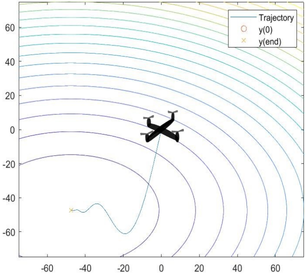
Performance Analysis of Source-Seeking Algorithms with Integral Quadratic Constraints

Adwait Datar

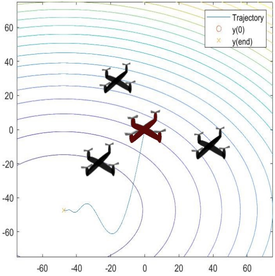
PhD Workshop, 2021
Technical University of Hamburg

28^{th} Feb, 2022

Source-seeking Problem



PhD workshop Sept 2021



PhD workshop Feb 2022

Assumptions (Informal)

- Field is differentiable and convex
- Local gradients available at some leader agents
- Connectivity assumptions on interconnection graphs

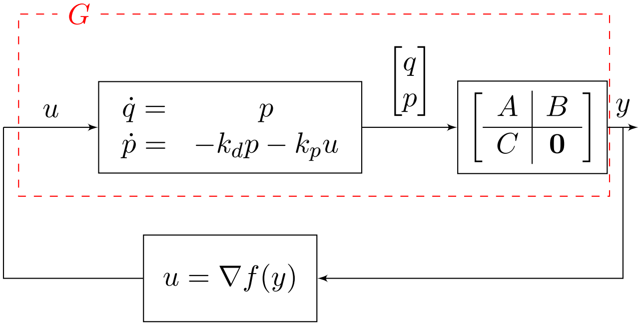
Outline

Review: Single agent case

Extension: Multiple agents

Numerical results

Control Architecture: Single Agent



$$
\begin{aligned}
\dot{\eta}(t) &= A_G \eta(t) + B_G u(t), \qquad \eta(0) = \eta_0 \\
y(t) &= C_G \eta(t) \\
u(t) &= \nabla f(y(t)),
\end{aligned}
$$

Loop in the deviation variables: Single agent

Equilibrium

$$
\begin{aligned}
0 &= A_G \eta_* + B_G u_* = A_G \eta_* \\
y &= C_G \eta_* \\
u_* &= \nabla f(y_*) = 0
\end{aligned}
\tag{1}
$$

Loop in the transformed variables

$$
\begin{aligned}
\dot{\tilde{\eta}}(t) &= A_G \tilde{\eta}(t) + B_G \tilde{u}(t), \qquad \tilde{\eta}(0) = \eta_0 - \eta_* \\
\tilde{y}(t) &= C_G \tilde{\eta}(t)
\end{aligned}
\tag{2}
$$

and

$$
\tilde{u}(t) = \nabla f(\tilde{y}(t) + y_*)
\tag{3}
$$

Main Results: Single Agent

- Let y_* minimizes $f \in \mathcal{S}(m, L)$ [1]

[1] $m||y_1 - y_2||^2 \leq (\nabla f(y_1) - \nabla f(y_2))^T (y_1 - y_2) \leq L||y_1 - y_2||^2$

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- $\mathbb{P} = \left\{ \begin{bmatrix} \mathbf{0} & \begin{bmatrix} H & -P_3 \\ -P_1^T & \mathbf{0} \end{bmatrix} \\ * & \mathbf{0} \end{bmatrix} : \mathsf{LMI}(H, P_1, P_3) < 0 \right\}$

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Theorem 1

If $\exists \mathcal{X} > 0, P \in \mathbb{P}$ such that

$$\begin{bmatrix} \mathcal{A}^T\mathcal{X} + \mathcal{X}\mathcal{A} + 2\alpha\mathcal{X} & \mathcal{X}\mathcal{B} \\ \mathcal{B}^T\mathcal{X} & \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathcal{C}^T \\ \mathcal{D}^T \end{bmatrix} (P \otimes I_d) \begin{bmatrix} \mathcal{C} & \mathcal{D} \end{bmatrix} \leq 0,$$

