Polynomials Cheat Sheet

Factoring Quadratics

Standard Form

A quadratic equation in standard form:

$$ax^2 + bx + c = 0$$

Quadratic Formula

The solutions (roots) of the quadratic equation are given by the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant:

$$D = b^2 - 4ac$$

- If $D > 0$, there are two real and distinct roots. - If $D = 0$, there is one real root (a repeated root). - If $D < 0$, there are two complex conjugate roots.

Example:
Solve $x^2 - 4x + 3 = 0$.
Compute the discriminant:

$$D = (-4)^2 - 4(1)(3) = 16 - 12 = 4$$

Since $D > 0$, there are two real roots.

$$x = \frac{-(-4) \pm \sqrt{4}}{2(1)} = \frac{4 \pm 2}{2}$$

Thus, $x = 3$ or $x = 1$.

Difference and Sum of Squares

- **Difference of Squares**:

$$a^2 - b^2 = (a + b)(a - b)$$

Example:

$$x^2 - 9 = (x + 3)(x - 3)$$

- **Sum of Squares**:
Note: $a^2 + b^2$ cannot be factored over real numbers.

Perfect Square Trinomials

- **Perfect Square Trinomial**:

$$a^2 \pm 2ab + b^2 = (a \pm b)^2$$

Examples:

$$x^2 + 6x + 9 = (x + 3)^2$$

$$x^2 - 10x + 25 = (x - 5)^2$$

Factoring Quadratics with Leading Coefficient

When $a \neq 1$, factor using methods such as:

- Trial and Error - AC Method

AC Method Steps:

1. Multiply a and c to get ac. 2. Find two numbers m and n such that $m \times n = ac$ and $m + n = b$. 3. Rewrite the quadratic as $ax^2 + mx + nx + c$. 4. Factor by grouping.

Example:

Factor $2x^2 + 7x + 3$.

1. $a = 2$, $b = 7$, $c = 3$. 2. $ac = 2 \times 3 = 6$. 3. Find $m = 6$, $n = 1$ (since $6 \times 1 = 6$ and $6 + 1 = 7$). 4. Rewrite: $2x^2 + 6x + x + 3$. 5. Factor by grouping:

$$(2x^2 + 6x) + (x + 3)$$
$$2x(x + 3) + 1(x + 3)$$
$$(2x + 1)(x + 3)$$

Factoring by Grouping

Used when there are four terms.

Example:

Factor $x^3 + 3x^2 + 2x + 6$.

1. Group terms: $(x^3 + 3x^2) + (2x + 6)$. 2. Factor out GCF from each group:

$$x^2(x + 3) + 2(x + 3)$$
$$(x^2 + 2)(x + 3)$$

Completing the Square

Used to solve quadratics or rewrite them in vertex form.

Steps:

1. Ensure $a = 1$. If not, divide both sides by a. 2. Move c to the other side. 3. Add $\left(\frac{b}{2}\right)^2$ to both sides. 4. Factor the perfect square trinomial.

Example:

Complete the square for $x^2 + 6x + 5$.

1. $a = 1$. 2. Move c: $x^2 + 6x = -5$. 3. Add $\left(\frac{6}{2}\right)^2 = 9$:

$$x^2 + 6x + 9 = -5 + 9(x + 3)^2 = 4$$

Decomposing Higher-Order Polynomials

Synthetic Division

Used to divide polynomials by binomials of the form $x - r$.

Steps:

1. Write the coefficients. 2. Bring down the leading coefficient. 3. Multiply by r and add to next coefficient. 4. Repeat.

Example:

Divide $f(x) = 2x^3 - 3x^2 + 4x - 5$ by $x - 2$.

Set up synthetic division with $r = 2$:

$$\begin{array}{c|cccc} 2 & 2 & -3 & 4 & -5 \\ & & 4 & 2 & 12 \\ \hline & 2 & 1 & 6 & 7 \end{array}$$

So, the quotient is $2x^2 + x + 6$ with a remainder of 7:

$$f(x) = (x - 2)(2x^2 + x + 6) + 7$$

Since the remainder is not zero, $x - 2$ is not a factor, and the term $\frac{7}{x-2}$ remains in the expression.

Note: If the remainder is zero, $x - r$ is a factor of the polynomial.

Asymptotes

Vertical Asymptotes

Occur where the denominator equals zero (for rational functions), and the numerator does not equal zero.

Example:

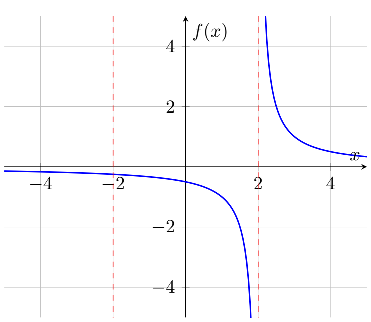
Find vertical asymptotes of $f(x) = \frac{x+2}{x^2-4}$.

Set denominator to zero:

$$x^2 - 4 = 0 \implies x = \pm 2$$

Vertical asymptotes at $x = -2$ and $x = 2$.

Graph:



Horizontal Asymptotes

Determined by the degrees of the numerator and denominator polynomials.

- **Degree of numerator < degree of denominator**: Horizontal asymptote at $y = 0$.
- **Degrees equal**: Horizontal asymptote at $y = \frac{\text{leading coefficient of numerator}}{\text{leading coefficient of denominator}}$.
- **Degree of numerator > degree of denominator**: No horizontal asymptote; may have an oblique (slant) asymptote.

Example:

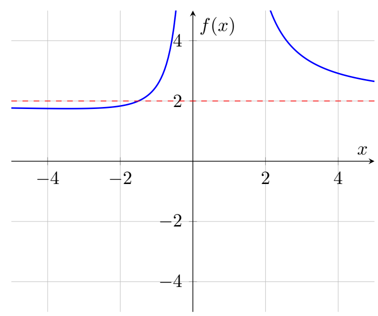
Find horizontal asymptote of $f(x) = \frac{2x^2+3}{x^2-x}$.

Degrees are equal ($n = 2$), so:

$$y = \frac{2}{1} = 2$$

Horizontal asymptote at $y = 2$.

Graph:



Identifying Cubic Graphs Based on Roots

For a cubic polynomial $f(x) = a(x - r_1)(x - r_2)(x - r_3)$:

- The graph crosses the x-axis at roots r_1, r_2, r_3. - The behavior at each root depends on multiplicity.

Example:

Given $f(x) = (x + 2)(x - 1)^2$:

- Roots at $x = -2$ (multiplicity 1) and $x = 1$ (multiplicity 2). - At $x = -2$, the graph crosses the x-axis. - At $x = 1$, the graph touches and turns around (due to multiplicity 2).

Graph: