Gradient-based Cooperative Control of quasi-Linear Parameter Varying Vehicles with Noisy Gradients

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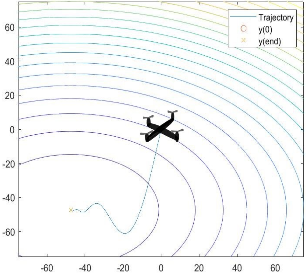
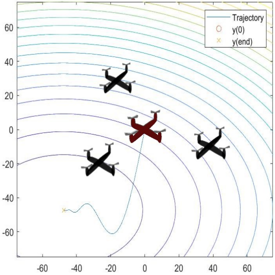
Source-seeking Problem



Single agent

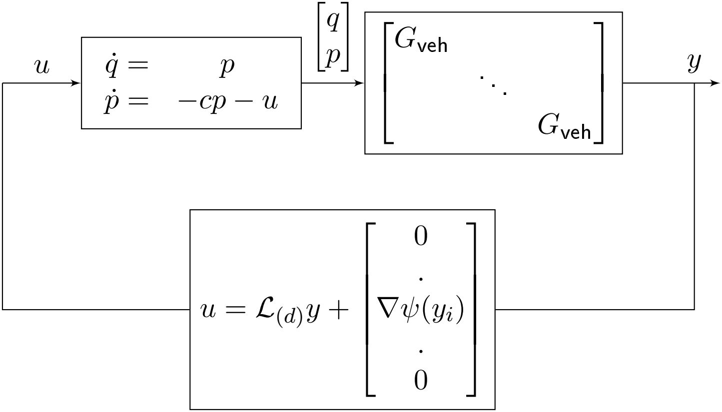


Multiple agents

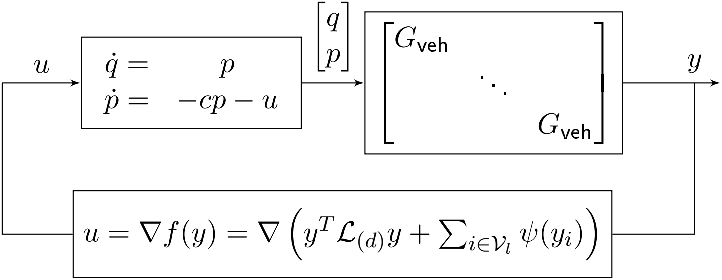
Problem Setting

- Field is differentiable and convex
- *Noisy* gradients available at some leader agents
- Agent dynamics governed by a quasi-LPV model

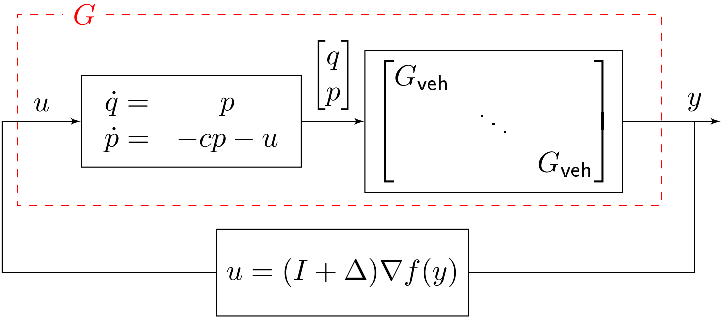
Robust Analysis Problem with Formation Control



Robust Analysis Problem with Formation Control



Robust Analysis Problem with Formation Control



Assume

1. $||\Delta|| \leq \delta$
2. $0 \prec m \cdot I \preceq \nabla^2 f(y) \preceq L \cdot I$ (Notation: $f \in \mathcal{S}(m, L)$)

Control Architecture: Multiple agents

$$
\begin{aligned}
\dot{\eta}(t) &= \hat{A}_G(\rho(t))\eta(t) + \hat{B}_G(\rho(t))u(t), \qquad \eta(0) = \eta_0, \\
y(t) &= \hat{C}_G(\rho(t))\eta(t).
\end{aligned}
$$

where, notation $\hat{X} = I_N \otimes X$ and $x = [x_1^T, \ldots, x_N^T]^T$ is used.