then, $||y(t) - y_*(t)|| \leq \kappa e^{-\alpha t}$ holds for all $t \geq 0$

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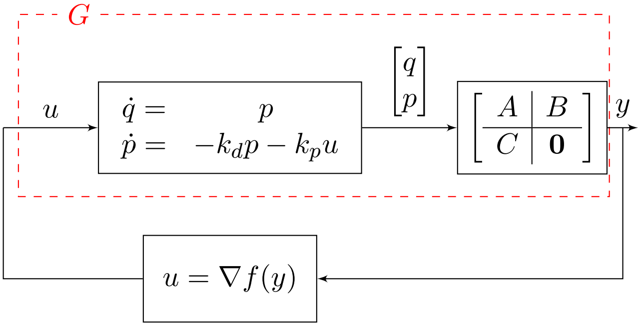
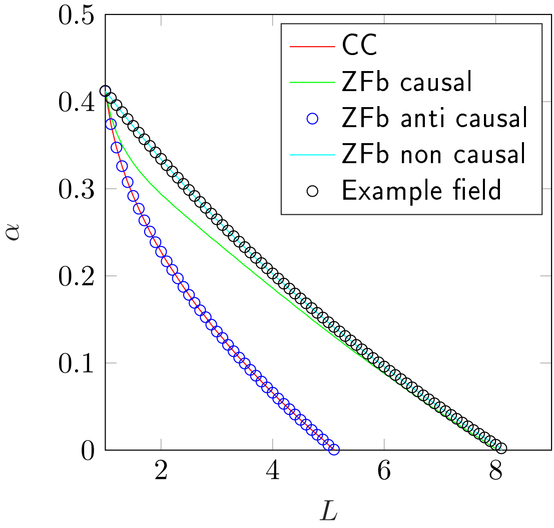
Numerical results for a single quadrotor



Setup

- Linearized quadrotor model + LQR local controller
- Questions:
  - How robust is the given controller for different fields?
  - How do we select the PD gains for the pre-filter?
  - How conservative are the estimates on the rates of convergence?

Robustness w.r.t scalar field for a single quadrotor



Control Architecture: Multiple agents

$$
\begin{aligned}
\dot{\hat{\eta}}(t) &= \hat{A}_G \hat{\eta}(t) + \hat{B}_G \hat{u}(t), \qquad \hat{\eta}(0) = \hat{\eta}_0, \\
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\end{aligned}
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where, notation $\hat{X} = I_N \otimes X$ and $\hat{x} = [x_1^T, \ldots, x_N^T]^T$ is used.

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Formation control law with gradient-based forcing term

$$\hat{u} = \mathcal{L}_{(d)}(\hat{y} - \hat{r}) + \begin{bmatrix} u_{\psi_1} \\ \vdots \\ u_{\psi_N} \end{bmatrix}. \tag{4}$$

$$u_{\psi_i}(t) = \begin{cases} \nabla \psi(y_i), & \text{if } i \in \mathcal{V}_l, \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

Control Architecture: Multiple agents

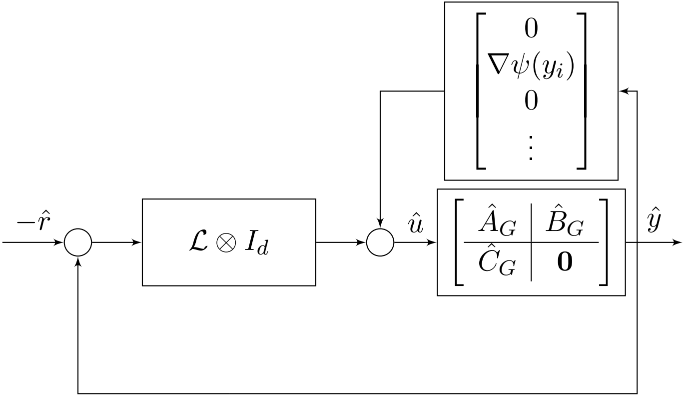

Figure 2: Control architecture

Formulation as a robust control problem

Definition

For a given graph $\mathcal{G}$ of order N (with its corresponding laplacian $\mathcal{L}$), the leader set $\mathcal{V}_l$, a scalar field ψ and a given formation reference vector $\hat{r} \in \mathbb{R}^{Nd}$, define a function $f : \mathbb{R}^{Nd} \to \mathbb{R}$ by

$$f(\hat{y}) = \frac{1}{2}(\hat{y} - \hat{r})^T \mathcal{L}_{(d)}(\hat{y} - \hat{r}) + \sum_{v_i \in \mathcal{V}_l} \psi(y_i). \tag{6}$$

