show that for a (p,C) smooth function m we have

$$\sup_{x\in[-a,a]^d} |m(x) - P(x)| \le c_1 \cdot \frac{1}{M^p}$$ wit $P(x)$ being a local convex comb. of TP of m.

$p = q + s \qquad 0 < s \le 1$

Beweis:

Nach dem Satz über Lagrangsche Form des Restglieds ex. ein $\xi \in [x, x_{i_k}]$, so dass

$$m(x) = T_{x_{i_k}, q-1}[m(x)] = \sum_{\substack{j_1,\dots,j_d \in \{0,\dots,q\} \\ j_1+\dots+j_d \le q-1}} \frac{1}{j_1!\cdots j_d!} \frac{\partial^{j_1+\dots+j_d} m(x_{i_k})}{\partial^{j_1} x^{(1)} \cdots \partial^{j_d} x^{(d)}} (x^{(1)} - x_{i_k}^{(1)})^{j_1} \cdots (x^{(d)} - x_{i_k}^{(d)})^{j_d}$$

$$+ \sum_{\substack{q \le j_1,\dots,j_d \le q \\ j_1+\dots+j_d \le q}} \frac{1}{j_1!\cdots j_d!} \cdot \frac{\partial^{j_1+\dots+j_d} m(\xi_{i_k})}{\partial^{j_1} x^{(1)} \cdots \partial^{j_d} x^{(d)}} \underbrace{(x^{(1)} - x_{i_k}^{(1)})^{j_1} \cdots (x^{(d)} - x_{i_k}^{(d)})^{j_d}}_{\le \frac{2a}{M}}$$

Δ's vgl. 8 da $m(x) = \sum_{k=1}^{(M+1)^d} m(x) \prod_{j=1}^{d} \left(1 - \frac{M}{2a} |x^{(j)} - x_{i_k}^{(j)}|\right)_+$

$$\Rightarrow \quad |m(x) - P(x)| \le \sum_{k=1}^{(M+1)^d} |m(x) - p_{i_k}| \prod_{j=1}^{d} \left(1 - \frac{M}{2a} |x^{(j)} - x_{i_k}^{(j)}|\right)_+$$

$$\le \left(\frac{2a}{M}\right)^q \| \underbrace{\xi - x_{i_k}}_{\le \frac{2a}{M}} \|^s \cdot C \underbrace{\sum_{k=1}^{(M+1)^d} \prod_{j=1}^{d} \left(1 - \frac{M}{2a} |x^{(j)} - x_{i_k}^{(j)}|\right)_+}_{=1}$$

fest.

$$\overset{q+s=p}{=} C \cdot \left(\frac{2a}{M}\right)^p$$

$\square$