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Weighted fuzzy time series models for TAIEX forecasting

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Abstract

This study proposes weighted models to tackle two issues in fuzzy time series forecasting, namely, recurrence and weighting. It is argued that recurrent fuzzy relationships, which were simply ignored in previous studies, should be considered in forecasting. It is also recommended that different weights be assigned to various fuzzy relationships. In previous studies, these fuzzy relationships were treated as if they were equally important, which might not have properly reflected the importance of each individual fuzzy relationship in forecasting. The weighted models are compared with the local regression models in which weight functions also play an important role. Both models are different by nature, but certain theoretical backgrounds in local regression models are adopted. By using the Taiwan stock index as the forecasting target, the empirical results show that the weighted model outperforms one of the conventional fuzzy time series models.

Keywords: Local regression models; Recurrence; Stock market; Weight schemes

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1. Introduction

Different approaches have been adopted for solving time series problems, including conventional and fuzzy approaches. Tanaka et al. [1–3] applied linear programming to solve fuzzy regression, and Watada applied fuzzy regression to solve fuzzy time series [4]. More recently, Tseng et al. extended fuzzy regression to autoregressive integrated moving average (ARIMA) analyses [5,6].

Meanwhile, Song and Chissom have proposed novel definitions for fuzzy time series [7]. Following these definitions, fuzzy time series models have been proposed for various applications, such as enrollment [8–13], stock indices [14,15,9,16–20], business cycles [21], and temperature forecasting [22]. Meanwhile, seasonal models [23,24], higher order models [25,17], bivariate models [14], and type 2 models [18,19], etc., have also been proposed.

The fuzzy time series model proposed by Song and Chissom [11] consists of two major processes: (1) fuzzification and (2) the establishment of fuzzy relationships and forecasting. One study has pointed out that during the fuzzification process, different lengths of intervals will result in various forecasting results, and it has been proposed that the effective lengths of intervals be used [16]. The forecasting results that were based on the effective lengths of intervals were found to outperform those based on arbitrary ones.

In establishing fuzzy relationships and forecasting, the way in which the appropriate fuzzy relationships are established is critical. In this study, there are two issues in this process that we intend to resolve: recurrence and weighting. The recurrent fuzzy relationships were ignored in previous studies [8–12,17]. Such ignorance resulted in there being missing information and hence the forecasting results were unsatisfactory. In addition, treating each fuzzy relationship as being of equal importance, as in previous studies [8–12,16,17,20], might not have properly reflected actual needs. Weighted fuzzy time series models are proposed here to resolve both problems.

Weighted fuzzy time series models appear quite similar to the weight functions in local regression models; however, both are different. The local regression models [26,27] focus on fitting using a small portion of the data, while the fuzzy relationships in weighted fuzzy time series models are established using the possible data from the whole of the database. Nevertheless, some of the theoretical backgrounds from the local regression models are applied to support the rationale for using weighted fuzzy time series models.

The goal of this study is to propose weighted fuzzy time series models to improve fuzzy time series forecasting, with the daily stock index in Taiwan being used in the empirical analysis. To that end, this paper is organized as follows: Section 2 reviews the previous fuzzy time series models. Section 3 discusses the rationale behind using weighted models. In Section 4, the weighted fuzzy time series models are introduced. Section 5 elaborates on the use of weighted models in forecasting. Section 6 evaluates the models' performance and Section 7 concludes the paper.

2. Review of fuzzy time series studies

2.1. Definitions

In Ref. [7], Song and Chissom defined a fuzzy time series as follows:

Definition 1. Let $Y(t)$ $(t = \ldots, 0, 1, 2, \ldots)$, a subset of real numbers, be the universe of discourse on which fuzzy sets $f_i(t)$ are defined. If $F(t)$ is a collection of $f_1(t)$, $f_2(t), \ldots, F(t)$ is called a fuzzy time series defined on $Y(t)$.

Definition 2. Suppose $F(t)$ is caused by $F(t-1)$ only. The relationship is expressed as $F(t) = F(t-1)*R(t, t-1)$, where $R(t, t-1)$ is the fuzzy relationship between $F(t)$ and $F(t-1)$; * represents an operator.

