Time Dilation in Special Relativity

Lorentz Factor

The Lorentz factor (γ) describes how time dilates for an object moving at velocity v relative to an observer:

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

where: - v = velocity of the moving object, - c = speed of light ($c = 299{,}792{,}458\,\mathrm{m/s}$).

Time Dilation Formula

The relationship between time intervals measured in different frames:

$$\Delta t = \gamma \Delta t'$$

- $\Delta t'$ = time interval measured in the **moving frame** (also known as *proper time*), - Δt = time interval measured in the **stationary frame**.

Deriving Time Difference

To find the time difference ($\Delta\tau$) between the two frames:

$$\Delta\tau = \Delta t - \Delta t' = \gamma\Delta t' - \Delta t' = \Delta t'(\gamma - 1)$$

Solving for Time in Moving Frame

Given a desired time difference ($\Delta\tau$):

$$\Delta t' = \frac{\Delta\tau}{\gamma - 1}$$

Calculations at Various Speeds

Case 1: $v = 80\%\, c$

Calculate Lorentz Factor:

For $v = 0.8c$:

$$\gamma = \frac{1}{\sqrt{1 - (0.8)^2}} = \frac{1}{\sqrt{0.36}} = \frac{1}{0.6} \approx 1.6667$$

Find Time in Moving Frame for $\Delta\tau = 60\,\mathrm{s}$:

$$\Delta t' = \frac{60\,\mathrm{s}}{1.6667 - 1} = \frac{60\,\mathrm{s}}{0.6667} \approx 90\,\mathrm{s}$$

Calculate Time in Stationary Frame:

$$\Delta t = \gamma\Delta t' = 1.6667 \times 90\,\mathrm{s} = 150\,\mathrm{s}$$

Verify Time Difference:

$$\Delta\tau = \Delta t - \Delta t' = 150\,\mathrm{s} - 90\,\mathrm{s} = 60\,\mathrm{s}$$

Case 2: $v = 90\%\, c$

Calculate Lorentz Factor:

For $v = 0.9c$:

$$\gamma = \frac{1}{\sqrt{1 - (0.9)^2}} = \frac{1}{\sqrt{0.19}} \approx 2.2942$$

Find Time in Moving Frame for $\Delta\tau = 60\,\mathrm{s}$:

$$\Delta t' = \frac{60\,\mathrm{s}}{2.2942 - 1} \approx 46.3615\,\mathrm{s}$$

Calculate Time in Stationary Frame:

$$\Delta t = \gamma\Delta t' \approx 2.2942 \times 46.3615\,\mathrm{s} \approx 106.3615\,\mathrm{s}$$

Verify Time Difference:

$$\Delta\tau = \Delta t - \Delta t' = 106.3615\,\mathrm{s} - 46.3615\,\mathrm{s} = 60\,\mathrm{s}$$

Case 3: $v = 99\%\, c$

Calculate Lorentz Factor:

For $v = 0.99c$:

$$\gamma = \frac{1}{\sqrt{1 - (0.99)^2}} \approx 7.0888$$

Find Time in Moving Frame for $\Delta\tau = 60\,\mathrm{s}$:

$$\Delta t' = \frac{60\,\mathrm{s}}{7.0888 - 1} \approx 9.8581\,\mathrm{s}$$

Calculate Time in Stationary Frame:

$$\Delta t = \gamma\Delta t' \approx 7.0888 \times 9.8581\,\mathrm{s} \approx 69.8581\,\mathrm{s}$$

Verify Time Difference:

$$\Delta\tau = \Delta t - \Delta t' = 69.8581\,\mathrm{s} - 9.8581\,\mathrm{s} = 60\,\mathrm{s}$$

Energy Requirements

Relativistic Kinetic Energy

The kinetic energy (KE) required to accelerate an object to a relativistic speed is:

$$KE = (\gamma - 1)mc^2$$

where: - m = rest mass of the object, - γ = Lorentz factor, - c = speed of light.

Calculating Energy for the DeLorean

Assuming the DeLorean has a mass $m = 1{,}230\,\text{kg}$ (approximate mass of a DeLorean car).

