Verifying an Effect-Based Cooperative Concurrency Scheduler in Iris

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Table of Contents

1. Introduction (WIP)

- As a motivation for the work: program verification, **safety** and why we care about it.
- Iris is a new separation logic which allows proving safety for programs using mutable shared state.
- Many programs nowadays user user-level concurrency to handle a big number of tasks. As an example for OCaml 5 there exists the Eio library which provides concurrency primitives using effect handlers.
- Effect handlers are a versatile concept which allow a modular treatment of effects, the implementation in form of a handler is separated from the code using the effect, and it's more lightweight than monads. Give a simple example of state.
  - The biggest upside is that they are more composable than monads which often require rewriting of parts of the program into monadic style
  - In theory effect can be tracked by the type system, although OCaml 5 does not do that yet.
  - Explain the concept of **effect safety** here.
  - Mention that continuations can only be invoked once? (not really necessary info)
- We want to verify some parts of the Eio library but the standard Heaplang language for Iris does not support effect handlers.
  - Hazel is an Iris language formalizing effect handlers using protocols.
  - Since OCaml 5 allows both effect handlers and mutable shared state we had to add a multi-threaded semantics to Hazel.
- Inherent part of a scheduler is liveness, because it is responsible for running all fibers to completion. Unfortunately it is hard to prove liveness properties in Iris so we just focus on safety and effect safety.

1.1. The Eio Library (WIP)

- Library for cooperative concurrency in OCaml 5.
- Implements switching between tasks using effect handlers.
- A fiber is a normal OCaml function which may perform effects that are handled by a scheduler.
- Each scheduler is only responsible for a single thread, more can be spawned.
- It offers abstractions to operating system resources to fibers, e.g. network, filesystem, timers etc.
- It also offers synchronization and message pasing constructs like mutexes & channels which are specialized to handle fibers, i.e. a mutex does not suspend the system-level thread, but the fiber.

1.2. Focus and Structure of the Thesis

Eio aims to be the standard cooperative concurrency library for OCaml 5 so it includes many functions for structured concurrency of fibers (e.g. `Fiber.{first, any, both, all}` which run two or more fibers and combine their results), support for cancelling fibers, abstractions for operating system resources, a different scheduler implementation per OS, and

synchronization constructs like promises and mutexes. But for this work we restrict ourselves to verifying the safety and effect safety of Eio's core functionalities: 1. Running fibers in a "common denominator" scheduler that does not interact with any OS resources, 2. awaiting the result of other fibers using the *promise* synchronization construct, 3. and spawning new schedulers to run fibers in another thread.

Below we show the simplified module hierarchy of the concepts we focus on.
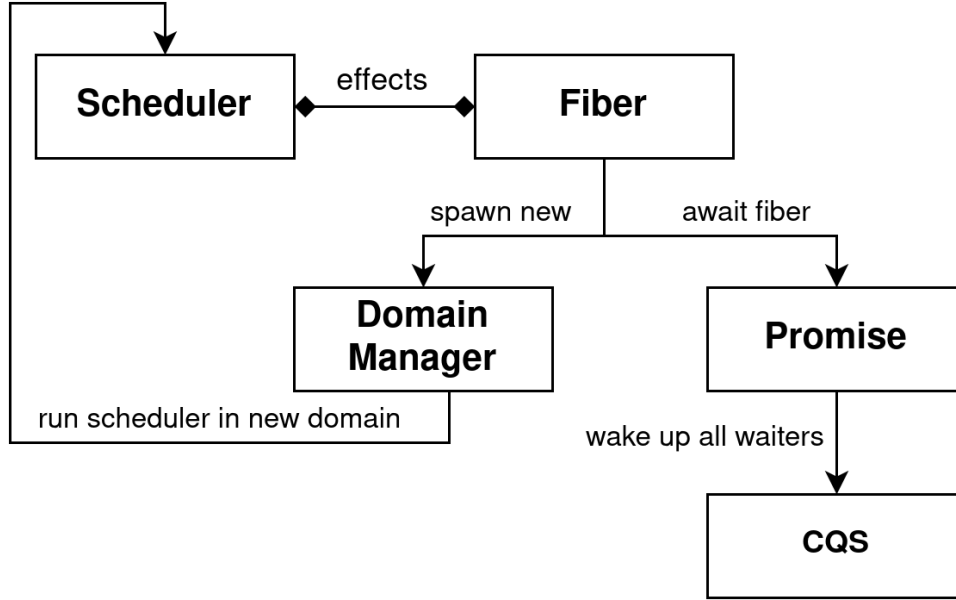
Fibers can fork off new fibers using the Fork effect and suspend execution using the Suspend effect, which are both handled by the scheduler. This is all discussed in [ref, section 2.2]. Promises are built on top of a *CQS* datastructure, which is a kind of lock-free condition variable that is used by fibers to suspend execution until a promise is fulfilled. The specification of promises is discussed in [ref, section 2.3]. The CQS specification is already verified using Iris, but Eio uses a custom implementation for which we had to adapt the proof and we discuss this process in [ref, section 3]. Fibers in Eio also have access to *thread-local variables* by performing an effect, which is discussed in [ref, section 4]. They are thread-local in the sense that they are shared between all fibers of one scheduler. Finally, we discuss our addition of multi-threading to the Hazel operational semantics in order to model running schedulers in different threads. This turned out to be technically trivial so we only discuss it in the appendix and take a multi-threaded semantics and support for Iris' *shared invariants* as a given.

