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The granular extension of Sugeno-type fuzzy models based on optimal allocation of information granularity and its application to forecasting of time series

Wei Lu [a,*], Liyong Zhang [a], Witold Pedrycz [b,c,d], Jianhua Yang [a], Xiaodong Liu [a]

[a] School of Control Science and Engineering, Dalian University of Technology, Dalian City, People's Republic of China
[b] Department of Electrical & Computer Engineering, University of Alberta, Edmonton T6R 2V4 AB, Canada
[c] Department of Electrical and Computer Engineering, Faculty of Engineering, King Abdulaziz University, Jeddah 21589, Saudi Arabia
[d] Systems Research Institute, Polish Academy of Sciences, Warsaw, Poland

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ABSTRACT

The Sugeno-type fuzzy models are used frequently in system modeling. The idea of information granulation inherently arises in the design process of Sugeno-type fuzzy model, whereas information granulation is closely related with the developed information granules. In this paper, the design method of Sugeno-type granular model is proposed on a basis of an optimal allocation of information granularity. The overall design process initiates with a well-established Sugeno-type numeric fuzzy model (the original Sugeno-type model). Through assigning soundly information granularity to the related parameters of the antecedents and the conclusions of fuzzy rules of the original Sugeno-type model (i.e. granulate these parameters in the way of optimal allocation of information granularity becomes realized), the original Sugeno-type model is extended to its granular counterpart (granular model). Several protocols of optimal allocation of information granularity are also discussed. The obtained granular model is applied to forecast three real-world time series. The experimental results show that the method of designing Sugeno-type granular model offers some advantages yielding models of good prediction capabilities. Furthermore, those also show merits of the Sugeno-type granular model: (1) the output of the model is an information granule (interval granule) rather than the specific numeric entity, which facilitates further interpretation; (2) the model can provide much more flexibility than the original Sugeno-type model; (3) the constructing approach of the model is of general nature as it could be applied to various fuzzy models and realized by invoking different formalisms of information granules.

1. Introductory notesintroduction

Granular computing [1,2,26,44] has emerged as a unified and coherent platform of constructing, describing, and processing information granules. At present, it has become one of the fastest growing information-processing paradigms in the domain of computational intelligence and human-centered systems. Granular computing concentrates on processing information granules and constructively develops a holistic view at the discipline and incorporates the existing technologies and formalisms of sets (interval analysis) [21], fuzzy sets [47,28,32], rough sets [22,24,23], shadowed sets [25], probabilistic sets [11,12] and alike. It identifies the essential commonalities between the diversified problems and technologies used there, which could be cast into a unified framework known as a granular world. Essentially, granular computing is a fully operational processing entity that interacts with the external world (that could be another granular or numeric one) by collecting necessary granular information and returning the outcomes of the granular computing (being either granular or numeric).

In the process of developing the human-centered system model, information granules, information granulation and information granularity, which are closely related with granular computing, play a vital role. The underlying concept of information granules exhibits far reaching implications by giving rise to generic, semantically meaningful entities that are crucial to the perception of real world, modeling its phenomena and supporting decision-making processes. Information granulation is an inherent and omnipresent activity of human beings carried out with intent of gaining a better insight into a problem under consideration and arriving at its efficient solution. In particular, information granulation aims at

* Corresponding author. Tel.: +86 41184706161; fax: +86 41184706161.
E-mail address: luwei@dlut.edu.cn (W. Lu).

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transforming the problem at hand into several smaller and therefore manageable tasks. Information granulation comes with the process of abstraction of data and derivation of knowledge from information. The ensuing information granules can be formed by exploiting a certain underlying formalism such as intervals, fuzzy sets, rough sets, shadowed sets or alike. Zadeh [46] presented a general view of fuzzy sets viewed as examples of information granules: $g = (x \textit{ is } G) \textit{ is } \lambda$, where x is a variable of a universe of discourse U, G is a convex fuzzy subset of U, and λ is the probability of x belonging to the subset G. Pedrycz and Vukovich [32] also showed a model of generalization and specialization of fuzzy information granules. Design and development of information granules can be supported by some methods such as various clustering algorithm including the well-known Fuzzy C-means clustering [3], the principle of justifiable information granularity [29] and so on. Information granularity can help us focus on the most suitable level of detail of problems to be solved. The number of information granules implies a certain level of information granularity. The level of abstraction (generality) associates with information granularity. In summary, information granularity is concerned with the size (specificity) of information granules. Depending upon the formalism of information granules used in the process of information granulation, information granularity can be described and quantified in different ways as the size of the granule (its granularity is expressed by counting the number of elements embraced by the granule for discrete case), the length of the granule (for continuous case) or alike.

As fuzzy rules-based models can realize nonlinear numeric mapping from the space of input variables to the space of output variables, the Sugeno-type models, which were introduced by Takagi, Sugeno and Kang [42,39,40], have enjoyed a visible position in system modeling. The Sugeno-type models give rise to more sophisticated yet more complex rule-based systems, where the rules are equipped with conclusions being formed as local regression models instead of the fuzzy consequent that normally appears in the Mamdani models [18]. Designing a Sugeno-type model generally comprises two fundamental phases, namely structure development and parameter estimation. Structure development includes determination of the number of rules and the antecedent parameters of rules. There are diverse design methods such as fuzzy grids [13], fuzzy trees [19], fuzzy clustering [41,45,7,43,34,9] to support structure development of Sugeno-type models. Once the number of rules and the antecedent parameters have been determined, the Sugeno-type model is viewed as a collection of local linear models. Parameters of the linear models associated with each rule are obtained by pseudo-inversion or by applying the recursive least square method [7]. Alternatively, the antecedent parameters may only be considered as initial estimates and then all parameters of Sugeno-type model including the antecedent parameters and the consequent parameters can be further optimized by gradient-descent algorithm, back-propagation algorithm [17] or global optimization algorithms such as genetic algorithm (GA) [36,8,6], particle swarms algorithm (PSO) [5,15,16,20] and others.

In a nutshell, the Sugeno-type model is associated with fuzzy rules that have a certain format with a functional-type consequent. In this way the Sugeno-type model decomposes the input space into subspaces by forming a series of fuzzy sets and then approximates the system in each subspace by a simple linear regression model. From the point of view of granular computing, information granules and information granulation inherently emerge in the process of designing Sugeno-type model — if fuzzy sets used to divide the input space are envisioned as fuzzy information granules [46,47] which serve as basic building blocks of Sugeno-type model, the process of fuzzy partition input space can be regarded as the process of fuzzy information granulation [46,47] to the input space. Albeit, the

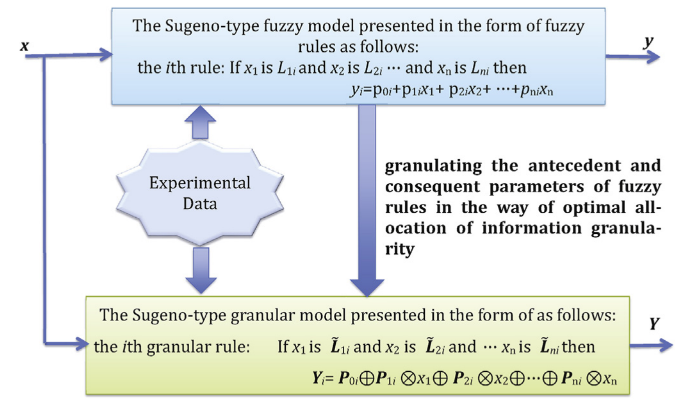


Fig. 1. The schematic diagram of extending the Sugeno-type fuzzy model to its granular model by an optimal allocation of information granularity.

Sugeno-type fuzzy model implies the idea of information granules and information granulation, in essence the fuzzy model is a nonlinear numeric model — any numeric input to such a model produces a numeric output. From the view of perception, information granules permeate almost all human endeavors [44]. The granular model in which inputs are numeric entities or granules yet the output is granules, which facilitates further interpretation [30]. The essence of the underlying idea of granular modeling is to substantially augment (elevate) the existing fuzzy models by forming granular fuzzy models. The ultimate intent is to make such models in rapport with the system (data) under consideration. Based on the above, one of the motivations of our research is to extend the Sugeno-type fuzzy model to its granular model (granular counterpart) — here the output of the resulting Sugeno-type granular model is an information granule rather than a single numeric entity.

From the perspective of system modeling, the construction of system model depends on quite large experimental data. There is no any kind of models including the Sugeno-type models which can fully capture these data without generating any modeling error meaning that the output of the model is equal to the output data for all inputs forming the training data [27]. To quantify this lack of accuracy, we give up on the precise numeric model and make the model granular by admitting granular parameters and allocating a predetermined level of information granularity to related parameters so that the granular model obtained in this way "*cover*" as many training data as possible. In short, under a certain level of granularity the output of the granular model obtained by granulating the parameters of existing numeric model covers the data. Bearing in mind this idea, another motivation behind this research is that in the process of extending the Sugeno-type fuzzy model to its granular model, information granularity is regarded as crucial design asset and when taken into full account it helps establish a better rapport of the resulting Sugeno-type granular model with the system under modeling and offers certain flexibility to the resulting Sugeno-type granular model.

In this paper, we develop a comprehensive design process for extending the Sugeno-type fuzzy model to the Sugeno-type granular model through granulating the antecedent and consequent parameters of fuzzy rules in the Sugeno-type model (i.e., granulating the supports of membership functions and the respective coefficients of each local regression model, which leads to generate the granular membership degree and the granular local regression models) in an optimal allocation (distribution) of information granularity. Fig. 1 illustrates this design process, where x is the numeric input of the Sugeno-type fuzzy model and the extended granular model, y is the numeric output of the Sugeno-type fuzzy model, and $\boldsymbol{Y}$ is the output of the formed Sugeno-type granular model, which is expressed as a certain formalisms. In addition, in Fig. 1, L_{ki} is the linguistic label of the antecedent fuzzy set A_{ki} in the Sugeno-type fuzzy model, $\widetilde{\boldsymbol{L}}_{ki}$ is the linguistic label of the antecedent fuzzy set $\widetilde{\boldsymbol{A}}_{ki}$ in the Sugeno-type granular model, where $k = 1, 2, \cdots, n$. Note that $\widetilde{\boldsymbol{A}}_{ki}$ ($k = 1, 2, \cdots, n$) is the granular counterpart of the antecedent fuzzy set A_{ki} in the Sugeno-type fuzzy model (i.e. $\widetilde{\boldsymbol{A}}_{ki}$ is an interval-valued fuzzy set), which can be formed by granulating the supports of fuzzy set A_{ki} according to an optimal allocation of information granularity. Similarly, $\boldsymbol{P}_{ji}$ ($j = 0, 1, \cdots, n$) is the granular counterpart of the jth coefficient p_{ji} of local linear regression model corresponding to the ith fuzzy rule. The symbols "$\oplus$" and "$\otimes$" underline a fact that processing involves information granules rather than plain numeric entities.