Let $F(t-1) = A_i$ and $F(t) = A_j$. The relationship between $F(t)$ and $F(t-1)$ (referred to as a fuzzy logical relationship, FLR, in [8,11,12]) can be denoted by

$$A_i \to A_j ,$$

where A_i refers to the left-hand side (LHS) and A_j to the right-hand side (RHS) of the FLR.

These FLRs can be further grouped to establish the fuzzy relationships. Various models have been proposed to establish fuzzy relationships, which are subsequently discussed in detail.

2.2. Lengths of intervals

In addition to establishing proper fuzzy relationships, the fuzzification process is also critical to fuzzy time series forecasting. In many previous fuzzy time series models, the lengths of the intervals were all equal and were also arbitrary, such as 1000 for enrollment forecasting [8–12]. However, one recent study has demonstrated that the lengths of the intervals could greatly affect the forecasting results [16]. Average- and distribution-based lengths have thus been proposed. The results of the forecasts that used the effective lengths of the intervals have been shown to improve upon most of the forecasting results that used arbitrary ones. Hence, this study also applies both lengths of intervals to the empirical analysis.

Both lengths of intervals are exemplified as follows. In the previously mentioned recent study, a time series was given as 30, 50, 80, 120, 100, and 70. The average-based length of intervals can be calculated as follows: (1) Calculate the first differences, which are 20, 30, 40, 20, and 30. Their average is then 28. (2) Take half of the average as the length, which is 14. (3) According to the length (in Step 2), the base for the length of the intervals is determined to be 10 by a Map Table. (4) Round down the length 14 by the base 10, which is 10. So 10 is chosen as the average-based length of intervals.

The distribution-based length can be calculated as follows. (1) Calculate the first differences (in the same way as for average-based differences), to arrive at an average of 28. (2) From a Map Table, the base for the length of intervals is 10. (3) The number of first differences larger than 30 is 1. The number of first differences larger than 20 is 3. (4) Because 20 is the largest length that we can have, which is still smaller than at least half of the first differences, 20 is chosen as the length of the intervals.

2.3. A conventional fuzzy time series model

Chen's model is one of the conventional models with simple calculations as well as better forecasting results [8]. It is used as an example in the illustration that follows:

Step 1: Defining the universe of discourse and intervals for observations.

According to the problem domain, the universe of discourse for observations, $U = [starting, ending]$, is defined. After the length of the intervals, l, is determined, the U can be partitioned into equal-length intervals $u_1, u_2, \ldots, u_b$ [8,10–13]. Each interval u_d can be calculated as $[starting + (d-1) \times l,\ starting + d \times l]$ and its corresponding midpoint m_d as $\frac{1}{2} \times [starting + (d-1) \times l + starting + d \times l]$, where $d = 1, 2, \ldots$.

Step 2: Defining fuzzy sets for observations.

Each linguistic observation, A_i, can be defined by the intervals $u_1, u_2, u_3, \ldots, u_b$. $A_i = f_{Ai}(u_1)/u_1 + f_{Ai}(u_2)/u_2 + \cdots + f_{Ai}(u_b)/u_b$. Following [8], each A_i can be represented as follows: if $i = 1, \ldots, A_i = \cdots + 0/u_{i-2} + 0.5/u_{i-1} + 1/u_i + 0.5/u_{i+1} + 0/u_{i+2} + \cdots$.

Step 3: Fuzzifying observations.

Each datum can be mapped to a fuzzy set. As in [8,12], a datum is fuzzified to A_j if the maximal degree of membership of that datum is in A_j.

$$\text{fuzzify}(actual_t) = A_j \text{ if } f_{actual_t}(A_j) = \max[f_{actual_t}(A_z)] \quad \text{for all } z\,,$$

where $z = 1, \ldots,$ $actual_t$ is the datum at time t, and $f_{actual_t}(A_z)$ is the degree of membership of $actual_t$ under A_z.

Step 4: Establishing FLRs.

As in Definition 4, two consecutive fuzzy sets A_i (at $t-1$) and A_{j1} (at t) can be used to establish an FLR as $A_i \to A_{j1}$.

Step 5: Establishing fuzzy relationships.

