Time Dilation in Special Relativity

Lorentz Factor

The Lorentz factor (γ) describes how time dilates for an object moving at velocity v relative to an observer:

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

where: - v = velocity of the moving object, - c = speed of light ($c = 299,792,458\,\mathrm{m/s}$).

Time Dilation Formula

The relationship between time intervals measured in different frames:

$$\Delta t = \gamma \Delta t'$$

- $\Delta t'$ = time interval measured in the **moving frame** (proper time), - Δt = time interval measured in the **stationary frame**.

Time Difference

The time difference ($\Delta\tau$) between the two frames:

$$\Delta\tau = \Delta t - \Delta t' = \Delta t'(\gamma - 1)$$

Calculations

We will calculate the Lorentz factor and time difference for velocities ranging from 0 to c.

Example Velocities

Let's consider the following velocities:

$$v = 0.1c,\ 0.5c,\ 0.9c,\ 0.99c,\ 0.999c$$

Calculations for Each Velocity

1. Calculate γ:

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

2. Assume $\Delta t' = 1\,\mathrm{s}$, calculate Δt and $\Delta\tau$:

$$\Delta t = \gamma \Delta t'$$

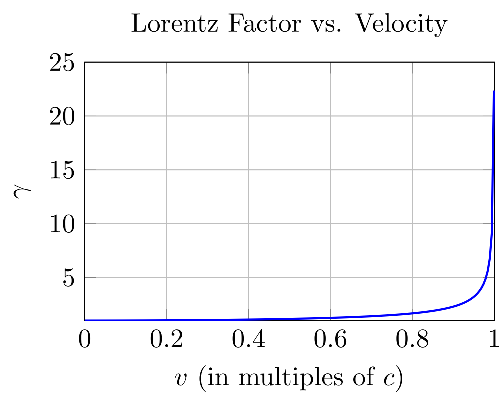
$$\Delta\tau = \Delta t - \Delta t' = (\gamma - 1)\Delta t'$$

Tabulated Results

v	v/c	γ	Δt (s)	$\Delta\tau$ (s)
$0.1c$	0.1	1.0050	1.0050	0.0050
$0.5c$	0.5	1.1547	1.1547	0.1547
$0.9c$	0.9	2.2942	2.2942	1.2942
$0.99c$	0.99	7.0888	7.0888	6.0888
$0.999c$	0.999	22.3663	22.3663	21.3663

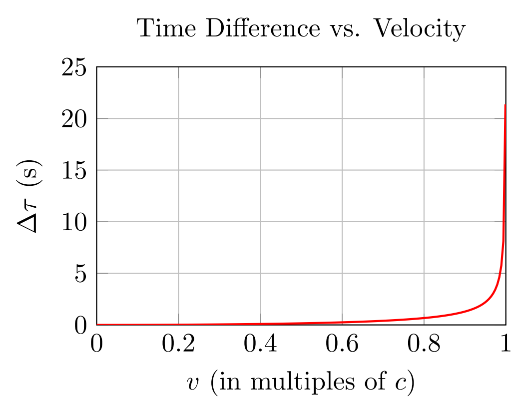
Plots of Lorentz Factor and Time Difference

Plot of v vs. γ



Plot of v vs. $\Delta\tau$

Assuming $\Delta t' = 1\,\mathrm{s}$.



Energy Requirements

Relativistic Kinetic Energy

The kinetic energy (KE) required to accelerate an object to a relativistic speed is:

$$KE = (\gamma - 1)mc^2$$

where: - m = rest mass of the object, - γ = Lorentz factor, - c = speed of light.

Calculations for the DeLorean

Assuming the DeLorean has a mass $m = 1,230\,\text{kg}$.

