Probabilistic Roadmaps-Spline based Trajectory Planning For Wheeled Mobile Robot

Priyanka Sudhakara, Velappa Ganapathy and Karthika Sundaran
School of Computing, SRM University,
Kancheepuram-603203, Tamil Nadu, India
{priyanka.k@ktr.srmuniv.ac.in, ganapathy.v@ktr.srmuniv.ac.in, karthika.su@ktr.srmuniv.ac.in}

***Abstract*— This paper proposes a trajectory planning of a mobile robot using Probabilistic Roadmaps [PRM] with spline technique. In the proposed method, the PRM method is combined with the spline technique to generate smooth trajectories, which are important for curved path navigation of a wheeled mobile robot. The PRM technique is expanded to a cubic Ferguson's curve, a spline based method. Trajectories that are been generated using the proposed PRM-Spline combination satisfies direction constraints approach on both source and target positions. This makes the proposed algorithm remarkably unlike from other trajectory planning algorithms. As a result, the paths produced by the mobile robots are sub-optimal, dynamically and geometrically feasible, and satisfy direction constraints approaches. Simulation results that are performed affirm these PRM-Spline technique properties and manifest the proposed algorithm validity, implying that it can be efficiently used in trajectory planning of wheeled mobile robot operating in real-time environments.**

Keywords*— *Probabilistic Roadmaps [PRM]; Spline Curve; Trajectory Planning; Mobile Robots; Obstacle Avoidance

I. Introduction

Trajectory planning of a mobile robot is a vital issue in robotics to attain autonomy. Presently, in the area of robotics, significant amount of works has been spent on trajectory planning algorithms and have been proposed [1]. Diverse varieties of algorithms have been proposed for producing a feasible trajectory. Especially, trajectory planning methods based on sampling like Rapidly Exploring Random Trees (RRT) and Probabilistic Roadmaps (PRM) [2] attain considerable demand in motion planning communities. They deal and overcome the computational problems of deterministic motion planning approaches by circumventing the representation of obstacles that are explicit in the scenario. The probabilistic approach to motion planning originates from 1990 where Barraquand and Latombe showed that randomization could be a powerful tool for planning in high dimensional spaces [3]. Later, the probabilistic roadmaps [4] (known as PRM) were introduced, which in contrast to the work by Barraquand created a reusable data structure that could be used for multiple queries. A significant drawback of the PRM based planners is that they may require a very large number of samples to connect components through a narrow passage. This issue is discussed in [5]. Different strategies for biasing the random sampling have been suggested. Boor et. al. [6] propose a Gaussian sampling strategy inspired from image processing.

In this paper, Probabilistic Roadmaps-Spline based algorithm which utilizes cubic Ferguson's spline technique is proposed. Unlike other trajectory planning techniques, the proposed algorithm can produce an optimal and smoothened trajectory that could be utilized in mobile robotics and it is asymptotically optimal. Section 2 describes the optimal Probabilistic Roadmaps-Spline based trajectory planning followed by Section 3 describing simulation results and conclusions are thereafter.

II. Optimal Probabilistic Roadmaps-Spline for Trajectory Planning

A. Probabilistic Roadmaps(PRM):

Other sampling-based trajectory planning methods [7]-[12] like Probabilistic roadmaps (PRM) provides a global trajectory planning, to capture the free configuration space connectivity, by utilizing sampling techniques. In specific, constructing the roadmaps using PRM approach resembles a web of roadmaps acquired by sampling and connecting a large number connecting neighboring configurations through collision-free trajectories. Thenceforth, roadmap performs various tasks like target seeking, guiding the robot or homing [13], [14]. In [15], Cubic Bezier curves are been applied to PRMs to shepherd non-holonomic robots to the target while maintaining the whole configuration.

We propose a Probabilistic Roadmaps-Spline based Trajectory Planning algorithm which utilizes cubic Ferguson's spline method for mobile robot trajectory planning. The proposed algorithm is based on the PRM technique and can efficiently solve complicated trajectory planning issues in greatly constrained scenarios. The PRM technique cannot generate smoothened trajectories. Hence, to generate such trajectories, combination of the cubic spline technique based on cubic Ferguson's curve with the PRM method, which eases the production of smoothened curved trajectories for wheeled mobile robots. In this combinational algorithm, the boundaries that are created by PRM technique can be linked to make a smoothened ultimate trajectories that have direction flow. The direction, position and the robots' path angle of the goal signifies the nodes of the tree structure.

