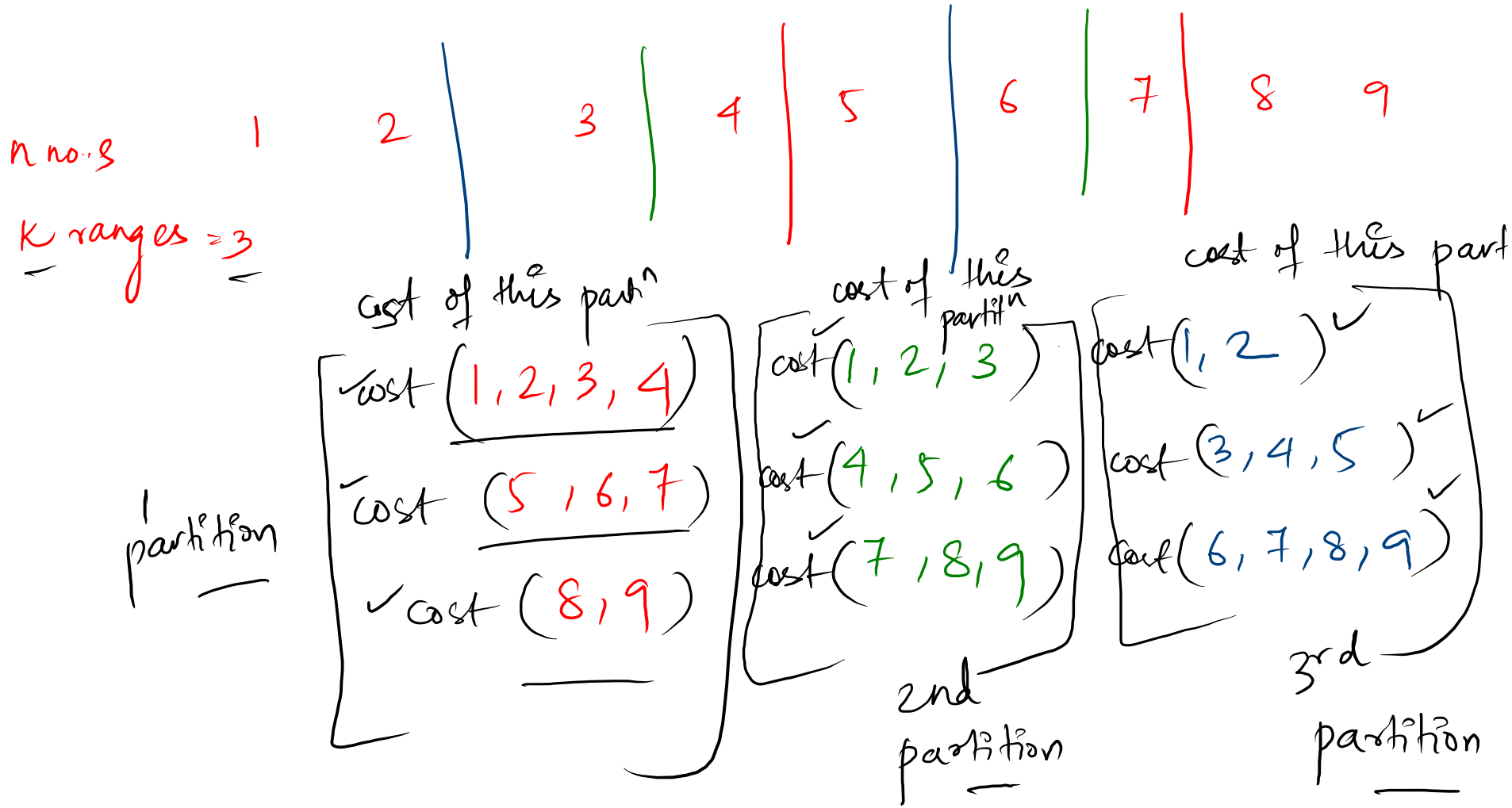
Integer Partition w/o Rearrangement

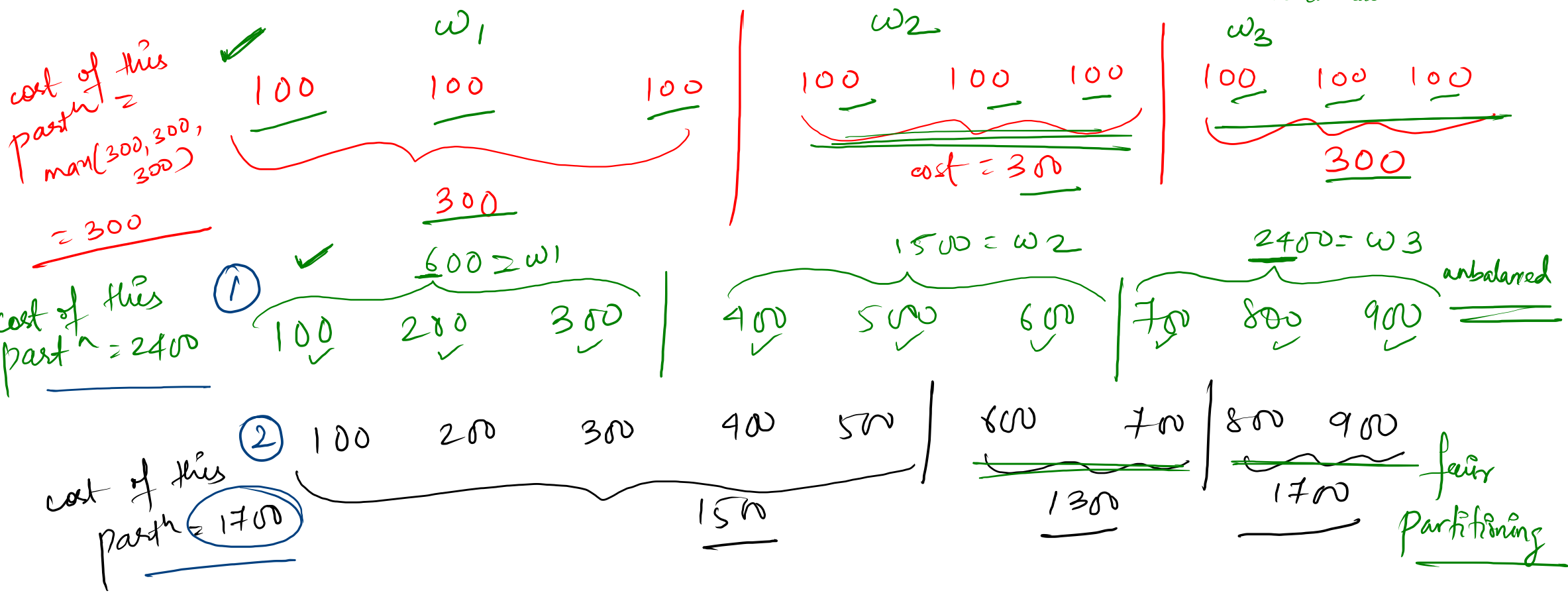


minimize the max partn cost across different partitions.

max cost from all its parts

