

Problem 1.

For the context given by the following table:

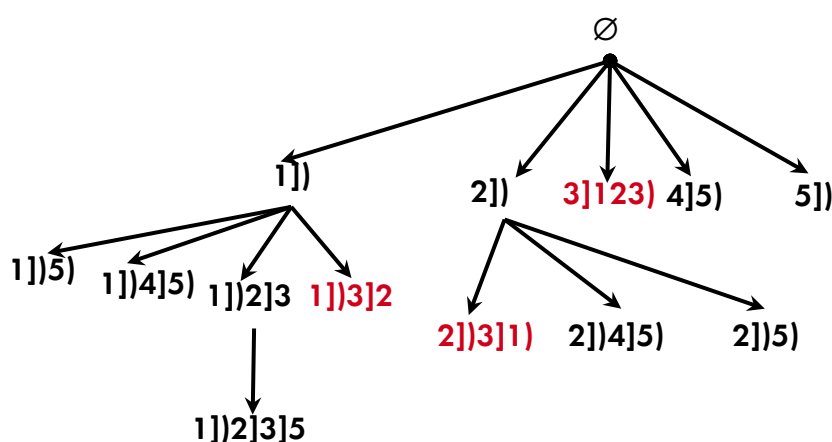
CONSTRUCT

1. All concepts using CbO;
2. Diagram of the concept lattice;
3. Generator cover of implications and proper premise base of implications;
4. Generator basis of association rules with $\text{conf} \geq \frac{1}{3}$ and $\text{supp} \geq \frac{1}{4}$ (using concept lattice diagram).

| | a | b | c | d |
|---|---|---|---|---|
| 1 | | x | x | x |
| 2 | x | x | x | |
| 3 | | x | x | |
| 4 | x | | | x |
| 5 | x | x | | x |

Solution:

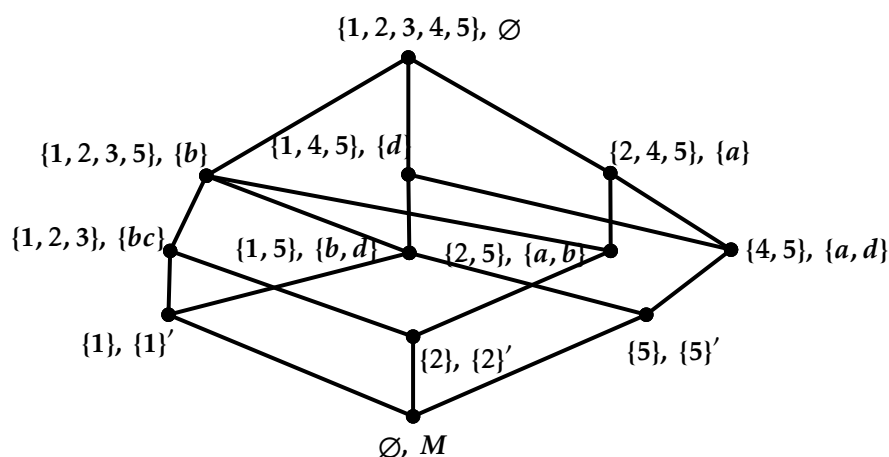
1. CbO



All concepts

- $(\emptyset, \{a, b, c, d\})$;
- $(1, \{b, c, d\})$;
- $(2, \{a, b, c\})$;
- $(5, \{a, b, d\})$;
- $(\{1, 5\}, \{b, d\})$;
- $(\{2, 5\}, \{a, b\})$;
- $(\{4, 5\}, \{a, d\})$;
- $(\{1, 2, 3\}, \{b, c\})$;
- $(\{1, 4, 5\}, \{d\})$;
- $(\{2, 4, 5\}, \{a\})$;
- $(\{1, 2, 3, 5\}, \{b\})$;
- $(\{1, 2, 3, 4, 5\}, \{\emptyset\})$;

2. Concept lattice:



3. Generation cover of implications ($\text{MinGen} \rightarrow \text{MinGen}''$):

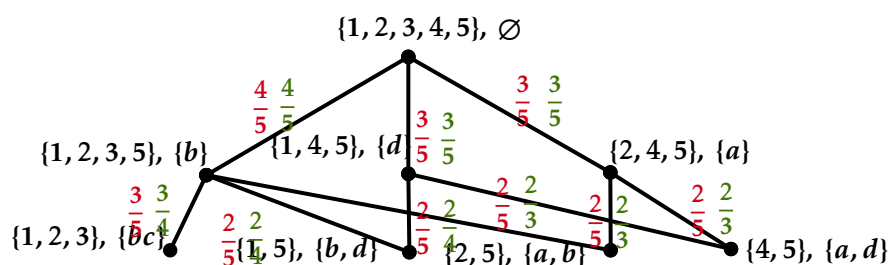
- $acd \rightarrow abcd$;
- $cd \rightarrow bcd$;
- $ac \rightarrow abc$;
- $c \rightarrow bc$.

And after simplifications:

- $acd \rightarrow b$
- $cd \rightarrow b$
- $ac \rightarrow b$
- $c \rightarrow b$

After removing duplicates: $c \rightarrow b$.

4. Computed conf and supp:



Finally:

- $0 \rightarrow a$;
- $0 \rightarrow b$;
- $0 \rightarrow d$;
- $b \rightarrow a$;
- $b \rightarrow c$;
- $b \rightarrow d$;
- $d \rightarrow a$;
- $d \rightarrow b$;
- $a \rightarrow b$;
- $a \rightarrow d$.

■

Problem 2.

Minimum (Duquenne-Guigues) basis of implications has the following form: $\{A \rightarrow A'' \mid A \text{ is a pseudointent}\}$, where pseudointents are defined recursively as follows:

Definition

A subset of attributes P is a pseudointent if

1. $P'' \neq P$;
2. $Q'' \subset P \forall Q \text{ s.t. } Q \subset P$.

| | a | b | c | d | e |
|---|---|---|---|---|---|
| 1 | u | v | t | r | q |
| 2 | t | t | q | r | q |
| 3 | q | t | q | t | t |
| 4 | r | q | q | u | u |
| 5 | q | v | t | r | t |

CONSTRUCT:

minimum basis of functional dependencies for the given many-valued context.

Hint: convert the many-valued context to a binary one with the same implications and compute the minimum (Duquennet-Guigues base).

Solution: Let me to convert the many-valued context to a binary one. To get the implication basis it requires to find all the nontrivial implications $A \rightarrow A''$, transform them to the minimal form and remove duplicates. So, the implications:

$$a \rightarrow ae \quad b \rightarrow bc \quad bd \rightarrow bcd \quad cd \rightarrow bcd$$

Hence, the basis:

$$a \rightarrow e \quad b \rightarrow c \quad bd \rightarrow c \quad cd \rightarrow b.$$

| | a | b | c | d | e |
|---|---|---|---|---|---|
| 1 | | | | × | × |
| 2 | | × | × | × | |
| 3 | | × | × | | |
| 4 | | | × | | |
| 5 | | | | × | |
| 6 | × | | | | × |