We build a graph of G = (N, V) using PRM [16] recording the free configuration space connectivity CS_{free}. N, a set of nodes that are free, is generated randomly using uniform sampling strategy. G is the arcs that are gathered in the set of

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V, are nodes of connecting pairs that are in collision-free trajectories. Usually, PRM continues in two phases: (i) construction state, and (ii) research state:

A. Construction state

Random nodes are first generated using uniform strategy in the free space ($free_{space}$) that are added to the graph G. Then, to choose a set of neighboring nodes for each newly generated node, a selecting strategy is utilized. The choice of discretization's parameters is the first issue that was met in the PRM's construction of trajectories. Specifically, N_{max} is the number of nodes and d_{min} is the distance in sampling that is the minimum distance between two nodes in the web. One of the most important part is the selection of the size of the web (N_{max}) to cover the maximum space in the $free_{space}$. A technique on computing the ratio between the $surface_{free}$ and the $surface_{total}$ of the environment is used to calculate N_{max} for a given d_{min}. The aim is to generate a number of random nodes; the ratio between the $surface_{free}$ and the $surface_{total}$ will be equal to the ratio between the number of $free_{nodes}$ and the total number of generated nodes.

$$\frac{Surface_{free}}{Surface_{total}} = \frac{number\ of\ free_{nodes}}{total\ number\ of\ nodes} \quad (1)$$

In the ratio between the $surface_{free}$ and the $surface_{total}$, N_{max} will be the integer part that includes a square of side d_{min}.

$$N_{max} = integer\left[\frac{Surface_{free}}{d_{min}^2 * \frac{\pi}{2}}\right] \quad (2)$$

For a given node q_i it is considered that the adjacent nodes of q_i and any node that is located at a distance lower or equal to d_{max}. d_{max} is the maximum distance between two neighboring nodes. In order to ensure a better connectivity to the web, the distance d_{max} is chosen proportionally direct to d_{min} ($d_{max} = 2\ d_{min}$). Collision detection decides whether a node belongs to the $free_{space}$ or not. It is also used to validate whether a given trajectory is included in the $free_{space}$ or not. The resulting web that contains N_{max} nodes where each nodes is connected to its neighboring nodes by segments.

B. Research state

In the research state, the graph G which was built in the construction state will be utilized to find the trajectories between a source point and a target point. In order to find the shortest trajectory that connects the source and the target nodes, an algorithm was utilized that has three steps:

The initial step is to discover the shortest trajectory by applying the A star algorithm [17] in the formerly constructed web. The trajectories that are computed may comprise uneven parts. In the second step, a process of choosing an alternate route is performed to remove the uneven parts of the computed trajectories. The trajectories that are acquired after the implementation of choosing the alternate route process may not be the shortest, to improve it further, a process of discretization is called in the final step that will split the trajectories into various parts with length greater than or equal to a predefined discretization per single step. The second and the final steps are repeated several times until the gain of choosing the alternative route will be negligible. The simulation results of PRM technique are done by [18].

B. Cubic Ferguson's Spline Technique:

Splines are a good choice for robot's trajectory movements for its ease of implementation and high degree of smoothness. Considering the start position P0 and the target position P1 of the spline and their corresponding tangent vector P\`0 and P\`1, we can seek Ferguson spline using the following equation in [19]:

$$X(a) = P_0\, f_1(a) + P_1\, f_2(a) + P_0' f_3(a) + P_1' f_4(a) \quad (3)$$

Where corresponding f1, f2, f3, f4 are Ferguson-multi-nomials and a ε [0, 1] is parameter which are described by

$$f_1(a) = 2a^3 - 3a + 1 \quad (4)$$
$$f_2(a) = 2a^3 - 3a^2 \quad (5)$$
$$f_3(a) = a(a-1)^2 \quad (6)$$
$$f_4(a) = a^2(a-1) \quad (7)$$

