CMPS 101
Homework Assignment 5

1. (This is the 2nd exercise on page 1 of the handout on recurrence relations.) Define the function $S(n)$ by the recurrence

$$S(n) = \begin{cases} 0 & n = 1 \\ S(\lceil n/2 \rceil) + 1 & n \geq 2 \end{cases}$$

Use the iteration method to show that $S(n) = \lceil \lg(n) \rceil$, and hence $S(n) = \Theta(\log(n))$.

2. Define $T(n)$ defined by the recurrence formula

$$T(n) = \begin{cases} 6 & 1 \leq n < 3 \\ 2T(\lfloor n/3 \rfloor) + n & n \geq 3 \end{cases}$$

   a. Use the iteration method to write a summation formula for $T(n)$.
   b. Use the summation in (a) to show that $T(n) = O(n)$
   c. Use the Master Theorem to show that $T(n) = \Theta(n)$

3. Use the Master theorem to find asymptotic solutions to the following recurrences.
   a. $T(n) = 7T(n/4) + n$
   b. $T(n) = 9T(n/3) + n^2$
   c. $T(n) = 6T(n/5) + n^2$
   d. $T(n) = 6T(n/5) + n\log(n)$
   e. $T(n) = 7T(n/2) + n^2$
   f. $S(n) = aS(n/4) + n^2$ (Note: your answer will depend on the parameter a.)

4. p.75: 4.3-2
The recurrence $T(n) = 7T(n/2) + n^2$ describes the running time of an algorithm *A*. A competing algorithm *B* has a running time given by $S(n) = aS(n/4) + n^2$. What is the largest integer value for a such that *B* is a faster algorithm than A (asymptotically speaking)? In other words, find the largest integer a such that $S(n) = o(T(n))$.

5. Let $T(n)$ satisfy the recurrence $T(n) = aT(n/b) + f(n)$, where $f(n)$ is a polynomial satisfying $\deg(f) > \log_b(a)$. Prove that case (3) of the Master Theorem applies, and in particular that the regularity condition necessarily holds.

6. Show that the number vertices of odd degree in any graph must be even. (Hint: Use the Handshake Lemma mentioned in the Graph Theory handout.)