Formation control law with gradient-based forcing term

$$
u = \mathcal{L}_{(d)}(y - r) + \begin{bmatrix} u_{\psi_1} \\ \vdots \\ u_{\psi_N} \end{bmatrix}. \tag{1}
$$

$$
u_{\psi_i}(t) = \begin{cases} \nabla \psi(y_i), & \text{if } i \in \mathcal{V}_l, \\ 0, & \text{otherwise.} \end{cases} \tag{2}
$$

qLPV modeling

Assume

There exists a function $g : \mathbb{R}^{n_\eta} \to \mathbb{R}^{n_\rho}$ such that
$\rho(t) = g(\eta(t)) \quad t \in [0, \infty)$ and there exists $c \geq 0$ such that

$$\mathcal{B}(\eta_*, c) = \{\eta : ||\eta - \eta_*|| \leq c\} \subset \{\eta \in \mathbb{R}^{n_\eta} : g(\eta) \in \mathcal{P}\} =: \mathcal{P}_g^{-1}$$

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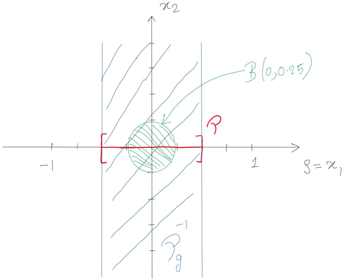
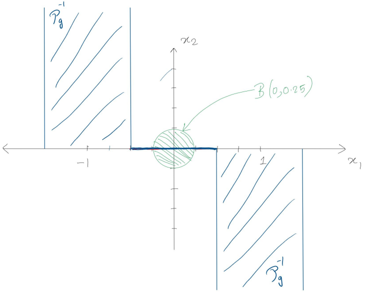
Examples with $\eta = [x_1 \quad x_2]^T$, $g : \mathbb{R}^2 \to \mathbb{R}$, $\mathcal{P} = [-0.5, 0.5]$

$\rho = x_1$



$\rho = x_1 + \text{sign}(x_2)$



Main Result

Theorem

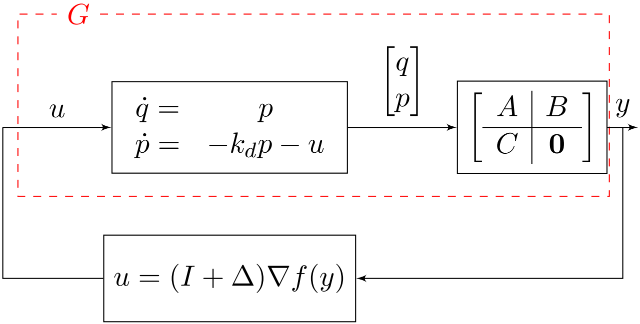
If there exist $\mathcal{X}_0 \succ 0, P \in \mathbb{P}$, $\lambda \geq 0$ such that

$$\begin{bmatrix} \mathcal{A}_0(\tilde{\rho})^T \mathcal{X}_0 + \mathcal{X} \mathcal{A}_0(\tilde{\rho}) + 2\alpha \mathcal{X}_0 & (*) \\ \mathcal{B}_0(\tilde{\rho})^T \mathcal{X}_0 & \mathbf{0} \end{bmatrix} + (*) \begin{bmatrix} P \otimes I_d & \\ & \lambda(M \otimes I_d) \end{bmatrix} \begin{bmatrix} \mathcal{C}_{10}(\tilde{\rho}) & \mathcal{D}_{10} \\ \mathcal{C}_{20} & \mathcal{D}_{20} \end{bmatrix} \preceq 0, \qquad (3)$$

holds for all $\tilde{\rho} \in \mathcal{P}$, then for all initial conditions η_0 such that $||\eta_0 - \eta_*|| < \frac{c}{\sqrt{\mathsf{cond}(\mathcal{X})}}$, the trajectories converge to the equilibrium exponentially at rate α.

