$\deg(\sigma)$ 1

$\deg(\sigma')$ 2

$\deg(\sigma'')$ 3

$(\sigma^3)' = 3\sigma^2\sigma'$

$\sigma''' = (\cancel{\ } a_1\sigma + a_2\sigma^2 + a_3\sigma^3)' = a_1\sigma' + 2a_2\sigma\sigma' + 3a_3\sigma^2\sigma'$

$P_1(x)$

$\vdots$

P_n

$N = \left\{ \begin{matrix} x_{1,1} & & \\ \vdots & & \\ \vdots & & \\ x_{n,1} & \dots & x_{n,n} \end{matrix} \right\} \cap (0,1)$

$\cancel{\ } (0,1) \setminus N \neq \emptyset$

$\cancel{\Rightarrow \exists x \in \mathbb{R} \quad \sigma(x) \notin}$

$\Rightarrow \exists y \in (0,1) \setminus N$

da $\mathrm{range}(\sigma) = (0,1)$ $\exists x \in \mathbb{R}: \sigma(x) = y$

$\Rightarrow$

$t_\sigma = x$

$\sigma^{\deg(\sigma^{(n-1)})+2}$

$\sigma^n = (\sigma^{(n-1)})' = \left(\sum_{k=1}^{\deg(\sigma^{(n-1)})} a_k\sigma^k\right)' = \sum_{k=1}^{\deg(\sigma^{(n-1)})} a_k k \sigma^{k-1}\sigma'$

$= \sum_{k=1}^{\deg(\sigma^{(n-1)})-1} \dots + a_{\deg(\sigma^{(n-1)})} \cdot \deg(\sigma^{(n-1)})$

$$\sigma^{(n)} = P_n(\sigma)$$

$$a_0 + a_1 X + a_2 X^2 + \dots$$

$P_n \neq 0 \quad n \in \mathbb{N}$

Sei $N = \deg(P_n)$

$P_n = a_0 + \dots + a_N \sigma^N$ mit $a_N \neq 0$.

$$\sigma^{(n+1)} = \left(\sigma^{(n)}\right)'$$

$$= \left(P_n(\sigma)\right)' = \left(\sum_{k=0}^{N} a_k \sigma^k\right)'$$

$$= \sum_{k=1}^{N} a_k \cdot k \, \sigma^{k-1} \sigma'$$

$$= \underbrace{\sum_{k=1}^{N-1} a_k \cdot k \, \sigma^{k-1} \sigma'}_{\deg \leq N} \underbrace{+ N a_N \sigma^{N-1} (1-\sigma)\sigma}_{\deg = N+1}$$