Question:

14. Show that if $a \neq c$, $a^x = c^q$ and $c^y = a^z$, then $xy = qz$. (AHSME 1951)

Correct Method

Using substitution (move cleverly):

Given: $a^x = c^q$ and $c^y = a^z$

From $a^x = c^q$, we can substitute into $c^y = a^z$:

$$c^y = a^z \tag{1}$$
$$(a^x)^{y/q} = a^z \tag{2}$$
$$a^{xy/q} = a^z \tag{3}$$

Since the bases are equal:

$$\frac{xy}{q} = z$$

Therefore:

$$xy = qz$$

OR Logarithm Approach:

Given: $a^x = c^q$ and $c^y = a^z$

Taking logarithms of both equations:

$$x \log a = q \log c \tag{4}$$
$$y \log c = z \log a \tag{5}$$

From the first equation:

$$\log c = \frac{x \log a}{q}$$

Substituting into the second equation:

$$y \cdot \frac{x \log a}{q} = z \log a \tag{6}$$
$$\frac{xy \log a}{q} = z \log a \tag{7}$$

Since $a \neq c$ (therefore $\log a \neq 0$):

$$\frac{xy}{q} = z$$

Therefore:

$$xy = qz$$