According to Fig. 1, the entire design process for extending the Sugeno-type fuzzy model to its granular model is now outlined as follows. A starting point of the overall design process is a well-established Sugeno-type fuzzy model. Subsequently, the antecedent and consequent parameters of fuzzy rules in the original Sugeno-type fuzzy model are granulated through an optimal allocation of information granularity such that the Sugeno-type granular model is formed, viz., any numeric input to the model generates a granular output. Here the levels of information granularity distributed throughout all fuzzy rules are assigned to the antecedent and consequent parameters associated with each fuzzy rule in an optimal fashion. The optimization of allocation of information granularity leads to transform the design process (viz., making the antecedent and consequent parameters of fuzzy rules become granules and finally forming the granular counterpart of the original Sugeno-type fuzzy model) into the optimization problem of some performance indices assessing the quality of the resulting Sugeno-type granular model. In this sense, the Sugeno-type granular model formed as a result of an optimal allocation of information granularity can be viewed as the generalization of the original Sugeno-type fuzzy model. It is worth noting that the Sugeno-type granular model is constructed on a basis of the already developed Sugeno-type fuzzy model (we are not concerned in what way the Sugeno-type fuzzy model has been developed). In addition, considering that the formed Sugeno-type granular model is applied to realize the interval prediction of time series, i.e., using the Sugeno-type granular model to forecast the change range of time series in the future time, in the entire design process we focus on *interval information granules*, that is intervals are used to represent information granules. The key features of the design process presented in this paper are now summarized as follows:

- Information granularity manifests throughout the entire design process of the Sugeno-type granular model. By soundly allocating information granularity, the parameters of original Sugeno-type fuzzy model become information granules.
- Compared with the Sugeno-type fuzzy model, the Sugeno-type granular model shows much more flexibility by admitting variable granularity.
- Output of the Sugeno-type granular model is an information granule, which facilitates further interpretation.
- The proposed approach of extending the Sugeno-type fuzzy model to its granular counterpart is of general nature as it could be applied to various fuzzy models and realized by invoking different formalisms of information granules.

This paper is structured as follows. In Section 2 we briefly recall the Sugeno-type fuzzy model and the underlying processing giving rise to granular outputs. Next, in Section 3, the design process for extending the Sugeno-type fuzzy model to the Sugeno-type granular model is elaborated along with the two main aspects mentioned-above, namely an optimal allocation of information granularity and the optimization of performance indices guiding the allocation process. Experimental studies in which the Sugeno-type granular model that is formed according to the design process presented is applied to realize interval forecasting of three real world time series are covered in Section 4. Finally, Section 5 presents some conclusions.

2. Preliminaries

The Sugeno-type fuzzy model has been applied successfully to a large number of problems [38,4,35,33,37]. In this section, we briefly recall the essence of the Sugeno-type fuzzy model, and then introduce the foundation of supporting arithmetic of the extended Sugeno-type granular model — interval operations.

2.1. The Sugeno-type fuzzy model

Let us consider a continuous multi-input single-output (MISO) system. Let $D = \langle X; Y \rangle$ be the set of input–output data pairs observed from the system, $X = \{x_1, x_2, \cdots, x_m\} \subset R^n$ be the set of n-dimensional input vectors, and $Y = \{y_1, y_2, \cdots, y_m\} \subset R$ be the set corresponding to the output vector. Here m is the number of input–output data pairs included in D. The set D is obtained by taking together with the set X and the set Y, which is denoted as $D = \{(x_k, y_k)\} \subset R^{n+1}$, $k = 1, 2, \ldots, m$. The Sugeno-type fuzzy model which consists of a set of "If-Then" fuzzy rules having the following form:

$$\begin{aligned}&\text{the } i\text{th rule } R^i: \text{ If } x_1 \text{ is } L_{1i} \text{ and } x_2 \text{ is } L_{2i} \text{ and} \cdots \text{and } x_n \text{ is } L_{ni}\\ &\qquad\text{then } y_i = f_i(x_1, x_2, \ldots, x_n)\end{aligned} \tag{1}$$

where x_j is the jth input variable of the model, L_{ji} is the linguistic label of an antecedent fuzzy set A_{ji}, and $f_i = p_{0i} + p_{1i}x_1 + \cdots + p_{ni}x_n$ is a local linear regression function of the input variables, $j = 1, 2, \ldots, n$.

The numeric output y of the Sugeno-type model considering the knowledge-base included k rules is determined as the weighted average of the individual rule output y_i ($i = 1, 2, \ldots, k$):

$$y = \frac{\sum_{i=1}^{k} \mu_i y_i}{\sum_{i=1}^{k} \mu_i} \tag{2}$$

with $\mu_i = T(A_{1i}(x_1), A_{2i}(x_2), \ldots, A_{ni}(x_n))$ being the matching degree between the antecedent part of the ith rule and the current inputs $x = (x_1, x_2, \ldots, x_n)$ and with T being a certain t-norm.

Generally, fuzzy sets used in the above Sugeno-type fuzzy model can be the form of the triangular-like, the trapezoid-like or the Gaussian-like membership functions. Furthermore, the consequent parts of the model can also be replaced by constants, i.e., $y_i = p_{0i}$ ($i = 1, 2, \ldots, k$). In this case, the model is called the "0-*order*" Sugeno-type model.

2.2. Interval operations

Interval numbers are used to represent a set of generic objects in the interval mathematics. For example, interval numbers $[a, b]$ are such that $a \le x \le b$. Let X and Y be the two interval numbers, say $X = [a, b]$ and $Y = [c, d]$. The algebraic operations are defined as follows [10]:

- Addition: $X + Y = [a + c, b + d]$
- Subtraction: $X - Y = [a - d, b - c]$
- Multiplication: $X \times Y = [min\{ac, ad, bc, bd\}, max\{ac, ad, bc, bd\}]$
- Division (excluding division by an interval containing 0)

$$\frac{X}{Y} = X \times \frac{1}{Y} \quad with \quad \frac{1}{Y} = \left[\frac{1}{d}, \frac{1}{c}\right]$$

- Mapping of intervals:
  for nondecreasing mapping, $f(X)$=$f([a, b])$=$[f(a), f(b)]$
  for nonincreasing mapping, $f(X)$=$f([a, b])$=$[f(b), f(a)]$

3. From the Sugeno-type fuzzy model to its granular counterpart: the design method based on an optimal allocation of information granularity

In this section, we focus on the design process for extending the Sugeno-type fuzzy model to its granular counterpart. The design is based on interval granule and an optimal allocation of information granularity. In what follows, the underpinning of the design process is first introduced, and then a way on how to allocate information granularity into a well-established Sugeno-type fuzzy model is detailed. Finally, the Sugeno-type granular model is presented.

3.1. The underpinning of extending the Sugeno-type fuzzy model to its granular counterpart

As noted above, a Sugeno-type granular model can be found by granulating parameters of a well-established Sugeno-type fuzzy model (it is also called the original Sugeno-type model) in an optimal allocation of information granularity. The allocation of information granularity and the interval granule are foundation of the entire design process. The available information granularity (more specifically, its level of granularity), which is treated as an important design asset, has to be distributed among the antecedent and consequent parameters of the original Sugeno-type model so that the output of the model "covers (includes)" as many of the experimental data as possible. In what follows, we take a Sugeno-type fuzzy model with two inputs and one output as a simple example to illustrate the fundamental of designing the granular counterpart of the model.

Let us consider a simple two-input one-output system, for a given set of input–output data pairs $D=\{(x_{1i}, x_{2i}); y_i\}$ $(i=1, 2, \ldots, n)$ including n samples, a well-established Sugeno-type fuzzy model including r fuzzy rules is provided as follows:

the ith rule R^i: If x_1 is L_{1i} and x_2 is L_{2i} then

$$y_i = p_{0i} + p_{1i}x_1 + p_{2i}x_2 \tag{3}$$

where $i=1, 2 \ldots r$, x_1 and x_2 are input variables of the model, L_{1i} and L_{2i} are respectively the linguistic labels of the antecedent fuzzy set A_{1i} and A_{2i}, and y_i is the numeric output of the ith rule. In order to obtain the granular counterpart of the model, a mechanism of allocating information granularity in which an admissible, predefined level of granularity ε in [0, 1] is allocated to related parameters in the original Sugeno-type model is adopted. In what follows, taking the granulation of parameters (the antecedent parameters and the consequent parameters) of the ith rules of the above-mentioned Sugeno-type model, we elaborate on how to form the granular counterparts of parameters of the original Sugeno-type model according to the above-mentioned mechanism.

For the antecedent of the ith rule of the above-mentioned Sugeno-type model (see Eq.(3)), assuming that the antecedent fuzzy set $A_{1i}=(a_{1i}, m_{1i}, b_{1i})$ and $A_{2i}=(a_{2i}, m_{2i}, b_{2i})$ are triangular fuzzy sets, where a_{1i} and a_{2i} are the lower bounds of support of the triangular fuzzy sets, b_{1i} and b_{2i} are the upper bounds of support of the triangular fuzzy sets, m_{1i} and m_{2i} are the core of the triangular fuzzy sets. The core of fuzzy set is regarded as the representative of data belonging to the fuzzy set. The level of the corresponding information granularity can be described and quantified through determining the sum of membership degree of data belonging to the fuzzy set. The adjustment of information granularity (the variable information granularity) can be realized by changing support of the fuzzy set. Based on this, for a given level of information granularity $\varepsilon_i \in [0, 1]$, it is allocated to the lower bound of support of the fuzzy set A_{1i}, i.e., the parameter a_{1i}. Thus the granular counterpart of the parameter can be generated in the form of intervals in the following way:

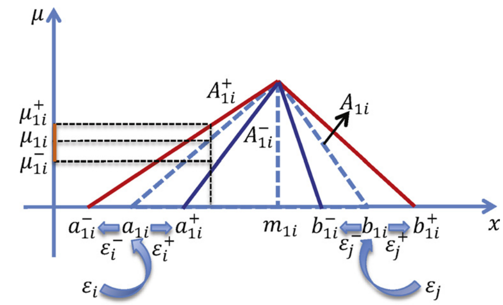


Fig. 2. Forming granular (interval-valued) membership degree with the aid of allocating information granularity to support of the triangular fuzzy set A_{1i}.

$$\begin{aligned} a_{1i}^- &= a_{1i} - \varepsilon_i^- \left|a_{1i}\right| \\ a_{1i}^+ &= a_{1i} + \varepsilon_i^+ \left|a_{1i}\right| \end{aligned} \tag{4}$$

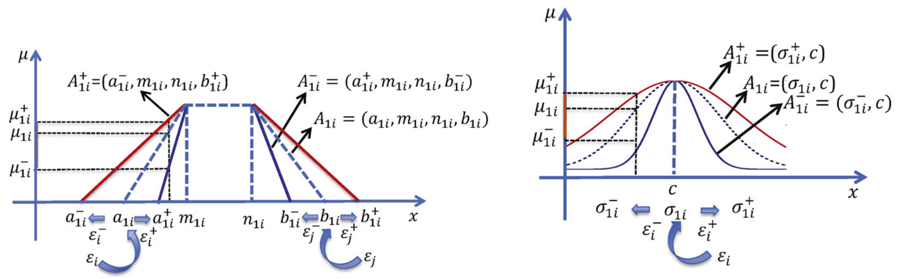
where $\varepsilon_i = \varepsilon_i^- + \varepsilon_i^+$. The resulting granular (interval) counterpart of the parameter a_{1i} is denoted as $G(a_{1i}) = [a_{1i}^-, a_{1i}^+]$. It is worth noting that in this notation we use the symbol $G(a_{1i})$ to highlight an operation of the granular counterpart produced around the value of the original numeric parameter a_{1i}. Similarly, if a level of granularity $\varepsilon_j \in [0, 1]$ is assigned to the parameter b_{1i} — the upper bound of support of fuzzy set A_{1i}, its granular counterpart $G(b_{1i}) = [b_{1i}^-, b_{1i}^+]$ can be also obtained in the same way, viz.,

$$\begin{aligned} b_{1i}^- &= b_{1i} - \varepsilon_j^- \left|b_{1i}\right| \\ b_{1i}^+ &= b_{1i} + \varepsilon_j^+ \left|b_{1i}\right| \end{aligned} \tag{5}$$

where $\varepsilon_j = \varepsilon_j^- + \varepsilon_j^+$.