The FLRs with the same LHSs can be grouped into an ordered FLRG by putting all their RHSs together as on the RHS of the FLRG [8]. For example, $A_i \to A_{j1}$, $A_i \to A_{j2}, \ldots, A_i \to A_{jk}$, can be grouped into an FLRG: $A_i \to A_{j1},\ A_{j2}, \ldots, A_{jk}$. However, the repeated FLRs are counted only once [8,11,12].

Step 6: Forecasting.

Suppose $F(t-1) = A_i$. The forecasting of $F(t)$ is conducted using the following rules:

Rule 1: If $A_i \to$, the forecast of $F(t)$ is equal to A_i.

Rule 2: If $A_i \to A_{j1}, A_{j2}, \ldots, A_{jk}$, the forecast of $F(t)$ is equal to $A_{j1}, A_{j2}, \ldots, A_{jk}$.

Step 7: Defuzzifying.

If the forecast of $F(t)$ is $A_{j1}, A_{j2}, \ldots, A_{jk}$, the defuzzified result is equal to the arithmetic average of the midpoints of $A_{j1}, A_{j2}, \ldots, A_{jk}$ [8].

$$\text{final}(t) = \text{defuzzified}(A_{j1}, A_{j2}, \ldots) = \frac{\sum_{p=1}^{k} m_{jp}}{k} ,$$

where final(t) is the final forecast.

3. Why weighted models?

There are two reasons why weighted fuzzy time series models are proposed. The first is to resolve recurrent fuzzy relationships, which were not properly handled in previous related studies. The other is to assign proper weights to various fuzzy relationships to reflect the differences in their importance. Both issues are elaborated. In addition, local regression models are compared with the weighted models.

3.1. Recurrence

In previous studies [8–12,17] the repeated FLRs were simply ignored when fuzzy relationships were established. To explain this, we use the FLRG [8]. Suppose there are FLRs in chronological order as follows:

$$\begin{aligned} &(t=1) \quad A_1 \to A_1 ,\\ &(t=2) \quad A_1 \to A_2 ,\\ &(t=3) \quad A_2 \to A_1 ,\\ &(t=4) \quad A_1 \to A_1 ,\\ &(t=5) \quad A_1 \to A_1 . \end{aligned} \tag{1}$$

In Eq. (1), four out of five FLRs have the same LHS, A_1, as in Eq. (2):

$$\begin{aligned} &(t=1) \quad A_1 \to A_1 ,\\ &(t=2) \quad A_1 \to A_2 ,\\ &(t=4) \quad A_1 \to A_1 ,\\ &(t=5) \quad A_1 \to A_1 . \end{aligned} \tag{2}$$

Following Ref. [8], these FLRs in Eq. (2) are used to establish an FLRG as

$$A_1 \to A_1, A_2 . \tag{3}$$

The occurrences of the same FLRs in Eq. (2) are regarded as if there were only one occurrence. In other words, the recent identical FLRs are simply ignored in Eq. (3).

The ignoring of recurrence, however, is questionable. The occurrence of a particular FLR represents the number of its appearances in the past. For instance, in Eq. (2), $A_1 \to A_1$ appears three times and $A_1 \to A_2$ only once. The recurrence can be used to indicate how the FLR may appear in the future.

Hence, to cover all of the FLRs, an approach to representing the fuzzy relationship is suggested below:

$$A_1 \rightarrow A_1, A_2, A_1, A_1 \ . \tag{4}$$

In the weighted models, the recurrence of each FLR should be taken into account. From this viewpoint, the FLRs of various recurrences are assigned different weights.

3.2. Weights

In previous studies [8,9,17,11,12] each FLR was treated as if it was of equal importance, which may not have reflected the real world situation. There are two possible ways of determining the weights. The first is to determine them based on domain know-how, which can be elicited from domain experts. The second is to determine them based on their chronological order. In the case of the latter, the FLRs for different time slots are usually considered to differ in importance. One way is to regard the recent FLRs as more important than the older ones; hence, higher weights are assigned to the recent ones.

To apply the chronologically-determined weights, we can simply assign different weights for each FLR incrementally, say 1, 2, 3, and 4:

$$\begin{aligned}
&(t=1) \quad A_1 \rightarrow A_1 \quad \text{with weight } 1, \\
&(t=2) \quad A_1 \rightarrow A_2 \quad \text{with weight } 2, \\
&(t=4) \quad A_1 \rightarrow A_1 \quad \text{with weight } 3, \\
&(t=5) \quad A_1 \rightarrow A_1 \quad \text{with weight } 4 \ .
\end{aligned} \tag{5}$$

As a result, the most recent FLR ($t=4$) is assigned the highest weight of 4, which means that the probability of its appearance in the near future is higher than in the case of the others. On the other hand, the most aged FLR ($t=1$) is assigned the lowest weight of 1, which means that the probability of its appearance in the near future is lower than in the case of the others.

