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Path Tracking Control of Lagrange Systems with Obstacle Avoidance

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Abstract: This paper addresses a path tracking problem with obstacle avoidance for Lagrange systems. The proposed method is based on field potential methods in combination with navigation functions for obstacle avoidance. First, it is shown that a simple combination of the navigation function with the conventional path tracking controller does not work. Therefore, in order to cope with this problem, a new feedback law is proposed for a path parameter which characterizes the reference path. It is proved that the proposed controller achieves both path following and collision avoidance. Moreover, since the method adopts bounded navigation functions, the proposed controllers generate bounded input signals even when target systems approach obstacles. Finally, an experimental evaluation is performed with a two-link manipulator to illustrate the effectiveness of the proposed method.

Keywords: Lagrange systems, obstacle avoidance, path tracking, robot manipulators.

1. INTRODUCTION

Autonomous navigation of robots in environments with obstacles has been an area of signif- icant interest for many years. The main issue concerning this topic has been point to point control of robots with obstacle avoidance. From early researches, off-line path planning methods have been proposed, which compute collision-free trajectories and corresponding inputs [1,2]. As for online methods, artificial potential function based methods [3] as well as path planning methods [4] have been developed. An advantage of artificial potential function based methods is that designed controllers consist of gradient vector fields based on environments, and those fields cause less computation burdens than many other path planning methods. However, these methods sometimes suffer from the problem of local minimums. To overcome this problem, Khatib et al. have proposed special potential functions called navigation functions (NFs) for obstacles in the form of star shapes [5,6] and in that of more complicated ones [7]. Navigation functions have the following advantages:

(ad1) Navigation functions have only one minimum at a desired position, and do not have local minimums at other positions.
(ad2) Since navigation functions are bounded and smooth, derived control inputs are bounded even in areas close to obstacles.
(ad3) Navigation functions are available for many obstacle shapes; thus it is easy to design corresponding potential functions.

Because the NF based method targets Lagrange systems, this method is effective for a large class of mechanical systems.

Besides point to point control, it is important for robots to track given reference paths in engineering applications. Ordinary trajectory tracking problems for Lagrange systems in environments without obstacles have been discussed in earlier works [8,9]. Following these works, trajectory time scaling methods have been developed to take account of available acceleration and velocity of robots [10-12]. In recent years, various approaches for trajectory tracking of Lagrange systems have been developed such as adaptive control [13,14], energy and passivity based control [15,16], and model predictive control [17,18]. On the other hand, a trajectory tracking problem in consideration of obstacle avoidance has been discussed in [19]. This method uses linearizing coordinate transformations determined by obstacle shapes and rate constraints ensuring velocity limits. However, derived controllers may produce large input signals when positions of target systems are close to obstacles because the inverse matrices of Jacobian matrices in the transformations are not well-defined on the boundaries of obstacles.

This paper proposes a path tracking controller for Lagrange systems with obstacle avoidance, whose input signal is bounded. In order to enjoy the advantages (ad1)-(ad3) of navigation functions, our strategy is to insert a navigation function into an ordinary path tracking controller. As proved later in this paper, it turns out that this method does not always guarantee the collision avoidance. To overcome this problem, we adopt a path tracking technique with speed assignment, which has been developed by [20,21]. In path tracking, reference paths are represented by two components: reference functions describing the geometric shapes of

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the reference paths and path parameters specifying the reference functions. The idea to guarantee collision avoidance is to adjust the introduced path parameters so as for the target system to move with assigned velocity if the obstacles are far enough, and to limit its velocity otherwise. This paper proposes a new feedback adjustment law for path parameters to ensure effective obstacle avoidance and path tracking.

This paper mainly focuses on static obstacles and reference paths which do not overlap with obstacles. Although other papers [22-24] deal with dynamic obstacles or reference paths overlapping with obstacles, no rigorous theoretical support is given. On the other hand, this paper provides a rigorous proof for the proposed method to achieve path tracking with collision avoidance based on the authors' paper [25]. Moreover, an experimental result with a 2-link manipulator shows the practical validity of the proposed method.

This paper is organized as follows: Section 2 describes the target system with the problem setting. Section 3 presents an existing obstacle avoidance method and an existing trajectory tracking method as preliminary. Section 4 points out a problem of the simple combination of these methods. Moreover, we propose a path tracking method with obstacle avoidance with a new feedback law for path parameters. Section 5 gives an experimental result with a 2-link manipulator. Section 6 concludes this paper. All proofs are found in the appendix.

The following notations are used. Let $\mathbb{R}$ and $\mathbb{R}_+$ be the sets of real numbers and non-negative real numbers, respectively. For a set $\mathcal{D}\subset\mathbb{R}^m$, $\breve{\mathcal{D}}$ and $\partial\mathcal{D}$ denote the inner set and boundary of $\mathcal{D}$, respectively. For a vector $x\in\mathbb{R}^m$, let $d(x,\mathcal{D})=\inf_{q\in\mathcal{D}}\|x-q\|$ be the distance between the vector x and the set $\mathcal{D}$.

2. PROBLEM SETTING

Consider the Lagrange system

$$M(q)\ddot{q}+(C(\dot{q},q)+D)\dot{q}=u, \tag{1}$$

where $q,\ u\in\mathbb{R}^m$ are the generalized coordinate and the input, respectively. The inertia matrix $M\colon\mathbb{R}^m\to\mathbb{R}^{m\times m}$ is a C^1 symmetric matrix function satisfying $\mu_1 I\le M(q)\le\mu_2 I,\ \forall q\in\mathbb{R}^m$ for some positive numbers μ_1 and μ_2. Assume that the Coriolis matrix function $C\colon\mathbb{R}^m\times\mathbb{R}^m\to\mathbb{R}^{m\times m}$ and M satisfies the equality

$$\dot{M}(q)=C(\dot{q},q)+C^\top(\dot{q},q), \tag{2}$$

which holds for Lagrange systems [26] in general. The damping matrix D is positive semi-definite. The kinetic energy of the system (1) is given by $K=\dot{q}^\top M(q)\dot{q}/2$. Assume that the system (1) does not include gravitational terms because they can be cancelled by a suitable additional input to u. An important example of this system is a robot manipulator as presented in Example 1.

The control objective considered in this paper consists of two tasks: *obstacle avoidance* and *path tracking*. Let a set $\mathcal{F}\subset\mathbb{R}^m$ be a workspace surrounded by obstacles in which the coordinate q should stay so as to avoid collision with obstacles. Assume that at the initial time the coordinate stays still in the workspace, that is $q(0)\in\mathcal{F}$ and $\dot{q}(0)=0$. Then, the obstacle avoidance task is represented as

$$q(0)\in\mathcal{F},\quad \dot{q}(0)=0\Rightarrow q(t)\in\mathcal{F},\ \forall t\ge 0. \tag{3}$$

For the path tracking task, the coordinate q is required to move along a reference path with an assigned velocity in a determined direction. The problem of path tracking has been studied by [20,21], which do not consider collision avoidance. The reference path and the assigned velocity on the path are represented by functions $\varphi(r)\in\mathbb{R}^m$ and $v_d(r)\in\mathbb{R}_+$, which are characterized by a path parameter $r\in\mathbb{R}$. Note that when the parameter r is fixed, the variables $\varphi(r)$ and $v_d(r)$ represent a desired position on the reference path and a desired velocity at this point, respectively. Then, the path tracking task is divided into geometric and dynamic tasks. The geometric task requires the coordinate q to reach and track the reference path $\varphi(r)\in\mathbb{R}^m$. By regarding the path parameter as a time-varying function $r(t)$, this problem is described as

$$q(t)\to\varphi(r(t))\ (t\to\infty). \tag{4}$$

For the dynamic task, the velocity $\|\dot{q}(t)\|$ should move with the assigned velocity $v_d(r(t))$ as

$$\|\dot{q}(t)\|\to v_d(r(t))\quad (t\to\infty). \tag{5}$$

Moreover, in order to determine the direction at which the coordinate q moves, the path parameter $r(t)$ has to increase (decrease) after a sufficiently large time T as

$$\dot{r}(t)>0\ (<0),\quad t\ge T. \tag{6}$$

The conditions (5) and (6) of the dynamic task are reduced to one condition as follows:

Proposition 1: Consider the system (1), a desired path $\varphi(r)$ and an assigned velocity $v_d(r)$ which are of class C^2 and C^1, respectively. Assume that these functions satisfy

$$\iota_1\le\left\|\frac{d\varphi}{dr}(r)\right\|\le\iota_2,\quad \left\|\frac{d^2\varphi}{dr^2}(r)\right\|\le\iota_3,\ \iota_4\le\|v_d(r)\|\le\iota_5 \tag{7}$$

for some positive constants ι_1, ι_2, ι_3, ι_4, ι_5 and all $r\in\mathbb{R}$. Assume that $\dot{q}(t)$, $u(t)$, $\dot{r}(t)$ and $\ddot{r}(t)$ are bounded as time-varying functions and that (4) is satisfied for a path parameter $r(t)$. Then, (5) and (6) are realized if

$$\dot{r}(t)\to \mathrm{v}(r(t))\ \text{as}\ t\to\infty \tag{8}$$

for the function $\mathrm{v}:\mathbb{R}\to\mathbb{R}_+$

$$\mathrm{v}(r)=v_d(r)\left\|\frac{d\varphi}{dr}(r)\right\|^{-1}\ \text{or}\ =-v_d(r)\left\|\frac{d\varphi}{dr}(r)\right\|^{-1}. \tag{9}$$

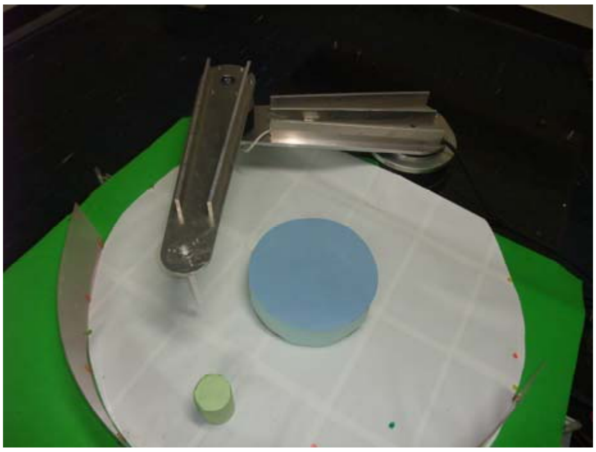

Fig. 1. Experimental setup of a 2-link manipulator.

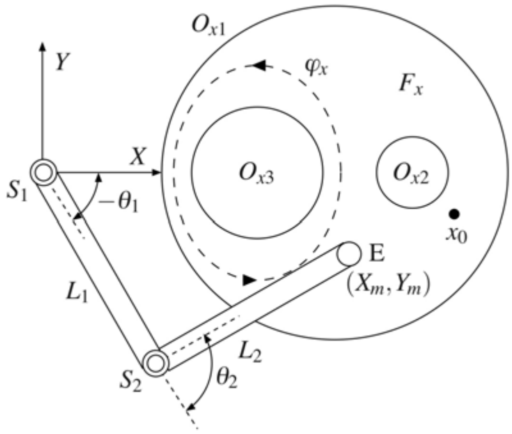

Fig. 2. Sketch of a 2-link manipulator.

Then, we adopt the condition (8) as the dynamic task instead of (5) and (6). Now, the problem in question is formulated follows:

Problem 1: For the system (1), a workspace $\mathcal{F} \subset \mathbb{R}^m$, a reference path φ and an assigned velocity ν under the following assumptions (as1)-(as3), design a bounded input $u(\dot{q},q,t)$ and a path parameter $r(\dot{q},q,t)$ such that the solution $(\dot{q}(t),\ q(t))$ of (1) from every initial state $(\dot{q}(0), q(0)) \in \{0\}\times \mathcal{F}$ satisfies the obstacle avoidance task (3) and the path tracking tasks (4) and (8).

(as1) Workspace $\mathcal{F}$ is compact and connected.

(as2) There exists a positive constant ε such that the reference path φ keeps distance ε from the boundaries of $\mathcal{F}$, that is
$\exists\, \varepsilon > 0$ s.t. $\varphi(r) \in \mathcal{F}_\varepsilon,\ \forall r \in \mathbb{R}_+$,
where $\mathcal{F}_\varepsilon = \mathcal{F} \cap \{\varphi : d(\varphi, \partial\mathcal{F}) \geq \varepsilon\}$.

(as3) The reference path $\varphi: \mathbb{R}_+ \to \mathcal{F}$ and the desired velocity $\nu: \mathbb{R}_+ \to \mathbb{R}_+$ are of class C^2 and C^1, respectively, and satisfy (7) and the following for some positive constants ι_1, ι_2, ι_3, ι_6 and ι_7

$$\iota_6 \leq \|\nu(r)\| \leq \iota_7, \quad \forall r \in \mathbb{R}. \tag{10}$$

Example 1: Consider a 2-link manipulator in a horizontal plane shown in Fig. 1, which is sketched in Fig. 2. Link L_1 rotates around a fixed axis and Link L_2 is jointed with L_1. Let S_1 and S_2 be the fixed axis and the joint of the links, to which torques τ_1 and τ_2 can be applied, respectively. Let m_i, l_i and I_i be the mass, length of L_i and the moment of inertia of L_i about S_i, respectively, and s_2 be the length between S_2 and the gravity point of L_2. The X-Y axes are set in the plane with the origin S_1. Let θ_1 be the angle between the X axis and Link L_1, and θ_2 be that between L_1 and L_2, counterclockwise. This system is described by (1) with the coordinate $q = [\theta_1\ \theta_2]^\top$, the input $u = [\tau_1\ \tau_2]^\top$, the inertia and Coriolis matrix functions

$$M = \begin{bmatrix} I_1 + I_2 + m_2 l_1^2 + 2m_2 l_1 s_2 \cos\theta_2 & * \\ I_2 + m_2 l_1 s_2 \cos\theta_2 & I_2 \end{bmatrix},$$

$$C = \begin{bmatrix} -m_2 l_1 s_2 \sin\theta_2 \dot{\theta}_2 & -m_2 l_1 s_2 \sin\theta_2 (\dot{\theta}_1 + \dot{\theta}_2) \\ m_2 l_1 s_2 \sin\theta_2 \dot{\theta}_1 & 0 \end{bmatrix},$$

where * is the same to the (2,1)th component, and $D = \mathrm{diag}(d_1, d_2)$, where d_1 and d_2 are the damping coefficients of S_1 and S_2, respectively. As shown in Fig. 1 this manipulator has a stick on the end of the hand, the end-effector, which is surrounded by obstacles, the black and blue cylinders, and cannot go out of the white round area. The space where the end-effector can move is the end-effector, the workspace and the reference path, respectively. Let $x = [X_m\ Y_m]^\top$ be the coordinate of E in the X-Y plane. Then, the mapping

$$x = f(q) = \begin{bmatrix} l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2) \\ l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2) \end{bmatrix},$$

transforms x to the generalized coordinate q. Note that there exist sets $\mathcal{D}_\theta$ and $\mathcal{D}_x \subset \mathbb{R}^2$ such that $f: \mathcal{D}_\theta \to \mathcal{D}_x$ is diffeomorphic. Assume that the workspace $\mathcal{F}_x$ is contained by $\mathcal{D}_x$, and the set

