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Q. ①

Reduce Grammar:

$S \rightarrow AB / CA$

$B \rightarrow BC / AB$

$A \rightarrow a$

$C \rightarrow aB / b$

→

① Production of B doesn't have any terminal in it. It is producing non-terminating strings. Hence, it should be reduced.

② Any production rule including B should be reduced.

∴ Reduced Grammar: $S \rightarrow CA$

$A \rightarrow a$

$C \rightarrow b$

Q. 3)

CFG → CNF

$S \rightarrow bA / aB$

$A \rightarrow bAA / aS / a$

$B \rightarrow aBB / bS / a$

→

① As there are no useless variables, null productions & unit productions, the given CFG is already reduced.

②

Prod.

New Prod.

$S \rightarrow bA$

$S \rightarrow XA$

~~$X \rightarrow b$~~

$S \rightarrow aB$

$S \rightarrow YB$

$Y \rightarrow a$

$A \rightarrow bAA$

$A \rightarrow XU$

$U \rightarrow AA$

$A \rightarrow aS$

$A \rightarrow YS$

$$B \rightarrow aBB$$

$$B \rightarrow YV$$

$$V \rightarrow BB$$

$$B \rightarrow bS$$

$$B \rightarrow XS$$

Combining all the new prod. rules, we get

$$S \rightarrow XA / YB$$

$$A \rightarrow XV / YS / a$$

$$B \rightarrow YV / XS / a$$

$$X \rightarrow b$$

$$Y \rightarrow a$$

\therefore This is the CNF Form.

Q.5)

Lets apply these rules one by one :

① start with ϵ

② 'a' is palindrome

③ assume $x = 'b'$

$\therefore axa$ will be a palindrome

$\therefore aba$ indeed is a palindrome

assume $x = 'aba'$

$\therefore axa \rightarrow abaabaaba$ should be a palindrome, which is true.

Hence, by following these rules palindrome can be defined.

Q.6)

$$L = \{a^n b^{n-3} \mid n \geq 3\}$$

$$\therefore L = \{a^3 b^0, a^4 b^1, a^5 b^2, \dots\}$$

$$S \rightarrow aaaX$$

$$X \rightarrow aXb / \epsilon$$

Q.8)

 $CFG \rightarrow CNF$ $S \rightarrow aSaA / A$ $A \rightarrow abA / b$

→

(1) Eliminate Unit Prod.

 $S \rightarrow A \quad S \rightarrow abA$ $S \rightarrow b$ \therefore Reduced Form : $S \rightarrow aSaA / abA / b$ $A \rightarrow abA / b$

(2)

Prod.

New Prod.

 $S \rightarrow aSaA$ $S \rightarrow XSXA$ $S \rightarrow PQ$ $X \rightarrow a$ $P \rightarrow XS$ $Q \rightarrow XA$ $S \rightarrow abA$ $S \rightarrow XYA$ $S \rightarrow ZA$ $Y \rightarrow b$ $Z \rightarrow XY$ $A \rightarrow abA$ $A \rightarrow ZA$

Combining these new rules, we get the CNF form :

 $S \rightarrow PQ / ZA / b$ $A \rightarrow ZA / b$ $P \rightarrow XS$ $Q \rightarrow XA$ $X \rightarrow a$ $Y \rightarrow b$ $Z \rightarrow XY$

Q.12) Let $L = \{ aab, aaaaabb, aaaaaabbb \dots \}$

$$\therefore S \rightarrow aasb / \epsilon$$

Q.13) $L =$ Do not contain 3 consecutive b's
 $\therefore L = \{ b, bb, ba, bab, babb, bbabb \dots \}$

$$\therefore S \rightarrow b / bb / Sa / Sab / Sabbb / \epsilon$$

Q.15) CFC \rightarrow CNF

$$S \rightarrow AB$$

$$A \rightarrow BSB / BB / b$$

$$B \rightarrow aAb / a$$

\rightarrow ① The given CFC is already in its reduced form.

② We need to convert it into CNF.

Prod.	New Prod.	
$S \rightarrow AB$	-	-
$A \rightarrow BSB$	$A \rightarrow XB$	-
	$X \rightarrow BS$	-
$B \rightarrow aAb$	$B \rightarrow YAZ$	$B \rightarrow PZ$
	$Y \rightarrow a$	$P \rightarrow YA$
	$Z \rightarrow b$	-

The CNF form is :

$$S \rightarrow AB$$

$$A \rightarrow XB / BB / b$$

$$B \rightarrow PZ / a$$

$$X \rightarrow BS$$

$$P \rightarrow YA$$

$$Y \rightarrow a$$

$$Z \rightarrow b$$

③ Now, CNF should be converted to CNF.