For both the domain- and chronologically-determined weights, different weight schemes can be used. On the other hand, based on the merits of these weighted models, the models used in previous studies (without the weight specification) can be regarded as models to which equal weights are assigned. Therefore, the weighted models are a general case of these models.

3.3. Local regression models

The concept of applying weighs to fuzzy time series models appears very similar to applying them to local regression models. Weight functions are applied in both models. In fact, both are different by nature. First, in local regression models, weight functions are used to assist in the fitting of data at t, by means of a small portion of data around t [26,27]. For example, in Fig. 1, the point at t can be modeled by the points around t, say $(t-2)$, $(t-1)$, t, $(t+1)$, and $(t+2)$. This is why this is referred to as local regression. However, from the conventional fuzzy time series model, the

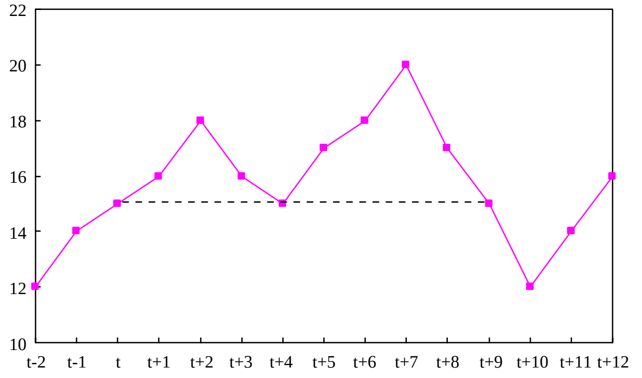

Fig. 1. Local regression model.

fuzzy relationships in fuzzy time series models are established using all of the data. In the same figure, there are various points with the same value as t, such as $t+4$ and $t+9$. FLRs (Step 4 in Chen's model) are established as shown below.

(Suppose the y-axis is represented by fuzzy sets.)

$$A_{15} \to A_{16} \quad (\text{from } t \text{ to } t+1)\,,$$
$$A_{15} \to A_{17} \quad (\text{from } t+4 \text{ to } t+5)\,,$$
$$A_{15} \to A_{12} \quad (\text{from } t+9 \text{ to } t+10)\,.$$

Then, an FLRG is established as

$$A_{15} \to A_{16}, A_{17}, A_{12}\,.$$

Second, the weight functions in the local regression models are based on the fitted point t, which possesses the highest weight, and the weight decreases for the point drifting away on both sides of t. However, the weight function is monotonic for weighted fuzzy time series, which means that the more recent the fuzzy set, the higher will be the weight.

Nevertheless, some of the theoretical background in local regression models can be applied to support the weighted models. In local regression models, when the point gets closer to the point for fitting, the weight gets higher. In essence, in the weighted models, the more recent the fuzzy set (the closer it is to the out-of-sample data), the higher will be the weight. Meanwhile, in local regression models, the sum of the weights is equal to 1, which is also applicable to the setting in the weighted models.

4. Weighted fuzzy time series models

Weighted fuzzy time series models are proposed based on the conventional fuzzy time series models. Step 5 in the conventional models is revised to handle the recurrence issue. Step 6 remains the same. Step 7 is revised to process the defuzzified matrix. Meanwhile, Step 8 is added to handle the weight assignment. Step 9 is added to calculate the final results.

Step 5: Establishing fuzzy relationships (revised).

The recurrent FLRs are taken into account by revising Step 5 in the conventional model. The example in Eq. (1) is used.

Example 1.1. There is one FLR with one LHS, A_2. So we have an FLRG as in Eq. (3): $A_2 \to A_1$.

Example 1.2. There are 4 FLRs with the same LHS, $A_1 \to A_1$, $A_1 \to A_2$, $A_1 \to A_1$, and $A_1 \to A_1$ in Eq. (2). These FLRs are used to establish another FLRG as in Eq. (4): $A_1 \to A_1, A_2, A_1, A_1$.

