

$$\begin{cases} \dot{x} = v \\ \dot{v} = u \end{cases}$$

$$u = L(r-x) - L v \quad \leftarrow \text{Protocol}$$

$$\begin{cases} \dot{x} = v \\ \ddot{x} = -Lx - Lv + Lr \end{cases}$$

With ZOH exact discretization,

$$x_{k+1} = \begin{bmatrix} I & T \cdot I \\ 0 & I \end{bmatrix} x_k + \begin{bmatrix} \frac{T^2}{2} \cdot I \\ T \cdot I \end{bmatrix} u_k$$

$$x_k = \begin{bmatrix} \hat{x}_k^1 \\ \hat{x}_k^2 \end{bmatrix} \begin{matrix} \leftarrow \text{pos} \\ \leftarrow \text{vel} \end{matrix}$$

$$u_k = k_p L (r_k - \hat{x}_k^1) - k_d L \hat{x}_k^2$$

$$x_{k+1} = \begin{bmatrix} I - \frac{k_p \cdot T^2}{2} L_k & T \cdot I - \frac{k_d \cdot T^2}{2} L_k \\ -k_p \cdot T \cdot L_k & I - k_d \cdot T \cdot L_k \end{bmatrix} x_k + \begin{bmatrix} \frac{k_p \cdot T^2}{2} \cdot L_k \\ T \cdot k_p \cdot L_k \end{bmatrix} r_k$$

MJLS for analysis & simulation