Calculations at Various Speeds

1. $v = 0.1c$: $\gamma = 1.0050$
2. $v = 0.5c$: $\gamma = 1.1547$
3. $v = 0.9c$: $\gamma = 2.2942$
4. $v = 0.99c$: $\gamma = 7.0888$
5. $v = 0.999c$: $\gamma = 22.3663$

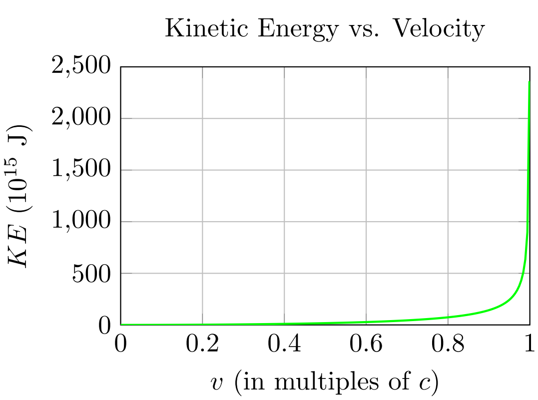
Calculate KE for each velocity:

$$KE = (\gamma - 1)mc^2$$

Tabulated Results

v	γ	KE (J)	KE (10^{16} J)
$0.1c$	1.0050	3.328×10^{16}	0.3328
$0.5c$	1.1547	1.066×10^{18}	10.66
$0.9c$	2.2942	2.205×10^{19}	220.5
$0.99c$	7.0888	6.583×10^{19}	658.3
$0.999c$	22.3663	2.078×10^{20}	2078

Plot of v vs. KE



Energy from Hydrogen Fusion

Energy Released per Fusion Reaction:

In the proton-proton chain reaction (dominant in stars like the Sun), four protons fuse to form a helium nucleus, releasing about 26.7 MeV per reaction.

$$E_{\text{fusion}} = 26.7\,\text{MeV} = 4.28 \times 10^{-12}\,\text{J}$$

Energy per Kilogram of Hydrogen:

Mass of four protons:

$$m_{\text{protons}} = 4 \times 1.6726 \times 10^{-27}\,\text{kg} = 6.6904 \times 10^{-27}\,\text{kg}$$

Number of reactions per kg of hydrogen:

$$N = \frac{1\,\text{kg}}{6.6904 \times 10^{-27}\,\text{kg}} \approx 1.495 \times 10^{26}$$

Total energy per kg of hydrogen:

$$E_{\text{total}} = N \times E_{\text{fusion}} = (1.495 \times 10^{26}) \times (4.28 \times 10^{-12}\,\text{J}) \approx 6.4 \times 10^{14}\,\text{J}$$

Calculating Mass of Hydrogen Needed

For $v = 0.9c$:

Energy required:

$$KE = 2.205 \times 10^{19}\,\text{J}$$

Mass of hydrogen needed:

$$m_{\text{H}} = \frac{KE}{E_{\text{total per kg}}} = \frac{2.205 \times 10^{19}\,\text{J}}{6.4 \times 10^{14}\,\text{J/kg}} \approx 34,453\,\text{kg}$$

For $v = 0.99c$:

Energy required:

$$KE = 6.583 \times 10^{19}\,\text{J}$$

Mass of hydrogen needed:

$$m_{\text{H}} = \frac{6.583 \times 10^{19}\,\text{J}}{6.4 \times 10^{14}\,\text{J/kg}} \approx 102,859\,\text{kg}$$

For $v = 0.999c$:

Energy required:

$$KE = 2.078 \times 10^{20}\,\text{J}$$

Mass of hydrogen needed:

$$m_{\text{H}} = \frac{2.078 \times 10^{20}\,\text{J}}{6.4 \times 10^{14}\,\text{J/kg}} \approx 324,688\,\text{kg}$$

Summary Table

v	KE (10^{19} J)	m_{H} (kg)	m_{H} (tonnes)
$0.9c$	2.205	34,453	34.5
$0.99c$	6.583	102,859	102.9
$0.999c$	20.78	324,688	324.7

Interpretation

- Accelerating the DeLorean to relativistic speeds requires immense amounts of energy. - Even with efficient hydrogen fusion, large masses of hydrogen would be needed. - For $v = 0.999c$, over 324 tonnes of hydrogen would be required.

Key Equations Summary

Lorentz Factor

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

Time Dilation

$$\Delta t = \gamma \Delta t'$$

Time Difference

$$\Delta \tau = \Delta t - \Delta t' = \Delta t'(\gamma - 1)$$

Relativistic Kinetic Energy

$$KE = (\gamma - 1)mc^2$$

Energy from Fusion

$$E_{\text{fusion per kg}} \approx 6.4 \times 10^{14} \text{ J/kg}$$