976***
	(-1.293)	(-2.077)	(-0.612)	(-0.396)	(-5.047)	(-2.710)
Model 2	0.941*	0.979*	0.877*	0.967*	0.969***	0.978***
	(-1.347)	(-1.386)	(-1.305)	(-1.624)	(-3.512)	(-2.766)
Model 3	0.990	1.008	0.903***	0.968***	1.026	1.002
	(-1.107)	(1.888)	(-3.881)	(-4.543)	(7.411)	(0.762)
Model 4	0.936*	0.965**	1.230	1.058	0.920***	0.987*
	(-1.296)	(-1.902)	(1.673)	(1.984)	(-6.197)	(-1.539)
Model 5	0.967*	0.979***	0.975	1.027	0.945***	0.982***
	(-1.264)	(-2.556)	(-0.282)	(1.393)	(-7.273)	(-2.453)
Model 6	0.936*	0.970**	0.869*	0.965**	0.967***	0.977***
	(-1.343)	(-1.856)	(-1.518)	(-1.800)	(-3.908)	(-3.092)
Model 7	0.902*	0.961**	1.018	1.011	0.932***	0.976***
	(-1.311)	(-1.778)	(0.167)	(0.393)	(-4.723)	(-2.217)
Model 8	0.921*	0.965***	0.840*	0.958**	0.966***	0.974***
	(-1.444)	(-2.042)	(-1.534)	(-1.854)	(-3.757)	(-3.024)

Notes: See Table 3.

Table 7: Success ratios of GRACH-MIDAS models with leverage effects

	h=1	h=5	h=22
Model 1	0.606***	0.617***	0.605***
	(10.870)	(12.070)	(10.740)
Model 2	0.614***	0.625***	0.611***
	(11.730)	(12.900)	(11.430)
Model 3	0.526*	0.524*	0.517**
	(1.454)	(1.574)	(1.735)
Model 4	0.549***	0.499	0.513*
	(4.946)	(-0.139)	(1.336)
Model 5	0.544***	0.511	0.553***
	(4.421)	(1.096)	(5.351)
Model 6	0.598***	0.615***	0.605***
	(10.020)	(11.860)	(10.780)
Model 7	0.596***	0.606***	0.592***
	(9.808)	(10.870)	(9.387)
Model 8	0.596***	0.599***	0.611***
	(9.766)	(10.150)	(11.380)

Notes: This table provides the ratio of the number of days that each asymmetric GARCH-MIDAS model forecast volatility better than the benchmark model of standard GARCH-MIDAS relative to total number of out-of-sample days. The typology of these models is given in Table 1. [36] method is used to examine whether the success ratio is significantly different from 0.5. We use one-side test and give the statistics with standard normal distribution in the parentheses. Asterisks *, ** and *** reject the null hypothesis of equal forecasting accuracy at 10%, 5% and 1% significance levels, respectively.