1.3. Contributions

To summarize our contributions, in this thesis we verify the **safety** and **effect safety** of a simplified model of Eio which serves as an extended case study on the viability of Hazel for verifying programs with effect handlers. This includes:

- The verification of the basic Eio fiber abstraction running on a common denominator scheduler.
- An adaptation of the existing verification of CQS to the customized version used by Eio.
- Adding multi-threading to Hazel's operational semantics, which shows we can reason about programs with multi-threading & effect handlers.

2. Verifying a Simplified Eio Scheduler

[TODO mention that all code examples are an OCaml rendering of the verified Hazel code, based on but not equal to the Eio code]

Cooperative concurrency schedulers are commonly treated in the literature on effect handlers [ref, Paolos dissertation, Dolan paper, Dan Leijen paper] because they are a lucid example for the usefulness of handling delimited continuations in this way. Generally, the scheduler contains an effect handler and a fiber is just a normal function. The fiber can yield execution by performing an effect, jumping to the effect handler (i.e. the scheduler) and providing it with the rest of the fiber's computation in the form of a continuation. The scheduler has a collection of continuations and by invoking one of them it schedules the next fiber. This approach is also used in Eio.

We can therefore use the simple cooperative concurrency scheduler case study from the dissertation of de Vilhena [ref] as a starting point for our verification work. In the following section we discuss the implementation of the simplified Eio model in more detail to give an intuition about what their specification and the logical state to prove them should look like. From this intuition we will derive a formalization in [ref, section 2.2].

2.1. Implementation

Let us first get an idea of how different components of the core Eio fiber abstraction interact by looking at their types. `Scheduler.run` is the main entrypoint to Eio and it is provided a function which represents the first fiber to be executed. `Fiber.fork_promise` is also provided a function which represents a fiber, but this one will be forked so that it runs concurrenctly. It returns a promise holding the eventual return value of the new fiber. The promise is thread-safe so any other fiber can use the `Promise.await` function to wait until the value is available. Common problems like deadlocks are not prevented in any way and are the responsibility of the programmer.

```
Scheduler.run : (() -> 'a) -> ()
Fiber.fork_promise : (() -> 'a) -> 'a Promise.t
Promise.await : 'a Promise.t -> 'a
```

We present code examples in a pseudo-OCaml 5 syntax because the concrete syntax of effect handlers is verbose. Instead, we use an overloading of the match syntax that is common in the literature which includes cases for handled effects.

```
(* declares an effect E that carries a value of type int and has a bool return value. *)
effect E : int -> bool
```

(* Matches on the expression e and evaluates the second branch if it performs the effect E.
  The continuation k captures the rest of the computation of e. *)
match e **with**
| v -> ...
| effect (E v) k -> ...

Scheduler.run   As mentioned above this is the main entry point to the Eio library. It
receives the main fiber as an argument and sets up the scheduler environment. The
run_queue is a queue of ready fibers and the next function pops one fiber (i.e. function)
and evaluates it. The inner fork function is called once on each fiber to execute it and
handle any performed effects.

Handling a Fork effect is simple because it just carries a new fiber to be executed so the
handler recursively calls the fork function.

Handling a Suspend effect may look complicated at first due to the higher-order register
function. This effect is used by fibers to suspend execution and through the register
function they can control under what conditions they should resume execution again. The
key point is that as long as the continuation k is not invoked, the fiber will not continue to
execute. So the waker callback is defined to "wake up" a fiber by placing its continuation
into the run queue and the register function can install it as a callback somewhere (or
even call it directly). Waking up should be possible across thread boundaries, which is
why the run queue in Eio needs to be thread-safe.

```
effect Fork : (() -> 'a) -> ()
type 'a waker : 'a -> ()
effect Suspend : ('a waker -> ()) -> 'a

let run (main : () -> 'a) : () =
  let run_queue := queue_create () in
  let rec fork fiber =
    match fiber () with
    | _ -> next run_queue
    | effect (Fork fiber) k ->
        queue_push run_queue (fun _ -> invoke k ());
        fork fiber
    | effect (Suspend register) =>
        let waker = fun v -> queue_push run_queue (fun _ -> invoke k v) in
        register waker;
        next run_queue
  in
  fork main
```

Fiber.fork_promise   This is the basic way to create a new fiber in Eio and the only one
we model in our case study. It will create a promise and spawn the provided function
as a new fiber using the Fork effect. When f is reduced to a value result, it will fulfill
the promise with that value and signal all fibers waiting for that result to wake up. The
major difference to the implementation of de Vilhena is that promises in Eio are entirely

handled by the fiber, and not in the effect handler code of the scheduler. This achieves a better separation of concerns and simplifies the logical state needed for the proof.

```
let fork_promise (f : () -> 'a) : 'a Promise.t =
  let p = Promise.create () in
  let fiber = fun () ->
    let result = f () in
    match Atomic.get p with
    | Done _ -> error "unreachable"
    | Waiting cqs ->
        Atomic.set p (Done result);
        CQS.signal_all cqs
  in
  perform (Fork fiber)
  p
```

Promise.await   This function is looks most complicated in our case study which is partly due to the Suspend effect and also due to the use of CQS functions. Promise.await p suspends execution of the calling fiber until p is fulfilled and then returns its value. The "suspend execution" part is handled by performing the Suspend effect because the scheduler will switch to the next fiber in its run queue. Then, the "until p is fulfilled" part is handled by CQS [ref, paper].