Once that the support of the triangular fuzzy set A_{1i} has been granulated through allocating a predefined level of granularity, in the sequel we let a_{1i}^+ and b_{1i}^- be the lower and upper bound of a new fuzzy set A_{1i}^-, respectively. Likewise, let a_{1i}^- and b_{1i}^+ be the lower and upper boundary of other new fuzzy set A_{1i}^+, respectively. Thus two new fuzzy sets — $A_{1i}^- = (a_{1i}^+, m_{1i}, b_{1i}^-)$ and $A_{1i}^+ = (a_{1i}^-, m_{1i}, b_{1i}^+)$ can be constructed. This implies that the fuzzy set A_{1i} is augmented to its granular counterpart (an interval-valued fuzzy set) $\tilde{\boldsymbol{A}}_{1i} = [A_{1i}^-, A_{1i}^+]$ so that the membership degree μ_{1i} generated by the fuzzy set A_{1i} is transformed into the interval-valued membership degree $\tilde{\mu}_{1i} = [\mu_{1i}^-, \mu_{1i}^+]$. The process of forming interval-valued membership degree $\tilde{\mu}_{1i}$ is presented in Fig. 2. For the triangular fuzzy set A_{2i}, the corresponding granular counterpart $\tilde{\boldsymbol{A}}_{2i} = [A_{2i}^-, A_{2i}^+]$ can be also formed in the same way. Note that if both A_{1i} and A_{2i} are the trapezoidal fuzzy sets, the Gaussian fuzzy sets or other types of fuzzy sets, we can also refer to the above method to obtain the corresponding granular counterparts. Fig. 3 shows the forming process of interval-valued membership degree when A_{1i} is the trapezoidal fuzzy set and the Gaussian fuzzy set, respectively.

For the consequent parameters of the ith rule of the original Sugeno-type model shown in Eq.(3), viz. coefficients of the local linear regression model — p_{0i}, p_{1i} and p_{2i}, their granular counterparts can be easily produced around the values of these numeric parameters in the way that is similar with granulating the antecedent parameters mentioned in the above. Let the values of level of information granularity allocated to the three consequent parameters be ε_i, ε_j and ε_k, respectively. The granular counterparts of the three parameters are easily calculated as follows:



(a) The case of the trapezoidal fuzzy set (b) The case of the Gaussian fuzzy set

Fig. 3. Building interval-valued membership degree in the case of the trapezoidal fuzzy set and the Gaussian fuzzy set.

- For the parameter p_{0i}, its granular counterpart $\boldsymbol{P}_{0i} = G(p_{0i}) = [p_{0i}^{-}, p_{0i}^{+}]$ is $p_{0i}^{-} = p_{0i} - \varepsilon_i^{-} |p_{0i}|$, $\quad p_{0i}^{+} = p_{0i} + \varepsilon_i^{+} |p_{0i}|$
- Similarly, for the parameters p_{1i} and p_{2i}, the corresponding granular counterparts $\boldsymbol{P}_{1i}$ and $\boldsymbol{P}_{2i}$ are given as:

$$\boldsymbol{P}_{1i} = G(p_{1i}) = [p_{1i}^{-}, p_{1i}^{+}] \text{ with } p_{1i}^{-} = p_{1i} - \varepsilon_j^{-} |p_{1i}|, \quad p_{1i}^{+} = p_{1i} + \varepsilon_j^{+} |p_{1i}|$$
$$\boldsymbol{P}_{2i} = G(p_{2i}) = [p_{2i}^{-}, p_{2i}^{+}] \text{ with } p_{2i}^{-} = p_{2i} - \varepsilon_k^{-} |p_{2i}|, \quad p_{2i}^{+} = p_{2i} + \varepsilon_k^{+} |p_{2i}|$$

Once the related parameters (including the antecedent and consequent parameters) of the ith rule of the original Sugeno-type model shown in Eq.(3) are granulated and the ensuing granular counterparts are formed, the fuzzy rule is transformed into the corresponding granular rule. If each rule associated with the original numeric Sugeno-type model (see Eq.(3)) is transformed into the corresponding granular rule in the same manner mentioned in the above, the original numeric Sugeno-type fuzzy model is transformed into (extended to) the corresponding Sugeno-type granular model presented in the form of following:

$$\text{the } i\text{th granularrule } GR^i: \text{ If } x_1 \text{ is } \tilde{\boldsymbol{L}}_{1i} \text{ and } x_2 \text{ is } \tilde{\boldsymbol{L}}_{2i} \text{ then } \boldsymbol{Y}_i = \boldsymbol{P}_{0i} \oplus \boldsymbol{P}_{1i} \otimes x_1 \oplus \boldsymbol{P}_{2i} \otimes x_2 \tag{6}$$

where $\tilde{\boldsymbol{L}}_{1i}$ and $\tilde{\boldsymbol{L}}_{2i}$ are the linguistic labels corresponding to the interval-valued fuzzy set $\tilde{\boldsymbol{A}}_{1i}$ and $\tilde{\boldsymbol{A}}_{2i}$. Note that here the symbols shown in circles ("⊕" and "⊗") highlight a fact that processing involves information granules (interval granules) rather than plain numeric entity.

More generally, let us consider the general form of the Sugeno-type fuzzy model including k fuzzy rules as shown in (1). In the model, if the total number of parameters to be granulated is h (where if fuzzy sets used in the antecedent of the model are the triangular fuzzy sets or the trapezoidal fuzzy sets, the value of h is $3kn+k$; if fuzzy sets used in the antecedent of the model are the Gaussian fuzzy sets, the value of h is $2kn+k$. Here n is the total number of the input variables of the model), for a given level of information granularity $\varepsilon \in [0, 1]$ being viewed as a design asset, it transforms those numeric parameters (these parameters include the support of fuzzy sets from antecedents of each rule of the original Sugeno-type model and the coefficients of the local regression polynomial from consequence of each rule of the original Sugeno-type model) into their granular counterparts such that the level of admissible information granularity ε is allocated to them in such a way a balance of levels of information granularity with $\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_h$ being the levels of information granularity is satisfied that is

$$h\varepsilon = \sum_{i=1}^{h} \varepsilon_i = \sum_{i=1}^{h} (\varepsilon_i^{-} + \varepsilon_i^{+}), \tag{7}$$

i.e.,

$$\varepsilon = \sum_{i=1}^{h} \varepsilon_i / h = \sum_{i=1}^{h} (\varepsilon_i^{-} + \varepsilon_i^{+}) / h, \tag{8}$$

which is led to the formation of Sugeno-type granular model. Concisely, the process of forming granular model based on the numeric parameters of the original Sugeno-type model is visualized in Fig. 4. Here the symbol $sup(A_{ij})$ expresses the support of fuzzy set A_{ij} corresponding to linguistic label L_{ij} and the symbol $G(A_{ij})$ expresses granulating operation to the support of fuzzy set A_{ij}, where $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, k$.

In the process of forming Sugeno-type granular model, the allocation of information granularity ε across all parameters to be granulated in the model is crucial to the performance of the resulting granular model. In the next subsection, we focus on how to allocate information granularity to each numeric parameter to be granulated in the model.

3.2. Optimal allocation of information granularity

The realization of optimal allocation of information granularity needs to consider two essential aspects. One is that a way of allocating information granularity to individual parameter to be granulated in the Sugeno-type fuzzy model, which is expressed in some form of protocols of management of information granules, and the other is that an optimization of the process of allocation of granularity realized in the presence of a certain performance index.

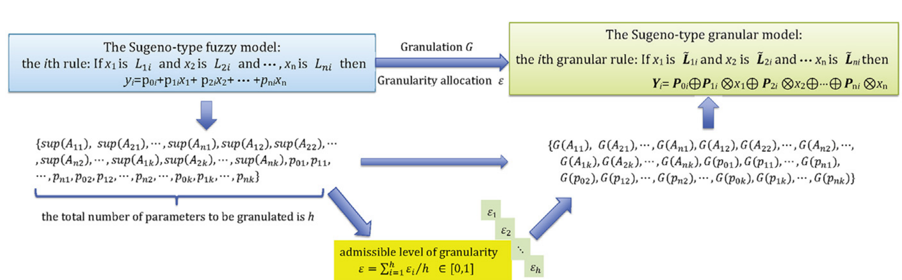


Fig. 4. The blueprint of the formation of the Sugeno-type granular model.

3.2.1. Protocols of allocation of information granularity

An allocation of the available information granularity ε to individual parameter of the original Sugeno-type fuzzy model can be realized in several different ways depending on how much diversity one would like to consider in the allocation process. In what follows, several main protocols of allocation of information granularity are presented as follows. Note that in the below-mentioned all protocols, the allocated information granularity meets the balance of granularity expressed by Eq.(7) that is $h\varepsilon$ (recall that "h" denotes the total number of parameters to be granulated in the original Sugeno-type fuzzy model).

