

show that for a (p, C) smooth function m we have

$$\sup_{x \in [-a, a]^d} |m(x) - p(x)| \leq C \cdot \frac{1}{M^p}$$

with $p(x)$ being a local convex comb of $\mathbb{P}_{\text{of } m}$

$$p = q + s \quad 0 \leq s \leq 1$$

Beweis:

Nach dem Satz über Lagrange'sche Form der Restglied ex. ein $\xi \in [x, x_{ik}]$, so dass

$$m(x) = T_{x_{ik}, q}[m(x)] = \sum_{\substack{j_1, \dots, j_d \in \{0, \dots, q\} \\ j_1 + \dots + j_d \leq q-1}} \frac{1}{j_1! \dots j_d!} \frac{\partial^{j_1 + \dots + j_d} m(x_{ik})}{\partial x_1^{j_1} \dots \partial x_d^{j_d}} (x^{(1)} - x_{ik}^{(1)})^{j_1} \dots (x^{(d)} - x_{ik}^{(d)})^{j_d}$$

$$+ \sum_{\substack{j_1, \dots, j_d \in \{0, \dots, q\} \\ j_1 + \dots + j_d \leq q}} \frac{1}{j_1! \dots j_d!} \frac{\partial^{j_1 + \dots + j_d} m(\xi)}{\partial x_1^{j_1} \dots \partial x_d^{j_d}} (x^{(1)} - x_{ik}^{(1)})^{j_1} \dots (x^{(d)} - x_{ik}^{(d)})^{j_d} \leq \frac{2a}{M}$$

$$\text{Es gilt: } m(x) = \sum_{k=1}^{(M+1)^d} m(x_k) \prod_{j=1}^d \left(1 - \frac{M}{2a} |x^{(j)} - x_{ik}^{(j)}|\right)_+$$

$$\Rightarrow |m(x) - p(x)| \leq \sum_{k=1}^{(M+1)^d} |m(x_k) - p(x_k)| \prod_{j=1}^d \left(1 - \frac{M}{2a} |x^{(j)} - x_{ik}^{(j)}|\right)_+$$

$$\leq \left(\frac{2a}{M}\right)^q \cdot \underbrace{\left\| \sum_{k=1}^{(M+1)^d} m(x_k) \right\|}_{\leq \frac{2a}{M}} \cdot \underbrace{\left(\sum_{k=1}^{(M+1)^d} \prod_{j=1}^d \left(1 - \frac{M}{2a} |x^{(j)} - x_{ik}^{(j)}|\right)_+ \right)}_{=1}$$

$$q+s=p$$

$$= C \cdot \left(\frac{2a}{M}\right)^p$$

□

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