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Overall close-loop system

$$\begin{aligned} \dot{\hat{\eta}}(t) &= \hat{A}_G \hat{\eta}(t) + \hat{B}_G \hat{u}(t), \qquad \hat{\eta}(0) = \hat{\eta}_0, \\ \hat{y}(t) &= \hat{C}_G \hat{\eta}(t), \\ \hat{u}(t) &= \nabla f(\hat{y}(t)). \end{aligned} \qquad (7)$$

Recall Theorem 1

$$\left[\begin{array}{c|c} \mathcal{A} & \mathcal{B} \\ \hline \mathcal{C} & \mathcal{D} \end{array}\right] = (\Pi \otimes I_N) \begin{bmatrix} \hat{G} \\ I_{Nd} \end{bmatrix} = \left[\begin{array}{c|c} A_\Pi \otimes I_N & B_\Pi \otimes I_N \\ \hline C_\Pi \otimes I_N & D_\Pi \otimes I_N \end{array}\right] \left[\begin{array}{c|c} \hat{A}_G & \hat{B}_G \\ \hline \hat{C}_G & \mathbf{0} \\ \mathbf{0} & I_{Nd} \end{array}\right]$$

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Theorem 1 applied to the overall networked system

If $f \in \mathcal{S}(m, L)$, y_* minimizes f and $\exists \mathcal{X} > 0, P \in \mathbb{P}$ such that

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Questions

1. How to characterize minimizers of f?
2. How to verify if $f \in \mathcal{S}(m, L)$?
3. Can we exploit this structure in the LMIs?

1. Minimizers of f

Recall definiteion of f

$$f(\hat{y}) = \frac{1}{2}(\hat{y} - \hat{r})^T \mathcal{L}_{(d)}(\hat{y} - \hat{r}) + \sum_{v_i \in \mathcal{V}_l} \psi(y_i).$$

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Assumptions

1. Let $\psi \in \mathcal{S}(m_\psi, L_\psi)$ for some $0 < m_\psi \leq L_\psi$ and let y_* minimize ψ.
2. For every node $v_i \in \mathcal{V}$, there is a node $v_j \in \mathcal{V}_l$ such that $\mathcal{G}$ contains an $i - j$ path.

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Separately consider the cases

1 Consensus $\hat{r} = \mathbf{0}, |\mathcal{V}_l| \geq 1$

2 Formation control with single leader $\hat{r} \neq \mathbf{0}$, $|\mathcal{V}_l| = 1$

3 Formation control with multiple leaders $\hat{r} \neq \mathbf{0}$, $|\mathcal{V}_l| > 1$

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Lemma (Consensus)

Let Assumptions 1 and 2 hold and let $\hat{r} = 0$. Then, $\hat{y}$ is the minimizer of f iff $\hat{y} = \mathbf{1}_N \otimes y_*$.

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Lemma(Formation with one leader)

Let Assumptions 1 and 2 hold and let $|\mathcal{V}_l| = 1$. Then, $\hat{y}$ is the minimizer of f iff $\hat{y}_i = y_*$ for $i \in \mathcal{V}_l$ and $y_j = y_* + (r_j - r_i)$ for all $j \in \mathcal{V}$.

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Conjecture (Formation with multiple leaders)

Let Assumptions 1 and 2 hold. The unique minimizer y_* of ψ lies in the convex hull of positions of leader agents.

2. Smoothness and convexity of f

Definition of grounded Laplacians

Define grounded Laplacians

$$\begin{aligned} \mathcal{L}_s &= \mathcal{L} + m_\psi E, \\ \mathcal{L}_b &= \mathcal{L} + L_\psi E, \end{aligned} \tag{8}$$

where, E is a diagonal matrix with the i^{th} diagonal entry equal to 1 if $i \in \mathcal{V}_l$ and equal to 0 otherwise.

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where, E is a diagonal matrix with the i^{th} diagonal entry equal to 1 if $i \in \mathcal{V}_l$ and equal to 0 otherwise.