- $\mathcal{P}_1$: uniform allocation of information granularity. This protocol is the simplest one. Each parameter to be granulated is treated in the same way. In essence, the protocol does not call for any optimization. All parameters to be granulated are replaced by intervals constructed with the use of the same value of ε. The intervals themselves are distributed symmetrically around the original values of the parameters. In terms of the protocol, for any a parameter p_i, it can be extended to its granular counterpart $\boldsymbol{P}_i$ around its numeric value, i.e., $\boldsymbol{P}_i = G(p_i) = [p_i - \varepsilon_- \left|p_i\right|, \quad p_i + \varepsilon_+ \left|p_i\right|] = [p_i - \frac{\varepsilon}{2}\left|p_i\right|, \quad p_i + \frac{\varepsilon}{2}\left|p_i\right|]$.
- $\mathcal{P}_2$: uniform allocation of information granularity with asymmetric position of intervals around the numeric parameter to be granulated. In this protocol, each parameter uses the same level of information granularity ε, but ε_- and ε_+ may be different. Let $\gamma \in [0, 1]$ be a level of asymmetry (asymmetry degree), for any a parameter p_i, we can obtain its granular counterpart $\boldsymbol{P}_i$ according to the following form: $\boldsymbol{P}_i = G(p_i) = [p_i - \varepsilon_- \left|p_i\right|, \quad p_i + \varepsilon_+ \left|p_i\right|] = [p_i - \gamma\varepsilon \left|p_i\right|, \quad p_i + (1-\gamma)\varepsilon \left|p_i\right|]$. If $\gamma = \frac{1}{2}$, the protocol degenerates into the protocol $\mathcal{P}_1$.
- $\mathcal{P}_3$: non-uniform allocation of information granularity with symmetrically distributed intervals of information granules. For the protocol, the different parameters are assigned the different level of information granularity, but here the $\varepsilon_{i-} = \varepsilon_{i+} = \frac{\varepsilon_i}{2}$. According to the protocol, for any parameter p_i, it has been assigned into the level of information granularity ε_i, thus its granular counterpart $\boldsymbol{P}_i$ can be obtained in the form of following way: $\boldsymbol{P}_i = G(p_i) = [p_i - \varepsilon_{i-} \left|p_i\right|, \quad p_i + \varepsilon_{i+} \left|p_i\right|] = [p_i - \frac{\varepsilon_i}{2}\left|p_i\right|, \quad p_i + \frac{\varepsilon_i}{2}\left|p_i\right|]$. If all parameters are assigned the same level of granularity namely $\varepsilon_1 = \varepsilon_2 = \cdots = \varepsilon_h = \varepsilon$, the protocol degenerates to the protocol $\mathcal{P}_1$.
- $\mathcal{P}_4$: non-uniform allocation of information granularity with asymmetrically distributed intervals of information granules. With regards to the protocol, the different parameters are associated with the different level of information granularity, and ε_{i-} and ε_{i+} are also different. Here $\varepsilon_{i-} + \varepsilon_{i+} = \varepsilon_i$. For any a parameter p_i and the level of information granularity ε_i being assigned into the parameter, its granular counterpart $\boldsymbol{P}_i$ can be obtained according to the protocol, viz. $\boldsymbol{P}_i = G(p_i) = [p_i - \varepsilon_- \left|p_i\right|, \quad p_i + \varepsilon_+ \left|p_i\right|] = [p_i - \gamma_i\varepsilon_i \left|p_i\right|, \quad p_i + (1-\gamma_i)\varepsilon_i \left|p_i\right|]$, where $\gamma_i \in [0, 1]$ is a level of asymmetry for parameter p_i. The protocol provides the highest level of flexibility.
- $\mathcal{P}_5$: random allocation of information granularity. This protocol is proposed as a reference method to demonstrate how much improvement of performance of formed granular model is achieved by optimizing allocation of granularity by using different protocols.

Each protocol presented before implies a certain way to realize allocation of information granularity. For protocols $\mathcal{P}_1$ and $\mathcal{P}_5$, we envision no optimization, while for protocols $\mathcal{P}_2$, $\mathcal{P}_3$ and $\mathcal{P}_4$, the allocation process has to be realized by exploiting some optimization techniques (swarm optimization or evolutionary techniques) since the problem is of high dimensionality. Here we note for the protocol $\mathcal{P}_2$, $\mathcal{P}_3$ and $\mathcal{P}_4$, whatever optimization techniques we use, they imply a certain content of a particle or a chromosome. The length of the corresponding string depends on the protocol. The length of the string which becomes longer with the increased level of flexibility of granularity allocation.

As noted so far, having considered all the components that in essence constitute the environment of allocation of information granularity, we can bring them together to articulate a formal optimization process.

3.2.2. Performance indices

Let us recall the purpose of our research is to construct the Sugeno-type granular model by extending numeric parameters of the original Sugeno-type fuzzy model to their granular counterparts. The output of resulting Sugeno-type granular model comes in an interval format, which has to be evaluated with regard to the numeric target. There are two intuitively appealing criteria to be considered — one is the coverage criterion and the other is the specificity criterion. The former reveals an extent to which the training data are "covered/included" by the output of the corresponding Sugeno-type granular model, whereas the latter indicates a level of specificity of the information granules (i.e. the length of interval granules) produced by the corresponding Sugeno-type granular model.

Assuming that $\{(\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), \ldots, (\mathbf{x}_n, y_n)\}$ is a input–output data collection for training the original Sugeno-type model. For any $\mathbf{x}_k$ ($k = 1, 2, \ldots, n$), the corresponding Sugeno-type granular model, which is obtained by granulating numeric parameters of the original Sugeno-type model in the case of the predetermined granularity level ε, returns an interval $\boldsymbol{Y}_k = [Y_k^-, Y_k^+]$, then the mathematical descriptions of the two criteria are presented as follows:

- the coverage criterion Q:

$$Q = \frac{1}{n}\sum_{k=1}^{n} f(k) \tag{9}$$

where

$$f(k) = \begin{cases} 1 & x_k \in \boldsymbol{Y}_k = [Y_k^-, Y_k^+] \\ 0 & x_k \notin \boldsymbol{Y}_k = [Y_k^-, Y_k^+] \end{cases}, \quad k = 1, 2, \ldots, n.$$

- the specificity criterion V_1:

$$V_1 = \frac{1}{n}\sum_{k=1}^{n} \left|Y_k^+ - Y_k^-\right| \tag{10}$$

Evidently the two criteria are in conflict — the number of the training data covered by the output of Sugeno-type granular model increases, the length of interval granules produced by the corresponding Sugeno-type granular model might increase and cause a decrease in the specificity. As usual, we expect to find a sound compromise between the two criteria. Based on this, we consider another form of the specificity criterion which is shown in the following:

- The another form of specificity criterion:

$$V_2 = e^{-V_1} = e^{-\frac{1}{n}\sum_{k=1}^{n}\left|Y_k^+ - Y_k^-\right|} \tag{11}$$

Thus, the following composite performance index can be considered in the process of optimizing allocation of information granularity:

$$\max: \quad F = Q \times V_2 \tag{12}$$

So far the entire design process of extending the Sugeno-type fuzzy model to its granular model is outlined as follows:

Step 1. Identify the numeric parameters of the Sugeno-type model with existing training data by exploiting a certain identification method, which leads to form the Sugeno-type fuzzy model (the original Sugeno-type model).

Step 2. Use a certain protocol of allocation information granularity to assign a predefined level of information granularity ε to related parameters of the original Sugeno-type fuzzy model, viz., the numeric parameters of the original Sugeno-type fuzzy model are granulated, which results in forming corresponding granular parameters (the interval-valued parameters). Hence the Sugeno-type granular model is formed.

It is noteworthy that in Step 2, if protocol $\mathcal{P}_2$, $\mathcal{P}_3$ or $\mathcal{P}_4$ is used, a certain optimization technology should be applied to adjust the allocation of granularity guiding by maximizing the performance index F, which leads to generate an optimized set of granularities. These granularities are used to form the optimal granular parameters (the optimal interval-valued parameters) of the original Sugeno-type fuzzy model according to the protocol $\mathcal{P}_2$, $\mathcal{P}_3$ and $\mathcal{P}_4$.

3.3. The Sugeno-type granular model based on the optimal allocation of information granularity

Now, the granular extension of the Sugeno-type fuzzy model including k rules as shown in Eq.(1) — the Sugeno-type granular model is formally presented as follows:

the ith granular rule GR^i :

$$\text{If } x_1 \text{ is } \tilde{\boldsymbol{L}}_{1i} \text{ and } x_2 \text{ is } \tilde{\boldsymbol{L}}_{2i} \text{ and} \cdots \text{and } x_n \text{ is } \tilde{\boldsymbol{L}}_{ni}$$
$$\text{then } \boldsymbol{Y}_i = \boldsymbol{P}_{0i} \oplus \boldsymbol{P}_{1i} \otimes x_1 \oplus \cdots \oplus \boldsymbol{P}_{ni} \otimes x_n \tag{13}$$

where x_j ($j = 1, 2, \ldots, n$) is the jth input variable of the model, $\boldsymbol{Y}_i$ ($i = 1, 2, \ldots, k$) is the granular output of the ith rule of the model, $\tilde{\boldsymbol{L}}_{ji}$ ($j = 1, 2, \ldots, n$; $i = 1, 2, \ldots, k$) is the linguistic label of the interval-valued fuzzy set $\tilde{\boldsymbol{A}}_{ji}$ (the interval-valued fuzzy set $\tilde{\boldsymbol{A}}_{ji} = [A_{ji}^-, A_{ji}^+]$ is the granular counterpart of the fuzzy set A_{ji}), which is formed by granulating the support of A_{ji} and results in formation of the granular membership degree $\tilde{\boldsymbol{\mu}}_{\tilde{\boldsymbol{A}}_{ji}}$, i.e. for any x_j belonging to fuzzy set A_{ji}, $\tilde{\boldsymbol{\mu}}_{\tilde{\boldsymbol{A}}_{ji}}(x_j) = [A_{ji}^-(x_j), A_{ji}^+(x_j)]$, and $\boldsymbol{P}_{mi} = [p_{mi}^-, p_{mi}^+]$ ($m = 0, 1, 2, \ldots, n$) is a granular parameter (an interval-valued parameter). Besides, the symbols "$\oplus$" and "$\otimes$" represent the interval addition operation and the interval multiplication operation, respectively.

The output $\boldsymbol{Y}$ of the augmented Sugeno-type granular model is determined as follows:

$$\boldsymbol{Y} = \left(\bigoplus_{i=1}^{k} \tilde{\mu}_i \otimes \boldsymbol{Y}_i \right) \oplus\!\!\!\!\div \left(\bigoplus_{i=1}^{k} \tilde{\mu}_i \right) \tag{14}$$

where $\tilde{\boldsymbol{\mu}}_i = \tilde{\boldsymbol{\mu}}_{\tilde{\boldsymbol{A}}_{1i}}(x_1) \otimes \tilde{\boldsymbol{\mu}}_{\tilde{\boldsymbol{A}}_{2i}}(x_2) \otimes \cdots \tilde{\boldsymbol{\mu}}_{\tilde{\boldsymbol{A}}_{ni}}(x_n)$ is the interval-valued matching degree between the antecedent part of the ith granular rule and the current system inputs $x = (x_1, x_2, \ldots, x_n)$, $\boldsymbol{Y}_i$ ($i = 1, 2, \ldots, k$) is the granular output of individual granular rule and the symbols "$\oplus$", "$\otimes$" and "$\oplus\!\!\!\!\div$ " represent granular (interval) operation that is interval addition operation, interval multiplication operation and interval division operation.

4. Experimental studies

In this section, we present a series of numeric experiments to illustrate the development and application of the proposed Sugeno-type granular model to forecasting time series. Fig. 5 shows the difference between the Sugeno-type granular model and the Sugeno-type fuzzy model.

4.1. The forecasting of time series based on the Sugeno-type fuzzy model

A sequence consisting of L observed data, say $\mathbf{x} = \{x(t)\}$, $t = 1, 2, \ldots, L$, ordered in time, is called a time series, where time variable t may be replaced by any other variable exhibiting some physical meaning. The problem of forecasting of time series has been a research area for many researchers. The aim of forecasting of time series is to exploit observed values of time series to extrapolate its future values. Mathematically, the forecasting of time series can be realized in the form of the mapping from the past d observed data with Δ interval in time that is $x(t-(d-1)\Delta)$, $x(t-(d-2)\Delta)$, ..., $x(t-\Delta)$, $x(t)$ to a forecasted future value $x(t+p)$, i.e.,

$$f: \{x(t-(d-1)\Delta), x(t-(d-2)\Delta), \ldots, x(t-\Delta), x(t)\} \rightarrow x(t+p). \tag{15}$$

As a rule-based model, the Sugeno-type fuzzy model can be also used to forecast time series by capturing relationship between the input and the output of the mapping. The general method of building the Sugeno-type fuzzy model to forecast time series is summarized as follows:

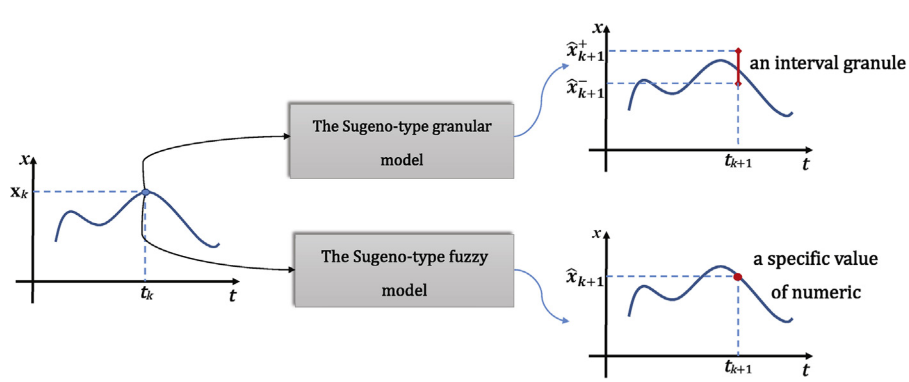


Fig. 5. Comparison of output of the Sugeno-type fuzzy model and its granular counterpart.

