Parallelizing Gauss-Seidel With MPI

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Motivation

- Direct method such as Gaussian elimination method are not robust for solving system of linear equation.(Computational time $\mathbf{O}(n^3)$ and memory requirement - off-chip memory bandwidth is a constraining resource in system performance).
- Iterative methods can be more robust
- We discuss here Gaus-seidel Method
- We can parallelize Gaus seidel method
- We use the successive over relaxation to aid convergence.

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Gauss-Seidel In a nut shell

Gauss-Seidel

$$x_i^{(k+1)} = \frac{1}{a_{ii}} \left(b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)} \right), \quad i = 1, 2, \ldots, n.$$

Successive Over relaxation (SOR) for Gauss-Seidel

$$x_i^{(k+1)} = (1-\omega) x_i^{k-1} + \frac{\omega}{a_{ii}} \left(b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)} \right)$$

$$i = 1, 2, \ldots, n.$$

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Gauss-Seidel In a nut shell

Gauss-Seidel VS SOR

- When $\omega = 1$. Then SOR = Gauss Seidel
- When $0 < \omega < 1$. SOR Under relaxation case
- When $1 < \omega < 2$. SOR Over relaxation case.

Sequential Gauss Seidel

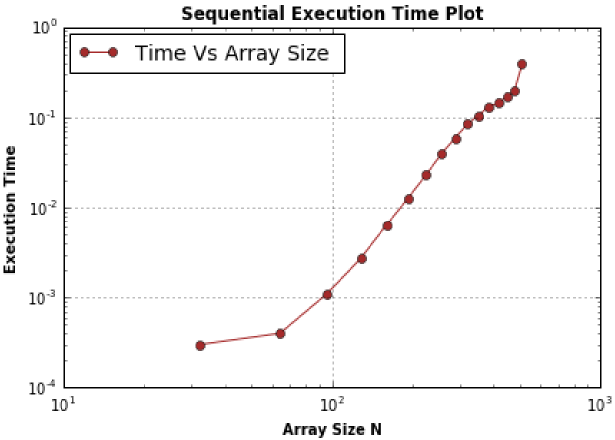


Figure: Sequential Gauss Seidel

Parallel Gauss-Seidel

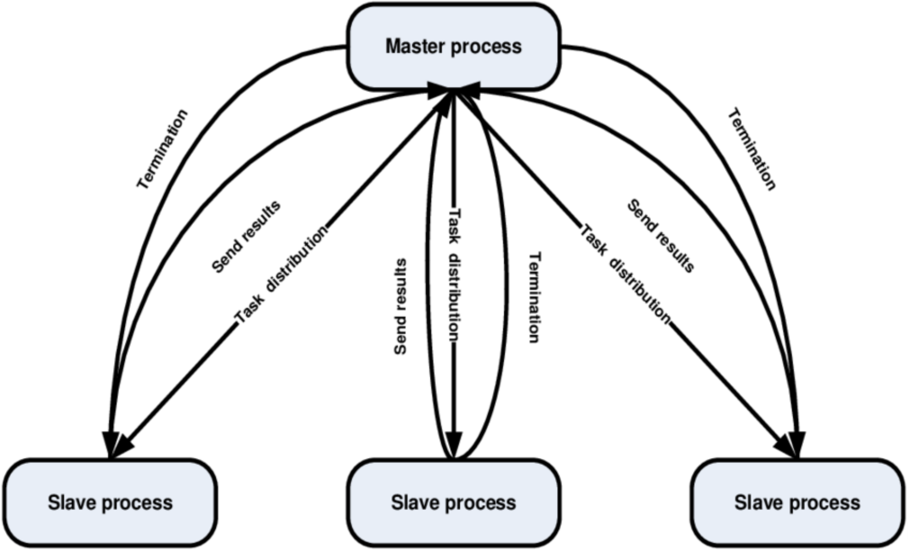
- Two methods to be compared:
  - Gauss Seidel
  - SOR Using Gauss-Seidel