Lemma

For constants $0 < m \leq L$, the following two statements are equivalent:

1. $f \in \mathcal{S}(m, L)$ for all $\psi \in \mathcal{S}(m_\psi, L_\psi)$,
2. $mI_N \preceq \mathcal{L}_b \preceq \mathcal{L}_s \preceq LI_N$

3. Decomposition

Define

$$\left[\begin{array}{c|c} \mathcal{A} & \mathcal{B} \\ \hline \mathcal{C} & \mathcal{D} \end{array}\right] = (\Pi \otimes I_N) \begin{bmatrix} \hat{G} \\ I_{Nd} \end{bmatrix} \text{ and } \left[\begin{array}{c|c} \mathcal{A}_0 & \mathcal{B}_0 \\ \hline \mathcal{C}_0 & \mathcal{D}_0 \end{array}\right] = \Pi \begin{bmatrix} G \\ I_d \end{bmatrix}$$

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$$\begin{bmatrix} \mathcal{A}^T \mathcal{X} + \mathcal{X}\mathcal{A} + 2\alpha\mathcal{X} & \mathcal{X}\mathcal{B} \\ \mathcal{B}^T \mathcal{X} & \mathbf{0} \end{bmatrix} + (*)(P \otimes I_{Nd}) \begin{bmatrix} \mathcal{C} & \mathcal{D} \end{bmatrix} \leq 0, \tag{9}$$

$$\begin{bmatrix} \mathcal{A}_0^T \mathcal{X}_0 + \mathcal{X}_0\mathcal{A}_0 + 2\alpha\mathcal{X}_0 & \mathcal{X}_0\mathcal{B}_0 \\ \mathcal{B}_0^T \mathcal{X}_0 & \mathbf{0} \end{bmatrix} + (*)(P \otimes I_d) \begin{bmatrix} \mathcal{C}_0 & \mathcal{D}_0 \end{bmatrix} \leq 0, \tag{10}$$

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Theorem

The following statements are equivalent:

1. $\exists \mathcal{X} > 0, P \in \mathbb{P}$ such that (9) is satisfied.
2. $\exists \mathcal{X}_0 > 0, P \in \mathbb{P}$ such that (10) is satisfied.

Final Analysis result

Definition

$$
\begin{aligned}
\Delta_{m,L} &= \{(\mathcal{G}, \mathcal{V}_l, \psi) : \psi \in \mathcal{S}(m_\psi, L_\psi), f \in \mathcal{S}(m, L)\} \\
&= \{(\mathcal{G}, \mathcal{V}_l, \psi) : \psi \in \mathcal{S}(m_\psi, L_\psi), mI_N \preceq \mathcal{L}_b \preceq \mathcal{L}_s \preceq LI_N\}
\end{aligned} \tag{11}
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Theorem

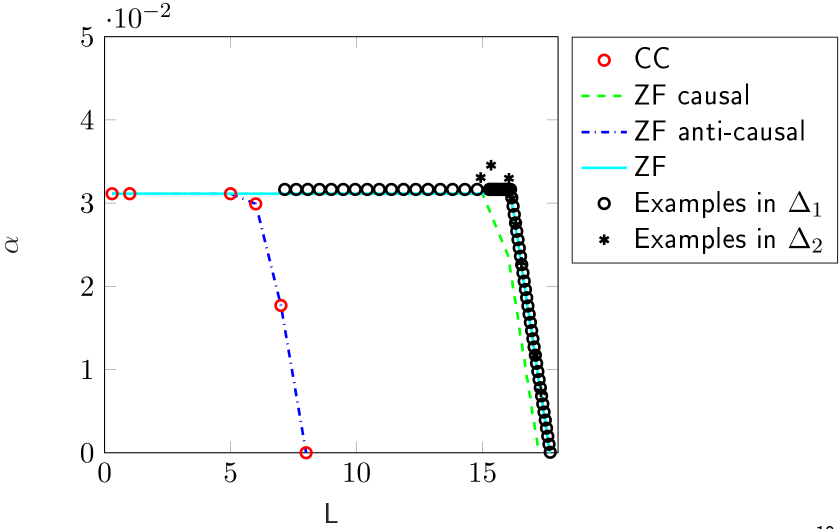
Let $(\mathcal{G}, \mathcal{V}_l, \psi) \in \Delta_{m,L}$ *for some* $0 < m \leq L$. *If* $\exists \mathcal{X}_0 > 0, P \in \mathbb{P}$ *such that*