CQS is an implementation of the observer pattern and functionally similar to condition variables[1] in languages like C++ (only lock-free), allowing fibers to register callbacks that will be called when a condition is signalled. In our case, we want to wait until p is fulfilled, so we register the waker function of the Suspend effect with CQS. The Fiber.fork_promise function then uses CQS.signal_all to call all registered wakers after it fulfills the promise.

Therefore, after the Suspend effect returns we know that the Done branch is reached.

Because CQS is lock-free, the register function must take care of possible interleavings where the promise is fulfilled in another thread. First, if CQS.register notices that there is a concurrent CQS.signal_all it will directly call the waker. Otherwise, the waker is registered but in fact the CQS.signal_all might have already finished before CQS.register even started. In this case the callback would be "lost" in the CQS, never to be called. To avoid this, register must check the state of the promise again, try to cancel the callback registration and call it directly.

```
let await (p: 'a Promise.t) : 'a =
  match Atomic.get p with
  | Done result -> result
  | Waiting cqs ->
    let register = fun waker ->
      let register_result = CQS.register cqs waker in
      match register_result with
      | None -> ()
      | Some register_handle ->
```

---

[1] https://en.cppreference.com/w/cpp/thread/condition_variable

```
        match Atomic.get p with
        | Done result ->
            if CQS.try_cancel register_handle
            then waker ()
            else ()
        | Waiting _ -> ()
    in
    perform (Suspend register);
    match Atomic.get p with
    | Done result -> result
    | Waiting _ -> error "unreachable"
```

The only **safety** concerns in the above implementations are Fiber.fork_promise expecting the promise to be unfulfilled and Promise.await expecting the promise to be fulfilled in the last match. In the next section we show how the former is addressed by defining a unique resource that is needed to fulfill a promise, and the latter is a consequence of the protocol of the Suspend effect.

2.2. Specifications

To prove specifications for an effectful program in Hazel we have to define not only the usual ghost state constructs to track program state but also protocols which describe the behavior of the program's effects. We take both from the simple cooperative concurrency scheduler case study from de Vilhena's dissertation and adapt them to our model of Eio.

Protocols First we look at the protocols for the Fork and Suspend effect.

$$
\begin{aligned}
isRegister\ reg\ &::=\ \forall waker.\ promiseInv\ \twoheadrightarrow \\
&\qquad (\forall v.\ P\ v \twoheadrightarrow ewp\ (waker\ v)\ \langle \bot \rangle\ \{\top\}) \twoheadrightarrow \\
&\qquad \triangleright ewp\ (reg\ waker)\ \langle \bot \rangle\ \{\top\} \\
Coop\ &::= Suspend\ \#\ !\ reg\ P\ (reg)\ \{isRegister\ reg\}. \\
&\qquad ?\ v\ (v)\{P\ v\} \\
&\qquad +\ FORK\ \#\ !\ e\ (e)\ \{\triangleright ewp\ (e\ ())\ \langle Coop \rangle \{\top\}\}. \\
&\qquad ?\ ()\{\top\}
\end{aligned}
$$

Fork stays almost the same but since promise handling is done entirely in the fibers, the scheduler does not interact with the return value so the *ewp* degenerates to a trivial postcondition.

The protocol for Suspend is entirely new. From the type of the Suspend effect we already know that some value can be transmitted from the party that calls the waker callback to the fiber that performed the effect. The protocol now expresses the same idea on the level

of resources. By appropriately instantiating P, the fiber can enforce that some condition holds before it can be signalled and then receive this resource when it continues execution. For example, in the Promise.await specification below, we ensure that the promise must be fulfilled before the effect returns by instantiating P with a resource that says the promise is fulfilled.

Logical State The most basic ghost state we track is wether a promise is fulfilled or not. The promise_waiting γ resource exists as two halves, one owned by the fiber and one by the invariant that tracks all promises. Both halves are combined when fulfilling the promise, yielding the persistent promise_done γ resource. It is not possible to have both a promise_waiting γ and a promise_done γ.

```
(* TODO convert to latex *)
Definition promiseInv : iProp Σ := (
 ∃ M, isPromiseMap M ∗
  ([∗ map] args ↦ Φ ∈ M, let '(p, γ, ε) := args in
    (∃ result,
      p ↦ Done result ∗
      promise_done γ ∗
      □ Φ result)
    ∨
     (∃ cqs,
      p ↦ Waiting cqs ∗
      is_cqs cqs ∗
      promise_waiting γ ∗
      resume_all_permit))
)%I.

Definition ready (k: val) : iProp Σ := (
 EWP (k #()) {{ _, ⊤ }}
)%I.
```

PromiseInv still tracks the state of all existing promises, which are either *Done* or *Waiting*. As long as the promise is not fulfilled, it holds and an instance of CQS so that wakers can be registered along with a resume_all_permit. This token is used to call the CQS.signal_all function which must only be called once. When the promise is fulfilled it instead holds a final value v satisfying some predicate Φ.

The *Ready* predicate now just expresses that *f* is safe to be executed. Compared to de Vilhena's formalization, the *Φ(v)* predicate was dropped because the scheduler does not interact with the return value of an effect; this has moved to the protocol of Suspend as *P(v)*. We were able to drop the other preconditions because they are now put into Iris shareable invariants since Eio supports multi-threading. Now *Ready* is neither recursive nor mutually recursive with *isPromise* anymore, which simplifies its usage in Iris.