Steps to Parallel GS

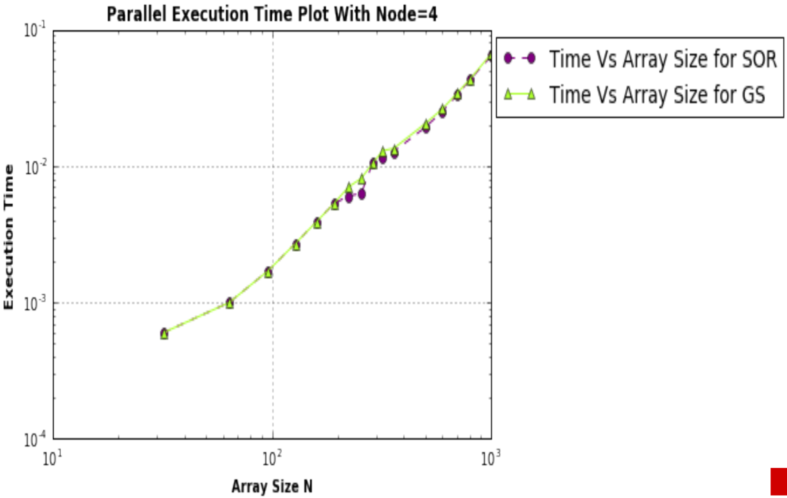
GS : Implementation in MPI .

- Master-Slave paradigm
- MS paradigm: computational task is divided into sub tasks with most of the processes used to compute the subtasks and a few processes (often just one process) managing the tasks
- Use a ID grid
- Load balance on the grid ($row = N/(number\_of\_processors)$)
- Slaves do local computations.
- Master gathers results from slaves
- Master update Values of unknown
- Scatter updated values
- Test for Convergence
- End loop or repeat.

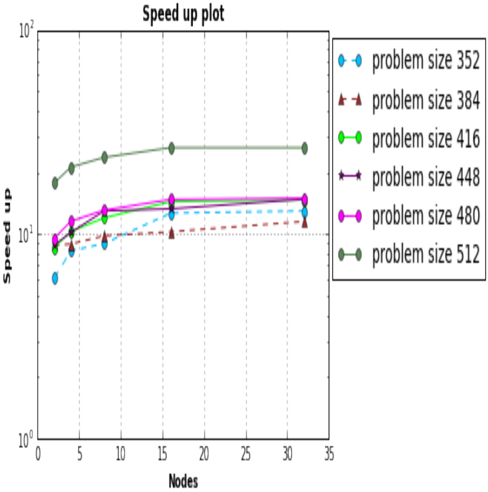
Simple LSTM and Backpropagation



Parallel GS plot: GS vs SOR.

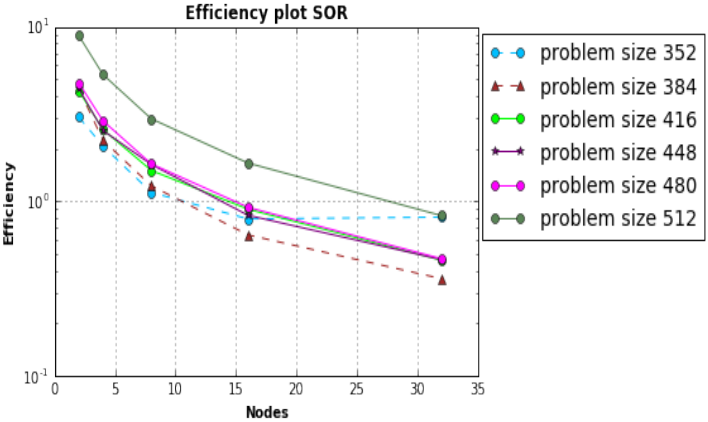


Speed up: for SOR.



Experimental results

Efficiency: for SOR.



