CMPS 101
Homework Assignment 2

1. p.50: 3.1-1
   Let $f(n)$ and $g(n)$ be asymptotically non-negative functions. Using the basic definition of Θ-notation, prove that $f(n)+g(n)=\Theta(\max(f(n),g(n)))$.

2. p.50: 3.1-3
   Explain why the statement "The running time of algorithm A is at least $O(n^2)$" is meaningless.

3. p. 50: 3.1-4
   Determine whether the following statements are true or false.
   a. $2^{n+1}=O(2^n)$
   b. $2^{2n}=O(2^n)$

4. p.58: 3-2abcdef
   Indicate, for each pair of expressions (A, B) in the table below, whether A is O, o, Ω, ω, or Θ of B. Assume that $k\geq 1$, $\varepsilon>0$, and $c>1$ are constants. Place 'yes' or 'no' in each of the empty cells below, and justify your answers.

	A	B	O	o	Ω	ω	Θ
a.	$\lg^k n$	n^{ε}					
b.	n^k	c^n					
c.	$\sqrt{n}$	$n^{\sin n}$					
d.	2^n	$2^{n/2}$					
e.	$n^{\lg c}$	$c^{\lg n}$					
f.	$\lg(n!)$	$\lg(n^n)$					

5. p.58: 3-4cdeh
   Let $f(n)$ and $g(n)$ be asymptotically positive functions (i.e. $f(n)>0$ and $g(n)>0$ for sufficiently large n.) Prove or disprove the following statements.

   c. Assume $\lg(g(n))\geq 1$ and $f(n)\geq 1$ for all sufficiently large n. Then $f(n)=O(g(n))$ implies $\lg(f(n))=O(\lg(g(n)))$.
   d. $f(n)=O(g(n))$ implies $2^{f(n)}=O(2^{g(n)})$.
   e. $f(n)=O((f(n))^2)$.
   h. $f(n)+o(f(n))=\Theta(f(n))$.

6. Let $f(n)=\Theta(n)$. Prove that $\sum_{i=1}^{n} f(i)=\Theta(n^2)$. (See the hint at bottom of p.4 of the handout on asymptotic growth rates.)

7. The last exercise in the handout entitled *Some Common Functions*.
   Use Stirling's formula to prove that $\binom{2n}{n}=\Theta\left(\frac{4^n}{\sqrt{n}}\right)$.