$$\mathcal{F} = \{q = f^{-1}(x) : x \in \mathcal{F}_x\} \subset \mathcal{D}_\theta$$

is defined in the joint coordinate space. Let $\varphi = f^{-1} \circ \varphi_x$ be a reference path in the set $\mathcal{F}$. Our objectives for this robot manipulator are as follow: *the end-effector E does not go out of the workspace $\mathcal{F}_x$, tracks the reference path φ_x and moves at a constant speed vd in a determined direction.* This objectives can be formulated in terms of Problem 1. First, if (3) is realized for the angles q and $\mathcal{F}$, then the end-effector's coordinate x stays in the workspace $\mathcal{F}_x$ and does not collide with obstacles. Next, if there exists a path parameter $r(t)$ satisfying (4) for the angles q and φ, the coordinate x tracks the reference path φ_x. Finally, from (9), the velocity $\|\dot{x}(t)\|$ converges to the desired speed v_d if (8) holds for

$$\nu(r) = v_d \left\| \frac{\partial(f^{-1})}{\partial x}(\varphi_x(r)) \frac{d\varphi_x}{dr}(r) \right\|^{-1}. \tag{11}$$

3. PRELIMINARY

This section presents two primitive methods of obstacle avoidance and trajectory tracking for Lagrange systems based on [7] and [8].

3.1. Obstacle avoidance via navigation functions

It is known that passivity based methods are effective to regulate Lagrange systems [26]. For point to point control to a desired point $q_d \in \mathbb{R}^n$, the PD-like controller

$$u = -\frac{\partial U_{q_d}}{\partial q}^{\mathrm{T}}(q) - D_1\dot{q} \tag{12}$$

is available for a potential function $U_{qd} : \mathbb{R}^m \to \mathbb{R}$ and an $m \times m$ positive definite matrix D_1, which is one of the passivity based controller. The PD-like controller (12) realizes the obstacle avoidance task (3) when the potential function replaces a navigation function, which is defined as follows [6]:

Definition 1: For a workspace $\mathcal{F}$ and a desired position $q_d \in \breve{\mathcal{F}}$, a navigation function $U_{qd}: \mathcal{F} \to [0,1]$ is of class C^2 such that it

(nf1) has the only one minimum extreme at q_d in $\mathcal{F}$,

(nf2) is admissible, that is uniformly takes the maximum value of 1 on the boundary of $\mathcal{F}$,

(nf3) is a Morse function, that is

$$\frac{\partial U_{q_d}}{\partial q}(q) = 0 \Rightarrow \det \frac{\partial^2 U_{q_d}}{\partial q^2}(q) \neq 0.$$

We briefly confirm that the obstacle avoidance (3) is achieved by the input (12) using a navigation function $U_{qd}(q)$. For the closed-loop system (1) with the input (12), let $H(\dot{q}, q) = K(\dot{q}, q) + U_{qd}(q)$ be a candidate for a Lyapunov function. Then, from (1) and (12), the derivative of H along the solution trajectory is given by

$$\dot{H} = -\dot{q}^{\mathrm{T}}(D+D_1)\dot{q}. \tag{13}$$

The function $\dot{H}$ in (13) is negative semi-definite according to the variable $(q, \dot{q})$ because the matrix $D+D_1$ is positive definite. Thus, H monotonically decreases, and $\dot{q}(t) \to 0 (t \to \infty)$ holds. Moreover, LaSalle's theorem guarantees the convergence $q(t) \to q_d$ from (nf1) and (nf3). Then, the following expressions hold for all time t

$$\begin{aligned} U_{qd}(q(t)) &\leq H(\dot{q}(t), q(t)) \leq H(\dot{q}(0), q(0)) \\ &= U_{qd}(q(0)) \leq 1, \end{aligned}$$

where the equation and the third inequality are from the assumptions in (3), and the second inequality is from (13). Thus, (3) holds because the solution $q(t)$ stays in the workspace $\mathcal{F}$ where $U_{qd}(q)$ is less than or equal to 1 from (nf2). Then, the solution $q(t)$ from almost every initial $q(0)$ in $\mathcal{F}$ realizes $q(t) \to q_d (t \to \infty)$ and the obstacle avoidance (3). See [6] for more details.

The way to construct navigation functions has been discussed in [6] and [7], which deal with obstacles in the forms of spheres and star shapes. In this paper, we assume that all obstacles are in the form of spheres and that the obstacles do not overlap each other. Note that our result can be easily generalized to star shape obstacles. Let l be the number of the obstacles. Let $\mathcal{O}_i$ be the set in $\mathbb{R}^m$ describing the ith obstacle. The first obstacle $\mathcal{O}_1$ is the wall surrounding the workspace $\mathcal{F}$ and the rest $\mathcal{O}_i$ $(i = 2,3,\ldots,l)$ are obstacles in the space $\mathbb{R}^m \setminus \mathcal{O}_1$. The workspace is given by $\mathcal{F} = \mathbb{R}^m \setminus \bigcup_{i \in \mathcal{L}} \mathcal{O}_i$, where $\mathcal{L} = \{1,2,\cdots,l\}$. The boundary $\partial\mathcal{O}_i$ $(i \in \mathcal{L})$ is in the form of circle with the radius $\rho_i > 0$ and the center coordinate $p_i \in \mathbb{R}^m$. Then, each obstacle is described as

$$\mathcal{O}_i = \{q \in \mathbb{R}^m : \mathcal{O}_i(q) < 0\}, \quad i \in \mathcal{L}$$

where $\mathcal{O}_i : \mathbb{R}^m \to \mathbb{R}$ are the functions

$$O_1(q) = -\|q - p1\|^2 + \rho_1^2 \tag{14}$$

$$O_i(q) = \|q - pi\|^2 - \rho_1^2 \quad (i = 2,3,\ldots,l). \tag{15}$$

Define the function $\Psi(q) = \lambda \prod_{i=1}^{l} O_i(q)$ for a positive constant λ, which satisfies

$$q \in \breve{\mathcal{F}} \Leftrightarrow \Psi(q) > 0 \quad \text{and} \quad q \notin \mathcal{F} \Leftrightarrow \Psi(q) < 0. \tag{16}$$

The inequalities in (16) mean that the sign of $\Psi(q)$ gives the information on whether q is inside or outside the workspace. Using $\Psi(q)$, a navigation function for a desired position $q_d \in \breve{\mathcal{F}}$ is given by

$$U_{q_d}(q) = \frac{\|q - q_d\|^2}{(\|q - q_d\|^{2\kappa} + \Psi(q))^{1/\kappa}} \tag{17}$$

for a sufficiently large natural number κ. There exists a natural number κ_0 such that (17) is a navigation function for any $\kappa (> \kappa_0)$.