no. of pages in a book

10 characters

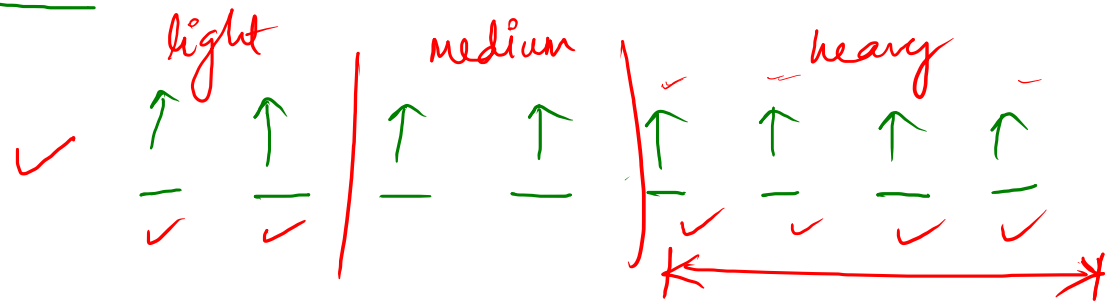


$S = \{ S_1, S_2 \ldots S_n \}, K$

non negative nos

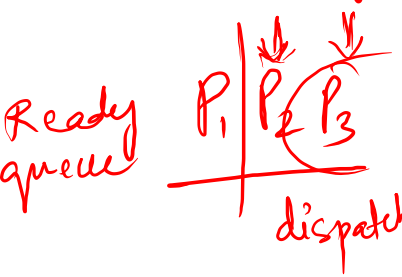
Total elapsed run time

11:00 → 3:00 ?