Step 1. Based on historical data of time series, construct the input–output data set in the form of following:

$$\{x(t-(d-1)\Delta), x(t-(d-2)\Delta), \ldots, x(t-\Delta), x(t); \quad x(t+p)\}$$

The constructed data set is divided into the training data subset and the testing subset. The former is used to construct the Sugeno-type fuzzy model whereas the latter is exploited to perform forecasting and evaluate the performance of the constructed models.

Step 2. Define fuzzy sets for each input variable to divide d dimensional input space, which can determine number of fuzzy rules included in the Sugeno-type fuzzy model. These defined fuzzy sets may be "*Small*", "*Middle*" or "*Large*" which is represented in terms of triangular, trapezoidal, Gaussian or any other types of membership functions. They are used to describe features of each input variable. In order to avoid substantial computing workload, as a method of generating information granules, fuzzy clustering is generally exploited to realize reduction of fuzzy rules in the design process of Sugeno-type fuzzy model [31].

Step 3. Identify the antecedent and consequent parameters on basis of the training data set, which results in the final formation of Sugeno-type fuzzy model. There are many available methods such as pseudo-inversion, the recursive least square, the gradient-descent algorithm, the back-propagation algorithm, the evolving algorithm, the swarms algorithm etc. to realize determination of these parameters.

Step 4. Based on the testing data set, use the formed Sugeno-type fuzzy model to realize prediction of time series.

4.2. The experimental setup

The objective of experiment is to illustrate the design method of the proposed Sugeno-type granular model, show its development, quantify the resulting performance and research impact of different protocols of allocation of information granularity on the formed Sugeno-type granular model. Three real-world time series — the Wolf's sunspot time series, the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) time series and the US civilian unemployment rates time series are used in experiment. For each time series, the experimental process is divided into three stages as follows:

Stage 1. According to Section 4.1, constructing the Sugeno-type fuzzy model of time series involved into experiments. In the stage, the standard version of Fuzzy C-means clustering (FCM) is exploited to design the structure of the Sugeno-type fuzzy model. The number of rules (the number of clusters) is determined by the trial-and-error method, and the selected type of fuzzy sets for antecedents of rules is Gaussian fuzzy set. Besides, the parametric identification of the Sugeno-type fuzzy model is realized by the combination of back-propagation and the least-squares method [14] in the stage. The performance of the Sugeno-type fuzzy model is evaluated by means of the mean squared error (MSE) that is commonly used.

Stage 2. Based on the training data subset, for a predefined level of information granularity $\varepsilon \in [0, 1]$, we start with the Sugeno-type fuzzy model constructed at Stage one to create the corresponding Sugeno-type granular model according to the above-mentioned design method (see Section 3). The stage is detailed as follows:

Table 1
The parameters of PSO algorithm for all experiments.

Description of parameters	Values
Population size	100
Acceleration constant	2.0
Acceleration constant	2.0
Inertial weight ξ	0.85
Initial positions	Random number
maximum number of iterations	1000
minimum objective function value	10^{-5}

Stage 2.1 Let the value of predefined level of information granularity ε first start with 0 and then gradually increase it by 0.01 until the value of 1 has been reached.

Stage 2.2 Considering some protocol mentioned in Section 3.2.1, for each available value of level of information granularity ε that is generated in Stage 2.1, it is allocated to related numeric parameters in the Sugeno-type fuzzy model which is constructed by Stage one, which results in that the corresponding numeric parameters are granulated. Thus, the granular counterpart of the Sugeno-type fuzzy model constructed by Stage one (the corresponding Sugeno-type granular model) is formed.

Note that for the protocol $\mathcal{P}_1$, $\mathcal{P}_5$, they do not require to use any optimization, whereas for the protocol $\mathcal{P}_2$, $\mathcal{P}_3$ or $\mathcal{P}_4$, some optimization technology is required. Based on this, in the process of overall experiments, the particle swarms optimization algorithm (PSO) is exploited when using the protocol $\mathcal{P}_2$, $\mathcal{P}_3$ or $\mathcal{P}_4$ to optimally assign level of information granularity. The values of parameters of the PSO algorithm used in experiments are reported in Table 1.

Stage 2.3 For each of protocols, record respectively the coverage index Q, the specificity index V_2 and the composite performance index (F) of Sugeno-type granular model formed on each available value of level of information granularity ε.

Stage 2.4 Evaluate global performance of the Sugeno-type granular model obtained under different protocols of allocation of information granularity. For some protocol, the global performance of the Sugeno-type granular model constructed can be quantified by aggregating the values of the composite performance index (F) over all levels of information granularity, namely

$$AUC = \int_0^1 F(\varepsilon)d\varepsilon. \tag{16}$$

The higher the AUC value, the higher the global performance of the developed granular model.

Stage 3. Based on the testing data subset, for each available value of level of information granularity ε generated in Stage 2.1, using the Sugeno-type granular model obtained in the case of using different protocols of allocation information granularity to perform interval prediction of time series, calculate respectively the coverage index Q, the specificity index V_2 and the composite performance index F and then assessing their global performance according to Eq.(16).

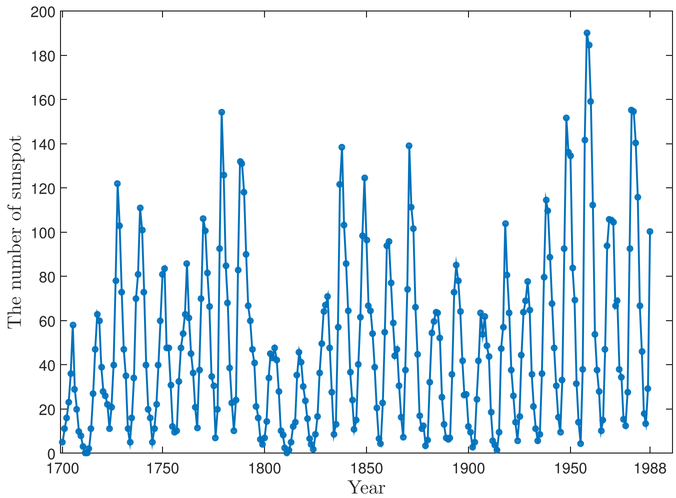


Fig. 6. The Wolf's sunspot time series.

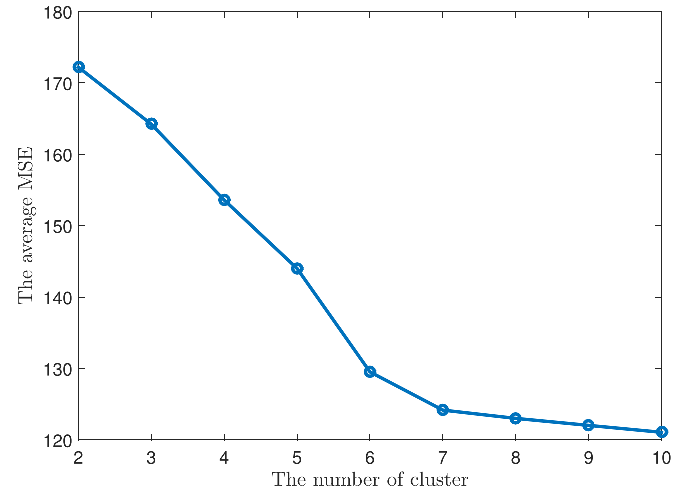


Fig. 7. The plot of average MSE versus the different number of clusters for the Wolf's sunspot time series.

4.3. *The Wolf's sunspot time series*

As a benchmark problem of fuzzy modeling and prediction, the Wolf's sunspot time series (from website: `https://datamarket.com/data/set/22wg/wolfs-sunspot-numbers-1700-1988#!ds=22wg&display=line`), which is shown in Fig. 6, is frequently used in many literatures. The time series consists of 289 observations, which is concerned with the annual number of sunspots from year 1700 to year 1988. In what follows, we take the time series as example to illustrate entire design process of the Sugeno-type granular model.

4.3.1. *Constructing the input–output data set*

Referring to (15), we let d, Δ and p be 4, 1 and 1, respectively. Based on history data of the sunspot time series, the input–output data set can be constructed in the following way:

$$\{x(t-3), x(t-2), x(t-1), x(t); x(t+1)\}$$

where t = 1703, 1705, ..., 1987. The resulting input–output data set consists of 285 input–output data pairs, in which the first 260 input–output data pairs are regarded as the training data and the latter 25 input–output data pairs are regarded as the testing data.

4.3.2. *Building the Sugeno-type fuzzy model*

According to Stage 1 described in Section 4.2, we build the Sugeno-type fuzzy model. It is worth noting that the performance of the constructed Sugeno-type fuzzy model is highly sensitive to the number of clusters (c). Here a sound number of clusters is determined by the trial-and-error method: in the process of building Sugeno-type fuzzy model, we start with c = 2 and increase it until the value of 10 is reached; for each value of c, let fuzzification factor m, another parameter of FCM, be 2. On the basis of the training data subset, we construct the Sugeno-type fuzzy model, use it to perform prediction and then evaluate its performance in terms of MSE. To assure high confidence in the experimental results, the above procedure is repeated 100 times. The resulting plot of the average MSE versus the different number of clusters are reported in Fig. 7.

From Fig. 7, we can see that the value of average MSE get substantially lower when increasing the number of clusters however the value slowly reduce when going beyond a certain number of clusters, say 8. Based on this, for the sunspot time series, the number of clusters is chosen to be 8 and the formalism of fuzzy sets used in the antecedent components is Gaussian fuzzy set, thus the constructed Sugeno-type fuzzy model totally includes 8 rules and 104 parameters (which is consisted of 64 antecedent parameters and 40 consequent parameters).

Table 2
The average value of AUC of the Sugeno-type granular model constructed by different granularity allocation protocols for the sunspot time series.