3.2. Path tracking in environments without obstacles

In this subsection, we consider an ordinary path tracking problem for the system (1) in environments without obstacles. Thus, consider the case where the path parameter $r(t)$ is assigned as time t in the path tracking task (4). Then, the reference path $\varphi(r(t))$ becomes just a time-varying function $\varphi(t)$. Let $\bar{q} = q - \varphi$ be the error between the coordinate q and the reference path φ, and the path tracking is achieved when $\bar{q}(t) \to 0$. Let U: $\mathbb{R}^m \times \mathbb{R}_+ \to \mathbb{R}_+$ be a potential function satisfying

$$k_1\|\bar{q}\|^2 \leq U(\bar{q}, t) \leq k_2\|\bar{q}\|^2, \quad \forall t \geq 0, \quad \forall \bar{q} \in \mathbb{R}^m \tag{18}$$

for some positive constants k_1 and k_2. Then, the path tracking is realized by the input

$$u = M(q)\ddot{\varphi} + C(\dot{q}, q)\dot{\varphi} - D\dot{\varphi} - \frac{\partial U}{\partial \bar{q}}^T(\bar{q}, t) - D_1\dot{\bar{q}} \tag{19}$$

with a positive definite matrix $D_1 \in \mathbb{R}^{m \times m}$, which is proposed in [8]. Let us confirm this fact by checking the stability of the closed-loop system of (1)

$$M(q)\ddot{q}+(C(\dot{q},q)+D+D_1)\dot{\bar{q}}+\frac{\partial U}{\partial \bar{q}}^T(\bar{q},t)=0, \tag{20}$$

which is called an error system. For the error system (20), consider the error function

$$\bar{H}(\dot{\bar{q}},\bar{q},q,t)=\frac{1}{2}\dot{\bar{q}}^T M(q)\dot{\bar{q}}+U(\bar{q},t) \tag{21}$$

as a candidate for a Lyapunov function. Using (2), the derivative of (21) is calculated as

$$\begin{aligned}\dot{\bar{H}}&=\dot{\bar{q}}^T M\ddot{\bar{q}}+\frac{1}{2}\dot{\bar{q}}^T\dot{M}\dot{\bar{q}}+\dot{U}\\&=-\dot{\bar{q}}^T(D+D_1)\dot{\bar{q}}+\frac{\partial U}{\partial t}(\bar{q},t).\end{aligned} \tag{22}$$

Thus, if $\partial U/\partial t \le 0$ always holds, $\dot{\bar{H}}$ is negative semi-definite. Then, $\bar{H}$ is a Lyapunov function, which guarantees the stability of the error system (20). Moreover, even if $\partial U/\partial t$ is not always negative semi-definite, but if this function is sufficiently smaller than U, the error system (20) can be locally exponentially stable. Note that the existing papers, e.g., [8], have never dealt with this case. This property is summarized as follows:

Lemma 1: For the Lagrange system (1), a reference path $\varphi(t)\in\mathbb{R}^m$ is given by a bounded C^2 function whose derivative is bounded. Assume that there exists a C^2 function $U:\mathbb{R}^m\times\mathbb{R}_+\to\mathbb{R}$ satisfying (18) and

$$k_3U(\bar{q},t)\le\frac{\partial U}{\partial \bar{q}}(\bar{q},t)\bar{q},\quad \forall t\le 0,\quad \forall \bar{q}\in\mathcal{D} \tag{23}$$

for some positive constants k_1, k_2 and k_3, where $\mathcal{D}\subset\mathbb{R}^m$ is an open set including the origin. Then, there exists a constant $\sigma_0>0$ such that if

$$\frac{\partial U}{\partial t}(\bar{q},t)\le\sigma U(\bar{q},t),\quad \forall\bar{q}\in\mathcal{D},\quad \forall t\ge 0, \tag{24}$$

holds for any positive numbers $\sigma(<\sigma_0)$, the error system (20) is locally exponentially stable.

4. PATH TRACKING WITH OBSTACLE AVOIDANCE

The combination of the obstacle avoidance and path tracking methods presented in Section 3 seems to be available to achieve both the path tracking and obstacle avoidance tasks. This idea yields the tracking controller (19) using a navigation function $U_d(q)$ instead of the original potential function $U(\bar{q},t)$. Subsection 4.1 shows that this strategy causes a problem such that the obstacle avoidance (3) is not achieved anymore. Then, to overcome this problem, Subsection 4.2 gives the idea of adjusting the path parameter r, and proposes an effective adjustment law for this parameter.

4.1. New condition for collision avoidance

The first idea is to utilize a navigation function in the tracking controller (19) as the potential function $U(\bar{q},t)$. For this purpose, we use a navigation function $U_{qd}(q)$ to assign the potential function as follows:

$$U(\bar{q},\varphi(r))=U_{q_d}(q)\big|_{q_d=\varphi(r),q=\bar{q}+\varphi(r)}, \tag{25}$$

whose variables are the tracking error $\bar{q}$ $(=q-\varphi(r))$ and the reference path $\varphi(r)$. If $U(q-\varphi(r),\varphi(r))$ is a navigation function for $\varphi(r)$ for any $r\in\mathbb{R}_+$ in the sense of (25), U is called a navigation function for the path φ. The following is an example of the navigation function for $\varphi(r)$ corresponding to (17).

$$U(\bar{q},\varphi(r))=\frac{\|\bar{q}\|^2}{(\|\bar{q}\|^{2\kappa}+\Psi(\bar{q}+\varphi(r)))^{1/\kappa}} \tag{26}$$

One may think that the tracking controller (19) with the navigation function $U(\bar{q},\varphi(r))$ in (26) realizes both the collision avoidance task (3) and the path tracking task (4). However, this is not the case. Namely, this controller cannot guarantee collision avoidance. In order to confirm this fact, consider the error function $\bar{H}(\dot{\bar{q}},\bar{q},q,r)$ in (21) whose potential function $U(\bar{q},t)$ is replaced by $U(\bar{q},\varphi(r))$ as a candidate for a Lyapunov function of the error system (20). By rewriting (22) with $\varphi(r)$ instead of q_d, the derivative of $\bar{H}$ along the trajectory is calculated as

$$\dot{\bar{H}}=-\dot{\bar{q}}^T(D+D_1)\dot{\bar{q}}+\frac{\partial U}{\partial\varphi}(\bar{q},\varphi(r))\frac{d\varphi}{dr}(r)\dot{r}. \tag{27}$$

The second term is not always negative, or does not converge to 0 because of the velocity assignment (8) for $\dot{r}$ and the assumption (10) on ν and $d\varphi/dr$. Thus, $\dot{\bar{H}}(t)$ is not negative semi-definite, which means that $\bar{H}$ is not a Lyapunov function. Moreover, it is unable to cancel the second term in (27) by any additional terms to the input u. As a result, the collision avoidance task (3) and the path tracking task (4) are not guaranteed by the tracking controller (19) even if the navigation function $U(\bar{q},\varphi(r))$ is used.

Now, we give up to guarantee the negative semi-definiteness of $\dot{\bar{H}}$. Before coming up with the main idea, we derive less conservative sufficient conditions for (3) and (4). Let the variable $g\in\mathbb{R}$ be the solution of the differential equation

$$\begin{aligned}\dot{g}&=-\dot{\bar{q}}^T D_2\dot{\bar{q}}+\frac{\partial U}{\partial\varphi}\frac{d\varphi}{dr}\dot{r},\\ g(0)&=U(\bar{q}(0),\varphi(r(0))),\end{aligned} \tag{28}$$

where $D_2\in\mathbb{R}^{m\times m}$ satisfies $D+D_1>D_2>0$. Then, a less conservative condition for (3) and (4) is given by

$$\dot{r}(0)=0,\quad g(t)\le 1,\quad \forall t\ge 0. \tag{29}$$

This fact is briefly explained as follows: from (27), (28) and (29), the expression

$$\bar{H}(t) = g(t) - \int_0^t \dot{\bar{q}}(t)^T (D + D_1 - D_2)\dot{\bar{q}}(t) \leq g(t) \leq 1 \quad (30)$$

holds. Then, the obstacle avoidance (3) is achieved from (nf2) because $U(t) \leq \bar{H}(t) \leq 1$ holds for any time t. Moreover, it can be shown that (29) yields

$$\dot{\bar{q}}(t) \to 0, \quad \frac{\partial U}{\partial \bar{q}}(t) \to 0 (t \to \infty). \quad (31)$$

Note that the second expression in (31) means that $q(t)$ converges to either of the reference path $\varphi(r)$) or a saddle point of U. To prevent $q(t)$ from converging to the latter, we let $\dot{r}$ to stop if q is close to obstacles. This condition is described as

$$U(\bar{q},\varphi(r)) \geq Um-\vartheta \Rightarrow \dot{r}=0 \quad (32)$$

for a positive ϑ and the minimum value U_m of U in the neighborhood of obstacles. Then, the path tracking task (4) is achieved for almost every initial position $q(0)$. From the above discussion, the following is derived:

Lemma 2: For the system (1), the workspace $\mathcal{F}$, the reference path φ and the desired velocity v, assume that there exists a navigation function U for φ. Then, the following statements hold with the scalar variable $g(t)$ and the input u given by (28) and (19), respectively.

