Classwork
Section 01B

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MATH 111T — Spring 2021

Classwork 1 (Week 3)

Problem 1. We have the groups $(\mathbb{R}, +)$ and $(\mathbb{R}^\times, \cdot\,)$, the latter has as a subgroup $(\mathbb{R}^\times_{>0}, \cdot\,)$. Give a homomorphism

$$\varphi : (\mathbb{R}, +) \to (\mathbb{R}^\times_{>0}, \cdot\,)$$

such that (a) φ is an isomorphism. (b) φ is not an isomorphism.

Problem 2. (a) For $(\mathbb{R}^\times, \cdot\,)$ as above, is $(\mathbb{R}^\times_{<0}, \cdot\,)$ a subgroup? (b) Consider the group $(\mathbb{C}^\times, \cdot\,)$, show $S \coloneqq \{z \in \mathbb{C} \;:\; |z| = 1\}$ is a subgroup.

Extra Problem. Consider the group $G = \{f : \mathbb{R} \to \mathbb{R} \;:\; f \text{ is bijective}\}$. This is a group under composition. Prove that the following set

$$H = \{f_r : \mathbb{R} \to \mathbb{R},\; f_r(x) = x^r \;:\; r = p/q,\; p, q \text{ are odd}\}$$

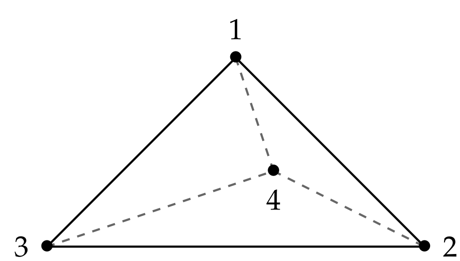
is a subset of G, and also a subgroup of G.

Why did we require p, q to be odd?

Classwork 2 (Week 5)

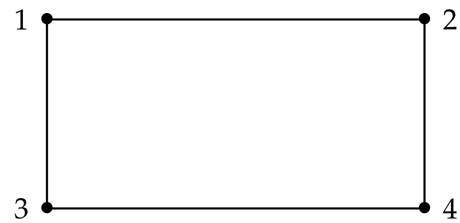
Problem 1. Consider the group $G = \mathbb{Z}/3\mathbb{Z} \times \mathbb{Z}/5\mathbb{Z}$, find an n and subgroup $H \leqslant S_n$ such that $G \cong H$.

Problem 2. Consider a regular tetrahedron, T



Carefully write down the cycles that correspond to the rotations of T. Which subgroup of S_4 can you identify it with?

Extra Problem. Consider the following rectangle R



Carefully write down the cycles that correspond to the symmetries of R. Which subgroup of S_4 can you identify it with?

(Warning: Rotating the rectangle by 90 degrees is not a symmetry of R as the resultant rectangle has a distinctly different shape than R.)

Classwork 3 (Week 7)

Problem 1. Consider a group G.

(a) For each $g \in G$ define a map
$$c_g : G \to G,\ h \mapsto ghg^{-1}$$
Call $\mathrm{Inn}(G) \coloneqq \{c_g \ : \ g \in G\}$. Prove that $\mathrm{Inn}(G) \leqslant \mathrm{Aut}(G)$, that is, show

(i) $\mathrm{Inn}(G) \subseteq \mathrm{Aut}(G)$.

(ii) $\mathrm{Inn}(G) \neq \varnothing$.

(iii) $c_g \circ c_h^{-1} \in \mathrm{Inn}(G)$, for any $g, h \in G$.

(b) Consider the function
$$\varphi : G \to \mathrm{Aut}(G),\ g \mapsto c_g$$
Show that

(i) φ is a group homomorphism.

(ii) Compute $\ker \varphi$; note that $\mathrm{im}\,\varphi = \mathrm{Inn}(G)$.

(iii) Use the first isomorphism theorem to write $\mathrm{im}\,\varphi = \mathrm{Inn}(G)$ as a quotient of G.