Step 6: Forecasting.

Example 2.1 (*Following Example 1.1*). Because $F(t-1) = A_2$, the forecast of $F(t)$ is A_1.

Example 2.2 (*Following Example 1.2*). On the other hand, because $F(t-1) = A_1$, the forecast of $F(t)$ is A_1, A_2, A_1, A_1.

Step 7: Defuzzifying (revised).

Suppose the forecast of $F(t)$ is $A_{j1}, A_{j2}, \ldots, A_{jk}$. The defuzzified matrix is equal to a matrix of the midpoints of $A_{j1}, A_{j2}, \ldots, A_{jk}$:

$$M(t) = [m_{j1}, m_{j2}, \ldots, m_{jk}] , \tag{6}$$

where $M(t)$ represents the defuzzified forecast of $F(t)$.

Example 3.1 (*Following Example 2.1*). Following Eq. (6), the defuzzified matrix is $M(t) = [m_1]$.

Example 3.2 (*Following Example 2.2*). The defuzzified matrix is $M(t) = [m_1, m_2, m_1, m_1]$.

Step 8: Assigning weights.

Suppose the forecast of $F(t)$ is $A_{j1}, A_{j2}, \ldots, A_{jk}$. The corresponding weights for $A_{j1}, A_{j2}, \ldots, A_{jk}$, say, $w_1, w_2, \ldots, w_k$, are specified. However, before forming the weight matrix with these $w_1, w_2, \ldots, w_k$, the weight matrix $W(t) = [w'_1, w'_2, \ldots, w'_k]$ should satisfy a condition:

$$\sum_{h=1}^{k} w'_h = 1 . \tag{7}$$

Hence, these weights $w_1, w_2, \ldots, w_k$ should be standardized. We then obtain the following weight matrix:

$$W(t) = [w'_1, w'_2, \ldots, w'_k] = \left[\frac{w_1}{\sum_{h=1}^{k} w_h}, \frac{w_2}{\sum_{h=1}^{k} w_h}, \ldots, \frac{w_k}{\sum_{h=1}^{k} w_h} \right] , \tag{8}$$

where w_h is the corresponding weight for A_{jh}.

In addition, the weight matrix is monotonic; hence, it also satisfies the following condition:

$$w_1 \leqslant w_2 \leqslant \cdots \leqslant w_k \, . \tag{9}$$

Following Eq. (9), an intuitive weight scheme is proposed as follows:

$$\begin{aligned} &w_1 = 1 , \\ &w_i = w_{i-1} + 1 \quad \text{for } i \geqslant 2 \, . \end{aligned} \tag{10}$$

Hence, the ith item in Eq. (8) can be rewritten as

$$\frac{w_i}{\sum_{h=1}^{k} w_h} = \frac{w_{i-1}+1}{\sum_{h=1}^{k} w_h} = \frac{(w_{i-2}+1)+1}{\sum_{h=1}^{k} w_h} = \cdots \frac{i}{\sum_{h=1}^{k} w_h} = \frac{i}{\sum_{h=1}^{k} h} \, . \tag{11}$$

In this case, Eq. (8) becomes

$$\begin{aligned} W(t) = [w'_1, w'_2, \ldots, w'_k] &= \left[\frac{w_1}{\sum_{h=1}^{k} w_h}, \frac{w_2}{\sum_{h=1}^{k} w_h}, \ldots, \frac{w_k}{\sum_{h=1}^{k} w_h} \right] \\ &= \left[\frac{1}{\sum_{h=1}^{k} h}, \frac{2}{\sum_{h=1}^{k} h}, \ldots, \frac{k}{\sum_{h=1}^{k} h} \right] . \end{aligned} \tag{12}$$

Example 4.1 (*Following Example 2.1*). When the forecast of $F(t)$ is A_1, the weight matrix is determined as $W(t) = \left[\frac{1}{1}\right]$.