$$\begin{bmatrix} -I & 0 \\ 0 & \delta^2 \end{bmatrix}$$

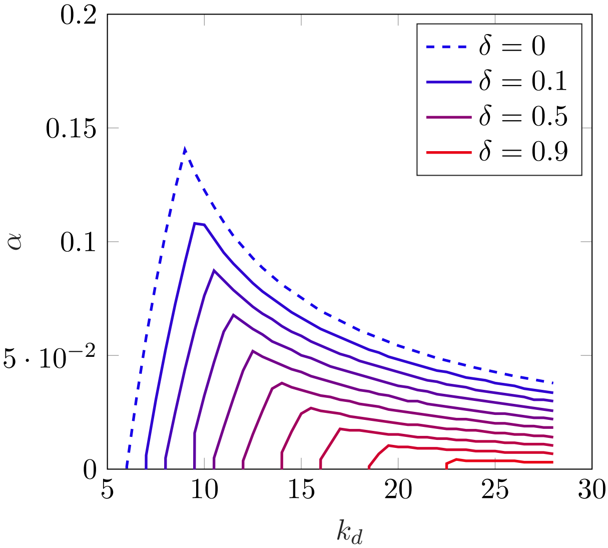
Numerical results for a single LTI quadrotor



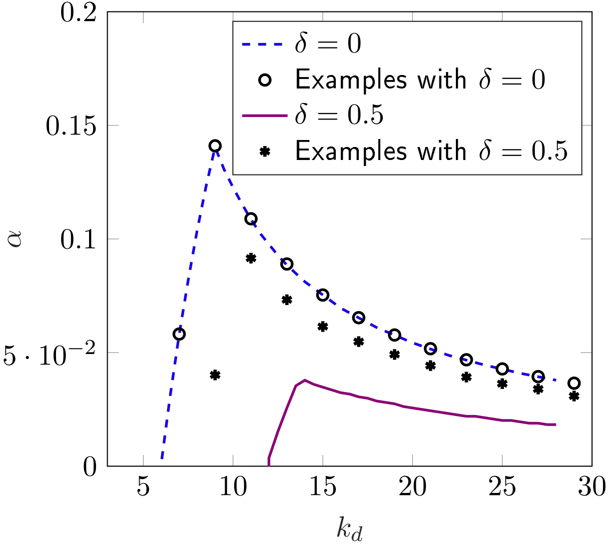
Setup

- Linearized quadrotor model + LQR local controller
- $||\Delta|| \leq \delta$

Designing k_d for largest convergence rate estimate α



Conservatism estimation by searching for examples



Non-linear Friction Model

Dynamics

$$m\ddot{x} + b|\dot{x}|\dot{x} = u_F, \tag{4}$$

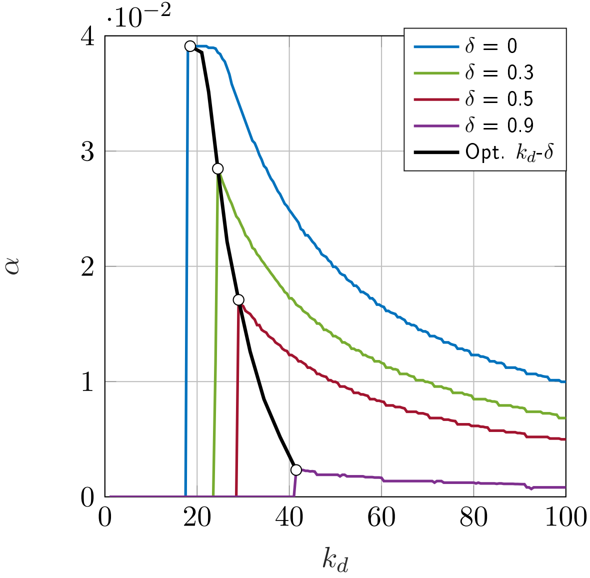
qLPV representation

$$\begin{aligned}
\begin{bmatrix} \dot{x}_{\mathsf{pos}} \\ \dot{x}_{\mathsf{vel}} \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ 0 & -\frac{b}{m}\rho \end{bmatrix} \begin{bmatrix} x_{\mathsf{pos}} \\ x_{\mathsf{vel}} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u_F \\
