Summations and Series

Summation Notation

The summation symbol Σ represents the sum of a sequence:

$$\sum_{i=m}^{n} a_i = a_m + a_{m+1} + \cdots + a_n$$

Example: Compute $\sum_{i=1}^{4} i$:

$$\sum_{i=1}^{4} i = 1 + 2 + 3 + 4 = 10$$

Arithmetic Series

An arithmetic series is the sum of terms in an arithmetic sequence, with a constant difference d.

General Form:

$$S_n = \frac{n}{2}(a + l) = \frac{n}{2}(2a + (n-1)d)$$

where:

- a: First term
- l: Last term
- d: Common difference
- n: Number of terms

Example: Sum of first 10 terms of $2, 5, 8, \ldots$:

$$a = 2, d = 3, n = 10$$
$$S_n = \frac{10}{2}(4 + 27) = 155$$

Geometric Series

A geometric series is the sum of terms in a geometric sequence, with a constant ratio r.

Finite Sum:

$$S_n = a\frac{1 - r^n}{1 - r}, \quad r \neq 1$$

Infinite Sum: If $|r| < 1$:

$$S = \frac{a}{1 - r}$$

General term:

$$a_n = ar^{n-1}$$

Example: Sum of first 5 terms of $3, 6, 12, \ldots$:

$$a = 3, r = 2, n = 5$$
$$S_n = 3\frac{1 - 32}{1 - 2} = 93$$

Partial Sums of Infinite Series

The partial sum S_N is the sum of the first N terms:

$$S_N = \sum_{n=1}^{N} a_n$$

For geometric series:

$$S_N = a\frac{1 - r^N}{1 - r}, |r| < 1$$

As $N \to \infty$, the sum approaches:

$$S = \frac{a}{1 - r}$$

Example: Sum of first 4 terms of $1, \frac{1}{2}, \frac{1}{4}, \ldots$:

$$a = 1, r = \frac{1}{2}, N = 4$$
$$S_N = \frac{1 - (\frac{1}{2})^4}{1 - \frac{1}{2}} = \frac{15}{8}$$

Conversions and Other Formulas

Converting Recursive to Explicit Form

Arithmetic:

$$a_n = a + (n-1)d$$

Geometric:

$$a_n = ar^{n-1}$$

Product Series Notation

The product notation Π represents the product of terms:

$$\prod_{i=m}^{n} a_i = a_m \times a_{m+1} \times \cdots \times a_n$$

Example: Compute $\prod_{i=1}^{4} i$:

$$\prod_{i=1}^{4} i = 1 \times 2 \times 3 \times 4 = 24$$

Infinite Product

An infinite product converges if the limit exists:

$$\prod_{i=1}^{\infty} a_i = \lim_{n\to\infty} \prod_{i=1}^{n} a_i$$

Example: Evaluate the infinite product $\prod_{n=1}^{\infty} \frac{1}{2^n}$:

$$\prod_{n=1}^{\infty} \frac{1}{2} = \lim_{n\to\infty} \frac{1}{2^n} = 0$$