Example 4.2 (*Following Example 2.2*). When the forecast of $F(t)$ is A_1, A_2, A_1, A_1, the weights are specified as follows: $w_1 = 1$, $w_2 = 2$, $w_3 = 3$, $w_4 = 4, \ldots$, by following Eq. (10). From Eq. (12), the weight matrix is determined as

$$\begin{aligned} W(t) &= \left[\frac{1}{1+2+3+4}, \frac{2}{1+2+3+4}, \frac{3}{1+2+3+4}, \frac{4}{1+2+3+4} \right] \\ &= \left[\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10} \right] . \end{aligned}$$

Step 9: Calculating results.

In the weighted model, the final forecast is equal to the product of the defuzzified matrix and the transpose of the weight matrix:

$$\begin{aligned} \text{final}(t) = M(t) \times W(t)^T &= [m_1, m_2, m_1, m_1] \times [w'_1, w'_2, \ldots, w'_k]^T \\ &= [m_1, m_2, m_1, m_1] \times \left[\frac{1}{\sum_{h=1}^{k} h}, \frac{2}{\sum_{h=1}^{k} h}, \ldots, \frac{k}{\sum_{h=1}^{k} h} \right]^T , \end{aligned} \tag{13}$$

where $\times$ is the matrix product operator, and $M(t)$ is a $1 \times k$ matrix and $W(t)^T$ is a $k \times 1$ matrix, respectively.

Example 5.1 (*Following Examples 3.1 and 4.1*).

$$\mathrm{final}(t) = M(t) \times W(t)^T = [m_1] \times \left[\tfrac{1}{1}\right]^T .$$

Example 5.2 (*Following Examples 3.2 and 4.2*).

$$\mathrm{final}(t) = M(t) \times W(t)^T = [m_1, m_2, m_1, m_1] \times \left[\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}\right]^T .$$

5. TAIEX forecasting

To illustrate how these four models perform fuzzy time series forecasting, some data from the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) are used.

Step 1: Defining the universe of discourse and intervals for observations.

According to the TAIEX for 1990, the universe of discourse for observations, U, is defined as [2500, 13 000]. The length of the intervals determined by the average-based length of intervals [16] is 100. The U can be partitioned into equal-length intervals $u_1, u_2, u_3, \ldots, u_{105}$, and the midpoints of these intervals are $m_1, m_2, m_3, \ldots, m_{105}$, respectively. Based on Eq. (3), the intervals are set as $u_1 = [2500, 2600], u_2 = [2600, 2700], u_3 = [2700, 2800], \ldots, u_{105} = [12\,900, 13\,000]$. Using Eq. (4), the midpoints are set as $m_1 = 2550, m_2 = 2650, m_3 = 2750, \ldots, m_{105} = 12950$.

Step 2: Defining fuzzy sets for observations.

Each linguistic observation, A_i, can be defined by the intervals $u_1, u_2, u_3, \ldots, u_{105}$, as follows:

$$\begin{aligned} A_1 = {} & 1/u_1 + 0.5/u_2 + 0/u_3 + 0/u_4 + 0/u_5 + 0/u_6 \\ & + \cdots + 0/u_{103} + 0/u_{104} + 0/u_{105} , \end{aligned}$$

$$\begin{aligned} A_2 = {} & 0.5/u_1 + 1/u_2 + 0.5/u_3 + 0/u_4 + 0/u_5 + 0/u_6 \\ & + \cdots + 0/u_{103} + 0/u_{104} + 0/u_{105} , \end{aligned}$$

$$\begin{aligned} A_3 = {} & 0/u_1 + 0.5/u_2 + 1/u_3 + 0.5/u_4 + 0/u_5 + 0/u_6 \\ & + \cdots + 0/u_{103} + 0/u_{104} + 0/u_{105} , \end{aligned}$$

$$\ldots$$

$$\begin{aligned} A_{104} = {} & 0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + 0/u_5 + 0/u_6 \\ & + \cdots + 0.5/u_{103} + 1.0/u_{104} + 0.5/u_{105} , \end{aligned}$$

$$\begin{aligned} A_{105} = {} & 0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + 0/u_5 + 0/u_6 \\ & + \cdots + 0/u_{103} + 0.5/u_{104} + 1.0/u_{105} . \end{aligned}$$

Table 1
Fuzzy sets for TAIEX

Date	TAIEX	Fuzzy set
8/22/1990	3506.80	A_{11}
8/23/1990	3333.46	A_9
…		
9/3/1990	3574.89	A_{11}
9/4/1990	3600.75	A_{12}
…		
9/13/1990	3520.11	A_{11}
9/14/1990	3480.47	A_{10}
…		
10/24/1990	3519.41	A_{11}
10/26/1990	3316.36	A_9

