Lemma 2

L $\sigma : \mathbb{R} \to [0,1]$ be 2-admissible according to def. 2.

$\Rightarrow$ for any $R > 0$, $a > 0$ the nn

$$f_{mult}(x,y) = \frac{R^2}{4\sigma''(t_\sigma)} \cdot \left(\sigma\left(\frac{2(x+y)}{R} + t_\sigma\right) - 2\sigma\left(\frac{x+y}{R} + t_\sigma\right) - \sigma\left(\frac{2\cdot(x-y)}{R} + t_\sigma\right) + 2\cdot\sigma\left(\frac{x-y}{R} + t_\sigma\right)\right)$$

$\Rightarrow \forall x \in [-a, a]$

$$|f_{mult}(x,y) - x\cdot y| \le \frac{20\,\|\sigma'''\|_\infty \cdot a^3}{3\cdot|\sigma''(t_\sigma)|} \cdot \frac{1}{R}$$

Beweis:

f_{sq} aus Lemm 1 mit

$$|f_{sq}(x) - x^2| \le \frac{\frac{5}{3}\cdot\|\sigma'''\|_\infty \cdot a^3}{|\sigma''(t_\sigma)|} \cdot \frac{1}{R}$$

da

for $x \in [-2a, 2a]$

$$f_{mult}(x,y) = \frac{1}{4}\left(f_{sq}(x+y) - f_{sq}(x-y)\right)$$

$$x\cdot y = \frac{1}{4}\left((x+y)^2 - (x-y)^2\right)$$

$$|f_{mult}(x,y) - x\cdot y| = \frac{1}{4}\cdot\left|f_{sq}(x+y) - f_{sq}(x-y) - (x+y)^2 + (x-y)^2\right|$$

$$\overset{\Delta\text{'s Ungl.}}{\le} \frac{1}{4}\left|f_{sq}(x+y) - (x+y)^2\right| + \frac{1}{4}\left|(x-y)^2 - f_{sq}(x-y)\right|$$

$$\overset{\text{Lemma 1}}{\le} 2\cdot\frac{1}{4}\cdot\frac{40}{3}\ \ldots$$

da $-2a \le x \le 2a$

$\Rightarrow x^3 \le 8a^3$ (kommt aus Beweis von Lemma 1 b) & $8\cdot 5 = 40$