Scheduler.run The interesting part about the scheduler specification is that it proves **effect safety** of the runtime, i.e. no matter what a fiber does it will not crash the scheduler due to an unhandled effect. However, the specification only talks about effect

safety and not about handling fibers correctly in any other way, e.g. regarding fairess of scheduling or just not dropping fibers. For example, a trivial function which ignores the main argument and immediately returns satisfies the same specificaiton. For a scheduler it would be desirable to prove these properties, too, but since they are liveness properties it is hard to do in Iris and not a focus of this thesis.

```
Lemma ewp_run (main : val) :
  EWP main #() <| Coop |> {{ ⊤ }} -∗
    EWP run main <| ⊥ |> {{ ⊤ }}.
```

The proof proceeds as follows: - Creating the run queue also returns a persistent resource which is used for all later calls to the next function. - For the inner fork function we use Löb induction since it is called recursively. - Since it is a deep effect handler we need to satisfy the deep-handler predicate for the Coop protocol. - The Fork case just recurses in the fork function so we use the induction hypothesis. - For the Suspend case we define the waker function and prove P v -∗ EWP waker () <| ⊥ |> {{ ⊤ }} in order to call the register function on it. This holds by construction of the waker function because P v is a precondition for invoking the fiber's continuation k.

Fiber.fork_promise   For this specification, the *PromiseInv* argument is needed to interact with promises and the *ewp* proves that the new fiber is safe to execute and obeys the Coop protocol. In return, the caller gets a promise that will eventually hold a value satisfying the predicate Φ.

```
Lemma ewp_fork_promise (f: val) Φ :
  promiseInv ∗ EWP f #() <| Coop |> {{v, □ Φ v}}
⊢
  EWP (fork_promise f) <| Coop |> {{ y,
    ∃ (p: loc), ⌜ y = #p ⌝ ∗ isPromise p Φ}}.
```

The proof proceeds as follows: - First, a new promise is created, which updates the *PromiseInv* invariant and yields one half of the promise_waiting γ resource for that new promise. - We define the actual fiber and prove its *ewp*. - Evaluating f yields a value satisfying Φ as given by the *ewp*. - Because we own promise_waiting γ the second branch of the match can be ruled out. Now the *PromiseInv* invariant is accessed to update the promise state to Done. This consumes both halves of the promise_waiting γ resource and yields a promise_done γ. We also take out the resume_all_permit. - We use this permit along with promise_done γ to call CQS.resume_all. promise_done γ is persistent so it can be used to call all wakers. - Using the *ewp* for the wrapped f we can perform a Fork effect. - Since the promise will be fulfilled with a value satisfying Φ we have the isPromise p Φ that we must return.

Promise.await   The register function which is passed to the Suspend effect is complex enough that we spin out its specification. The implementation of Promise.await is very different to the original but stil satisfies the same specification. *PromiseInv* and *IsPromise* are both needed to interact with the promise's state.

```
Lemma ewp_await_make_register (p: loc) (cqs: val) Φ:
  isPromise p Φ ∗ is_cqs cqs
```

⊢
 EWP (make_register #p cqs) <| ⊥ |> {{register,
  ∀ (waker: val), (promise_done γ -∗ EWP waker #() <| ⊥ |> {{ ⊤ }}) -∗
  promiseInv -∗
   ▷ EWP register waker <| ⊥ |> {{ ⊤ }} }}.

Lemma ewp_await (p: loc) Φ :
 promiseInv ∗ isPromise p Φ ⊢
  EWP await #p <| Coop |> {{v, □ Φ v}}.

The proof of the Promise.await specification proceeds as follows: - For the first match on the promise state we don't have any resources to constrain the possible results. - If the promise is already fulfilled we can take the Φ v and return that. - If it is not fulfilled, then we get access to a CQS instance and can make the register function using the *IsPromise* and is_cqs resources. - Using the *ewp* for the register function we can invoke the Suspend effect and set P _ := promise_done γ. - As a result we now have the promise_done γ resource and when we match on the promise again, the unfulfilled case can be ruled out. - So now we can take the Φ v and return it.

The proof of the make_register specification follows directly from the specifications of the CQS functions, which are explained in further detail in the next chapter.

3. Verifying Eio's Customized CQS (WIP)

CQS [ref, paper] (for CancellableQueueSynchronizer) is an implementation of a synchronization primitive that allows execution contexts to wait until signalled. Its specification is already formally verified in Iris, which we adapted to use in our case study. The nature of a CQS execution context is kept abstract but it is assumed that they support stopping execution and resuming with some value. This is because CQS is designed to be used in the implementation of other synchronization constructs (e.g. mutex, barrier, promise, etc.) which take care of actually suspending and resuming execution contexts as required by their semantics.

In the case of Eio an "execution context" is an Eio fiber but nevertheless CQS works across multiple threads, so fibers can use CQS to synchronize with fibers running in another thread. Eio implements a custom version of CQS adapted from the paper [ref, paper] in the Broadcast module, which in turn is used in the implementation of the *promise* synchronization construct. In this chapter we describe the behavior of Eio's *customized CQS*, highlight differences to the *original CQS*, and explain how we adapted the verification of the original CQS for our case study. If something applies to both the customized and original version we just use the term *CQS*. After having presented the adapted specification for the Broadcast module we can then explain the implementation of the Promise module which we kept abstract in section 1.

Operations of CQS

The original CQS supports three operations that are interesting to us. In a *suspend operation* the requesting execution context wants to wait until signalled. It places a

handle to itself in the datastructure and is expected to stop execution afterwards. But before it actually stops execution it can use the *cancel operation* to try to cancel the *suspend operation*. Finally, a *resume operation* can be initiated from a different execution context. It takes one handle out of the datastructure and uses it to signal the original execution context that it can resume execution. This fails if the *suspend operation* (and thereby the handle) had already been cancelled.

These operations enable a single execution context to wait until it is signalled by another. Eio's customized CQS supports an additional operation called the *resume-all operation*. As the name implies, it is a *resume operation* that applies to all currently saved handles. This operation was added so that **all** fibers waiting on a promise can be signalled when the promise is fulfilled.

To understand the operations it is helpful to view them in the context of their Eio implementation. Here, what we called the "handle" to an execution context is the `waker` callback resulting from a fiber performing a `Suspend` operation. We recall that if the `waker` callback is invoked, its fiber is placed into the scheduler's run queue and will therefore resume execution. We show the operations' OCaml types and also how the operations are used in the outer synchronization construct (i.e. an Eio *promise*).

An interaction with CQS as described in [ref, paper] is always guarded by first accessing some atomic variable. In the case of Eio, the atomic variable holds the state of the promise, which can either be `Unfulfilled cqs` – holding a customized CQS instance – or `Fulfilled v` – holding the final value `v` of the associated fiber.

- If the promise is already fulfilled with a value, a requesting fiber immediately returns that value.
- If the promise is not yet fulfilled, a requesting fiber will perform a `Suspend` effect in order to stop execution and use the *suspend operation* to wait until the promise is fulfilled.
- Optionally, it can also use the *cancel operation* afterwards.
- The fiber that is associated with the promise will fulfill it with a value and then use the *resume-all operation* to signal all waiting fibers that they can now retrieve the value.

suspend/resume_all/cancel operations can race
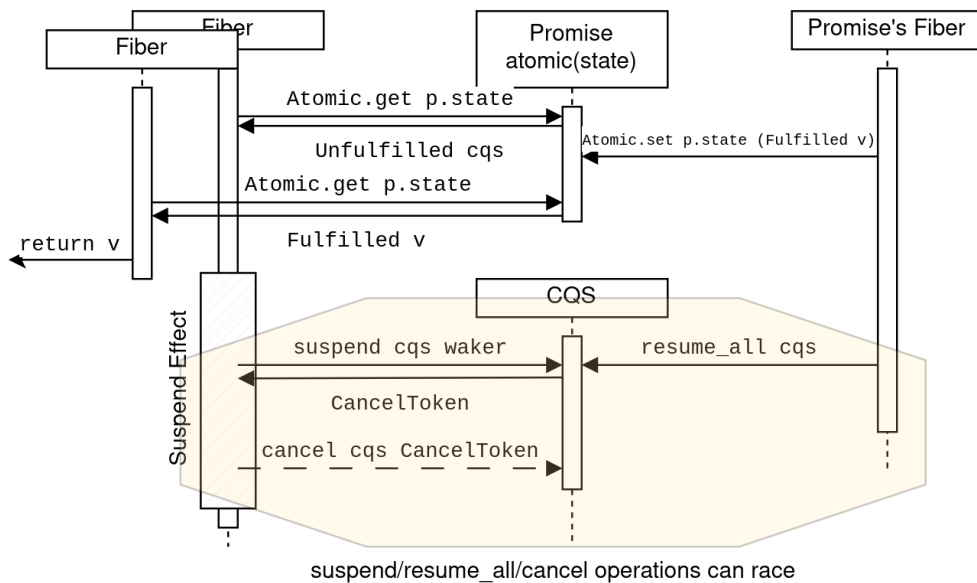
It is important to note that since CQS is lock-free and fibers can run on different threads there can be a race between concurrent *suspend*, *cancel* and *resume-all operations*. Possible interleavings and the necessity of the *cancel operation* are explained in section [ref, Promise Implementation]. This example illustrates that a CQS instance always acts as a thread-safe store for cancellable callbacks. More precisely, it is a FIFO queue but a *resume-all operation* dequeues all elements at once.

That CQS is "just" a store for cancellable callbacks is also reflected in the rather barebones types of the operations as implemented in OCaml. A CQS instance can be created and shared between different threads. New callbacks are inserted using the suspend function, yielding an optional request value. If suspend returns None the callback has already been invoked due to a concurrent resume_all. A request value can then be used to cancel the insertion, signifying that a fiber can only cancel its own callback. The resume_all function (logically) consumes the CQS, which will become more clear when we present the specifications in [ref, Verification of the Broadcast module]

type t
type request

val create : unit -> t
val suspend : t -> (unit -> unit) -> request option
val cancel : request -> bool
val resume_all : t -> unit

Implementation and Logical Interface of CQS

CQS is implemented as a queue of *cells* with two pointers pointing to the beginning and end of the active cell range, the *suspend pointer* and the *resume pointer*. Cells not

reachable from either pointer are garbage collected but their logical state is still tracked. There is a stack of operations for manipulating these pointers to implement the higher-level functionality but they are not part of the public API so we do not focus on them. Each cell is a container for one handle and the logical state of the queue tracks the logical state of all existing cells shown in figure [ref, below].