Case 1: $v = 80\%\,c$

Calculate Lorentz Factor:
Already calculated: $\gamma \approx 1.6667$
Calculate Kinetic Energy:

$$KE = (\gamma - 1)mc^2 = (1.6667 - 1) \times 1{,}230\,\text{kg} \times (299{,}792{,}458\,\text{m/s})^2$$

$$KE \approx 0.6667 \times 1{,}230\,\text{kg} \times (8.9875 \times 10^{16}\,\text{m}^2/\text{s}^2)$$

$$KE \approx 0.6667 \times 1{,}230\,\text{kg} \times 8.9875 \times 10^{16}\,\text{J/kg}$$

$$KE \approx 0.6667 \times 1.1065 \times 10^{20}\,\text{J} \approx 7.3765 \times 10^{19}\,\text{J}$$

Result:
Approximately 7.38×10^{19} joules of energy are required.

Case 2: $v = 90\%\,c$

Calculate Lorentz Factor:
Already calculated: $\gamma \approx 2.2942$
Calculate Kinetic Energy:

$$KE = (2.2942 - 1) \times 1{,}230\,\text{kg} \times c^2$$

$$KE \approx 1.2942 \times 1{,}230\,\text{kg} \times 8.9875 \times 10^{16}\,\text{J/kg}$$

$$KE \approx 1.2942 \times 1.1065 \times 10^{20}\,\text{J} \approx 1.4323 \times 10^{20}\,\text{J}$$

Result:
Approximately 1.43×10^{20} joules of energy are required.

Case 3: $v = 99\%\,c$

Calculate Lorentz Factor:
Already calculated: $\gamma \approx 7.0888$
Calculate Kinetic Energy:

$$KE = (7.0888 - 1) \times 1{,}230\,\text{kg} \times c^2$$

$$KE \approx 6.0888 \times 1{,}230\,\text{kg} \times 8.9875 \times 10^{16}\,\text{J/kg}$$

$$KE \approx 6.0888 \times 1.1065 \times 10^{20}\,\text{J} \approx 6.7389 \times 10^{20}\,\text{J}$$

Result:
Approximately 6.74×10^{20} joules of energy are required.

Understanding the Energy Scale

For perspective:
- The total annual energy consumption of the entire world is about 5×10^{20} joules. - Accelerating the DeLorean to $99\%\,c$ requires more energy than the world's annual energy consumption.

Instructions to Calculate Energy Required

To calculate the energy required to accelerate an object to a given velocity:

1. **Calculate Lorentz Factor** (γ):

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

2. **Compute Kinetic Energy** (KE):

$$KE = (\gamma - 1)mc^2$$

3. **Plug in the Mass and Speed of Light**:
   - m = mass of the object (in kg),
   - $c = 299{,}792{,}458\,\text{m/s}$.
4. **Calculate and Interpret the Result**:
   - The result is in joules (J).
   - Compare with known energy scales for perspective.

Example Calculation

Given: - $v = 95\%\,c$ - $m = 1{,}230\,\text{kg}$
Step 1: Calculate Lorentz Factor

$$\gamma = \frac{1}{\sqrt{1 - (0.95)^2}} \approx 3.2026$$

Step 2: Compute Kinetic Energy

$$KE = (3.2026 - 1) \times 1{,}230\,\text{kg} \times c^2$$

$$KE \approx 2.2026 \times 1{,}230\,\text{kg} \times 8.9875 \times 10^{16}\,\text{J/kg}$$

$$KE \approx 2.2026 \times 1.1065 \times 10^{20}\,\text{J} \approx 2.4361 \times 10^{20}\,\text{J}$$

Result:
Approximately 2.44×10^{20} joules of energy are required.

Key Equations Summary

Lorentz Factor

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

Time Dilation

$$\Delta t = \gamma \Delta t'$$

Time Difference

$$\Delta \tau = \Delta t - \Delta t' = \Delta t'(\gamma - 1)$$

Solving for Time in Moving Frame

$$\Delta t' = \frac{\Delta \tau}{\gamma - 1}$$

Relativistic Kinetic Energy

$$KE = (\gamma - 1)mc^2$$