Prod.	New Prod.	
$X \rightarrow BS$	$X \rightarrow PZS$	$X \rightarrow aS$
	$X \rightarrow YAZS$	
	$X \rightarrow aAZS$	
$P \rightarrow YA$	$P \rightarrow aA$	
$B \rightarrow PZ$	$B \rightarrow aAZ$	
$A \rightarrow XB$	$A \rightarrow aAZSB$	$A \rightarrow aAZB$
$A \rightarrow BB$	$A \rightarrow aSB$	$A \rightarrow aB$
$S \rightarrow AB$	$S \rightarrow aAZSBB$	$S \rightarrow aSBB$
	$S \rightarrow aAZBB$	$S \rightarrow aBB$
	$S \rightarrow bB$	

The CNF Form is :

$$S \rightarrow aAZSBB / aSBB / aAZBB / aBB / bB$$

$$A \rightarrow aAZSB / aSB / aAZB / aB / b$$

$$B \rightarrow aAZ / a$$

$$X \rightarrow aAZS / aS$$

$$Y \rightarrow a$$

$$Z \rightarrow b$$

$$P \rightarrow aA$$

Q.17) $S \rightarrow aAS / a$

$$A \rightarrow SbA / SS / ba$$

Derivation tree for "aabbaa"

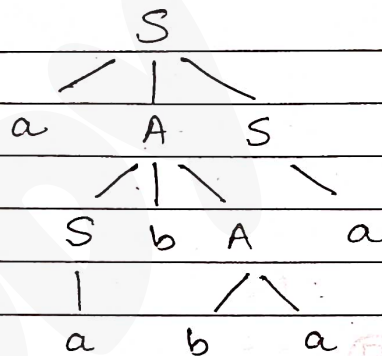
$$\therefore S \rightarrow aAS$$

$$\therefore S \rightarrow a(SbA)(a)$$

$$\therefore S \rightarrow a(a)b(ba)a$$

$$\therefore S \rightarrow aabbaa$$

Derivation Tree :



Q.18)

$S \rightarrow XbbaaX / aX$

$X \rightarrow Xa / Xb / \epsilon$

LMD & RMD for "abaabb"

LMD :

$S \rightarrow aX$

$S \rightarrow a(Xb)$

$S \rightarrow a(Xb)b$

$S \rightarrow a(Xa)bb$

$S \rightarrow a(Xa)abb$

$S \rightarrow a(Xb)aabb$

$S \rightarrow abaabb$

RMD :

$S \rightarrow aX$

$S \rightarrow a(Xb)$

$S \rightarrow a(Xb)b$

$S \rightarrow a(Xa)bb$

$S \rightarrow a(Xa)abb$

$S \rightarrow a(Xb)aabb$

$S \rightarrow abaabb$

AS there was only one variable in the production rule that we used, LMD & RMD are same.

$$Q.28) L = \{a^n b^{m+n} c^{2m} \mid n > 0, m > 0\}$$

$$\therefore L = \{ab^2c^2, a^2b^3c^2, a^2b^4c^4, a^3b^5c^4, \dots\}$$

lets assume L is context-free language.

$$\text{For } S = ab^2c^2 \quad (n=1, m=1),$$

$$\text{assume } u = a$$

$$v = b$$

$$x = b$$

$$y = c$$

$$z = c$$

$$S = uv^i xy^i z \quad \text{for } i > 0 \quad \text{should be in } L.$$

$$\begin{aligned} \therefore S_1 &= uv^2 xy^2 z \\ &= abbbccc \\ &= ab^3c^3 \end{aligned}$$

$$\text{But, } S_1 \neq a^n b^{m+n} c^{2m}$$

$$\therefore ab^3c^3 \notin L$$

\therefore According to pumping lemma,
 L is not a context-free language.

$$Q.29) L = \{ a^n b^{2n} c^n \mid n \geq 1 \}$$

$$\therefore L = \{ ab^2c, a^2b^4c^2, a^3b^6c^3, \dots \}$$

Let's assume L is a context-free language.

$$\text{For } S = a^2b^4c^2 \quad (n=2),$$

$$\text{assume } u = a$$

$$v = ab$$

$$x = bb$$

$$y = bc$$

$$z = c$$

$S = uv^ixy^iz$ should be in L for $i > 0$

$$\begin{aligned} \therefore S_1 &= uv^2xy^2z \\ &= a(abab)bb(bcbcb)c \\ &= aababbbcbcbcc \\ &= a^2bab^4cbc^2 \end{aligned}$$

$$\text{But, } S_1 \neq a^n b^{2n} c^n \\ a^2bab^4cbc^2 \notin L$$

According to pumping lemma,
 L is not a context-free language.