Flockng Algorithms for Source-Seeking Scenarios: From Double Integrators to General Non-Linear Vehicles

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Source seeking problem: Abstraction

- A group of N agents (e.g underwater robots) are deployed in the region of interest
- Each agent has the following properties:
  - Absolute position measurement (e.g GPS)[1]
  - Communication with agents within a fixed distance
  - Concentration measurement sensor
  - Computation capabilities
- **Problem:** Design distributed control algorithms that cause the agents to flock towards the source (location with the highest concentration) in a cooperative manner.

[1] This assumption can be removed as long there is relative displacement measurement

Source seeking problem as optimization

- Look at the source seeking problem as a minimization problem
- Momentum methods[2] have been used in optimization to speed up or dampen the oscillations
- Naturally have momentum of the mobile robots
- For theoretical analysis: Use continuous-time versions of optimization methods to study the assymptotic properties[3]

[2] Polyak,1964

[3] Redont et al., 2002

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Defining the velocity variable v, the dyanmics can be written as

$$\begin{aligned} \dot{x} &= v \\ \dot{v} &= -k_1 \nabla^2 f(x) v - k_2 \nabla f(x), \end{aligned} \tag{4}$$

Flocking dynamics

- Every particle interacts with every other particle based on an interaction field V which depends only on the distance between them
- An example of a potential field is the gravitational field $V(z) = \frac{mMG}{||z||}$
- Flocking dynamics[4]:

$$\dot{q} = p$$
$$\dot{p} = -\nabla V(q) - \hat{L}(q)p - cp$$

[4] Olfati-Saber, 2004

Flocking towards the source

- Consider now an underlying source field $\psi : \mathbf{R}^m \to \mathbf{R}$ which is smooth enough and convex.
- Add a forcing term to the flocking dynamics motivated from the discussion on optimization
- Flocking dynamics with source-seeking:

$$\dot{q} = p$$
$$\dot{p} = -\nabla V(q) - \hat{L}(q)p - cp - k_1 \nabla^2 \Psi(q) p - k_2 \nabla \Psi(q)$$

Stability(Convergence) Analysis

Flocking dynamics with source-seeking:

$$\dot{q} = p$$
$$\dot{p} = -\nabla V(q) - \hat{L}(q)p - cp - k_1 \nabla^2 \Psi(q) p - k_2 \nabla \Psi(q)$$

Theorem 1 For twice differentiable convex fields $\psi : \mathbf{R}^m \to \mathbf{R}$, the above flocking dynamics are stable i.e the trajectories remain bounded. Moreover, for all initial conditions $(q(0), p(0))$, the trajectories converge asymptotically to the set

$$\mathcal{W} := \{(q^*, 0) | \nabla V(q^*) + k_2 \nabla \Psi(q^*) = 0\}.$$

Corollary 1 Assume, the field $\psi : \mathbf{R}^m \to \mathbf{R}$ is strictly convex, quadratic and has a unique minimum located at q_s, we have that $\lim_{t \to \infty} q_c(t) = q_s$.

Towards Experiments: Modeling agents as double integrators

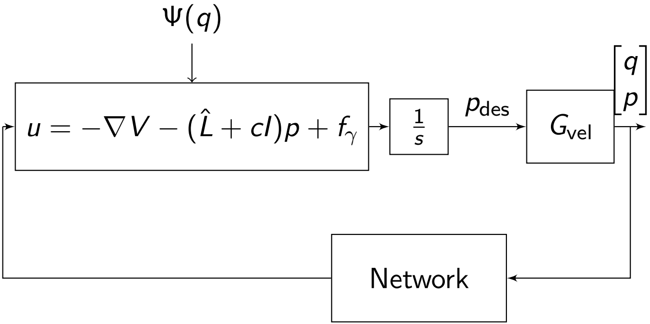
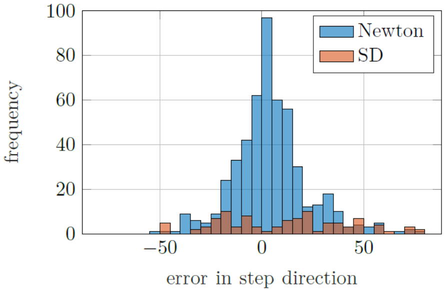


Figure 1: Control Architecture

- The flocking loop is runs at 20Hz
- G_{vel}: closed loop velocity tracking loop runs at 100Hz
- Initial tests show that the velocity controller is fast enough $\implies$ approximate G_{vel} by single integrator dynamics.

Towards Experiments: $\psi(q_i)$, $\nabla\psi(q_i)$, $\nabla^2\psi(q_i)$



- Corrupt the field strength measurement by $+/-$ 5 percent of the field strength.
- Estimate gradient by collecting local data and solving a least squares problem
- Estimate the hessian by collecting data from neighbors and solving another least squares problem
- Heuristic: Use Steepest Descent until the estimated hessian is positive definite.