Protocols	The training data subset	The testing data subset
$\mathcal{P}_1$	0.3133 ± 0.0031	0.2924 ± 0.0060
$\mathcal{P}_2$	0.5509 ± 0.0042	0.4728 ± 0.0057
$\mathcal{P}_3$	0.5903 ± 0.0027	0.5215 ± 0.0031
$\mathcal{P}_4$	0.6279 ± 0.0035	0.5742 ± 0.0026
$\mathcal{P}_5$	0.4413 ± 0.0086	0.4297 ± 0.0082

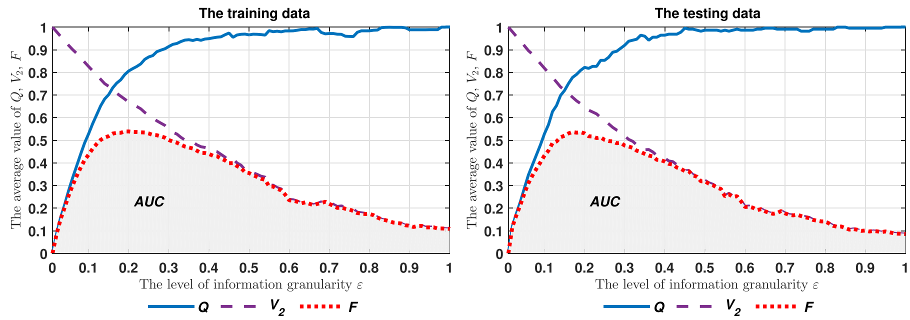
Note that for the Sugeno-type fuzzy model constructed in this stage, it totally consists of 72 parameters to be granulated, in which 32 parameters are parameter σs in Gaussian membership function used in antecedents and 40 parameters are the coefficients of polynomials used in conclusions (refer to Section 3.1 for details).

4.3.3. *Developing the Sugeno-type granular model and assessing its global performance*

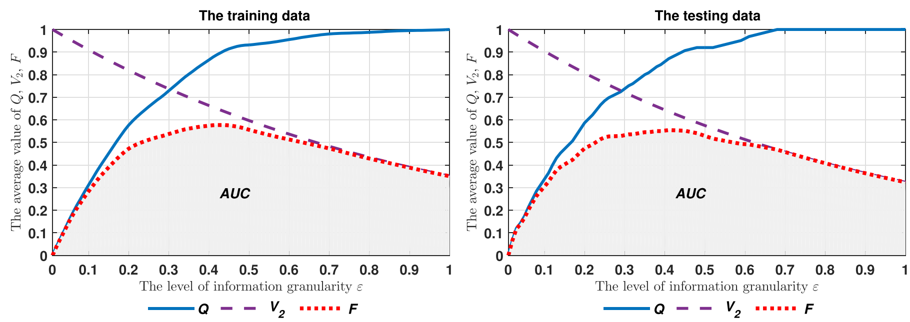
According to Stage 2 of experimental setup (see Section 4.2), the different protocols (refer to Section 3.2.1 for details) are exploited to optimally allocate information granularity ε to each parameter to be granulated in the Sugeno-type fuzzy model constructed above, which results in forming of the Sugeno-type granular models. Subsequently, we apply the Sugeno-type granular models which are formed by using the different protocols to perform interval prediction of time series and assess their performance (including the coverage index Q, the specificity index V_2, the composite performance index F and the global performance index AUC). In order to assure high confidence in the experimental results, for each protocol of allocating granularity, the above-mentioned experimental process is repeated 50 times. Table 2 reports the global performance of the Sugeno-type granular model constructed by using different protocols of allocating information granularity in the form of the average AUC value and the standard deviation. Besides, the plots of the average value of each of the above-mentioned performances index versus different level of information granularity ε are also illustrated in Figs. 8 and 9.

4.3.4. *Analysis*

(1) *Analysis of impact of different level of information granularity on the performance of the Sugeno-type granular model developed on basis of protocols which do not require using optimization technology*: Fig. 8 clearly shows relations between the performance indices of



(a) The case of using protocol $\mathcal{P}_1$ (not using optimization)



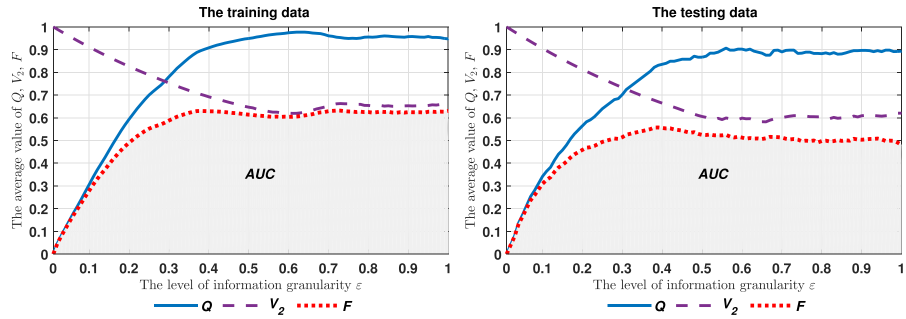
(b) The case of using protocol $\mathcal{P}_5$ (not using optimization)

Fig. 8. The plot of the average values of performance indices of the Sugeno-type granular model constructed by respectively using protocol $\mathcal{P}_1$ and $\mathcal{P}_5$ versus different level of information granularity for the sunspot time series.

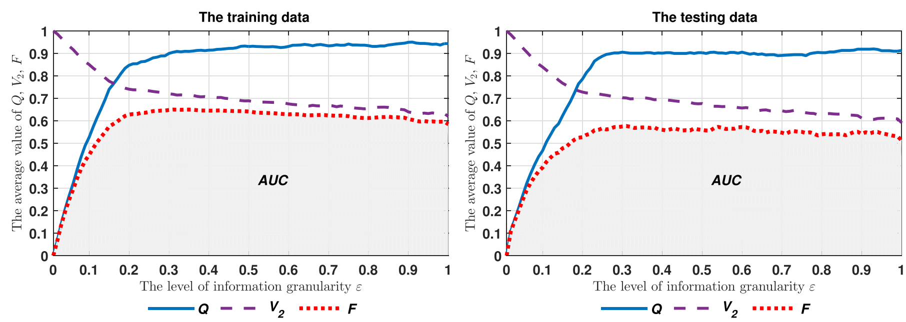
the Sugeno-type granular model obtained when using the protocol $\mathcal{P}_1$ and $\mathcal{P}_5$ to assign information granularity and the level of information granularity ε used in this process — for the coverage index (Q), its average value increases with increment of level of information granularity, when the value of level of information granularity is 1, the average value of corresponding coverage index can reach 1, viz. the output of corresponding Sugeno-type granular model completely covers historical data of time series; for the specificity index (V_2), its average value decreases with increment of level of information granularity, i.e. the interval length of output of the Sugeno-type granular model constructed gradually increase with increasing level of information granularity; for the composite performance index (F), the value of F substantially increases when increasing the value of level of information granularity however the value begins to decline when going beyond a certain value of level of information granularity. For example, considering the protocol $\mathcal{P}_1$ (see Fig. 8(a)), for the training and testing data set, when moving the value of level of information granularity ε from 0 to 0.19, the value of F get gradually higher and whereas when the value of level of information granularity ε exceeds 0.19, the value of F begins to decline. At the moment, the value of level of information granularity 0.19 becomes a “knee point”; for the protocol $\mathcal{P}_5$ (see Fig. 8(b)), the same situation is also appeared when the “knee point” is 0.41. The emergence of this phenomenon attributes to two reasons: one is that the values of F is synthesis results to the two conflict performance index — the coverage index Q and the specificity index V_2 and another is that the protocol $\mathcal{P}_1$ and $\mathcal{P}_5$ do not require using any optimization in the process of assigning information granularity.

(2) *Analysis of impact of different level of information granularity on the performance of the Sugeno-type granular model developed on basis of protocols which require using optimization technology*: Fig. 9 also clearly presents relations between the corresponding coverage index Q, specificity index V_2 and composite performance index F of the Sugeno-type granular model constructed on basis of the protocol $\mathcal{P}_2$, $\mathcal{P}_3$ and $\mathcal{P}_4$ and the level of information granularity ε — the average value of each of these performance indices presents “the saturation effect”, viz. the average value of each of these performance indices increases with the increment of the level of information granularity whereas the value becomes constant nearly (it is said to be “saturated”) when going beyond some level of granularity. For example, let us consider protocol $\mathcal{P}_4$, from Fig. 9(c), we clearly see that the values of the average coverage index Q, the specificity index V_2 and the composite performance index F are significant increase when the level of information granularity ε is increased from 0.0 to 0.16 — for training data subset, when the value of ε is 0, the average values of Q, V_2 and F are all 0 and when the value of ε is 0.16, the average values of Q, V_2 and F are 0.9538, 0.7082 and 0.6803, respectively. Whereas for testing data subset, the average values of Q, V_2 and F are all also 0 when the value of ε is 0 and the average values of Q, V_2 and F are respectively 0.9100, 0.7003 and 0.6491 when the value of ε is 0.16. When the level of information granularity is greater than 0.16, the value of each of these performance indices is constant for the training and testing data subset. At this point, the value of level of information granularity 0.16 becomes a “knee point”. For other protocols, there are also emergence of similar situation. The main reason resulting in this phenomenon is the protocol $\mathcal{P}_2$, $\mathcal{P}_3$ and $\mathcal{P}_4$ require using optimization technology to optimally allocate information granularity to parameters of the original Sugeno-type fuzzy model in the process of developing the Sugeno-type granular model. Besides, it is worth noting that when the level of information granularity is 0, the Sugeno-type granular model developed degenerates into the original Sugeno-type fuzzy model.

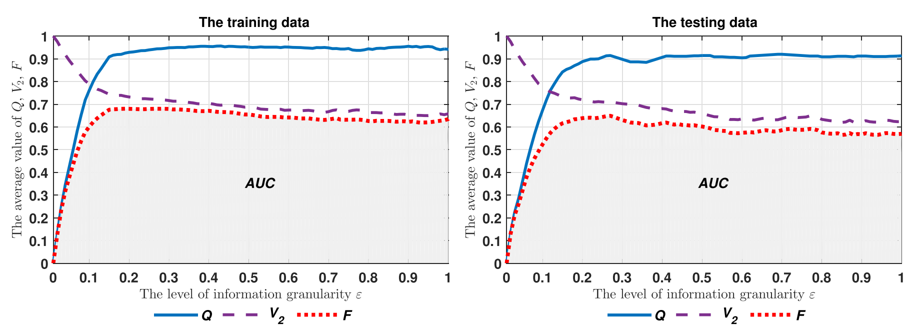
(3) *Analysis of global performance of the Sugeno-type granular model developed on basis of different protocols of allocating information granularity*: from Table 2, Figs. 8 and 9, we can see that



(a) The case of using protocol $\mathcal{P}_2$ (using optimization)



(b) The case of using protocol $\mathcal{P}_3$ (using optimization)



(c) The case of using protocol $\mathcal{P}_4$ (using optimization)

Fig. 9. The plot of the average values of performance indices of the Sugeno-type granular model constructed by respectively using protocol $\mathcal{P}_2$, $\mathcal{P}_3$ and $\mathcal{P}_4$ versus different level of information granularity for the sunspot time series.

the average value of *AUC* of the Sugeno-type granular model constructed according to the protocol $\mathcal{P}_4$ is slightly higher than the one obtained by the protocol $\mathcal{P}_3$. However, both are higher than the ones produced by other three protocols. The reason resulting in the findings is that the protocol $\mathcal{P}_4$ can provide highest level of flexibility, which results in more effective usage of information granularity. Fig. 10 shows the forecasted results of the Sugeno-type granular model which are constructed in the case of using the protocol $\mathcal{P}_4$ and the level of information granularity $\varepsilon = 0.16$. Under this case, the obtained value of coverage index, the specificity index and the composite performance index are $Q(0.16) = 0.92$, $V_2(0.16) = 0.73$ and $F(0.16) = 0.67$, respectively.