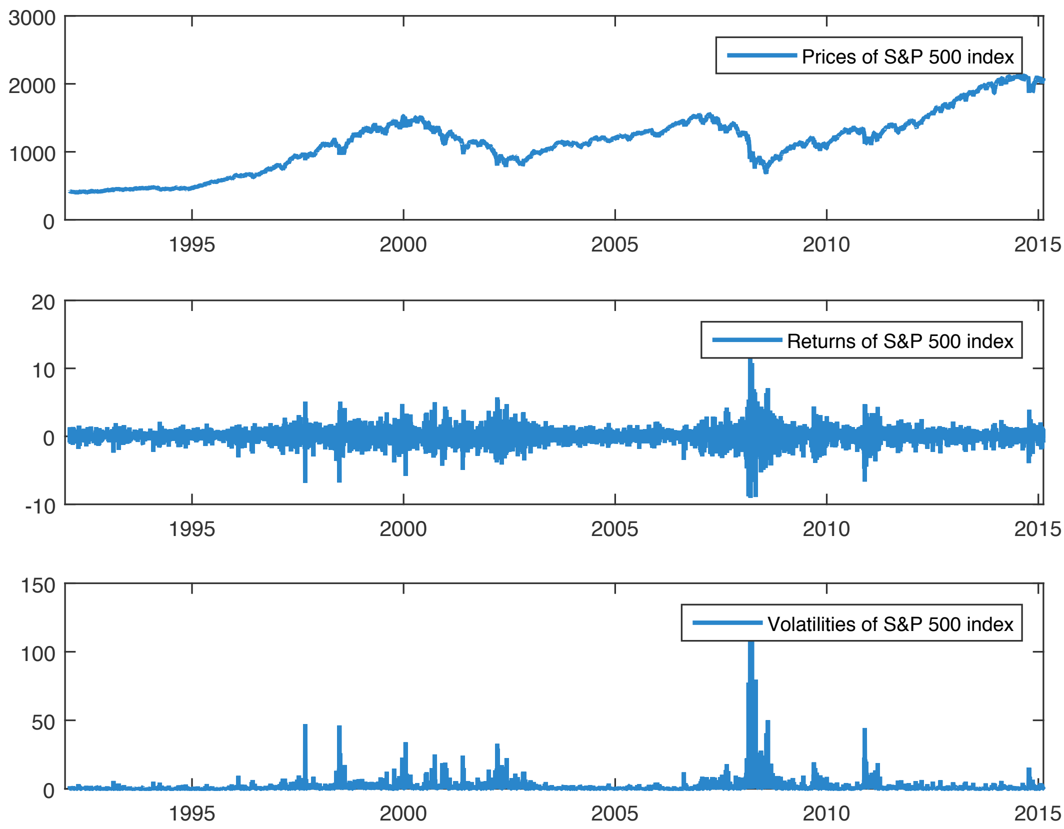

Figure 1: Prices, returns and volatilities of S&P 500 index

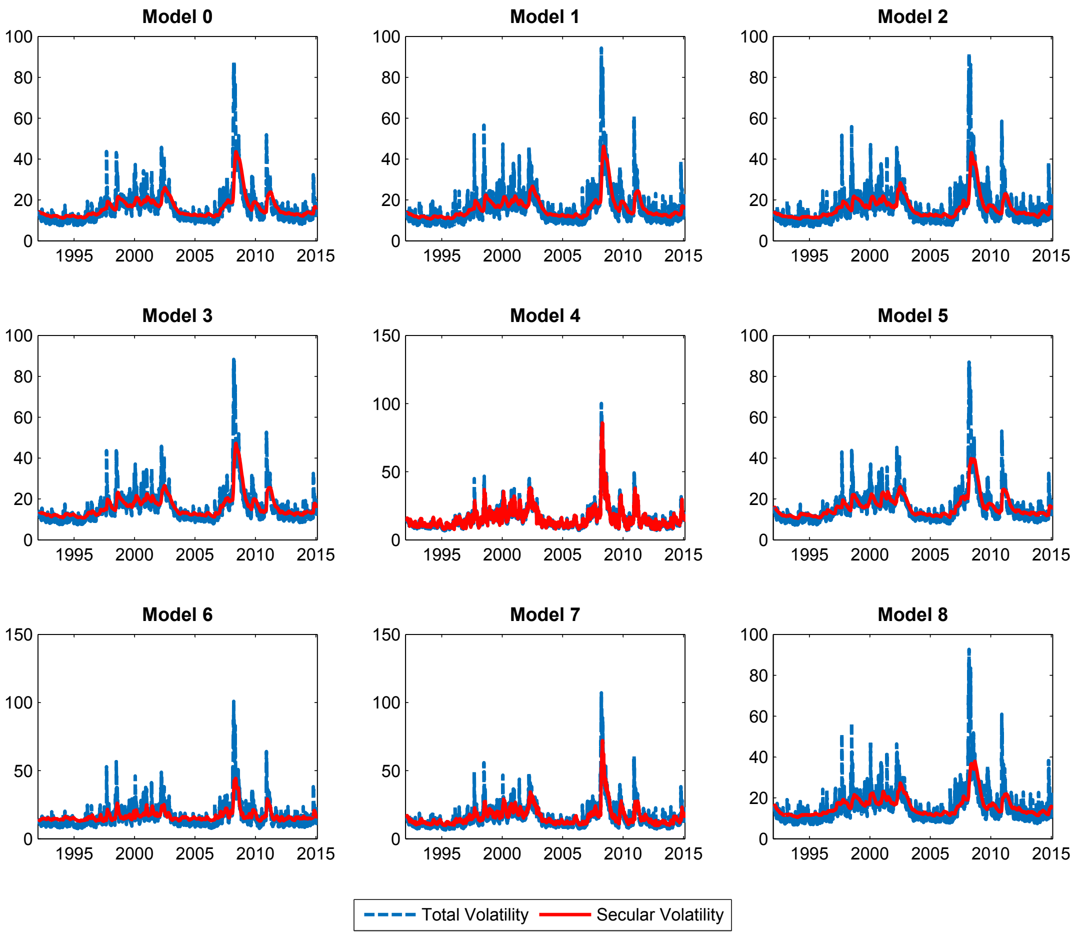

Figure 2: The estimated total volatility and secular volatility.