Indoor experimental setup

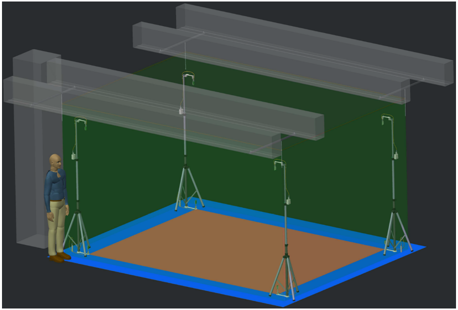


Figure 2: LPS Setup

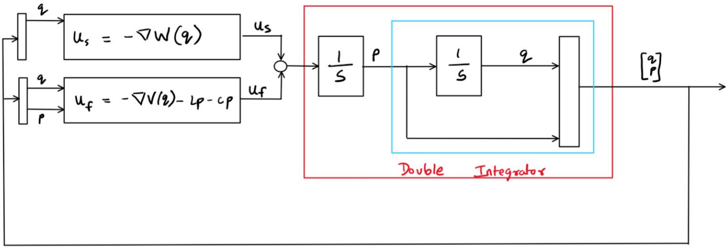
Figure 3: Crazyflie 2.1

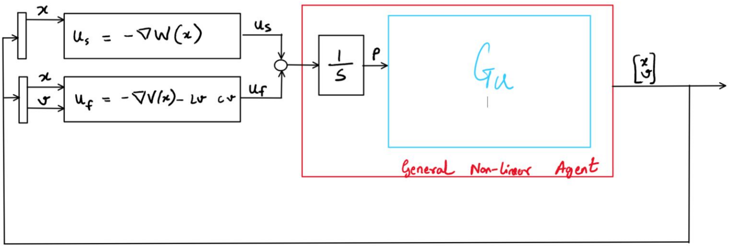
- Indoor Positioning System called the Loco Position System (LPS) and Crazyflie 2.0 developed by the company bitcraze[5]
- Implemented a peer-to-peer communication protocol [6]

[5] https://www.bitcraze.io/products/crazyflie-2-1/

[6] Paulsen, 2018.Master thesis.

Double integrator agents to general non-linear agents





Double integrator agents to general non-linear agents

Dynamics with double integrator:

$$\begin{aligned}\dot{q} &= p \\ \dot{p} &= -\nabla V(q) - \hat{L}(q)p - cp - k_1 \nabla^2 \Psi(q) p - k_2 \nabla \Psi(q)\end{aligned}$$

Dynamics with general agents:

$$\begin{aligned}\begin{bmatrix} x \\ v \end{bmatrix} &= G_{\text{cl}} p \\ \dot{q} &= p \\ \dot{p} &= -\nabla V(x) - \hat{L}(x) v - c v - k_1 \nabla^2 \Psi(x) v - k_2 \nabla \Psi(x)\end{aligned}$$

Double integrator agents to general non-linear agents

Core Idea:

Under

- a Lipschitz condition on the underlying scalar field
- a Lipschitz condition on the interaction field
- L_2 bound on the tracking error of the local closed loop, G_{cl}

the dynamics for general agents could be written as:

$$\dot{q} = p$$
$$\dot{p} = -\nabla V(q) - \hat{L}(q)p - cp - k_1 \nabla^2 \Psi(q) p - k_2 \nabla \Psi(q) + d$$
$$||d_T|| \leq K ||p_T||_{L_2} \forall T$$

- Standard dissipativity based arguments to show stability (Boundedness of trajectories)
- Assymptotic stability not shown !

Double integrator agents to general non-linear agents

A speculative idea

- IQCs can be used efficiently to get upper bounds on robust exponential decay rates[7]
- Can be possibly applied to LPV systems to obtain exponential decay rates under parameter variation
- These rates could be then used with a singular perturbation and a regular perturbation argument to prove asymptotic stability of the overall system
- The linear analogue would be to consider the slowest eigen values of the closed loop agent dynamics

[7] Boczar, R., Lessard, L., Packard, A. and Recht, B., 2017. Exponential stability analysis via integral quadratic constraints. arXiv preprint arXiv:1706.01337.

Other speculative ideas

- Adapt the flocking algorithm coefficients online based on the measured error such the $\dot{E} < 0$. (Borrowing ideas from adaptive control)
- Similar to ideas in [8], design local tracking control for passivity and use the fact the interconnections of passive systems is passive.

[8] Chopra, N. and Spong, M.W., 2006. Passivity-based control of multi-agent systems. In Advances in robot control (pp. 107-134). Springer, Berlin, Heidelberg.

Thank you