Lemma 4

$$(1-\frac{M}{2a}|x|)_+ = \max\{\frac{M}{2a}\cdot x+1,0\} - 2\max\{\frac{M}{2a}\cdot x,0\} + \max\{\frac{M}{2a}\cdot x-1,0\}$$

Beweis:

li.S. $(1-\frac{M}{2a}|x|)_+ = \max\{1-\frac{M}{2a}|x|,0\} = \max\{1-\frac{M}{2a}\max\{-x,x\},0\}$

Fall 1:

$x < -\frac{M}{2a}$

li.S. $\max\{1+\frac{M}{2a}x,0\}$

re.S.

$\max\{\frac{M}{2a}\cdot x+1,0\} - \underbrace{2\cdot 0 + \max\{\frac{M}{2a}\cdot x-1,0\}}_{=0,\ da\ x<0}$ ✓

Fall 2:

$-\frac{M}{2a} \le x \le 0$

li.S. $\max\{1+\frac{M}{2a}x,0\}$ ✓

re.S. $\max\{\frac{M}{2a}x+1,0\}$

Fall 3:

$0 < x \le \frac{M}{2a}$

li.S. $\max\{1-\frac{M}{2a}x,0\}$

re.S. $\max\{\frac{M}{2a}x+1,0\} - 2\cdot\max\{\frac{M}{2a}x,0\} + \max\{\frac{M}{2a}x-1,0\}$

Fall $x\cdot\frac{M}{2a} \le 1$

li.S. $\max\{1-\frac{M}{2a}x,0\} = 1-\frac{M}{2a}x$

re.S. $\max\{\frac{M}{2a}x+1,0\} - 2\max\{\frac{M}{2a}x,0\}$

$= \frac{M}{2a}x+1-2\frac{M}{2a}x = 1-\frac{M}{2a}x$

Falls $x\cdot\frac{M}{2a} > 1$

li.S. $=0$

re.S. $\cancel{\frac{M}{2a}x}+\cancel{1}-\cancel{2\cdot\frac{M}{2a}x}+\cancel{\frac{M}{2a}x}-\cancel{1}=0$

Fall 4:

$\frac{M}{2a} < x$ li.S. $\max\{1-\frac{M}{2a}x,0\}$

re.S. $\max\{\frac{M}{2a}x+1,0\} - 2\max\{\frac{M}{2a}x,0\} + \max\{\frac{M}{2a}x-1,0\}$

Falls $\frac{M}{2a}x \le 1$

l.S. $1-\frac{M}{2a}x$

r.S. $\frac{M}{2a}x+1-2\frac{M}{2a}x \quad +0$

$= 1-\frac{M}{2a}x$

Falls $\frac{M}{2a}x > 1$

l.S. $=0$

$\frac{M}{2a}x+1-2\frac{M}{2a}x+\frac{M}{2a}x-1=0$