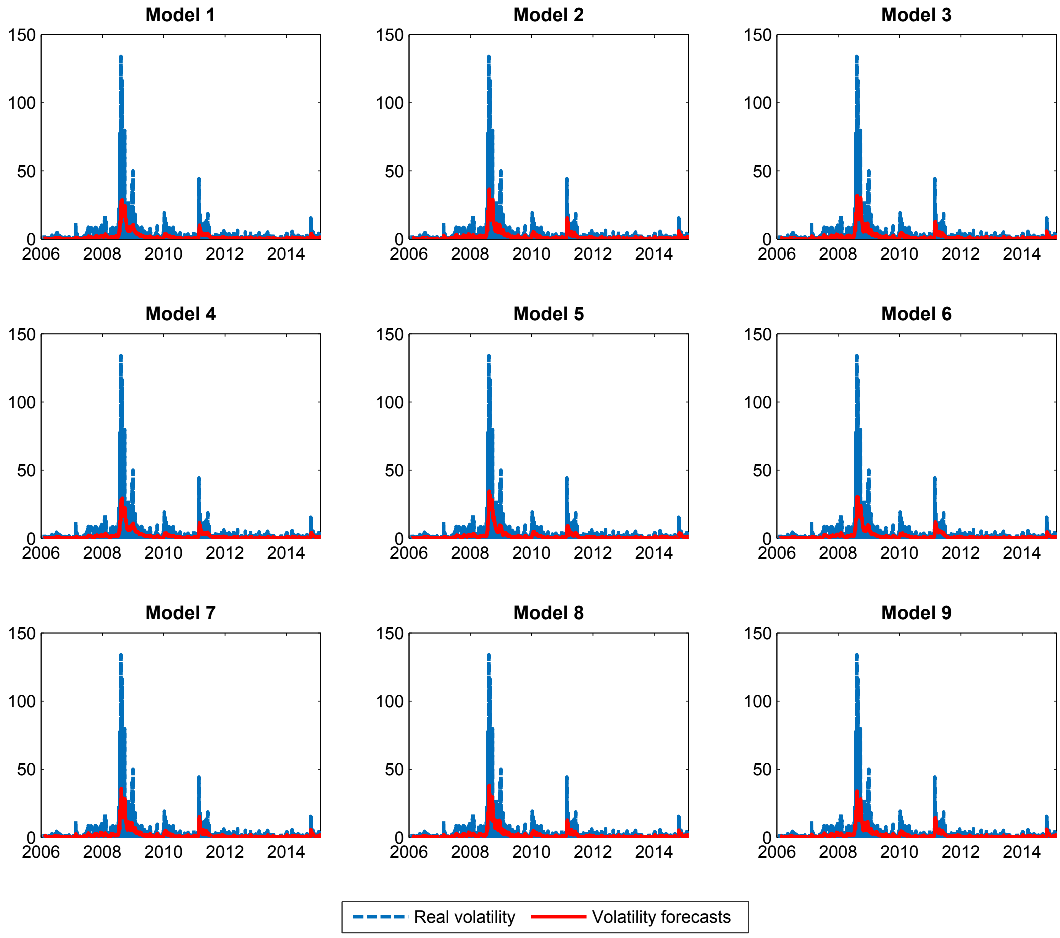

Figure 3: 1-day-ahead forecasts of GARCH-MIDAS and its asymmetric extensions.

