# Group Research Meeting

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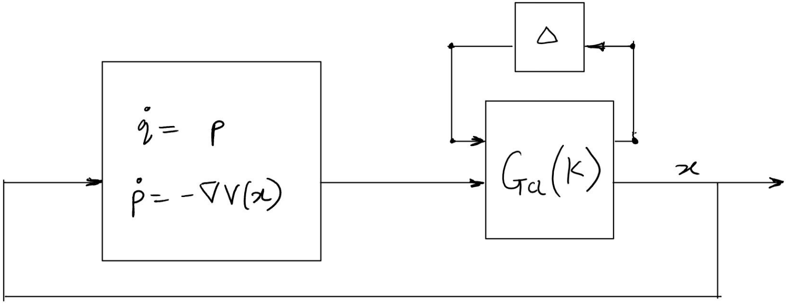
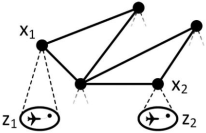
# Problem




- Coupled loop
- Two step-synthesis procedure:
  1. Design a local tracking exponentially stabilizing controller (with performance measures specified by the exponents)
  2. Use these as inputs to design the network dynamics
- Analysis via
  - rho-IQCs for exponential stability of tracking dynamics
  - Singular perturbation theory

# Time-scale separation[1]




$$\dot{x} = -(L(\mathcal{G}) \otimes I)\,(I \otimes M)\,z \;= -(L(\mathcal{G}) \otimes M)z$$
$$\varepsilon \dot{z} = g(x, z)$$

- If the time-scales are separated enough, we can study(and use the vast literature on)
  - simplified network dynamics
  - robust/LPV controller synthesis for uncertain/non-linear agent dynamics

[1] [2019] Awad ,Chapman , Schoof , Narang-Siddarth, and Mesbahi

# Updates

- Vidyasagar: Review singular perturbation for Linear systems
- Khalil:
  - Understand stability analysis
  - Exponential stability relaxes the conditions on Lyapunov functions
- Ongoing work:
  - Arrive at LMIs to search for Lyapunov functions
  - Check the conservatism for simple examples

# Problem 2: Stochastic version of the IQC result

**Theorem 7.16** *Suppose that $M : \mathscr{L}_{2e}^k \to \mathscr{L}_{2e}^l$ is causal and bounded, $\Delta : \mathscr{L}_{2e}^l \to \mathscr{L}_{2e}^k$ is causal, $\Sigma$ is a bounded quadratic form, $\mathscr{D} \subset \mathscr{L}_{2e}^l$, and that*

- *there exist $\varepsilon > 0$ and $m_0$ such that*

$$\Sigma \begin{pmatrix} w_T \\ M(w)_T \end{pmatrix} \leq -\varepsilon \|w_T\|^2 + m_0 \ \text{for all} \ T > 0, \ w \in \mathscr{L}_2^k; \tag{7.3.2}$$

- *there exists $\delta_0$ with*

$$\Sigma \begin{pmatrix} \Delta(z)_T \\ z_T \end{pmatrix} \geq -\delta_0 \ \text{for all} \ T > 0, \ z \in M(\mathscr{L}_2^k) + \mathscr{D}. \tag{7.3.3}$$

*Then there exist $\gamma > 0$ and $\gamma_0$ such for any $d \in \mathscr{D}$ with response $z \in \mathscr{L}_{2e}^l$ satisfying (7.3.1):*

$$\|z_T\|^2 \leq \gamma^2 \|d_T\|^2 + \gamma\gamma_0 \ \text{for all} \ T > 0. \tag{7.3.4}$$

*If $M$ is linear one can choose $\gamma_0 = m_0 + \delta_0$.*

- $\Delta$ is characterized by (7.3.3). Can we relax this to a stochastic version?

**Updates:**

- Understand the proofs and subtleties involved in hard and soft IQCs

Thank you