multicore processors

heavy



8 cores

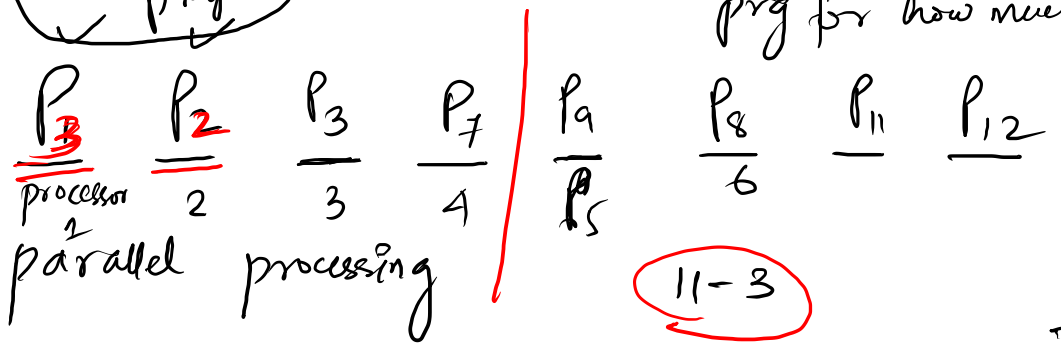
"core"

extremely bad heavy progms

11:00 – 3:00 ⇒ which processor ran which prg for how much time?

4096

8192



parallel processing

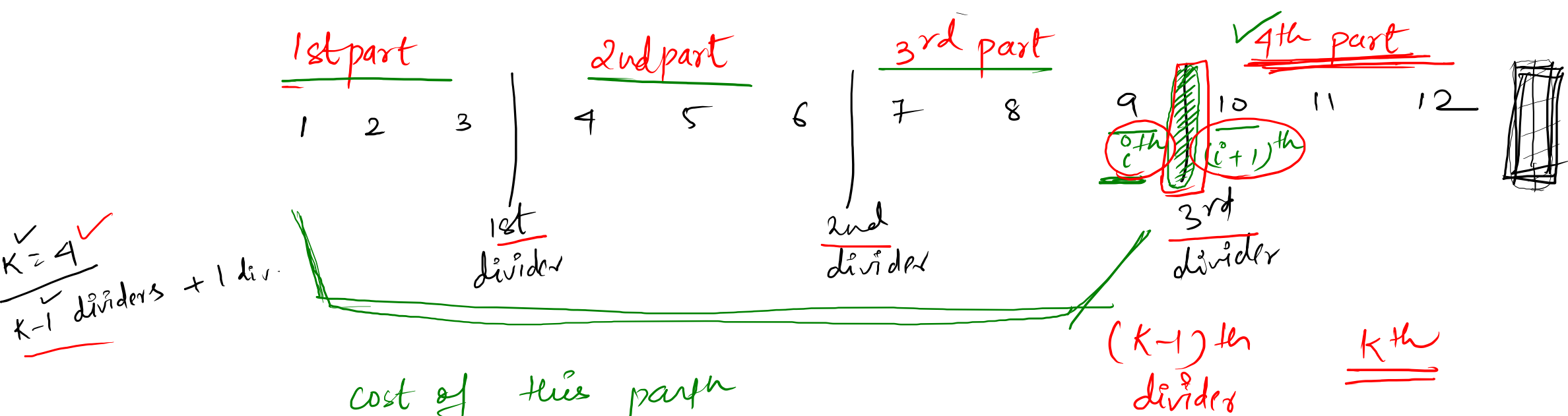
11-3

11-12 ???

12-3 idle

11-2 idle

2-3



cost of this partn

= max(max cost so far , cost of the last partn)

M[0, K]

