Vector Notation and Representation

Unit Vector Notation

Vectors can be represented in different notations:

- **Angle Bracket Notation**:
A vector $\vec{V}$ in two dimensions can be written as:

$$\vec{V} = \langle V_x, V_y \rangle$$

- **Column Vector Notation**:

$$\vec{V} = \begin{bmatrix} V_x \\ V_y \end{bmatrix}$$

- **Cartesian Coordinates**:
Using the unit vectors $\hat{i}$ and $\hat{j}$:

$$\vec{V} = V_x\hat{i} + V_y\hat{j}$$

Where:
- V_x is the component of the vector along the x-axis. - V_y is the component of the vector along the y-axis. - $\hat{i}$ and $\hat{j}$ are unit vectors in the x and y directions, respectively.

Zero-Length and Parallel Vectors

- **Zero-Length Vector (Zero Vector)**:
A vector with zero magnitude and no specific direction.

$$\vec{0} = \langle 0, 0 \rangle$$

Properties:
- Adding the zero vector to any vector leaves the original vector unchanged.

$$\vec{V} + \vec{0} = \vec{V}$$

- **Parallel Vectors**:
Two vectors are parallel if they have the same or opposite directions.
Conditions for Parallelism:
- Vectors $\vec{A}$ and $\vec{B}$ are parallel if:

$$\vec{A} = k\vec{B}$$

where k is a scalar constant.
- If $k > 0$, vectors are in the same direction. - If $k < 0$, vectors are in opposite directions.
- **Component Proportionality:**

$$\frac{A_x}{B_x} = \frac{A_y}{B_y}$$

If the ratios of the corresponding components are equal (and finite), the vectors are parallel.

Position Vectors and Vector Difference

If A and B are points in the Cartesian plane with position vectors $\vec{a}$ and $\vec{b}$ respectively, then the vector from point A to point B is given by:

$$\vec{AB} = \vec{b} - \vec{a}$$

This represents the displacement from point A to point B.
Example:
Let point A have coordinates $(2, 3)$ and point B have coordinates $(5, 7)$.
- Write the position vectors:

$$\vec{a} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

- Compute $\vec{AB}$:

$$\vec{AB} = \vec{b} - \vec{a} = \begin{bmatrix} 5 \\ 7 \end{bmatrix} - \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 5-2 \\ 7-3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

- The vector from A to B is $\vec{AB} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$.

- The magnitude of $\vec{AB}$ is:

$$|\vec{AB}| = \sqrt{(3)^2 + (4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

- The direction (angle) of $\vec{AB}$ with respect to the horizontal axis is:

$$\theta = \arctan\left(\frac{4}{3}\right) \approx 53.13^\circ$$

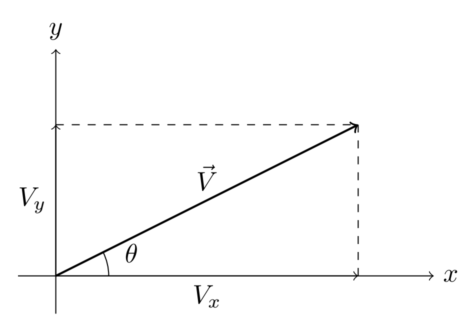
Vector Decomposition and Addition

Decomposing Vectors into Components

Any vector $\vec{V}$ can be decomposed into its horizontal and vertical components using trigonometric functions.

Given a vector of magnitude V at an angle θ with respect to the horizontal axis:

$$V_x = V\cos\theta$$
$$V_y = V\sin\theta$$



Example: A vector has a magnitude of $V = 5$ units and makes an angle of $\theta = 30^\circ$ with the horizontal axis. Find its components.

$$V_x = V\cos\theta = 5\cos 30^\circ = 5 \times \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{2} \approx 4.33$$
$$V_y = V\sin\theta = 5\sin 30^\circ = 5 \times \frac{1}{2} = \frac{5}{2} = 2.5$$

Adding Vectors Using Components

To add two vectors, decompose each vector into its components, add the corresponding components, and then recombine the resultant components to find the resultant vector.

Given two vectors $\vec{A}$ and $\vec{B}$:

$$\begin{aligned} A_x &= A\cos\alpha \\ A_y &= A\sin\alpha \\ B_x &= B\cos\beta \\ B_y &= B\sin\beta \end{aligned}$$

The resultant vector $\vec{R} = \vec{A} + \vec{B}$ has components:

$$\begin{aligned} R_x &= A_x + B_x \\ R_y &= A_y + B_y \end{aligned}$$

The magnitude and direction of $\vec{R}$ are:

$$\begin{aligned} R &= \sqrt{R_x^2 + R_y^2} \\ \theta_R &= \arctan\left(\frac{R_y}{R_x}\right) \end{aligned}$$

Example: Add vector $\vec{A}$ of magnitude $A = 10$ units at $\alpha = 45°$ and vector $\vec{B}$ of magnitude $B = 8$ units at $\beta = 120°$.

Step 1: Decompose vectors into components.

$$\begin{aligned} A_x &= 10\cos 45° = 10 \times \frac{\sqrt{2}}{2} = 5\sqrt{2} \approx 7.07 \\ A_y &= 10\sin 45° = 10 \times \frac{\sqrt{2}}{2} = 5\sqrt{2} \approx 7.07 \\ B_x &= 8\cos 120° = 8 \times \left(-\frac{1}{2}\right) = -4 \\ B_y &= 8\sin 120° = 8 \times \frac{\sqrt{3}}{2} = 4\sqrt{3} \approx 6.93 \end{aligned}$$

Step 2: Add components.

$$\begin{aligned} R_x &= A_x + B_x = 7.07 + (-4) = 3.07 \\ R_y &= A_y + B_y = 7.07 + 6.93 = 14 \end{aligned}$$

Step 3: Calculate resultant magnitude and direction.

$$R = \sqrt{(R_x)^2 + (R_y)^2} = \sqrt{(3.07)^2 + (14)^2} \approx \sqrt{9.42 + 196} \approx \sqrt{205.42}$$

$$\theta_R = \arctan\left(\frac{14}{3.07}\right) \approx 77.5°$$

So, the resultant vector has a magnitude of approximately 14.33 units and makes an angle of 77.5° with the horizontal axis.

Summary

- Decompose vectors into their x and y components using cos and sin functions. - Add the corresponding components to find the resultant components. - Calculate the magnitude and direction of the resultant vector from its components.