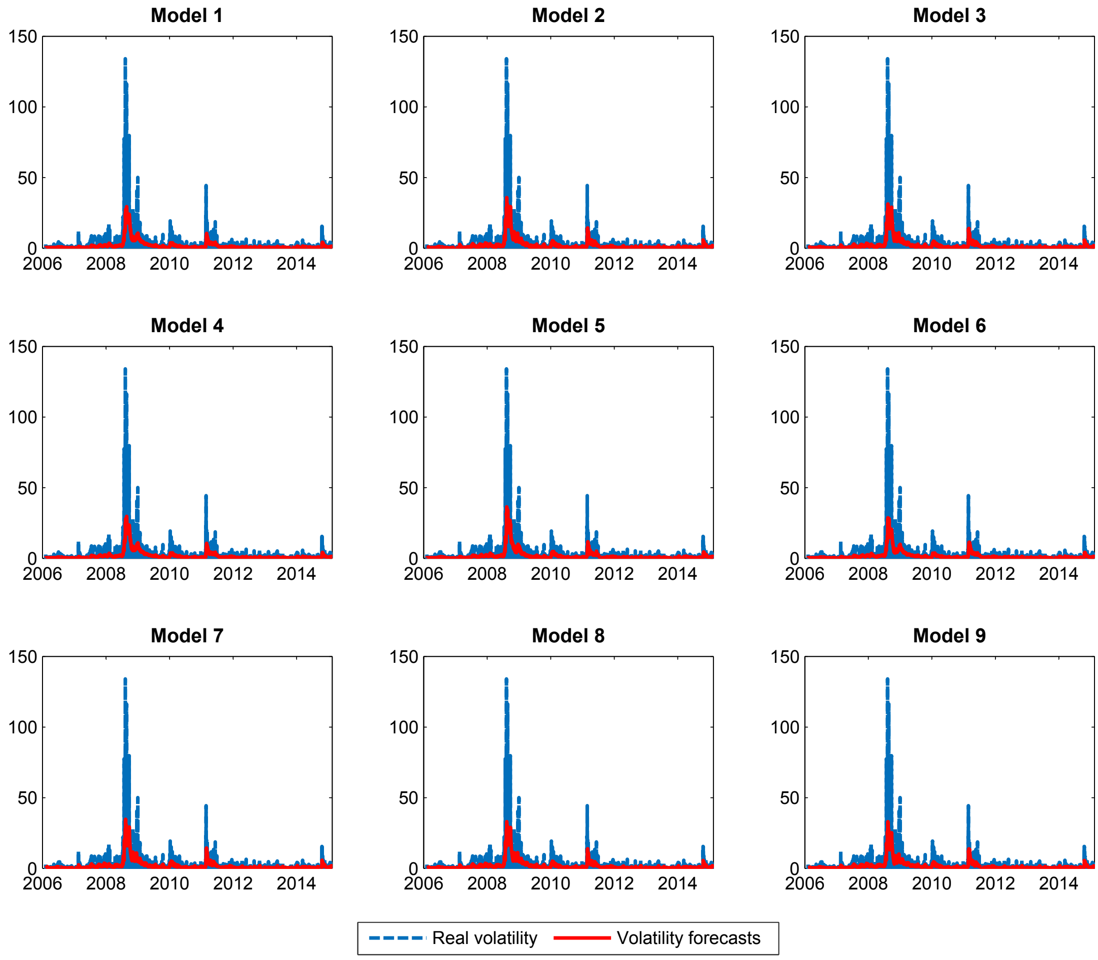

Figure 4: 5-day-ahead forecasts of GARCH-MIDAS and its asymmetric extensions.

