Differentiation

Definition

The derivative of a function $f(x)$ at a point x is defined as:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Power Rule

$$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$$

Example:

$$f(x) = x^3 \Rightarrow f'(x) = 3x^2$$

Constant Rule

$$f(x) = c \Rightarrow f'(x) = 0$$

Example:

$$f(x) = 2 \Rightarrow f'(x) = 0$$

Basic Derivatives

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}\ln x = \frac{1}{x} \quad \text{for } x > 0$$

Derivative of Exponential Functions

For $f(x) = a^x$, where $a > 0$ and $a \neq 1$:

$$\frac{d}{dx}a^x = a^x \ln a$$

Example:

$$\frac{d}{dx}2^x = 2^x \ln 2$$

Derivative of Logarithmic Functions

For $f(x) = \log_a x$, where $a > 0$ and $a \neq 1$:

$$\frac{d}{dx}\log_a x = \frac{1}{x \ln a}$$

Example:

$$\frac{d}{dx}\log_2 x = \frac{1}{x \ln 2}$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\csc x = -\csc x \cot x$$

Example:

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

Product Rule

If $f(x) = u(x) \cdot v(x)$:

$$f'(x) = u'v + uv'$$

Quotient Rule

If $f(x) = \frac{u(x)}{v(x)}$:

$$f'(x) = \frac{u'v - uv'}{v^2}$$

Chain Rule

If $f(x) = h(g(x))$, then:

$$f'(x) = h'(g(x)) \cdot g'(x)$$

Example:

$$f(x) = (2x^3 + x)^4$$

Then:

$$f'(x) = 4(2x^3 + x)^3 \cdot (6x^2 + 1)$$

Integration

Definition of Integration

The definite integral of a function $f(x)$ over the interval $[a, b]$ is defined as the limit of a Riemann sum:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

where: - $\Delta x = \frac{b-a}{n}$ is the width of each subinterval, - x_i^* is a sample point in the i-th subinterval $[x_{i-1}, x_i]$.

The integral represents the accumulation of the quantity $f(x)$ over the interval $[a, b]$.

Alternatively, the indefinite integral (antiderivative) of a function $f(x)$ is a function $F(x)$ such that:

$$\int f(x)\,dx = F(x) + C$$

where: - $F'(x) = f(x)$, - C is the constant of integration.

Power Rule

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (\text{for } n \neq -1)$$

Constant Rule

$$\int c\,dx = cx + C$$

Example:

$$\int 5x^2\,dx = 5\int x^2\,dx = 5 \cdot \frac{x^3}{3} = \frac{5x^3}{3} + C$$

Sum Rule

$$\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$

Basic Integrals

$$\int e^x\,dx = e^x + C$$

$$\int a^x\,dx = \frac{a^x}{\ln a} + C \quad (\text{for } a > 0,\ a \neq 1)$$

Example:

$$\int 2^x\,dx = \frac{2^x}{\ln 2} + C$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int \ln x\,dx = x\ln x - x + C$$

$$\int \log_a x\,dx = x\log_a x - \frac{x}{\ln a} + C$$

Example:

$$\int \log_2 x\,dx = x\log_2 x - \frac{x}{\ln 2} + C$$

Integrals of Trigonometric Functions

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$

$$\int \tan x\,dx = -\ln|\cos x| + C$$

$$\int \cot x\,dx = \ln|\sin x| + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C$$

$$\int \sec^2 x\,dx = \tan x + C$$

$$\int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\int \csc x \cot x\,dx = -\csc x + C$$

Example Calculations

1.
$$\int x^2\,dx = \frac{x^3}{3} + C$$

2.
$$\int 3^x\,dx = \frac{3^x}{\ln 3} + C$$

3.
$$\int \ln x\,dx = x\ln x - x + C$$

4.
$$\int \frac{1}{x\ln a}\,dx = \log_a x + C$$

5.
$$\int \sec^2 x\,dx = \tan x + C$$

6.
$$\int \tan x\,dx = -\ln|\cos x| + C$$