The number of active cells n (i.e. the length of the queue) is tracked by the logical resource `cqs_state n`. In normal usage of CQS, the atomic variable of the outer synchronization construct would encode the length of the queue in its value and keep this resource in an associated invariant. Logically changing the length of the queue is done using *enqueue* and *dequeue registration* operations when opening this invariant.

As we saw before, however, for promises the exact length of the queue is irrelevant because the *resume-all operation* will always set the length to 0. So in the adapted proof we keep the `cqs_state n` resource in the invariant of CQS itself. As a consequence we also move the *enqueue* and *dequeue registration* out of the public API because they are now done internally.

Verification of the **Broadcast** Module

In the following we describe the specifications we proved for the three operations `suspend`, `cancel` and `resume_all` of Eio's `Broadcast` module, in which points they differ from the specifications of the original CQS operations, and what changes we did to the internal logical state of CQS to carry out the proofs.

The first major change was replacing the future-based interface of the suspend operation with a callback-based interface. In the original CQS, performing a suspend operation returns a new future, which is also inserted as the handle into the queue. The execution context can then use the future to stop execution because it is assumed there is a runtime that allows suspending execution until the completion of a future. But Eio cannot use this interface because it uses the customized CQS to *build* the runtime that allows fibers to suspend until the completion of a promise. As explained above, Eio implements CQS with a callback-based interface where the fiber performing the suspend operation passes in a callback as the handle and afterwards implicitly stops execution. Performing a resume operation analogously invokes the callback, instead of completing the future.

This changes the logical state of CQS only slightly. The original CQS tracked the state of the future for each cell and managed *futureCancellation* and *futureCompletion* tokens. In the customized CQS we analogously track the state of the callback for each cell and manage *callbackInvokation* and *callbackCancellation* tokens.

For all three operations, the Eio implementation differs from the implementation already verified in the original CQS (i.e. some reordered instructions or a slightly different control flow) and they have different specifications as discussed below. However, the specifications of the underlying operations for manipulating the cell pointers are modular enough to allow us to prove the new specifications for `suspend` and `cancel`. Note that the presented specifications are cleaned up for readability.
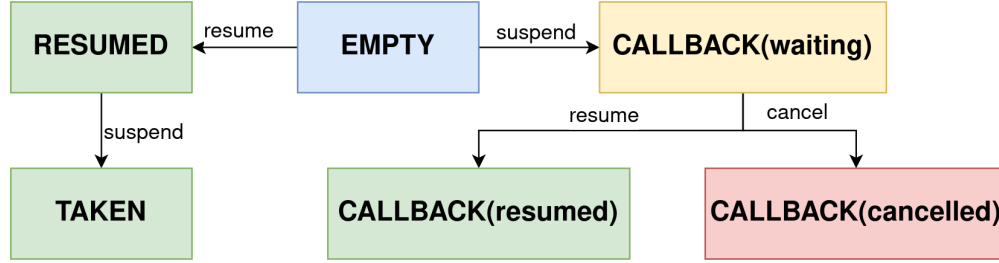
Aside: Implementation of resume_all
Eio implements resume_all by atomically increasing the *resume pointer* by some number n, instead of just 1 like in the original re

Because of technical differences between the infinite array implementation in the CQS mechanization & the infinite array impleme
Instead, I actually defined resume_all simply as a loop over a resume operation.
Since resume_all is only called once I posit that this verification is still valid but I still want to verify Eio's resume_all and remove this

The logical state of an individual cell is changed by the functions according to the following diagram.

```
  RESUMED  ←—resume—  EMPTY  —suspend→  CALLBACK(waiting)
     |                                          |
  suspend                     resume            |       cancel
     ↓                           ↓              ↓
   TAKEN              CALLBACK(resumed)    CALLBACK(cancelled)
```

create   Creating a CQS instance requires inv_heap_inv which is an Iris propositions that we are in a garbage-collected setting. It creates an is_cqs γ q which is a persistent resource that shows the value q is a CQS queue, along with a collection of ghost names we summarize with γ. The resource cqs_state n mentioned above is now kept inside is_cqs γ q. It also returns the unique resource resume_all_permit γ, which is held by the enclosing promise and allows calling the resume_all function once.

Theorem create_spec:
  {{{ inv_heap_inv }}}
    newThreadQueue #()
  {{{ γ q, RET q; is_cqs γ q ∗ resume_all_permit γ }}}.

suspend   For a *suspend operation* the *suspend permit* from the original CQS is not needed anymore since we do the *enqueue registration* internally. The is_waker resource is defined as V' -∗ EWP k () {{ ⊤ }} and represents the permission to invoke the callback k. We instantiate V' with promise_state_done γp so that the callback transports the knowledge that the promise has been fulfilled. is_waker is not persistent because the callback must be invoked only once and it might be accessed from a different thread.

The suspend function will advance the *suspend pointer* to allocate a new cell in the **EMPTY** logical state. If there is a concurrent call to resume_all which changed the cell to the **RESUMED** logical state before this function can CAS the callback into the cell, the callback is invoked immediately and NONEV is returned. In this case, the state of the cell will be set to **TAKEN**. Otherwise the callback is saved in the cell, which is advanced to the **CALLBACK(waiting)** logical state and a is_suspend_result resource is returned as the cancellation permit.

