

proof Lemma 3:  $x \in [-a, a]$

$$77. f_{id}(x) \in [-2a, 2a]$$

$$|f_{id}(x)| = |f_{id}(x) - x + x| \leq \cancel{|f_{id}(x) - x|} + |x| \leq |f_{id}(x) - x| + |x|$$

$$\leq \frac{\|\sigma''\|_{\infty} \cdot a^2}{2 \cdot |\sigma'(\epsilon_{\sigma, id})|} \cdot \frac{1}{R} + |x|$$

$$\leq \frac{\|\sigma''\|_{\infty} \cdot a^2}{2 \cdot (\sigma'(\epsilon_{\sigma, id})) \cdot \|\sigma''\|_{\infty} \cdot a} + |x|$$

$$= a + |x|$$

$$\leq 2a$$

$$\Rightarrow |f_{id}(x)| \leq 2a \Leftrightarrow f_{id}(x) \leq 2a$$

$$-f_{id}(x) \leq 2a$$

$$\Leftrightarrow -2a \leq f_{id}(x) \leq 2a$$

$$\Leftrightarrow f_{id}(x) \in [-2a, 2a]$$