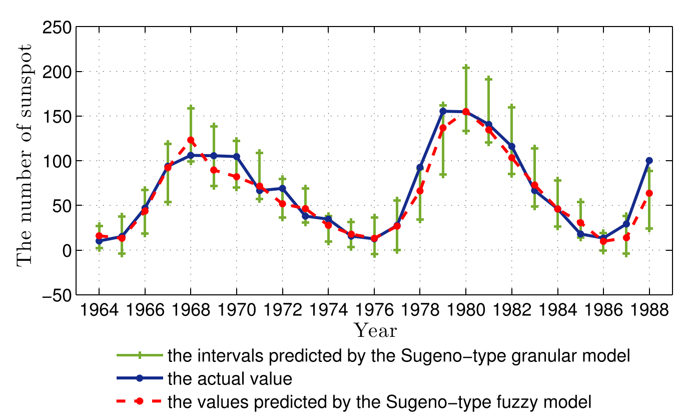


Fig. 10. The comparison plot of the forecasted intervals obtained by using the protocol $\mathcal{P}_4$ and the actual values for the Wolf's sunspot time series ($\varepsilon = 0.16$, $Q(0.16) = 0.92$, $V(0.16) = 0.73$, $F(0.16) = 0.67$).

4.4. TAIEX time series

The TAIEX time series shown in Fig. 11 consists of daily values of Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) from January 4, 1992 to December 27, 1992, which consists

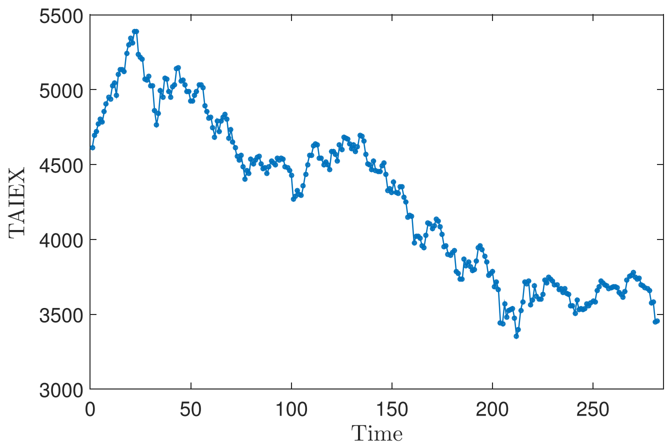


Fig. 11. The TAIEX time series.

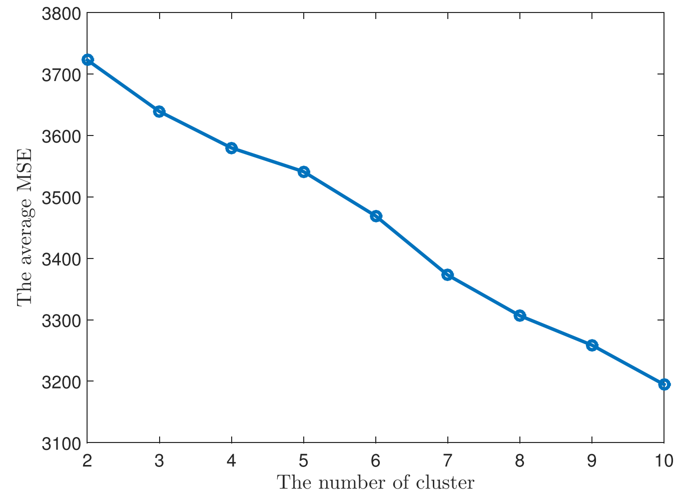
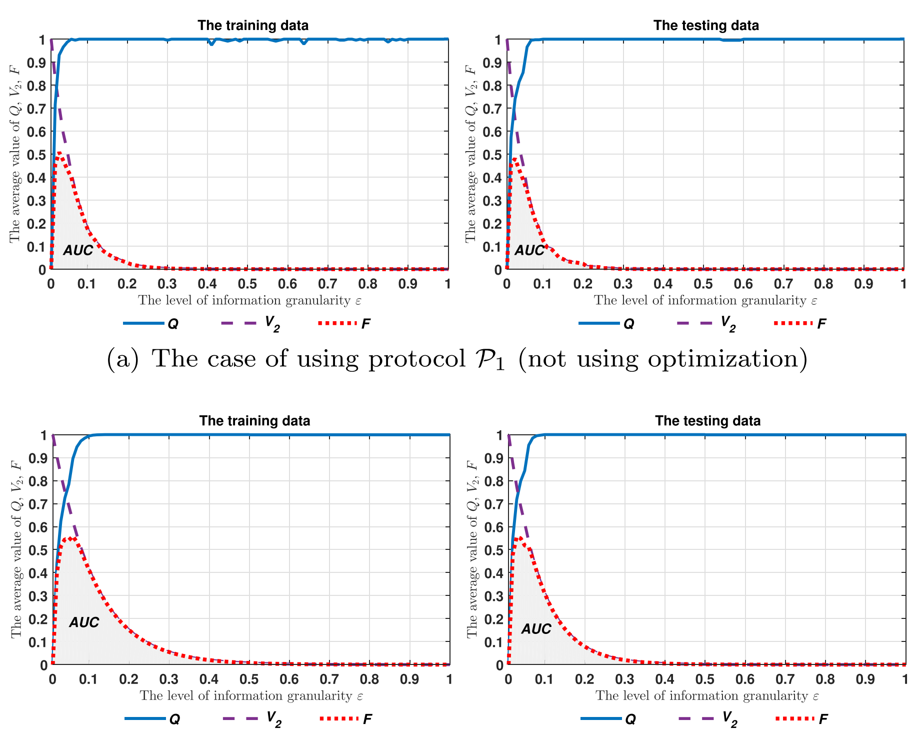


Fig. 12. The plot of performance of the constructed Sugeno-type fuzzy model versus the number of clusters for the TAIEX time series.

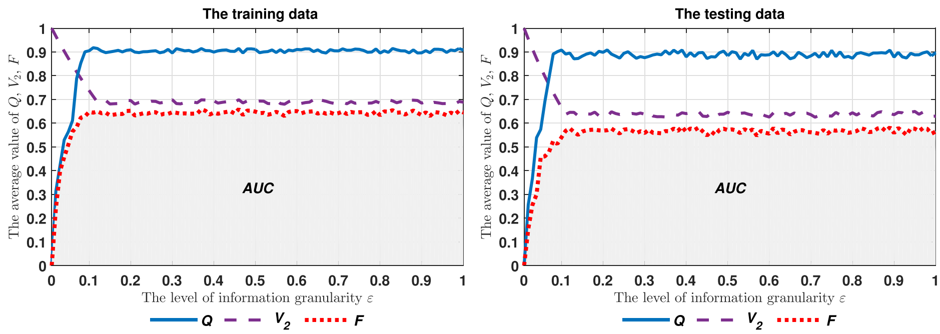
of 282 data. For Eq.(15), we let d, Δ and p be 4, 1 and 1, respectively, i.e. we exploit the value of TAIEX time series at $t-3$, $t-2$, $t-1$ and t time moment to forecast the value of TAIEX time series at $t+1$ time moment, which produces an input–output data set including 278 input–output data pairs. In the input–output data set, the first 228 data pairs are used to construct the Sugeno-type fuzzy model and its granular model, and the latter 50 data pairs are used to evaluate performance of the two models. The constructed Sugeno-type fuzzy model consists of four inputs and one output. In the process of constructing the Sugeno-type fuzzy model, FCM in which parameter m is set to 2 is used to extract rules on basis of the training data. The plot of performance of the constructed Sugeno-type fuzzy model quantified in terms of MSE versus the number of clusters is presented in Fig. 12. From Fig. 12, we can see that the performance of the Sugeno-type fuzzy model increases with increment of the number of clusters. Considering the compromise between the accuracy of model and the computing complexity of model, we select the number of cluster as 10. Thus, the constructed Sugeno-type fuzzy model totally includes 10 rules and 130 parameters (which is consisted of 80 antecedent parameters and 50 consequent parameters). Similar to the sunspot time series, these parameters can be identified by the combination of back-propagation and the least-squares method [14]. Referring to the experimental process of the Wolf's sunspot time series, we start with the constructed Sugeno-type fuzzy model to granulate its 90 parameters (including 40 parameter σs in Gaussian membership function used in antecedents and 50 coefficients of polynomials used in conclusions) according to different information granularity allocation protocols which is mentioned in Section 3.2, which results in the forming of corresponding granular model. All experimental results are reported in Figs. 13, 14 and Table 3.



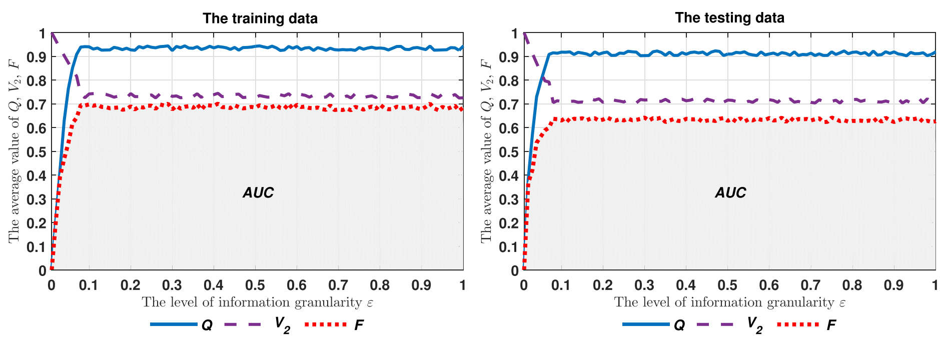
(a) The case of using protocol $\mathcal{P}_1$ (not using optimization)

(b) The case of using protocol $\mathcal{P}_5$ (not using optimization)

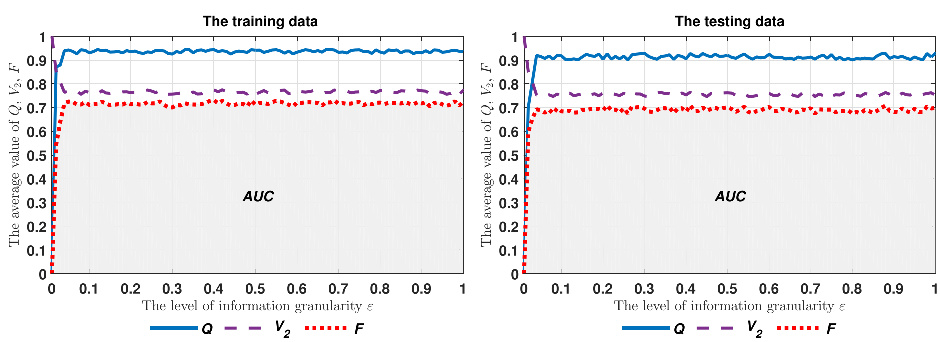
Fig. 13. The plot of the average values of performance indices of the Sugeno-type granular model constructed by respectively using protocol $\mathcal{P}_1$ and $\mathcal{P}_5$ versus different level of information granularity for the TAIEX time series.