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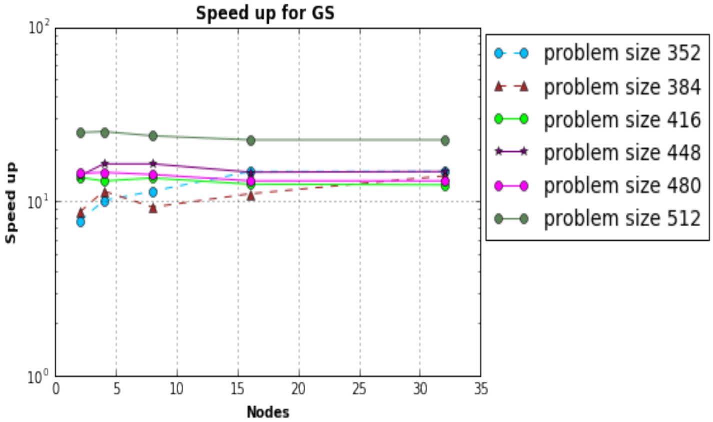

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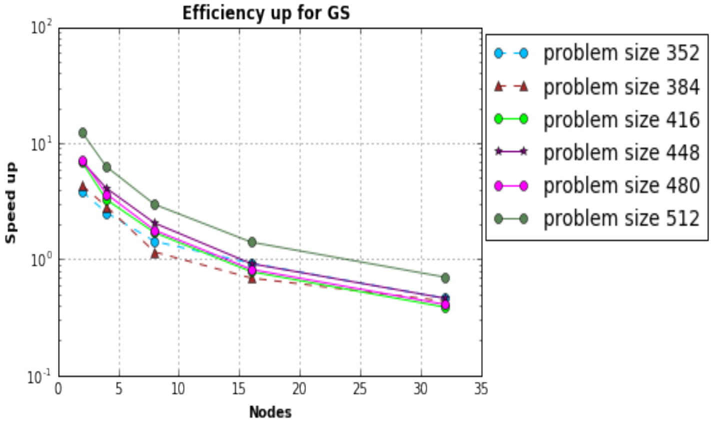

Improving The Approach

- Using 2D grid and communicators (*MPI_cart*).
- Hybrid approach

Conclusions

- Convergence becomes a problem in large problem size
- SOR provides better efficiency and speed for large problem size than normal GS
- Communication cost is incurred communicating results back and forth among processors.
- Hybrid approach can offer imporve performance

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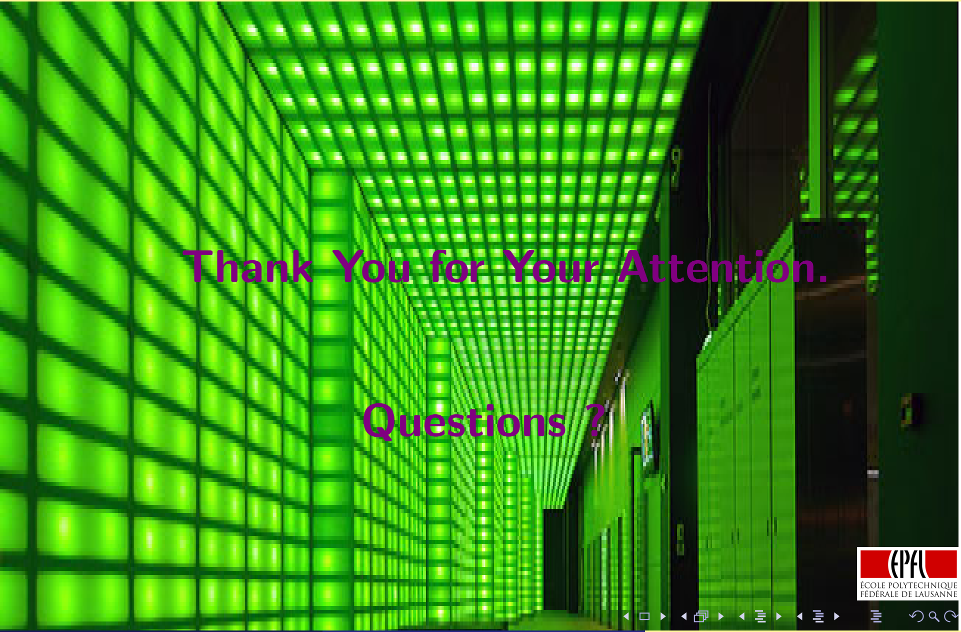
Conclusions



Thank You for Your Attention.

Questions ?

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