**CMPS 101**
**Homework Assignment 4**

1. Define $T(n)$ defined by the recurrence formula

$$T(n) = \begin{cases} 6 & 1 \le n < 3 \\ 2T(\lfloor n/3 \rfloor) + n & n \ge 3 \end{cases}$$

   Use induction to show that $\forall n \ge 1 : T(n) \le 6n$, and hence $T(n) = O(n)$. (Hint use strong induction with two base cases: $n = 1$ and $n = 2$.)

2. Let $T$ be a tree with $n$ vertices and $m$ edges. Prove that $m = n - 1$ by induction on $m$.

3. Let $G$ be an acyclic graph with $n$ vertices, $m$ edges and $k$ connected components. Use the result of the preceding problem to prove that $m = n - k$. (Hint: apply the preceding result to each of the $k$ trees composing $G$.)

4. Use the iteration method to find an exact solution to the recurrence:

$$T(n) = \begin{cases} 1 & 1 \le n < 3 \\ 2T(\lfloor n/3 \rfloor) + 5 & n \ge 3 \end{cases}$$