Equations (3) through (7) simply the source position P0 and target position P1 can be acquired by X(0) and X(l). Values of the positions P\`0 and P\`1 are acquired by derivative substitution.

$$f_1'(a) = 6a^2 - 6a \quad (8)$$
$$f_2'(a) = -6a^2 + 6a \quad (9)$$
$$f_3'(a) = 3a^2 - 4a + 1 \quad (10)$$
$$f_4'(a) = 3a^2 - 2a \quad (11)$$

By using the equations (1), (2) and (8 - 11), we can deduce $P_0' = X'(0)$ and $P_1' = X'(1)$.

3.3.1 Fitness Function and Food Coding: To clarify the issues of trajectory planning, mathematical notations of Ferguson spline are showed in 2d space as follows:

$$r(a) = \big(x(a), y(a)\big) = b_0 + b_1 a + b_2 a^2 + b_3 a^3 \quad (12)$$

Where

$$\begin{cases} b_0 = 2P_0 - 2P_1 + P_0' + P_1' \\ b_1 = -3P_0 + 3P_1 - P_0' + P_1' \\ b_2 = P_0' \\ b_3 = P_0 \end{cases} \quad (13)$$

Each spline is defined only by points P0 and P1 and vectors P_0' and P_1'. In equation (12) and (13), the string that contain two neighboring splines share the corresponding vector and one of the terminal points. The whole trajectory that is defined by total number of variables in 2D space is 4(n+ 1), where *n* is the number of splines in the string.

A prime part of a bio-inspired computing technique is to find a better fitness function for evaluation. The functions global minimum could be compatible to smooth paths that are short and safe. Long distance tracing of the path and the paths that causes obstacle collisions are penalized using two function parts in [19]. The fitness function is followed as in equation (14):

$$f = f_1 + \alpha f_2 \quad (14)$$

Where α is a weight factor that adjusts the proportion of the length. f_1 is defined by following equation (15):

$$f_1 = \frac{L}{L_{min}} \quad (15)$$

Where

L is the path length and

L_{min} is Euclidian distance between source and target point. f_2 is defined by following equation (16):

$$f_2 = \begin{cases} L, & d_{min} > d_{safe} \\ e^{(D\_(safe+1)/d\_(min+1))+1} & d_{min} \leq d_{safe} \end{cases} \quad (16)$$

Where d_{safe} is a constant that determine the influence of the obstacles.

III. Simulation Results

To validate the performance of our proposed Probabilistic Roadmaps-Spline combinational method, we carried out several simulations in five different varied environments. The trajectory planning scenario utilized in this paper comprised of resolution of original map: 500×500, fixed source point and target points for all four varied scenarios at [10, 10] and [490, 490] respectively. Fig. 1 to 5 illustrates the simulation results for five different environments. Simulation results (a), (b) and (c) shows the trajectory traced by the robot using PRM technique and (d) shows the trajectory traced using Ferguson's spline technique applied to PRM and the results are been tabulated in table 1. Table 1 shows the time of travel taken by the robot to reach the goal and the path length traversed by the robot from starting point