(pt1) If (29) holds, then the obstacle avoidance (3) is achieved, and $\| \dot{\bar{q}}(t) \|^2 \leq 2/\mu_1$ holds for any time t.

(pt2) Moreover, if $\dot{r}$ and $\ddot{r}$ are bounded, then the input (19) is bounded, and (31) holds.

(pt3) Moreover, if there exists a positive constant ϑ satisfying (32), where

$$U_m = \min_{q \in \varepsilon \cap \mathcal{F}, \varphi \in \mathcal{F}_\varepsilon} U(q-\varphi,\varphi)$$

$$\varepsilon = \{q : \partial U/\partial \bar{q} \ (q-\varphi, \varphi) = 0\} \setminus \{\varphi\},$$

then the path tracking (4) is achieved for almost every initial position $q(0) \in \mathcal{F}$.

4.2. New adjustment law for path parameter

Lemma 2 reduces Problem 1 to designing the path parameter r which satisfies (8), (29) and (32) and whose 1st and 2nd derivatives are bounded. For the navigation function (26), we assign an adjustment law for the path parameter r as the dynamics

$$\dot{r} = v(r)\beta(U)(1-e^{-\gamma_1 t})\frac{1-g}{(1-g)+\gamma_2 U^{\gamma_3}}, \quad (33)$$

where γ_1 and γ_2 are positive constants, $\gamma_3 < 1$ is a positive constant, and $\beta : [0,1] \to \mathbb{R}$ is a C^1 function satisfying

$$\beta(0)=1, \beta(U)=0 \text{ for } U \in [\gamma_4,1] \quad (34)$$

for a positive constant $\gamma_4 < 1$. Let us briefly confirm that this r satisfies the above requests. First, the numerator of the last term in (33) makes $r(t)$ slow down when $g(t)$ is close to 1, which implies the inequality in (29). Thus, the obstacle avoidance (3) is obtained from (pt1) in Lemma 2. Next, the input (19) is bounded from (pt2) because $\dot{r}$ and $\ddot{r}$ are bounded as shown in the proof of Theorem 1. Finally, $\beta(U)$ satisfying (34) guarantees (32), then the path tracking (4) is achieved from (pt3). Then, $U(t)$ and $e^{-\gamma_1 t}$ converge to 0, and $\dot{r}(t)$ converges to the desired velocity $v(r(t))$, which means the velocity assignment (8). Our main result is given as follows:

Theorem 1: Consider the system (1), the workspace $\mathcal{F}$, the reference path φ and the desired velocity v satisfying (as1)–(as4). Assume that (26) is a navigation function for φ with any $\kappa(>\kappa_0)$ where κ_0 is a natural number. Then, there exists a natural number $\bar{\kappa} (\geq \kappa_0)$ such that for any natural number $\kappa(>\bar{\kappa})$, the dynamic controller which consists of (19), (28) and (33) solves Problem 1 for almost every initial position $q(0) \in \mathcal{F}$. Here, D_1 and D_2 are m×m matrices satisfying $D+D_1>D_2>0$, γ_1 and γ_2 are positive constants, γ_3 is a constant satisfying $0.5 \leq \gamma_3 < 1$ and β is a C^1 function satisfying (34) for a positive $\gamma_4(<1)$.

As for the convergence rate of the error $\bar{q}$, the error system (20) is locally exponentially stable. The following theorem presents this property, which is directly derived from Lemma 1.

Theorem 2: Consider the workspace $\mathcal{F}$ and the reference path φ. Assume that a navigation function $U(\bar{q}, \varphi)$ is given by (26), that (3) holds and that $\dot{r}$ is bounded. Then, there exists a natural number $\tilde{\kappa}$ such that for any natural number $\kappa(>\tilde{\kappa})$, the error system (20) is locally exponentially stable with the controller given in Theorem 1.

Although Theorem 2 guarantees the locally exponential stability of the error system, we cannot arbitrarily choose the convergence rate of the state $(\dot{q}, q)$ with the controller proposed in Theorem 1. This is because the navigation function (26) have no design parameters corresponding to position gains. In order to adjust the convergence rate, we can multiply the navigation function $U(\bar{q}, t)$ by a positive constant k. The next corollary shows that even if we use the gained potential function $kU(\bar{q}, t)$, obstacle avoidance and path tracking are realized.

Corollary 1: For a positive constant $k \geq 1$, replace $U(\bar{q}, t)$ for $kU(\bar{q}, t)$, the condition $\gamma_4 <1$ for $\gamma_4< k$, $1-g$ in (33) for $k-g$, and $[\gamma_4,1]$ in (34) for $[\gamma_4,k]$ in Theorem 1. Then, the controller presented by Theorem 1 solves Problem 1 for almost every initial position $q(0) \in \mathcal{F}$ for any $\kappa(>\tilde{\kappa})$, where $\tilde{\kappa}$ is independent of k. Moreover, we can determine the local convergence rate of the error system (20) arbitrarily by the choice of the design parameter k.

5. EXPERIMENTAL RESULTS

Consider the 2-link manipulator and environment shown in Figs. 1 and 2. The mass, length and moment of inertia of the manipulator's links are m_1=12, m_2=2.2kg, l_1 = 0.50, l_2 = 0.50m and I_1 = 1.0, I_2 = 0.15kgm^2, the length of the gravity center is s_2 = 0.21m, and the damping coefficients of the joints are d_1 = 0.75, d_2=0.058kgm^2/s. Let $x(0)$=(0.92,−0.21)m be the initial position. The workspace $\mathcal{F}_x$ is described in Fig. 2, where the boundaries of $\mathcal{O}_{x1}$, $\mathcal{O}_{x2}$ and $\mathcal{O}_{x3}$ are given by the circles with the centers pi = (0.5,0), (0.5,0), (0.9,−0.08) and radii ρi = 0.5, 0.09, 0.25m. Let the reference path φ_x of the end-effector be the circle described by

$$\varphi_x(r) = 0.3[\cos(r)\ \sin(r)]^\top + [0.5\ 0.0]^\top[\mathrm{m}].$$

Our objective is to let the end-effector to move along the trajectory φ_x counterclockwise at the desired velocity v_d = 0.5. This control objective is described by the obstacle avoidance task (3), path tracking task (4) and (8) with the function ν in (11).

Let us design a controller to achieve these tasks. The navigation function is given in the form of (26) with $O_i(q) = O_{xi}(f(q))$ where $O_{xi}(x)$ is given by replacing q and i for x and xi, respectively, in (14) and (15). Note that $\mathcal{O}_i = \{q : O_i(q) < 0\}$ is a star shape set. The desired velocity $\nu(r)$ is given by the positive function in (8). The control input u consists of (19), (28) and (33) with the design parameters

$$\kappa = 8,\quad \lambda = 2^8,\quad D_1 = \mathrm{diag}(60,12),\quad D_2=\mathrm{diag}(30,6),$$
$$k=10,\quad \gamma_1=1,\quad \gamma_2=10,\quad \gamma_3=0.8,\quad \gamma_4=5,$$
$$\beta(U) = \begin{cases} (U-\gamma_4)^2/\gamma_4^2, & U < \gamma_4 \\ 0, & U \geq \gamma_4. \end{cases} \tag{35}$$

Theorem 1 and Corollary 1 guarantee that all the tasks (3), (4) and (8) are realized by using this controller. The designed controller is implemented in a personal computer (CPU: Pentium III 866MHz, Memory:192MB) with a low-pass filter 1/(0.005s+1) and a Coulomb friction compensation. The sampling time is 1 ms.