Theorem suspend_spec γ q k:
  {{{ is_cqs γ q ∗
    is_waker V' k }}}
    suspend q k
  {{{ v, RET v; ⌜v = NONEV⌝ v

∃ γk r, ⌜v = SOMEV r⌝ *
            is_suspend_result γ γk r k }}}.

cancel The specification of the *cancel operation* is a lot simplified compared to the original due to removed features. The is_suspend_result resource is used as a permission token and the r value is used to find the callback that should be cancelled.

If the callback had already been invoked by a concurrent call to resume_all (i.e. the logical state is **CALLBACK(resumed)**) the function returns false and no resources are returned to the caller. Otherwise, the permission to invoke the callback is returned and the cell is advanced to the **CALLBACK(cancelled)** logical state.

Theorem try_cancel_spec γ q γk r k:
 {{{ is_cqs γ q *
    is_suspend_result γ γk r k }}}
   cancel r
 {{{ (b: bool), RET #b; if b then is_waker V' k
                else True }}}.

resume_all The specification of the *resume-all operation* is also a lot simplified compared to the specification of the original *resume operation* because we removed multiple unused features. The resume_all_permit is a unique resource used to ensure the function can only be called once. The V' resource must be duplicable because it will be used to invoke multiple callbacks, which have V' as their precondition. It does not return any resources because its only effect is making an unknown number of fibers resume execution, which is not something we can easily formalize in Iris.

Theorem resume_all_spec γ q:
 {{{ is_thread_queue γ q *
    □ V' *
    resume_all_permit γ }}}
   resume_all q
 {{{ RET #(); True }}}.

Features Removed from Original CQS

The original CQS supports multiple additional features like a synchronous mode for suspend and resume, and also a smart cancellation mode. These features enlarge the state space of CQS and complicate the verification but are not used in Eio so when we ported the verification of CQS to our Eio case study we removed support for these features. This reduced the state space of a cell shown below (taken from the original paper) to something more manageable for us when adapting the proofs.
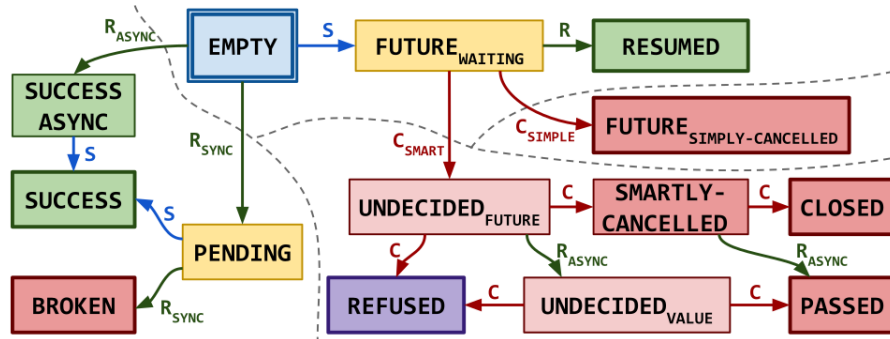
Figure 14: The state transition system for a single cell from the logical perspective.

Due to this, the part of the verification of the original CQS that we had to customize for Eio shrunk by approximately 1300 lines of Coq code from the original 3600 lines of Coq code, while there is an additional ~4000 lines of Coq code that we did not need to adapt.

4. Extending the Scheduler with Thread-Local Variables (WIP)

- How thread-local variables can be used.
- Explain the GetContext effect in Eio and how we model it in our scheduler.
- How we adapt our logical state to include GetContext. And explain that we need to parameterize the protocol to solve the issue of shared knowledge between the scheduler and fiber.

6. Evaluation (WIP)

7. Conclusion (WIP)

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- Retrofitting Parallelism onto OCaml

Appendix

A. Translation Table

Eio	Thesis	Mechanization
enqueue	waker callback	waker
f	register function	register
Fiber.fork_promise	Fiber.fork_promise	fork_promise
Promise.await	Promise.await	await
Broadcast & Cells	CQS	CQS

B. Towards A Multi-Threaded Scheduler

OCaml 5 added not only effect handlers but also the ability to use multiple threads of execution, which are called *domains* (in the following we use the terms interchangeably). Each domain in OCaml 5 corresponds to one system-level thread and the usual rules of multi-threaded execution apply, i.e. domains are preemtively scheduled and can share memory. Eio defines an operation to make use of multi-threading by forking off a new thread and running a separate scheduler in it. So while each Eio scheduler is only responsible for fibers in a single thread, fibers can await and communicate with fibers running in other threads.

In order for a fiber to be able to await fibers in another thread, the wakers_queue [note it will be in the Simple Scheduler section] from above is actually a thread-safe queue based on something called CQS, which we will discuss in detail in a later section.

Heaplang supports reasoning about multi-threaded programs by implementing fork and join operations for threads and defining atomic steps in the operational semantics, which enables the use of Iris *invariants*. In contrast, Hazel did not define any multi-threaded operational semantics but it contained most of the building blocks for using invariants. In the following we explain how we added a multi-threaded operational semantics and enabled the use of invariants.