(a) The case of using protocol $\mathcal{P}_2$ (using optimization)



(b) The case of using protocol $\mathcal{P}_3$ (using optimization)



(c) The case of using protocol $\mathcal{P}_4$ (using optimization)

Fig. 14. The plot of the average values of performance indices of the Sugeno-type granular model constructed by respectively using protocol $\mathcal{P}_2$, $\mathcal{P}_3$ and $\mathcal{P}_4$ versus different level of information granularity for the TAIEX time series.

Observing Figs. 13, 14 and Table 3, we can also get findings similar to the Wolf's sunspot time series: using the protocol $\mathcal{P}_4$ to optimally allocate information granularity can produce higher value of AUC (the global performance of the Sugeno-type granular model) than other four protocols. Besides, for any protocol which does require using optimization technology (i.e. $\mathcal{P}_2$, $\mathcal{P}_3$ or $\mathcal{P}_4$), the average coverage index (Q), the average specificity index (V_2) and the average composite performance index F, which are produced by the corresponding Sugeno-type granular model, also show the saturation effect. For example, for the protocol $\mathcal{P}_4$, when the value of level of information granularity is greater than 0.05, viz. the "knee point" is 0.05, the average coverage index (Q), the average specificity index (V_2) and the average composite performance index (F) enters the "saturated state" (see Fig. 14(c)). Fig. 15 presents the forecasted results of the corresponding Sugeno-type granular model which are constructed in the case of using the protocol $\mathcal{P}_4$ and the level of information granularity $\varepsilon = 0.05$. Under this case, the value of Q, V_2 and F are $Q(0.05) = 0.93$, $V_2(0.05) = 0.78$ and $F(0.05) = 0.73$, respectively.

Table 3
The value of AUC of the Sugeno-type granular model constructed by different granularity allocation protocols for the TAIEX time series.

Protocols	The training data subset	The testing data subset
$\mathcal{P}_1$	0.0412 ± 0.0006	0.0355 ± 0.0008
$\mathcal{P}_2$	0.6209 ± 0.0087	0.5733 ± 0.0009
$\mathcal{P}_3$	0.6735 ± 0.0007	0.6323 ± 0.0006
$\mathcal{P}_4$	0.7174 ± 0.0038	0.6886 ± 0.0026
$\mathcal{P}_5$	0.0834 ± 0.0008	0.0656 ± 0.0009

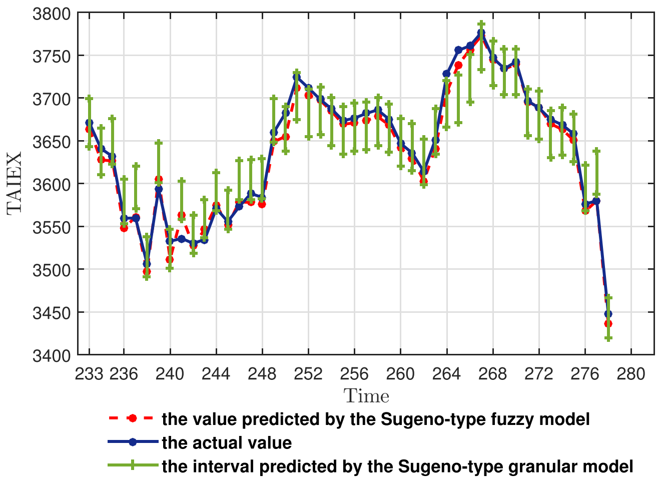


Fig. 15. The comparison plot of the forecasted intervals obtained by using the protocol $\mathcal{P}_4$ and the actual values for the TAIEX time series ($\varepsilon = 0.05$, $Q(0.05) = 0.93$, $V(0.05) = 0.78$, $F(0.05) = 0.73$).

4.5. The civilian unemployment rates (CUR) time series

The time series (from website: http://www.forecasts.org/data/data/UNRATE.htm) deals with monthly civilian unemployment rates in the USA from January 1, 1948 to December 1, 2013 and consists of 792 data, see Fig. 16.

For the time series, we let d, Δ and p be 2, 1 and 1, respectively. It leads to form an input–output data set including 790 input–output data pairs. In this input–output data set, the first 590 data pairs are used to construct the Sugeno-type fuzzy model and corresponding granular model, and the latter 200 data pairs are used to assess performance of the two models. The plot of the performance of the Sugeno-type fuzzy model versus the number of cluster is illustrated in Fig. 17, where the value of parameter m of FCM used is set to 2. We select the number of cluster as 10, which result in the forming of the Sugeno-type fuzzy model including entirely 10 rules and 70 parameters (in which 40 parameters are used in antecedents of rules and 30 parameters are used in conclusion of rules). Similar to the sunspot time series, these parameters can be also determined according to the combination of back-propagation and the least-squares method [14]. We use different allocation granularity protocols to optimally assign information granularity to related parameters of the constructed Sugeno-type fuzzy model. Here 50 parameters (including 20 parameter σs in Gaussian membership

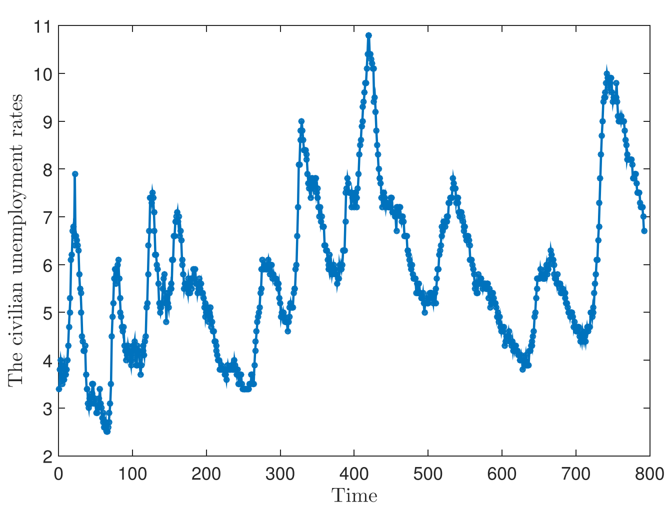


Fig. 16. The CUR time series.

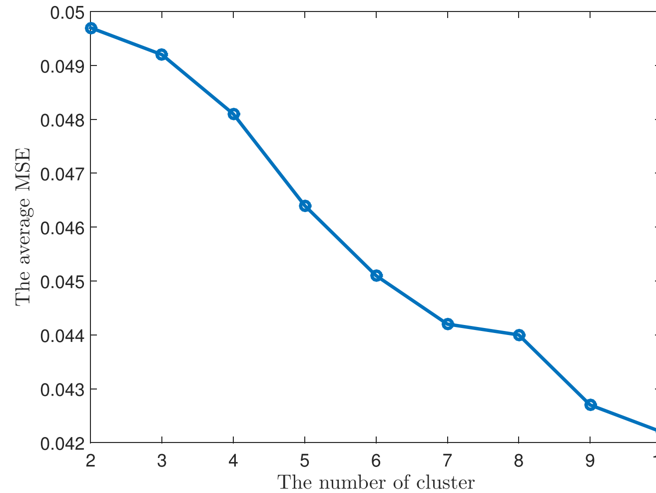


Fig. 17. The plot of performance of the constructed Sugeno-type fuzzy model versus the number of cluster for the CUR time series.

Table 4
The value of AUC of the Sugeno-type granular model constructed by the different granularity allocation protocols for the CUR time series.

Protocols	The training data subset	The testing data subset
$\mathcal{P}_1$	0.3520 ± 0.0051	0.3229 ± 0.0029
$\mathcal{P}_2$	0.5663 ± 0.0076	0.5279 ± 0.0018
$\mathcal{P}_3$	0.6232 ± 0.0022	0.5990 ± 0.0036
$\mathcal{P}_4$	0.6885 ± 0.0026	0.6492 ± 0.0038
$\mathcal{P}_5$	0.3903 ± 0.0021	0.3588 ± 0.0073

function used in antecedents and 30 coefficients of polynomials used in conclusions) are required to make granulation for forming corresponding granular model. Table 4 reports the value of AUC of the Sugeno-type granular model produced by different granularity allocation protocols. Similar to the above-mentioned two time series, the protocol $\mathcal{P}_4$ can also produce higher value of AUC than other four protocols. In the case of using the protocol $\mathcal{P}_4$, when the value of level of information granularity is greater than 0.12 (i.e. the "knee point" is 0.12), both the average coverage index (Q), the average specificity index (V_2) and the average composite performance index (F) of the constructed Sugeno-type granular model enter the "saturated state". Fig. 18 presents forecasted results of the corresponding Sugeno-type granular model. At the moment, the value of

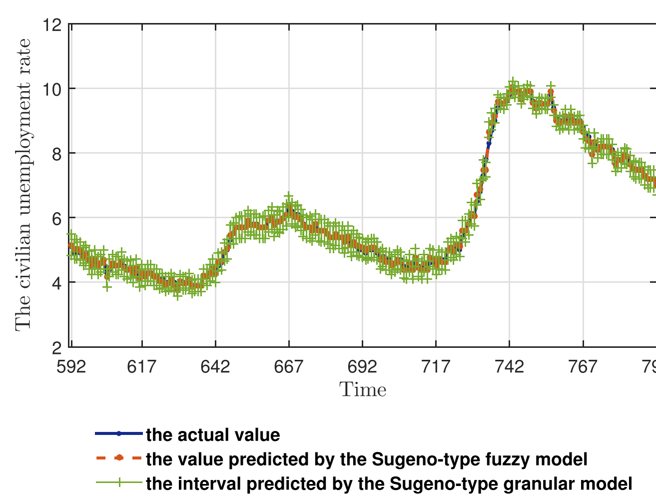


Fig. 18. The comparison plot of the predicted intervals obtained by using the protocol $\mathcal{P}_4$ and the actual values for the CUR time series ($\varepsilon = 0.12$, $Q(0.12) = 0.915$, $V(0.12) = 0.66$, $F(0.12) = 0.603$).

Q, V_2 and F are $Q(0.12) = 0.915$, $V_2(0.12) = 0.66$ and $F(0.12) = 0.603$, respectively.

5. Conclusions

In this paper, the design method of extending the Sugeno-type fuzzy model to its granular counterpart has been developed with the use of the optimal allocation of information granularity. The overall design process proceeds with a well-established Sugeno-type fuzzy model (the original Sugeno-type fuzzy model) and information granularity, which is regarded as an important and practically useful design asset, is fully exploited in the design process. The Sugeno-type granular model can be produced by information granularity optimally assigned to the corresponding of the model. Several protocols of optimal allocation of information granularity were presented. Three well-known time series from the real-world are considered to validate the feasibility and effectiveness of the proposed design method, whose results clearly identify the merits of the Sugeno-type granular model produced by our design method: (1) the output of the produced granular model is a granule (interval granule) instead of a numeric entity, which facilitates further interpretation; (2) the Sugeno-type granular model exhibits a higher level of flexibility than the original (numeric) Sugeno-type fuzzy model; (3) the proposed approach of extending the Sugeno-type fuzzy model to its granular model is of general nature as it could be applied to various fuzzy models and realized by invoking different formalisms of information granules.

Acknowledgements

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