$$\begin{bmatrix} \mathcal{A}_0^T \mathcal{X}_0 + \mathcal{X}_0 \mathcal{A}_0 + 2\alpha \mathcal{X}_0 & \mathcal{X}_0 \mathcal{B}_0 \\ \mathcal{B}_0^T \mathcal{X}_0 & \mathbf{0} \end{bmatrix} + (*)(P \otimes I_d) \begin{bmatrix} \mathcal{C}_0 & \mathcal{D}_0 \end{bmatrix} \leq 0, \tag{12}$$

is satisfied, then, $||y(t) - y_*(t)|| \leq \kappa e^{-\alpha t}$ *holds for all* $t \geq 0$.

Example 1: Conservatism

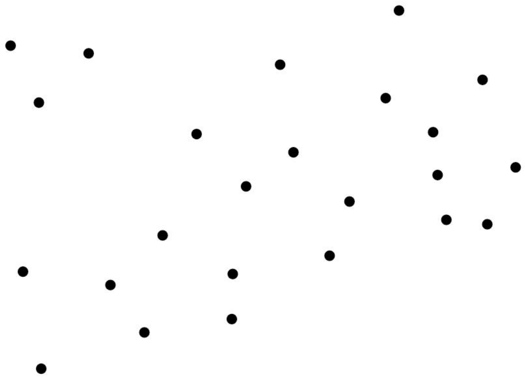
$$\Delta_1 = \{(\mathcal{G}, \mathcal{V}_l, \psi) : \mathcal{G} = \mathcal{G}^5_{\text{star}}, \mathcal{V}_l = \{1\}, \psi \in \mathcal{S}(2.02, L_\psi)\} \subset \Delta_{0.3,L}$$
$$\Delta_2 = \{(\mathcal{G}^{25}, \mathcal{V}_l, \psi) : \psi = 1.85||y - y_*||^2\} \subset \Delta_{m,L}$$



Example 2: Spectrum of Laplacian unknown

Assume:

- i At least one third of total number of agents are leaders
- ii Maximum degree of all agents is 2
- iii Every agent has an edge with at least one leader



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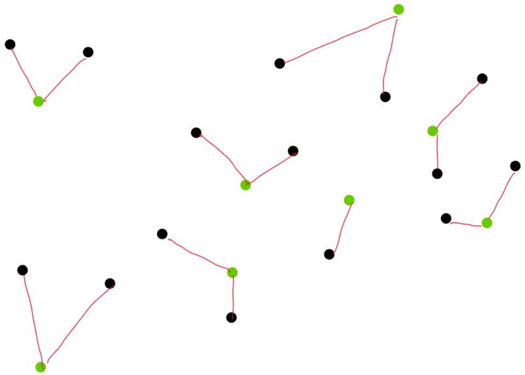

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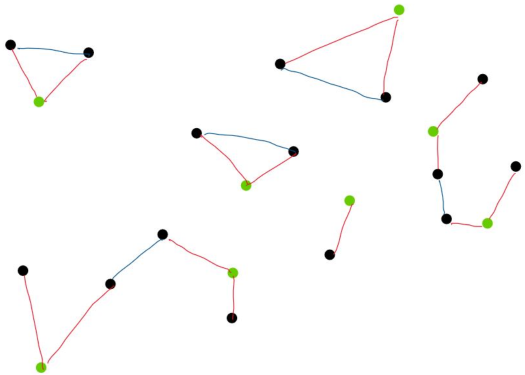
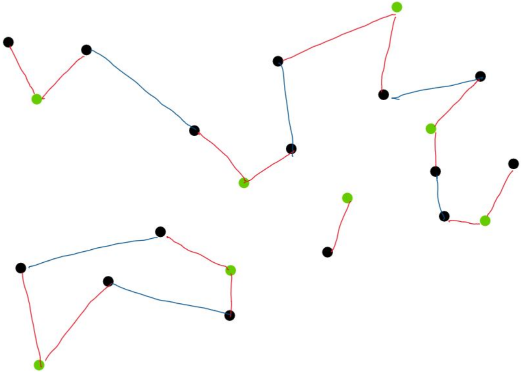


Example 2: Spectrum of Laplacian unknown

Assume:

- i At least one third of total number of agents are leaders
- ii Maximum degree of all agents is 2
- iii Every agent has an edge with at least one leader



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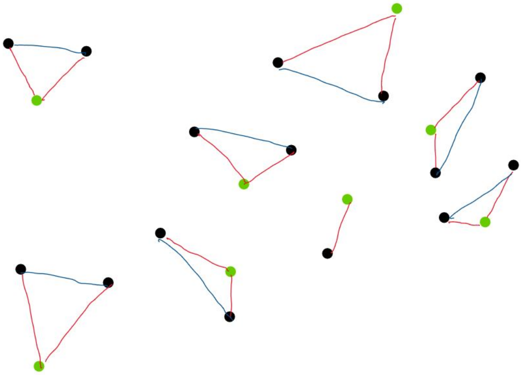

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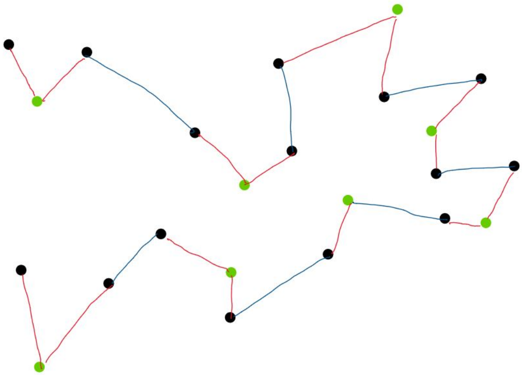

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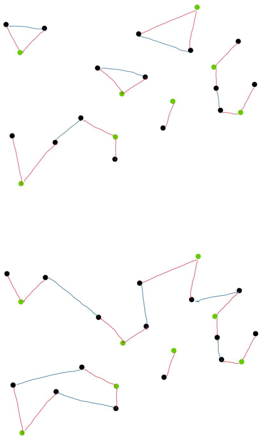


Example 2: Spectrum of Laplacian unknown

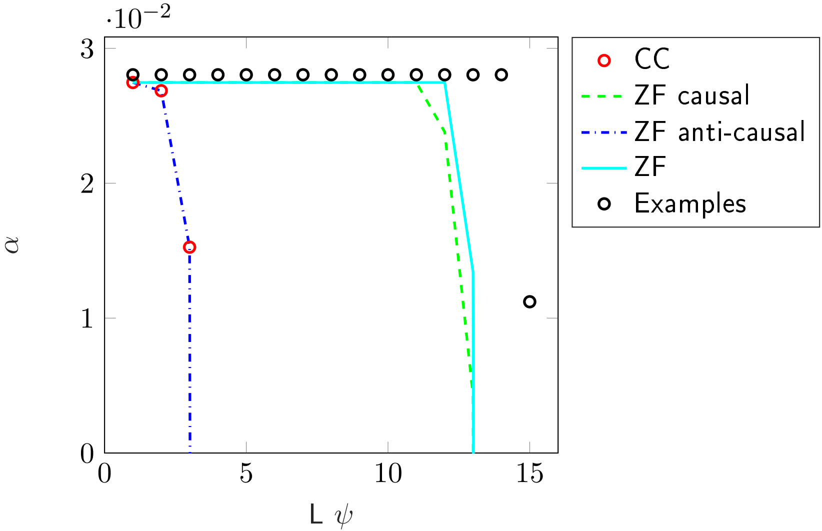
Setup

- Linearized quadrotor model + LQR tracking local controller

1. Conditions on the graph
   i. At least one third of total number of agents are leaders, i.e., $|\mathcal{V}_l| \geq |\mathcal{V}|$.
   ii. Maximum degree of all agents is 3
   iii. Every agent has an edge with at least one leader, i.e., For any $i \in \mathcal{V}$, there is a $j \in \mathcal{V}_l$ such that $(i,j) \in \mathcal{E}$
2. Let $\psi \in \mathcal{S}(1, L_\psi)$

- Let $\bar{\Delta} = \{(\mathcal{G}, \mathcal{V}_l, \psi) : (1), (2)\}$
- Can show: $\bar{\Delta} \subset \Delta_{m,L}$ with $m = 0.3$ and $L = L_\psi + 2 * 3$

Example 2: Spectrum of Laplacian unknown



Thank you