Fig. 3 illustrates the motion of the position x of the end-effector in an experiment. The solid curve depicts x from t = 0 to 20, and the dashed-dotted curves show the boundaries of the obstacles $\mathcal{O}_{xi}$. The workspace $\mathcal{F}_x$ is the region surrounded by the dashed-dotted curves where the solid curve exists. This figure shows that the end-effector of the manipulator stays in the workspace $\mathcal{F}_x$; thus, the obstacle avoidance (3) is realized. Moreover, it converges to the circular reference path φ_x and moves along the reference path φ_x counterclockwise after while. As shown in Fig. 4, the errors $\theta_1-\varphi_1(r)$ and $\theta_2-\varphi_2(r)$ between the angles of the joints and their desired ones converge to 0; thus the path tracking (4) is achieved. Fig. 5 depicts the time plot of the velocity $\|\dot{x}\|$ of the end-effector. The velocity reaches to the desired velocity v_d = 0.5, which implies that the velocity assignment (8) is realized. Fig. 6 shows the time plot of the input torques τ_1 and τ_2 of the motors, which are not so large even if the position x is close to the obstacles because they are guaranteed to be bounded from the property of the navigation function. This experimental result shows that all the assigned tasks (3), (4) and (8) are realized with the bounded input signals due to the proposed

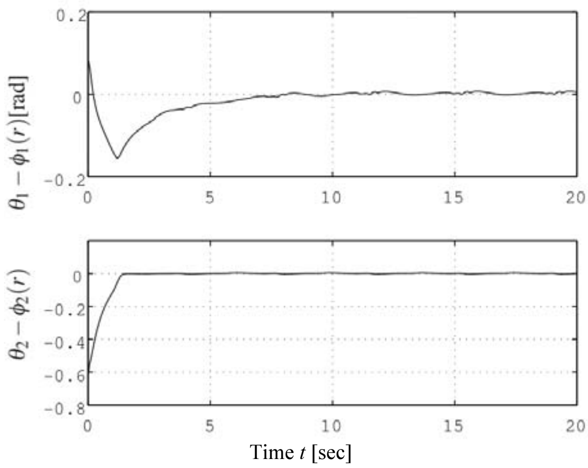

Fig. 4. Tracking errors between experimental and desired angles.

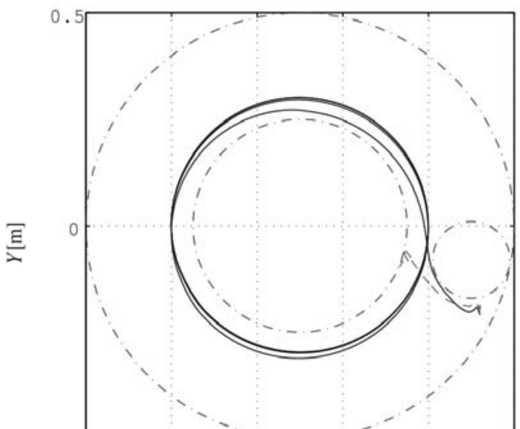

Fig. 3. Trajectory of end-effector in workspace.

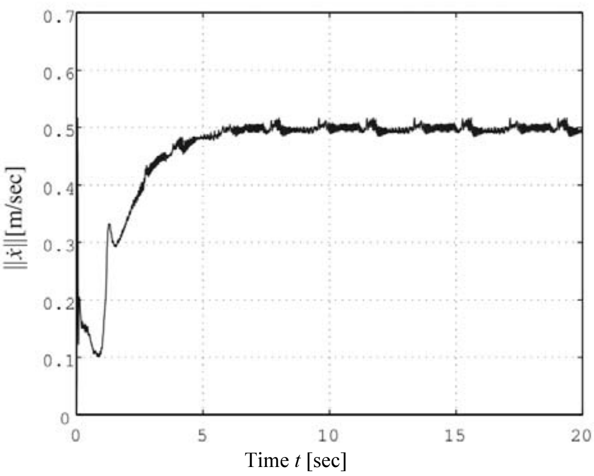

Fig. 5. Velocity of the end-effector.

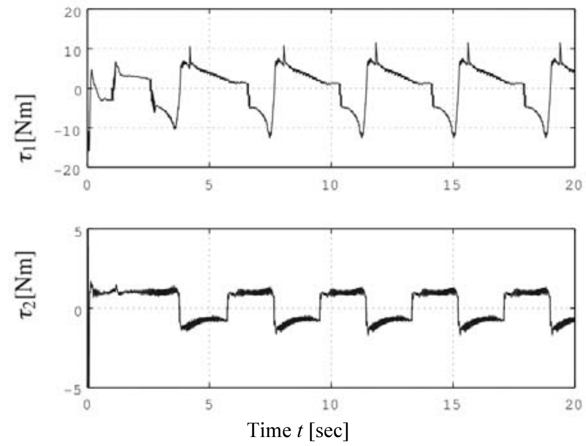

Fig. 6. Input torques.

controller. Note that the vibrations in Figs. 5 and 6 come from the motion properties of the motors and the difference operation of sensor signals.

Now, we compare the proposed controller with the ordinary tracking one without feedback of the path parameter r. The path parameter's velocity is directly assigned as $\dot{r} = \nu(r)$ without the adjustment law (33), and the controller (19) is employed with the same parameters to (35). The dashed curve in Fig. 3 describes the motion of x before t = 1.2, which shows that the end-effector crashes into one obstacle. This result shows that if $\dot{r}$ is not appropriately adjusted, the obstacle avoidance is not achieved because the error function $\bar{H}$ is possible to increase as mentioned in Section 4.1. These results illustrate the effectiveness of the proposed method.

6. CONCLUSION

This paper addressed a path tracking problem with obstacle avoidance for Lagrange systems. The proposed method was based on artificial potential function methods using navigation functions. In order to obtain the advantages of navigation functions, a navigation function was inserted into the ordinary tracking controller, which could not guarantee collision avoidance. The main contribution of this paper was to propose a feedback law for the path parameter which characterizes a given reference path, thereby ensuring effective obstacle avoidance and path tracking. This fact was proved theoretically. The proposed method has various merits inherited from navigation functions such that designed controllers generate bounded input signals and the corresponding potential functions are easily designed from shapes of obstacles. The experimental results with the 2-link manipulator showed that both obstacle avoidance and path tracking were achieved with bounded input signals by using the proposed method while the controller without adjusting the path parameter could not realize collision avoidance. These results illustrated the effectiveness of the proposed method. This method can be applied to various mechanical systems including mobile robots. The future work includes safe cruising of mobile robots surrounded by obstacles in more complicated shapes.

APPENDIX A

Proof of Proposition 1: From the first, second, fourth and fifth inequalities in (7), the function $\nu(r)$ given by (8) satisfies $\iota_4/\iota_2 \le \nu(r) \le \iota_5/\iota_1$. Therefore, (6) is fulfilled because (8) holds. From (4) and (8),

$$\left| \| \dot{q}(t) \| - vd(r(t)) \right| \le \left| \| \dot{q}(t) \| - \| \dot{\varphi}(r(t)) \| \right| \\ + \| \dot{\varphi}(r(t)) \| - vd(r(t)) \to 0$$

holds, which means (5). The proof is completed.