Aside: Memory Model in OCaml 5
In the OCaml 5 memory model, *atomic variables* are needed in order to access shared memory without introducing data races. Instead of modelling atomic variables in Hazel, we continue to use normal references because the multi-threaded operational semantics by definition defines all memory operations to be sequentially consistent. This seems to be the sta

Adding Invariants to Hazel

Invariants in Iris are used to share resources between threads. They encapsulate a resource to be shared and can be opened for a single atomic step of execution. During this step the resource can be taken out of the invariant and used in the proof but at the end of the step the invariant must be restored.

Hazel did already have the basic elements necessary to support using invariants. It defined a ghost cell to hold invariants and proved an invariant access lemma which allows opening an invariant if the current expression is atomic. In order to use invariant we only had to provide proofs for which evaluation steps are atomic. We provided proofs for all primitive evaluation steps. The proofs are the same for all steps so we just explain the one for Load.

Lemma ectx_language_atomic a e :
  head_atomic a e → sub_exprs_are_values e → Atomic a e.

```
Instance load_atomic v : Atomic StronglyAtomic (Load (Val v)).
Instance store_atomic v1 v2 : Atomic StronglyAtomic (Store (Val v1) (Val v2)).
...
```

An expression is atomic if it takes one step to a value, and if all subexpressions are already values. The first condition follows by definition of the step relation and the second follows by case analysis of the expression.

Since performing an effect starts a chain of evaluation steps to capture the current continuation, it is not atomic. For the same reason an effect handler and invoking a continuation are not atomic except in degenerate cases. Therefore, invariants and effects do not interact in any interesting way.

[TODO How we add support for the iInv tactic to use invariants more easily.]

Adding Multi-Threading to Hazel

To allow reasoning in Hazel about multi-threaded programs we need a multi-threaded operational semantics as well as specifications for the new primitive operations Fork, Cmpxcgh and FAA.

The language interface of Iris provides a multi-threaded operational semantics that is based on a thread-pool. The thread-pool is a list of expressions that represents threads running in parallel. At each step, one expressions is picked out of the pool at random and executed for one thread-local step. Each thread-local step additionally returns a list of forked-off threads, which are then added to the pool. This is only relevant for the Fork operation as all other operations naturally don't fork off threads.

```
(e, \sigma) ->_t (e', \sigma', es')
------------------------------------------------------------
(es_1 ++ e ++ es_2, \sigma) ->_mt (es_1 ++ e' ++ es_2 + es', \sigma')
```

Heaplang implements multi-threading like this and for Hazel we do the same thing. We adapt Hazel's thread-local operational semantics to include Fork, Cmpxchg and FAA operations and to track forked-off threads and get a multi-threaded operational semantics "for free" from Iris' language interface.

[TODO one of the proofs that the language interface requires was a bit tricky so include that.]

Additionally, we need to prove specifications for these three operations. Cmpxchg and FAA are standard so we will not discuss them here. The only interesting design decision in the case of Hazel is how effects and Fork interact. This decision is guided by the fact that in OCaml 5 effects never cross thread-boundaries. An unhandled effect just terminates the current thread. As such we must impose the empty protocol on the argument of Fork.

```
EWP e <| \bot |> { \top }
------------------------------------
EWP (Fork e) <| \Phi |> { x, x = () }
```

[TODO explain how that proof works.]

Using these primitive operations we can then build the standard CAS, Spawn, and Join operations on top and prove their specifications. For Spawn & Join we already need invariants as the point-to assertion for the done flag must be shared between the two threads.

Lemma spawn_spec (Q : val → iProp Σ) (f : val) :
  EWP (f #()) <| \bot |> {{ Q }} -∗ EWP (spawn f) {{ v, ∃ (l: loc), ⌜ v = #l ⌝ ∗ join_handle l Q }}.

Lemma join_spec (Q : val → iProp Σ) l :
  join_handle l Q -∗ EWP join #l {{ v, Q v }}.

Definition spawn_inv (γ : gname) (l : loc) (Q : val → iProp Σ) : iProp Σ :=
  ∃ lv, l ↦ lv ∗ (⌜lv = NONEV⌝ v
          ∃ w, ⌜lv = SOMEV w⌝ ∗ (Q w v own γ (Excl ()))).

Definition join_handle (l : loc) (Q : val → iProp Σ) : iProp Σ :=
  ∃ γ, own γ (Excl ()) ∗ inv N (spawn_inv γ l Q).

Note that for Spawn we must also impose the empty protocol on f as this expression will be forked-off.

This allows us to implement standard multi-threaded programs which also use effect handlers. For example, we can prove the specification of the function below that is based on an analogous function in Eio which forks a thread and runs a new scheduler inside it. Note that same as in Eio the function blocks until the thread has finished executing, so it should be called in separate fiber.

Definition spawn_scheduler : val :=
  (λ: "f",
    let: "new_scheduler" := (λ: <>, run "f") in
    let: "c" := spawn "new_scheduler" in
    join "c")%V.

Lemma spawn_scheduler_spec (Q : val -> iProp Σ) (f: val) :
  promiseInv -∗ EWP (f #()) <| Coop |> {{ _, True }} -∗
    EWP (spawn_scheduler f) {{ _, True }}.

The scheduler run and therefore also the spawn_scheduler function don't have interesting return values, so this part of the specification is uninteresting. What is more interesting is that they encapsulate the possible effects the given function f performs.

C. A Note on Cancellation

- That we tried to model cancellation but the feature is too permissive to give it a specification.
- There is still an interesting question of safety (fibers cannot be added to a cancelled Switch).
  But including switches & cancellation in our model would entail too much work so we leave it for future work.