Lemma 1

~~Beweis:~~ $R, a > 0 \quad \sigma: \mathbb{R} \to \mathbb{R}$

a) $\sigma \in C^2(\mathbb{R}) \quad t_{\sigma,id} \in \mathbb{R} \quad \sigma'(t_{\sigma,id}) \neq 0$

$$f_{id}(x) = \frac{R}{\sigma'(t_{\sigma,id})} \cdot \left(\sigma\left(\frac{x}{R} + t_{\sigma,id}\right) - \sigma(t_{\sigma,id})\right)$$

satisfies for any $x \in [-a,a]$:

$$|f_{id}(x) - x| \leq \frac{\|\sigma''\|_\infty \cdot a^2}{2 \cdot |\sigma'(t_{\sigma,id})|} \cdot \frac{1}{R}$$

b)
⋮

Beweis:

a) Taylorentwickl der Ordnung 2 in Entwicklungsstelle ξ

$$|f_{id}(x) - x|$$

ausklammern

$$= \left|\frac{R}{\sigma'(t_{\sigma,id})} \cdot \left(\sigma\left(\frac{\xi}{R} + t_{\sigma,id}\right) - \sigma(t_{\sigma,id}) + \frac{1}{R}\sigma'\left(\frac{\xi}{R} + t_{\sigma,id}\right)(x - \xi) + \frac{1}{2R^2}\sigma''\left(\frac{c}{R} + t_{\sigma,id}\right)(x-\xi)^2\right) - x\right|$$

$$= \left|\frac{R}{\sigma'(t_{\sigma,id})} \cdot \left(\overbrace{\sigma(t_{\sigma,id}) - \sigma(t_{\sigma,id})}^{=0} + \frac{x}{R}\sigma'(t_{\sigma,id}) + \frac{\cancel{x}^2}{R^2 2}\sigma''\left(\underbrace{\frac{c}{R} + t_{\sigma,id}}_{=c}\right)\right) - x\right|$$

$$= \left|\frac{R}{\sigma'(t_{\sigma,id})} \cdot \left(\frac{x}{R}\sigma'(t_{\sigma,id}) + \frac{x^2}{2R^2}\sigma''(c)\right) - x\right|$$

$$= \left|\sigma''(c) \cdot \frac{x^2}{2R\,\sigma'(t_{\sigma,id})} + x - x\right| \leq \frac{1}{2} \cdot \frac{\|\sigma''\|_\infty \cdot a^2}{\sigma'(t_{\sigma,id})} \cdot \frac{1}{R}$$

da $-a \leq x \leq a \Rightarrow a^2 \leq x^2 \leq a^2$ $\leftarrow$ ~~Absolute~~ Homogenität
Supremumsnorm $\|\sigma''(c)\| \leq \|\sigma''\|_\infty$