Proof of Lemma 2: Proof of (pt1): (3) is shown in the first paragraph of Section 4.2. The inequality $\| \dot{\bar{q}} \|^2 \le 2/\mu_1$ holds because $\dot{\bar{q}}^{\top} M \dot{\bar{q}}/2 \le 1$ holds from (21), (29) and (30).

Proof of the boundness of the input (19) in (pt2): Because all functions, e.g., M, C, are continuous for $\dot{q}, \dot{\bar{q}}, \ddot{\varphi}, \dot{\varphi} \in \mathbb{R}^n$, $q \in \mathcal{F}$ and $\varphi \in \mathcal{F}_{\mathcal{E}}$, we have only to show that the variables $\dot{q}, \dot{\bar{q}}, \ddot{\varphi}$ and $\dot{\varphi}$ are bounded. First, (pt1) guarantees the boundedness of $\dot{\bar{q}}$. Next, from (as3) and the boundedness of $\dot{r}$, the function $\dot{\varphi}$ is bounded, and so is $\dot{q}$. Moreover, $\ddot{\varphi}(r)$ is bounded from the boundedness of $\dot{r}$ and $\ddot{r}$, and (as3).

Proof of (31) in (pt2): The above discussion and (20) guarantee that $\ddot{\bar{q}}$ is bounded. Note that $\dot{\bar{q}} \in \mathcal{L}_2$ is shown later. Then, by using Barbalat's lemma for $h(t) = \dot{\bar{q}}_i(t)$, the first equation in (31) holds, where $\bar{q}_i$ denotes the ith component of $\bar{q}$. Moreover, from the boundedness of $\dot{q}$, $\dot{q}$, $\dot{\bar{q}}$ and $\ddot{\bar{q}}$, the variables q, $\dot{q}$, $\bar{q}$ and $\dot{\bar{q}}$ are bounded and uniformly continuous. Thus, from (20), $\ddot{\bar{q}}$ is uniformly continuous, which implies $\ddot{\bar{q}}(t) \to 0$ $(t \to \infty)$ from the first in (31) and Barbalat's lemma. Moreover, from (20), the second equation in (31) holds. Now, we will show that $\dot{\bar{q}} \in \mathcal{L}_2$. Because $M(q) \ge \mu_1 I$, we have only to show that

$$P(t) = \frac{1}{2}\int_0^t \dot{\bar{q}}(t)^T M(q(t)) \dot{\bar{q}}(t) dt$$

is bounded. From (21) and (30),

$$\dot{P} \le -(\lambda_{\min}(D + D_1 - D_2)/\mu_2)P + g - U \tag{A.1}$$

is derived, where $\lambda_{\min}(\cdot)$ denotes the minimum eigenvalue of a matrix. Let Σ be the differential equation given by replacing $\le$ with $=$ in (A.1). Let $P_m(t)$ be the solution of Σ with the initial state $P_m(0) = P(0) = 0$. Then, $P(t) \le P_m(t)$, $\forall t \ge 0$ holds from the comparison theorem. Because U and g are bounded, Σ is the linear system with the damping rate $\lambda_{\min}(D + D_1 - D_2)/\mu_2 > 0$ and the bounded disturbance $g - U$. Because this system is bounded-input bounded-output stable, Pm is bounded. From this fact, $P \le P_m$ and $P \ge 0$, P is bounded. Thus, $\dot{\bar{q}} \in \mathcal{L}_2$ holds.

Proof of (pt3): In order to show $q(t) \to \varphi(r(t))(t \to \infty)$, we have only to show that there exists a time t_0 such that

$$U(t) < U_m, \quad \forall t \geq t_0, \tag{A.2}$$

because $q(t)$ converges to $\varphi(r(t))$ or the set $\mathcal{E}$ from (pt2), and the convergence to the latter contradicts (A.2) from the definition of U_m. Let t_a be a time such that for a positive constant $\vartheta_0\ (<\vartheta)$,

$$\bar{K}(t) \leq \vartheta_0, \quad \forall t \geq t_a, \quad \bar{K} = \frac{1}{2}\dot{\bar{q}}^T M(q)\dot{\bar{q}} \tag{A.3}$$

holds. Such t_a necessarily exists because $\dot{\bar{q}}(t) \to 0$ from (pt2). Let t_b be a time such that

$$U(t_b) < Um - \vartheta. \tag{A.4}$$

In order to show that there exists such t_b, we assume that $U(t) \geq U_m - \vartheta$ always holds, and derive a contradiction. Then, because $\dot{r}=0$ from (32), $\varphi(r(t))$ does not move. Thus, from the discussion in Section 3.1., $q(t) \to \varphi(r(0))$ holds for almost all $q(0) \in \mathcal{F}$. This fact and (nf1) imply that $U(t) \to 0$, which contradicts that $U(t) \geq U_m - \vartheta$. Therefore, there exists a time t_b satisfying (A.4) for almost every initial coordinate $q(0)$. Now, we let t_0'' be t_b, and t_k, t_k' and t_k'' $(k = 1,2,\cdots)$ be the times such that

$$\begin{cases} U(t_k) = U(t''_k) = U_m - \vartheta, & U(t\mathrm{n}_k) > U_m - (\vartheta - \vartheta_0) \\ U(t) > U_m - \vartheta, & t \in (t_k, t''_k) \end{cases} \tag{A.5}$$

and $t''_{k-1} < t_k < t'_k < t''_k$. Because $\dot{U}$ is bounded from (22), the sequence $\{t_k\}$ has no accumulation points. Moreover, this sequence consists of finite elements as follows: Assume that this sequence has infinite elements, then there exists tc such that $t_c = \min\{t_k: t_k \geq t_a\}$. At the time $t \in (t_k, t''_k)$, $\dot{r}=0$ holds from (32), and then $\bar{H}(t)$ does not increase from (27). Thus, the expressions

$$U(t) \leq \bar{H}(t) \leq \bar{H}(t_k) = \bar{K}(t_k) + U_m - \vartheta \tag{A.6}$$

hold. Now, for $t_k(\geq t_c)$, from (A.3) and (A.6), the inequality $U(t) \leq U_m - (\vartheta - \vartheta_0)$ holds for $t \in (t_k, t''_k)$, which contradicts the existence of t'_k in (A.5). Thus, the sequence $\{t_k\}$ consists of finite elements. Then, the inequality $U(t) \leq U_m(\vartheta - \vartheta_0)$ holds for $t_0 = \max\{t_k\}$, which is (A.2). The proof is completed.

Proof of Theorem 1: We show the obstacle avoidance (3) and path following (4) for $\dot{r}$ given in (33) using Lemma 2. First, (29) holds, where the second expression holds because the second term in the right hand side of the upper expression in (28) is 0 when $g(t)=1$. Thus, (pt1) guarantees (3) and the boundedness of $\dot{\bar{q}}$. Assumption (as2) and the fact $|\dot{r}| \leq |v(r)|$, which is from (33), guarantee that $\dot{r}$ is bounded. For a natural number $\tilde{\kappa}$, $\ddot{r}$ is bounded for any $\kappa(>\tilde{\kappa})$ as shown later. From these facts, (pt2) guarantees (31) and the boundedness of the input (19). Finally, to use (pt3), we will show (32). Choose a positive $\vartheta\ (<1-\gamma_4)$, and for $\gamma = \gamma_4 + \vartheta\ (<1)$, there exists a natural number κ_1 such that $U_m \geq \gamma_4 + \vartheta$ for $\kappa > \kappa_1$ by using (A.8) in Lemma 3 for (25), which is given below. This fact and (34) give the expressions

$$U \geq U_m - \vartheta \Rightarrow U \geq \gamma_4 \Rightarrow \beta(U)=0 \Rightarrow \dot{r}=0,$$

which is just (32). Thus, the assumption of (pt3) is satisfied, and (4) holds for almost every $q(0) \in \mathcal{F}$. Finally, the velocity assignment (8) is achieved because of the convergences of $U(t)$ and $e^{-\gamma_1 t}$ to 0 in (33). Therefore, for $\bar{\kappa} = \max\{\kappa_0, \kappa_1, \tilde{\kappa}\}$, the solution $(\dot{q}(t), q(t))$ of (1) from every initial state $(\dot{q}(0), q(0)) \in \{0\} \times \mathcal{F}$ satisfies the control objectives (3), (4) and (8).