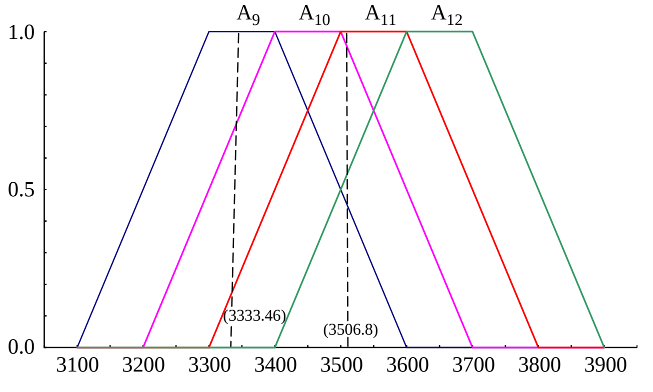

Fig. 2. Membership function for TAIEX.

Step 3: Fuzzifying observations.

Each TAIEX can be fuzzified into a fuzzy set. To facilitate the explanations that follow, some TAIEX are listed in Table 1 as an example. From Fig. 2, the TAIEX for 8/22/1990 was 3506.8; fuzzify(3506.8) = A_{11}. The TAIEX for 8/23/1990 was 3333.46; fuzzify(3333.46) = A_9.

Step 4: Establishing FLRs.

Based on the fuzzy sets in Step 3, the FLRs are established as follows.

$$A_{11} \to A_9, \ldots,$$

$$A_{11} \to A_{12}, \ldots,$$

$$A_{11} \to A_{10}, \ldots,$$

$$A_{11} \to A_9, \ldots .$$

Step 5: Establishing FLRGs.

The FLRs in Step 4 are used to establish an FLRG as

$$A_{11} \to A_9, A_{12}, A_{10}, A_9 \,.$$

Step 6: Forecasting.

If $F(t-1) = A_{11}$, the forecasting of $F(t)$ is equal to

$$A_9, A_{12}, A_{10}, A_9 \,.$$

Step 7: Defuzzifying (revised).

The corresponding defuzzified forecast is

$$M(t) = [m_9, m_{12}, m_{10}, m_9] \,.$$

Step 8: Assigning weights.

Following Eq. (12), the weight matrix becomes

$$\begin{aligned} W(t) &= \left[\frac{1}{1+2+3+4}, \frac{2}{1+2+3+4}, \frac{3}{1+2+3+4}, \frac{4}{1+2+3+4}\right] \\ &= \left[\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}\right] \,. \end{aligned}$$

Step 9: Calculating results.

The final forecast is calculated as $\text{final}(t) = M(t) \times W(t)^T$.

$$\begin{aligned} \text{final}(t) &= [m_9, m_{12}, m_{10}, m_9] \times \left[\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}\right]^T \,. \\ &= [3350, 3650, 3450, 3350] \times \left[\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}\right]^T = 3440 \,. \end{aligned}$$

Step 10: Evaluating performance.

The root mean squared error (RMSE) is used to measure performance:

$$\text{RMSE} = \sqrt{\frac{\sum_{t=1}^{s} (actual_t - \text{final}(t))^2}{s}} \,, \tag{14}$$

where there are s forecasts.

6. Empirical analyses

6.1. Data description and setup

To demonstrate the effectiveness of the weighted models, large amounts of data are needed. For this reason we use the daily TAIEX closing prices covering the period from 1990 to 1999. The data from January to October for each year are used to perform the estimation while those for November and December are used for forecasting.

