Trigonometry
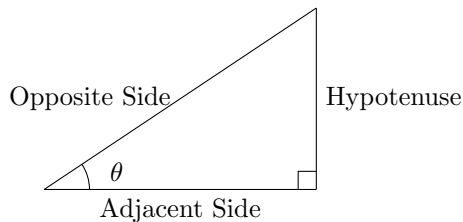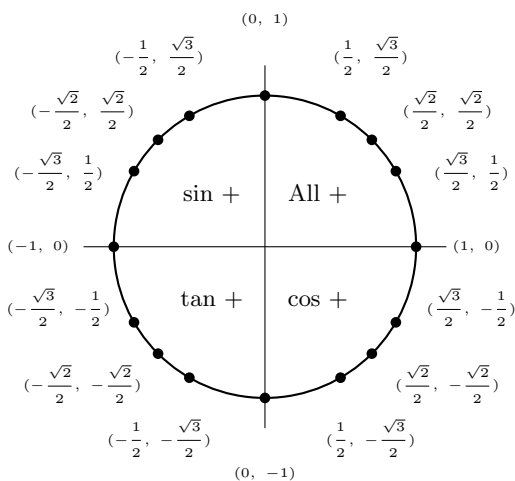

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}, \quad \cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}, \quad \tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \quad \csc\theta = \frac{1}{\sin\theta}, \quad \sec\theta = \frac{1}{\cos\theta}, \quad \cot\theta = \frac{1}{\tan\theta}$$

Unit Circle Diagram

The cosine is the x-value, the sine is the y-value

Radians to Degrees Conversion

$$180° = \pi \text{ radians}, \quad 1° = \frac{\pi}{180} \text{ radians}$$

For common angles, sine and cosine values can be calculated as:

$$\sin\theta = \frac{\sqrt{n}}{2}, \quad \cos\theta = \frac{\sqrt{4-n}}{2}$$

where n corresponds to the following angles:

$$
\begin{aligned}
n &= 0 \quad \text{for } \theta = 0°,\\
n &= 1 \quad \text{for } \theta = 30°,\\
n &= 2 \quad \text{for } \theta = 45°,\\
n &= 3 \quad \text{for } \theta = 60°,\\
n &= 4 \quad \text{for } \theta = 90°.
\end{aligned}
$$

Pythagorean Identity

$$\sin^2\theta + \cos^2\theta = 1$$

Example Calculation: If $\sin\theta = \frac{3}{5}$, then:

$$\cos\theta = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \frac{4}{5}$$
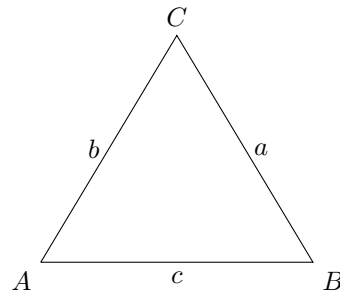
Addition Formulas

Sine Addition and Subtraction Formulas

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

Cosine Addition and Subtraction Formulas

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Inverse Formulation:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Example Calculation: If $a = 7$, $A = 30°$, and $B = 45°$, find b:

$$
\begin{aligned}
\frac{a}{\sin A} &= \frac{b}{\sin B}\\
b &= a \times \frac{\sin B}{\sin A}\\
&= 7 \times \frac{\sin 45°}{\sin 30°}\\
&= 7 \times \frac{\frac{\sqrt{2}}{2}}{\frac{1}{2}}\\
&= 7 \times \sqrt{2} \approx 9.9
\end{aligned}
$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Example Calculation: Given $b = 5$, $c = 7$, and $A = 60°$:

$$
\begin{aligned}
a^2 &= 5^2 + 7^2 - 2 \times 5 \times 7 \times \cos 60°\\
&= 25 + 49 - 70 \times \left(\frac{1}{2}\right)\\
&= 74 - 35\\
&= 39\\
a &= \sqrt{39} \approx 6.24
\end{aligned}
$$

Area of a General Triangle

$$\text{Area} = \frac{1}{2}ab\sin C$$

Triangle Inequality

For any triangle with sides a, b, and c:
$$a < b + c$$
$$b < a + c$$
$$c < a + b$$

Example: If $a = 3$, $b = 4$, then c must satisfy:

$$c < 3 + 4 \implies c < 7$$
$$c > |3 - 4| \implies c > 1$$

For any triangle with sides a, b, and c:
$$a < b + c$$
$$b < a + c$$
$$c < a + b$$

$$c < 3 + 4 \implies c < 7$$
$$c > |3 - 4| \implies c > 1$$