The rest of the proof is to show the boundedness of $\ddot{r}$ for any $\kappa(>\tilde{\kappa})$. Note that the denominator of the right hand side of (33) is more than or equal to a positive constant δ as shown later. Thus, if the derivative of every function composing (33) is bounded, then $\ddot{r}$ is bounded. The functions $\dot{v}(r)$, $\dot{\beta}(U)$ and $\dot{g}$ are bounded from (as2) and the boundedness of $\dot{r}$, $d\beta/dU$, $\dot{U}$ and $\dot{\bar{q}}$. Moreover, $|dU^{\gamma_3}/dt|$ is calculated as

$$\left|\frac{dU^{\gamma_3}}{dt}\right| = \frac{\gamma_3 |\dot{U}|}{U^{1-\gamma_3}} \leq \gamma_3 \frac{\|U_\phi\|\|\varphi_r \dot{r}\| + \|U_{\bar{q}}\|\|\dot{\bar{q}}\|}{U^{1-\gamma_3}}, \tag{A.7}$$

where $U_{\bar{q}} = \partial U(\bar{q},\varphi)/\partial\bar{q}$, $U_\phi = \partial U(\bar{q},\varphi)/\partial\varphi$ and $\varphi_r = d\varphi/dr$. Note that

$$\frac{\|U_\varphi\|}{U^{1-\gamma_3}} = \frac{\|\bar{q}\|^{2\gamma_3}\|\Psi_q\|/\kappa}{(\|\bar{q}\|^{2\kappa} + \Psi)^{\gamma_3/\kappa+1}},$$

$$\frac{\|U_{\bar{q}}\|}{U^{1-\gamma_3}} \leq \frac{2\Psi\|\bar{q}\|^{2\gamma_3-1} + \|\Psi_q\|\|\bar{q}\|^{2\gamma_3}/\kappa}{(\|\bar{q}\|^{2\kappa} + \Psi)^{\gamma_3/\kappa+1}},$$

where $\Psi_q = \partial\Psi(q)/\partial q$, have the bounded right hand sides from $\gamma_3 \geq 0.5$ and (A.14) in Lemma 4. Thus, the right hand side of (A.7) is bounded, and so is $\ddot{r}$. Finally, the denominator of the right hand side of (33) is positive because at least one of $U(t)$ and $1-g$ is always positive, which completes the proof.

Lemma 3: There exists a natural number κ_1 for any positive number $\gamma < 1$ such that for any natural number $\kappa(\geq \kappa_1)$, the function $U_{qd}(q)$ in (17) satisfies

$$U_{qd}(q) \geq \gamma, \quad \forall q \in \mathcal{F} \cap \mathcal{E}_0, \quad \forall\, q_d \in \mathcal{F} \tag{A.8}$$

for any natural number $\kappa(\geq\kappa_1)$, where $\mathcal{E}_0 = \{q: \partial U_{qd}/\partial q(q) = 0\} \setminus \{q_d\}$.

Proof: First, we calculate $\partial U_{qd}/\partial q(q-q_d)$ as

$$\frac{\partial U_{q_d}}{\partial q}(q-q_d) = KU(\bar{q},t), \quad K = \frac{2\Psi - \Psi_q \bar{q}/\kappa}{\|\bar{q}\|^{2\kappa} + \Psi}, \tag{A.9}$$

where $\Psi_q = \partial\Psi/\partial q$ and $\bar{q} = q - q_d$. From the relations $q \in \mathcal{E}_0 \Rightarrow KU_{qd}(q)=0$ and $q = q_d \Leftrightarrow U_{qd}(q)=0$, the relation $q \in \mathcal{E}_0 \Rightarrow K=0$ holds. Thus, the following relations hold:

$$(q \in \mathcal{F} \cap \mathcal{E}_0,\ q_d \in \mathcal{F}) \Rightarrow (q \in \mathcal{F},\ \ q_d \in \mathcal{F}, K = 0)$$

$$\Rightarrow \left(q \in \mathcal{F}, \;\; q_d \in \mathcal{F}_\varepsilon, \;\; \Psi = \frac{\Psi_q(q)(q-q_d)}{2\kappa} \right) \qquad \text{(A.10)}$$
$$\Rightarrow \Psi \le \frac{\gamma_0}{\kappa},$$

where $\gamma_0 = \max_{q\in\mathcal{F}, qd\in\mathcal{F}_\varepsilon} \left|\Psi_q(q)(q-q_d)\right|/2$, which is defined because of the continuousness of Ψ_q and the compactness of $\mathcal{F}$ and $\mathcal{F}_\varepsilon$. There exists a positive number $c > 0$ for any $q \in \mathcal{F}$ such that

$$\|q-\eta\| \le c\Psi(q), \quad \exists \eta \in \partial\mathcal{F}, \qquad \text{(A.11)}$$

for a certain η depending on q, because $q \in \mathcal{F}$ and $\eta \in \partial\mathcal{F}$ are not identical, and $\Psi(q)$ is positive for $q \in \mathcal{F}$. Therefore, for $q \in \mathcal{F}$, some $\eta \in \partial\mathcal{F}$ guarantees that

$$\|q - q_d\| \ge \|q_d - \eta\| - \|\eta - q\| \ge \varepsilon - c\frac{\gamma_0}{\kappa}, \qquad \text{(A.12)}$$

where the second inequality is from (A.10), (A.11) and $q_d \in \mathcal{F}_\varepsilon$. Moreover, from (17), (A.10) and (A.12),

$$U_{q_d}(q) \ge \frac{(\varepsilon - c\gamma_0/\kappa)}{((\varepsilon - c\gamma_0/\kappa)^{2\kappa} + \gamma_0/\kappa)^{1/\kappa}} \qquad \text{(A.13)}$$

holds for any $q \in \mathcal{F} \cap \mathcal{E}_0$ and $q_d \in \mathcal{F}_\varepsilon$. The right hand side of (A.13) converges to 1 as $\kappa \to \infty$. Thus, there exists a natural number κ_1 such that every natural number κ larger than it guarantees that the right hand side of (A.13) is larger than $\gamma (< 1)$. Then, (A.8) holds.

Lemma 4: For the function $\Psi(q)$, there exist positive numbers ε_1 and ε_2 such that

$$\varepsilon_1 \le \|q-q_d\|^{2\kappa} + \Psi(q) \le \varepsilon_2, \; \forall q \in \mathcal{F}, \; \forall q_d \in \mathcal{F}_\varepsilon. \qquad \text{(A.14)}$$

Proof of Lemma 4 and Theorem 2: See paper [25].

Proof of Corollary 1: This corollary is direct from Theorem 2 except that $\tilde{\kappa}$ is independent of k. We have to prove that two $\tilde{\kappa}$ s in Theorems 1 and 2 do not change by the replacement of U with kU. First, $\tilde{\kappa}$ in Theorem 1 is given by κ_1 in Lemma 3, which is determined so that the right hand side of (A.13) is larger than $\gamma (< 1)$. This relation does not change when we replace U_{qd} and $\gamma < 1$ with kU_{qd} and $\gamma < k$, respectively. $\tilde{\kappa}$ in Theorem 2 can be proved in the same manner. The proof is completed.

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