Table 2
Information for TAIEX forecasting

	1999	1998	1997	1996	1995	1994	1993	1992	1991	1990
Estimation	1/5–10/30	1/3–10/31	1/4–10/30	1/4–10/30	1/5–10/30	1/5–10/29	1/5–10/30	1/4–10/30	1/1–10/30	1/4–10/30
Forecasting	11/1–12/28	11/2–12/31	11/3–12/31	11/1– 12/31	11/1–12/30	11/1–12/31	11/2–12/31	11/2–12/29	11/1–12/28	11/1–12/27
Starting	5400	6200	6800	4600	4500	5100	3100	3300	3300	2500
Average	50	60	60	30	30	40	30	30	70	100
Distribution	40	40	50	20	30	30	20	20	50	90

Table 3
Forecasting RMSEs

	1999	1998	1997	1996	1995	1994	1993	1992	1991	1990
1. *Conventional models*										
Average	149	167	148	54	79	112	110	60	80	220
Distribution	159	159	149	52	79	132	105	60	79	270
2. *Weighted models*										
Average	142	151	133	54	70	135	105	67	61	227
Distribution	145	154	152	52	70	114	105	56	67	266

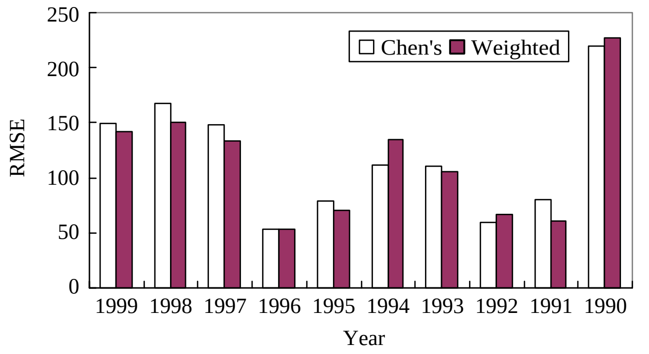

Fig. 3. Comparison of RMSEs by the average-based lengths of intervals.

In order to forecast the fuzzy time series, we need to determine the starting value for the universe of discourse (as in Step 1). To this end, we round down the minimal data for each year to the nearest hundred and set the value as the *starting*. We apply the average-based and distribution-based lengths of intervals as in Ref. [16]. All of the information is listed in Table 2.

6.2. Performance evaluation

After forecasting, the RMSEs obtained using the conventional and weighted fuzzy time series models, and covering the period from 1990 to 1999, are listed in Table 3. We find that only 4 out of 20 RMSEs (for both the average- and distribution-based lengths of intervals) in the weighted model are worse than their counterparts in Chen's model. The weighted model provides some improvement. The RMSEs for both models are also compared in Figs. 3 and 4 according to the different lengths of intervals, respectively.

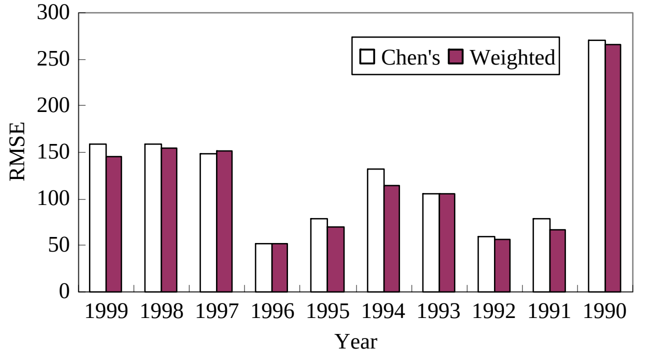

Fig. 4. Comparison of RMSEs by the distribution-based lengths of intervals.

7. Conclusions and future work

This study proposes weighted fuzzy time series models to resolve two issues in fuzzy time series forecasting: recurrence and weighting. The TAIEX from 1990 to 1999 is used as the forecasting target. Both average- and distribution-based lengths of intervals are chosen as the lengths of intervals. The forecasting results show that the weighted models outperform Chen's model. The weighted models can thus be applied to improve fuzzy time series forecasting.

The weighted fuzzy time series models appear similar to the local regression models. However, they are different in that the fuzzy relationships in the weighted models are established on the basis of all of the data. On the contrary, the fitting in the local regression models is based on only a portion of the data. Nevertheless, some of the theoretical background in the local regression models is applied in the weighted models. In addition, local regression models in relation to fuzzy time series remain an interesting topic worthy of further study.

As for the way in which the weights are specified, there can be various weight schemes for different applications. This study proposes a weight scheme similar to that of local regression models, where it is seen that the more recent ones have higher weights than the older ones. Others may be determined using methodologies such as neural networks or genetic algorithms. How to explore appropriate weight schemes for forecasting particular applications is also suggested as a future research topic.

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