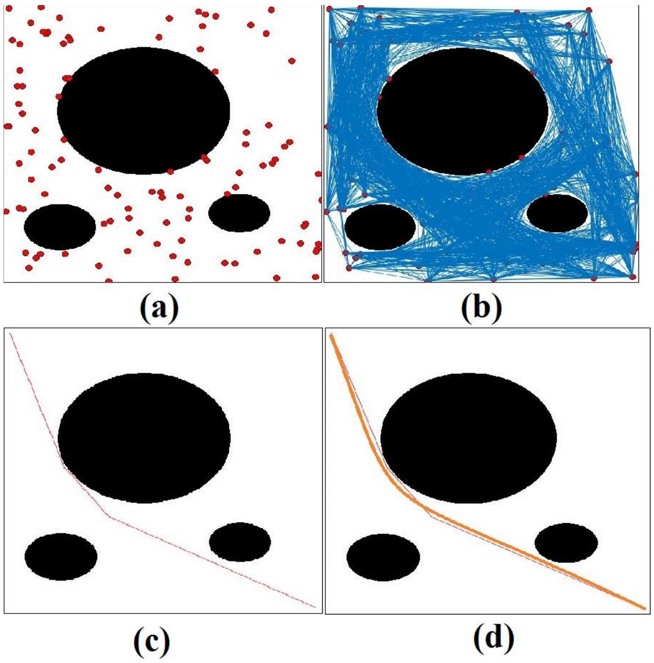

Fig. 1. Simulation Results 1: Trajectory Planning (a) to (c) using PRM technique, (d) after applying Ferguson's spline

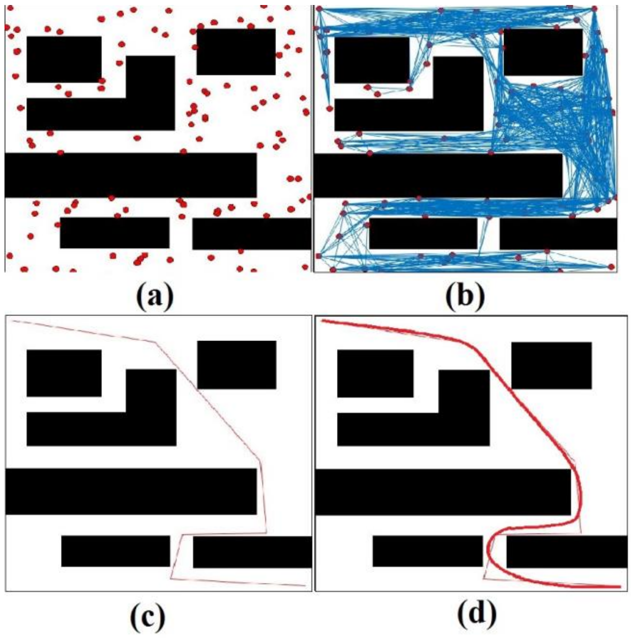

Fig. 2. Simulation Results 2: Trajectory Planning (a) to (c) using PRM technique, (d) after applying Ferguson's spline

to the target in the Simulation Results-1 to 5 using PRM technique and Probabilistic Roadmaps-Spline combinational method. From the simulation results 1 to 5, we have found that the time of travel by the robot to reach the goal after applying Ferguson spline is drastically reduced as compared to that of the ordinary PRM method.

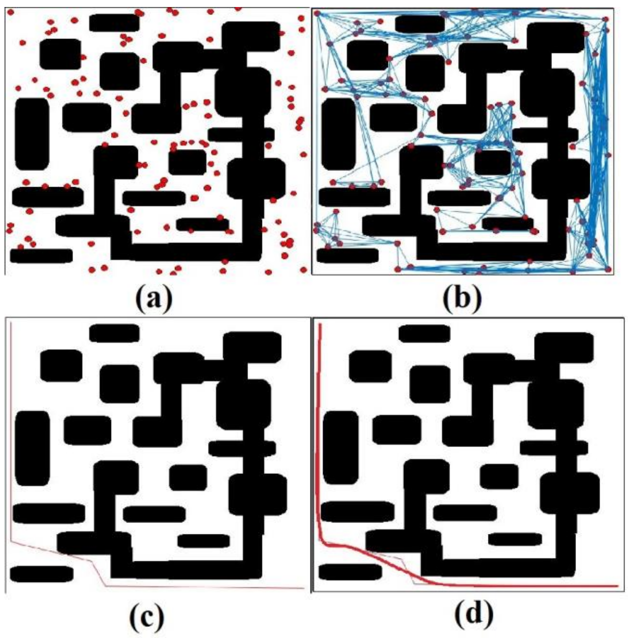

Fig. 3. Simulation Results 3: Trajectory Planning (a) to (c) using PRM technique, (d) after applying Ferguson's spline

