Group Research Meeting

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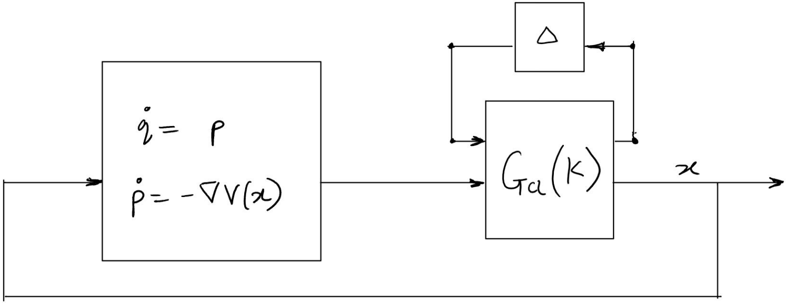
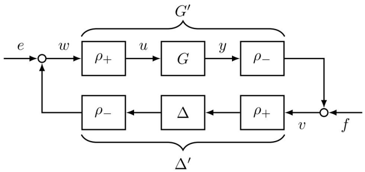
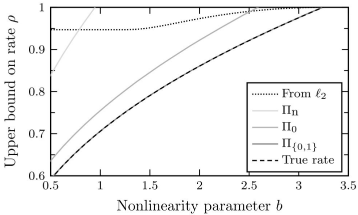
Problem



- Coupled loop
- Two step-synthesis procedure:
  1. Design a local tracking exponentially stabilizing controller (with performance measures specified by the exponents)
  2. Use these as inputs to design the network dynamics
- Analysis via
  - rho-IQCs for exponential stability of tracking dynamics
  - Singular perturbation theory

rho-IQCs[1]

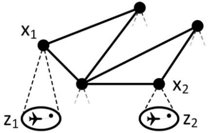




- L2/l2 gains give conservative bounds on the exponents
- Less conservative LMI formulations via rho-IQCs

[1] [2017] Boczar, Lessard, Packard, Recht

Time-scale separation[2]

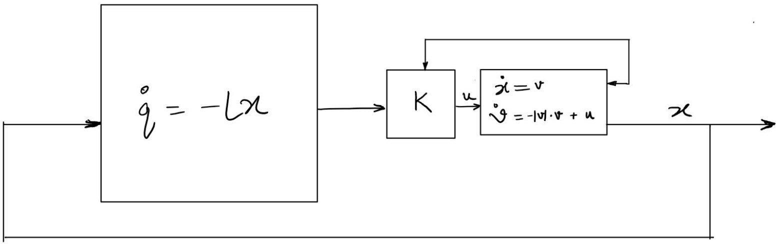


$$\dot{x} = -(L(\mathcal{G}) \otimes I)\,(I \otimes M)\,z \;= -(L(\mathcal{G}) \otimes M)z$$
$$\varepsilon \dot{z} = g(x, z)$$

- If the time-scales are separated enough, we can study(and use the vast literature on)
  - simplified network dynamics
  - robust/LPV controller synthesis for uncertain/non-linear agent dynamics

2[2019] Awad ,Chapman , Schoof , Narang-Siddarth, and Mesbahi

Starting Example



- First order consensus at the network level
- Possible options local agent dynamics:
  - LTI model with uncertain parameter
  - Damping coeff $b = |x|$ non-linearity (=sector non-linearity?)
  - Force saturation non-linearity (=sector non-linearity?)

Problem 2: Stochastic version of the IQC result

Theorem 7.16 *Suppose that* $M : \mathscr{L}_{2e}^k \to \mathscr{L}_{2e}^l$ *is causal and bounded,* $\Delta : \mathscr{L}_{2e}^l \to \mathscr{L}_{2e}^k$ *is causal,* Σ *is a bounded quadratic form,* $\mathscr{D} \subset \mathscr{L}_{2e}^l$, *and that*

- *there exist* $\varepsilon > 0$ *and* m_0 *such that*

$$\Sigma \begin{pmatrix} w_T \\ M(w)_T \end{pmatrix} \leq -\varepsilon \|w_T\|^2 + m_0 \text{ for all } T > 0,\ w \in \mathscr{L}_2^k; \tag{7.3.2}$$

- *there exists* δ_0 *with*

$$\Sigma \begin{pmatrix} \Delta(z)_T \\ z_T \end{pmatrix} \geq -\delta_0 \text{ for all } T > 0,\ z \in M(\mathscr{L}_2^k) + \mathscr{D}. \tag{7.3.3}$$

Then there exist $\gamma > 0$ *and* γ_0 *such for any* $d \in \mathscr{D}$ *with response* $z \in \mathscr{L}_{2e}^l$ *satisfying* (7.3.1):

$$\|z_T\|^2 \leq \gamma^2 \|d_T\|^2 + \gamma\gamma_0 \text{ for all } T > 0. \tag{7.3.4}$$

If M *is linear one can choose* $\gamma_0 = m_0 + \delta_0$.

- Δ is characterized by (7.3.3)
- Can we relax condition (7.3.3) and give a weaker implication of (7.3.4)

Thank you