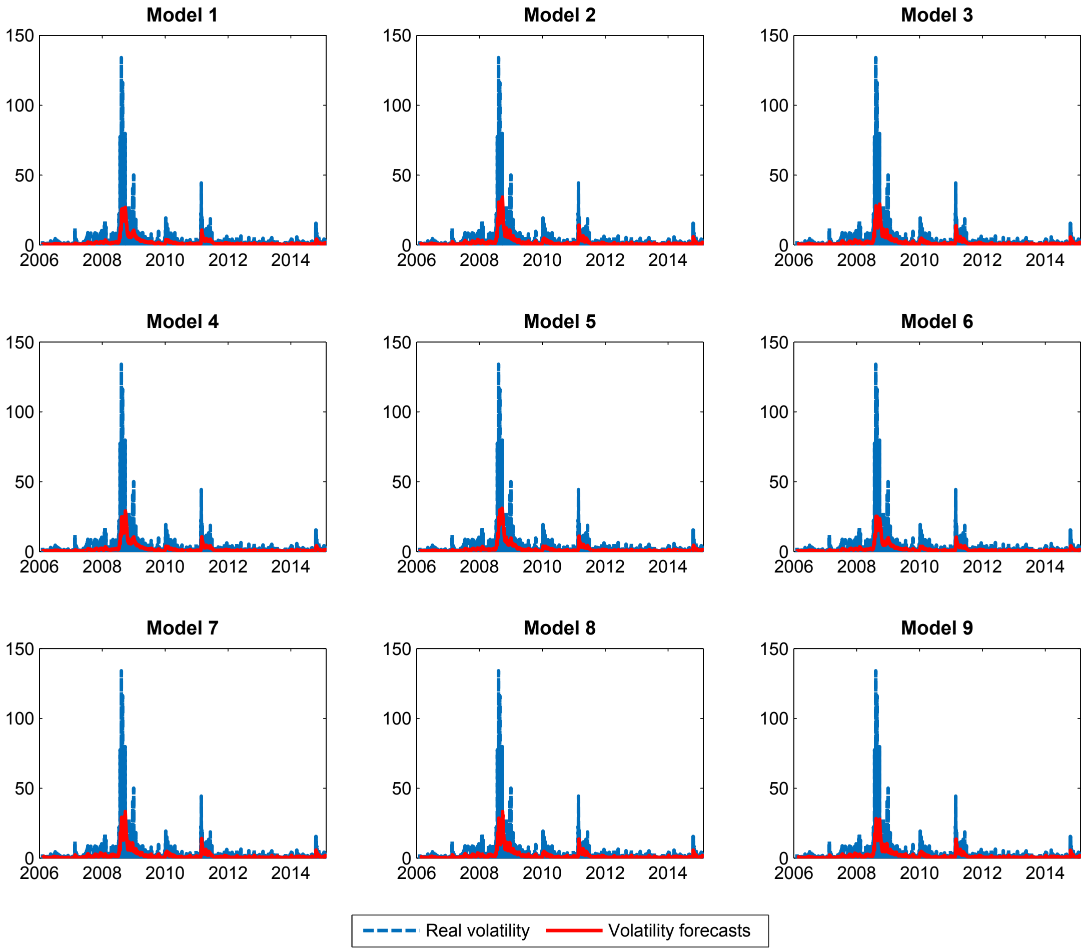

Figure 5: 22-day-ahead forecasts of GARCH-MIDAS and its asymmetric extensions

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Highlights

- We propose volatility models accounting for short-term and long-term asymmetry.
- Accounting for leverage effect can significantly improve predictive ability.
- Short-term leverage effect plays a more important role than long-term effect.
- The leverage effect is found in both short-term and long-term volatility.