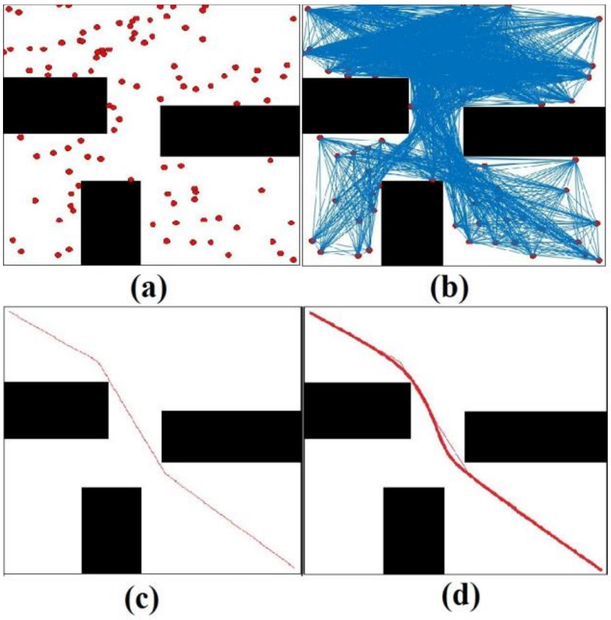

Fig. 4. Simulation Results 4: Trajectory Planning (a) to (c) using PRM technique, (d) after applying Ferguson's spline

As the rough turning points of the path are smoothed to curves, it will be easy for the robots to turn and move forward. This will reduce the time traversed by the robots to reach the goal as the robot doesn't want to stop while turning directions.

TABLE I. TIME TAKEN AND PATH LENGTH TRAVERSED BY THE ROBOT USING THE ORDINARY PRM METHOD AND PROBABILISTIC ROADMAPS-SPLINE COMBINATIONAL METHOD TO REACH THE TARGET (SIMULATION RESULTS 1 TO 5)

	Probabilistic Roadmaps (PRM) technique		**Probabilistic Roadmaps-Spline combinational method**	
	Time Taken for the robot to reach the Target (in sec)	***Path Length***	***Time Taken for the robot to reach the Target (in sec)***	***Path Length***
Simulation Results 1	6.388	722	4.789	621
Simulation Results 2	12.776	1083	10.003	944
Simulation Results 3	10.808	909	9.001	842
Simulation Results 4	5.204	695	4.106	601
Simulation Results 5	7.956	884	5.934	699

IV. Conclusion

In this paper, a trajectory planning approach for wheeled mobile robot is proposed. The proposed method combines the classic PRM technique with the cubic Ferguson's spline technique to generate trajectories that directs smoothness and continuation without any computation. To validate the capability of the proposed algorithm, simulations were processed and showed that the proposed method is

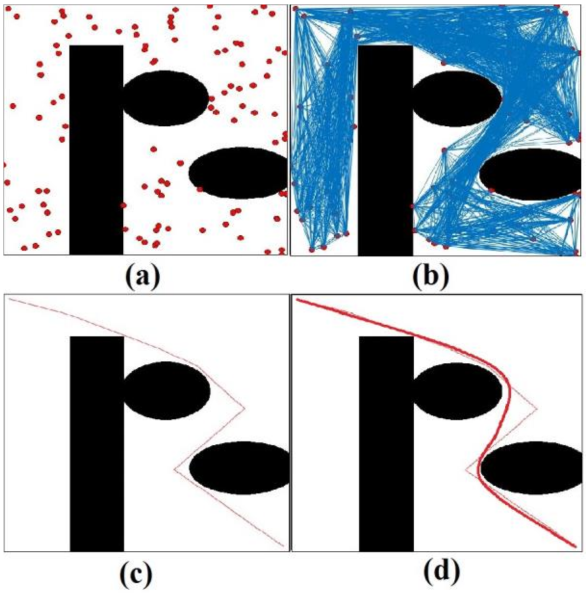

Fig. 5. Simulation Results 5: Trajectory Planning (a) to (c) using PRM technique, (d) after applying Ferguson's spline

feasible in both geometrical and dynamical environment. Like the classic PRM technique, our proposed combinational method is also optimally asymptotic and can satisfy direction limitations approach on both source and target locations. The applications of this proposed method relies on the cost function and can be utilized in various numbers of other fields.

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