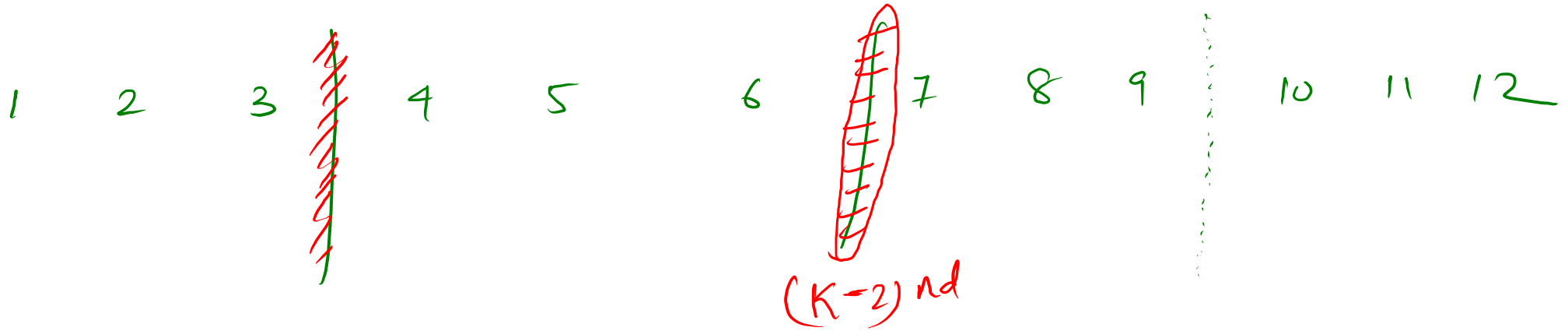
min (maxcost)

M[i, K-1]

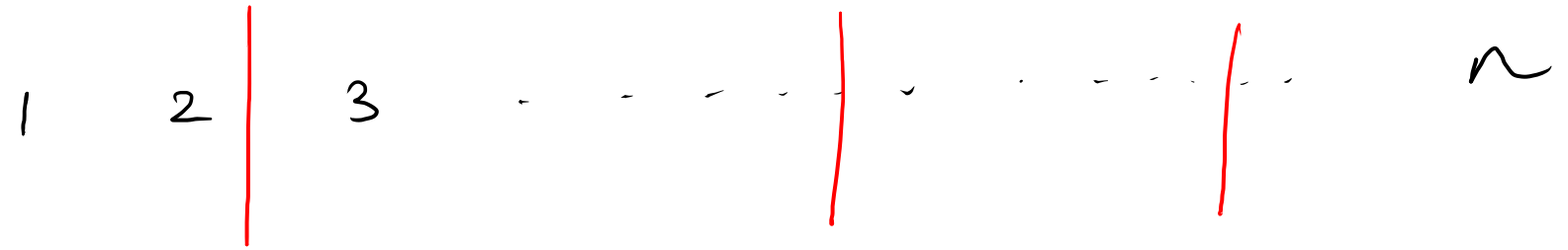
$\sum_{j=i+1}^{\hat{}} s_j$

K-1 ranges

Kth range



cost of the partn = max(max cost so far, Cost of partitioning rest of the elements)



K

cost so far

cost of the rest

M[n, K]

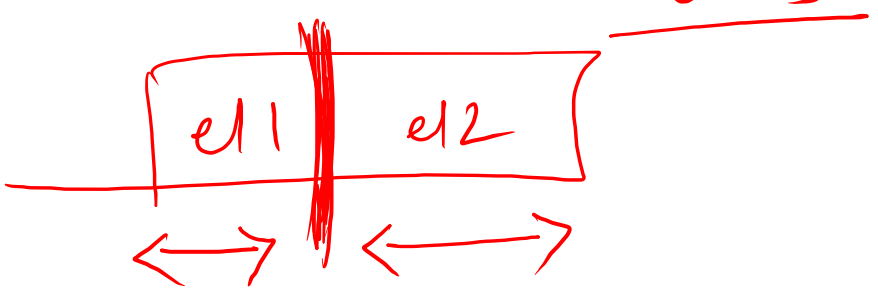
M[n-1, k-1]

M[1, 1] ⇒

M[2, 5]

M[10, 5]

M[2,1]



$$M[n,K] = \min_{i=1}^{n} \max \left(M[i, K-1] , \sum_{j=i+1}^{n} S_j \right)$$

if $K=1 \Rightarrow M[n,1] \Rightarrow$ part^n n elements into 1 part

$$\Rightarrow \sum_{i=1}^{n} S_i$$

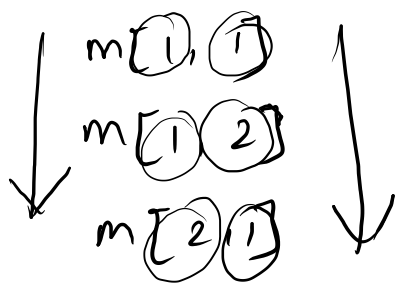
if $n=1 \Rightarrow M[1,K] \Rightarrow$ part^n 1 element into K parts

$$\Rightarrow S[1]$$

1 2 9

values/costs m

dividers d



solve the base cases

⇓

obtain costs of partning elements into 1–K ranges

s, n, K

n · K values

T(n) = time taken to compute 1 value * K · n.