y &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_{\mathsf{pos}} \\ x_{\mathsf{vel}} \end{bmatrix}
\end{aligned} \tag{5}$$

$$\rho(t) = |x_{\mathsf{vel}}(t)|,$$

Synthesize an LPV controller on $\mathcal{P} = [0, 5]$

Designing k_d for largest convergence rate estimate α



Noise can destabilize the dynamics

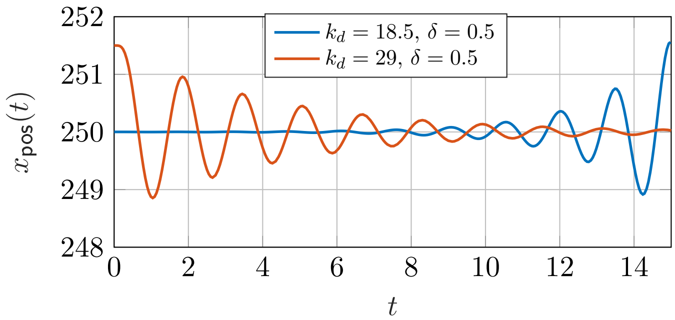
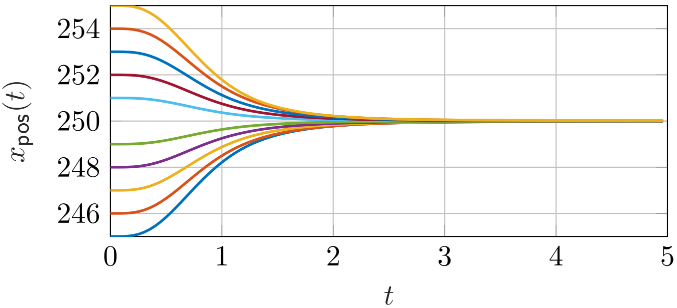


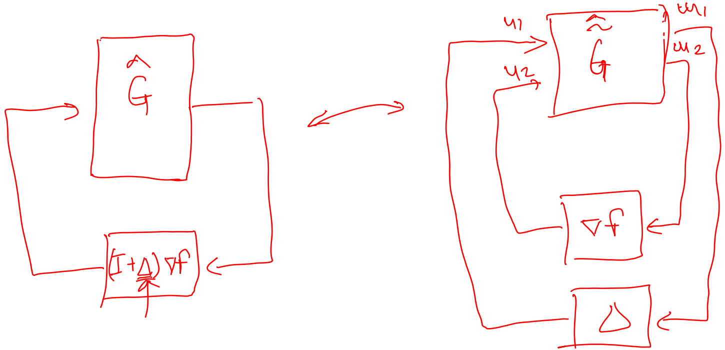
Figure 1: Single agent with noisy gradient and different values of k_d optimized for $\delta = 0$ and $\delta = 0.5$ respectively.

Consensus at Source With $\delta = 0.5$



Thank you

Main idea Behind Proof



$$\text{LMI} \Rightarrow \dot{V} + \underbrace{(*)\, M_1 \begin{bmatrix} u_1 \\ w_1 \end{bmatrix}}_{\geq 0} + \underbrace{(*)\, M_2 \begin{bmatrix} u_2 \\ w_2 \end{bmatrix}}_{\geq 0} \leq 0$$

$$\Rightarrow \dot{V} \leq 0$$