

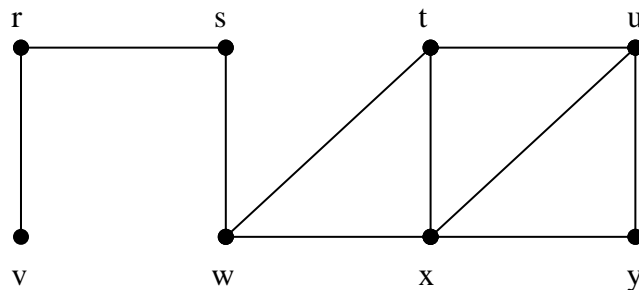
CMPS 101

Homework Assignment 6

1. Show that any connected graph G satisfies $|E(G)| \geq |V(G)| - 1$. Hint: use induction on the number of edges.
2. Let G be a graph on n vertices, A its adjacency matrix (as described in the Graph Theory handout), and let $d \geq 0$ be an integer. Show that the ij^{th} entry in A^d is the number of walks in G of length d from vertex i to vertex j . (Hint: use weak induction on d starting at $d = 0$.)

3. p. 601: 22.2-2

Show the d and π values that result from running breadth-first search on the undirected graph below using the following vertices as source. For each source, show the order in which vertices are added to the Queue, and show the state of the BFS tree after execution completes. Assume adjacency lists are processed in alphabetical order.



- a. Let vertex u be the source
 - b. Let vertex w be the source
 - c. Let vertex v be the source
4. p. 602: 22.2-4
What is the running time of BFS if we represent its input graph by an adjacency matrix and modify the algorithm to handle this form of input?
 5. p. 602: 22.2-7
There are two types of professional wrestlers: “good guys” and “bad guys.” Between any pair of professional wrestlers, there may or may not be a rivalry. Suppose we have n professional wrestlers and we have a list of r pairs of wrestlers for which there are rivalries. Give an $O(n + r)$ -time algorithm that determines whether it is possible to designate some of the wrestlers as good guys and the remainder as bad guys such that each rivalry is between a good guy and a bad guy. If it is possible to perform such a designation, your algorithm should produce it. (Hint: figure out how to use BFS to solve this problem.)