Q1

M, D

$S = \{1, 1, 1, 1, 1, 1, 1, 1, 1\}$ (1st, 2nd, 3rd)

$n = 9, \quad K = 3$

$p[0] = 0$

$p[1] = p[0] + s[1] = 0 + 1 = 1$

$p[2] = p[1] + s[2] = 1 + 1 = 2$

$p[3] = p[2] + s[3] = 2 + 1 = 3$

$p[4] = p[3] + s[4] = 3 + 1 = 4$

$p[5] = p[4] + s[5] = 4 + 1 = 5$

$p[6] = p[5] + s[6] = 5 + 1 = 6$

$p[7] = p[6] + s[7] = 6 + 1 = 7$

$p[8] = p[7] + s[8] = 8$

$p[9] = 9$

$M[i][1] = p[i]$

$M[1][1] = p[1] = 1$

$M[2][1] = p[2] = 2$

$M[3][1] = p[3] = 3$

$M[4][1] = p[4] = 4$

$M[5][1] = p[5] = 5$

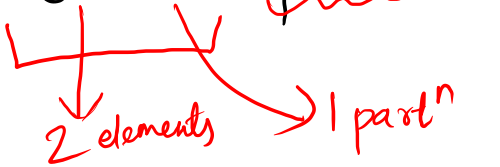
$M[6][1] = p[6] = 6$

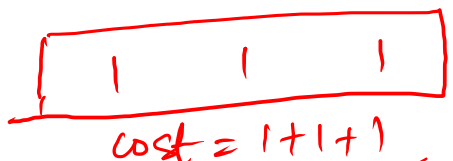
$M[7][1] = p[7] = 7$

$M[8][1] = 8$

$M[9][1] = 9$

$M[1][1] = 1, \; M[1][2] = 1, \; M[1][3] = 1$





$p[3] = p[2] + s[3]$

$= p[1] + s[2] + s[3]$

$= p[0] + s[1] + s[2] + s[3]$

$i = 2$, $j = 2$, $m[2][2] = \infty$, $x = 1$ to $i-1 = 1$

max cost of dividing 2 elements into 2 parts

$cost = \max(m[x][j-1], p[i] - p[x])$

$m[x][j-1]$: cost so far "1 to $i-1$" elements into $j-1$ ranges

$p[i] - p[x]$: cost of the rest of the seq

$(p[1] + p[2] + \cdots + p[x]) + p[x+1] \cdots p[i] - \cdots$

$(p[1] + p[2] + \cdots + p[x]) \approx \sum_{j=i}^{n} s_j$

$cost = \max(m[1][1], p[2] - p[1])$

cost so far: el 1 | el 2

c_1 c_2

$= \max(1, 1) = 1$

$m[2][2] = 1$

$d[2][2] = 1$

$s[x] + p[i] - p[1]$

$= s[2]$

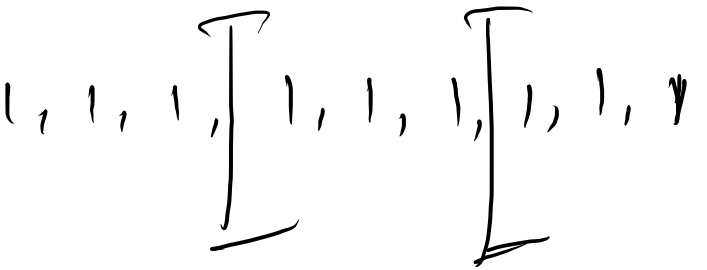
M	k				D	k		
n	1	2	3		n	1	2	3
1	1	1	1	← m[i][j]	1	–	–	–
1	2	1	1		1	–	1	1
1	3	2	1		1	–	1	2
1	4	2	2		1	–	2	2
1	5	3	2		1	–	2	3
1	6	3	2		1	–	3	4
1	7	4	3		1	–	3	4
1	8	4	3		1	–	4	5
1	9	5	3		1	–	4	6

m[i,1]

m[n,k]

⇓

max cost of any part^n !



i=2, j=3 $\quad m[2][3] = \infty$

$n = 1$ to $2-1 = 1 \Rightarrow$ cost $= \max(m[1][2], p[2]-p[1])$

$m[1][2] \rightarrow 1$

$= 1$

$m[2][3] = 1$

$d[2][3] = 1$

$= s[2]+s[3]$

$= p[1]+s[2]+s[3]-p[1]$

$p[2]+s[3]-p[1]$

el1, el2

divider placement

$m[3][2] = \infty$

i=3, j=2

$n = 1$ to $i-1 = 1, 2.$

when $n=1 \Rightarrow$ cost $= \max(m[1][1], p[3]-p[1])$

$= 2 \Rightarrow m[3][2] = 2$, $d[3][2] = 1$

when $n=2 \Rightarrow$ cost $= \max(m[2][1], p[3]-p[2])$

$\sum_{j=i+1}^{n} s_j = s[2]+s[3]$

2 → no change to m & d

$p[2]+s[3]-p[2]$

$s[3]$

$i=3, j=3$ $\quad m[3][3] = \infty$, $x = 1$ to $i-1 = 1$ to 2

$x=1 \Rightarrow$ cost $= \max(m[1][2], p[3]-p[1]) = 2 \Rightarrow m[3][3] = 2$

$d[3][3] = 1$

$x=2 \Rightarrow$ cost $= \max(m[2][2], p[3]-p[2]) = 1 \Rightarrow m[3][3] = 1$

$d[3][3] = 2$

$rp(s, d, n, k) = rp(s, d, 9, 3) \Rightarrow$ ~~$rp(s, d, 6, 2)$~~ ✓ ✓

$pb(s, 7, 9)$

~~$rp(s, d, 8, 1)$~~ ✓

~~$pb(s, 4, 9)$~~ ✓

M	k		
n	1	2	3
1	1	1	1
2	3	2	2
3	6	3	3
4	10	6	4
5	15	9	6
6	21	11	9
7	28	15	11
8	36	21	15
9	45	24	17

D	k		
n	1	2	3
1	–	–	–
2	–	1	1
3	–	2	2
4	–	3	3
5	–	3	4
6	–	4	5
7	–	5	6
8	–	5	6
9	–	6	7

rp(s, d, 9, 3) ⇒ rp(s, d, 7, 2) ⇒ rp(s, d, 5, 1)

pb(s, 8, 9) pb(s, 6, 9)

pb(s, 1, 9)

(1, 2, 3, 4, 5) (6, 7) (8, 9) → n

{6, 7} {8, 9} → n } }

{8, 9} → n

$\Theta(n \cdot n \cdot K)$

$= \Theta(Kn^2)$ ← no. of elements

↑ no. of ranges

$p[0] = 0$

$p[1] = p[0] + s[1] = 1 = m[1,1]$

$p[2] = p[1] + s[2] = 3 = m[2,1]$

$p[3] = p[2] + s[3] = 6 = m[3,1]$

$p[4] = p[3] + s[4] = 10 = m[4,1]$

$p[5] = p[4] + s[5] = 15 = m[5,1]$

$p[6] = 21 = m[6,1]$

$p[7] = 28 = m[7,1]$

$p[8] = 36 = m[8,1]$

$p[9] = 45 = m[9,1]$

$S = \{ \underset{1st}{1}, \underset{2nd}{2}, \underset{3rd}{3}, \underset{4th}{4}, \underset{5th}{5}, \underset{6th}{6}, \underset{7th}{7}, \underset{8th}{8}, \underset{9th}{9} \}$

$m[1][1] = 1$, $m[1][2] = 1$, $m[1][3] = 1$

$i = 2-1$ $j = 2$ $\quad m[2][2] = \infty$, $x = 1$ to $2-1 = 1$ to $1 = 1$

$cost = \max(m[1][1], p[2] - p[1])$

$= \max(1, 2) = 2 \Rightarrow m[2][2] = 2$

$d[2][2] = 1$

$i = 2$ $j = 3$, $x = 1$ to $1 = 1$

$m[2][3] = \infty$, $cost = \max(m[2][2], p[2] - p[1])$

$= \max(1, 2) = 2 \Rightarrow m[2][3] = 2$

$d[2][3] = 1$

Assignment

M, D matrices

$\{1, 2, 3, \ldots, 9\}$

$i = 2, j = 2$

$n = 1$ to $i-1$

$i = 2$ to n

$j = 2$ to k

—

—

for $n = 1$ to $i-1$

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←

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(5 marks)

Deadline

Sunday 5:00 PM

PDF of snaps.