THEOREM

IF L IS A CFL THEN THERE EXISTS A PDA (NON-DETERMINISTIC) THAT RECOGNIZES IT.

L CFL IMPLIES THERE EXISTS A CFG G_L THAT WILL GENERATE IT.

WE WILL DESIGN A NON-DETERMINISTIC PDA THAT WILL CHECK IF ITS INPUT STRING CAN BE GENERATED BY G_L.

NON-DETERMINISM ALLOWS US TO GUESS THE DERIVATION IF ONE EXISTS!

BRANCHES CORRESPOND TO APPLICATION OF RULES.

HAVE TO KEEP TRACK OF INTERMEDIATE-STRING AS PROCESS GOES ON. CAN NOT ALL BE IN STACK, (SINCE CAN ONLY ACCESS TOP OF STACK.)

NOTE CAN DISCARD ANY TERMINALS ON LEFT THAT HAVE BEEN MATCHED.

WORK WITH FIRST NON TERMINAL I.E. LEFT MOST.

PLACE $ AND START VAR ON STACK

DO FOREVER

IF TOP OF STACK A VAR
POP IT AND PUSH A RULE FOR IT.
(NON-DETERMINISTICALLY!)

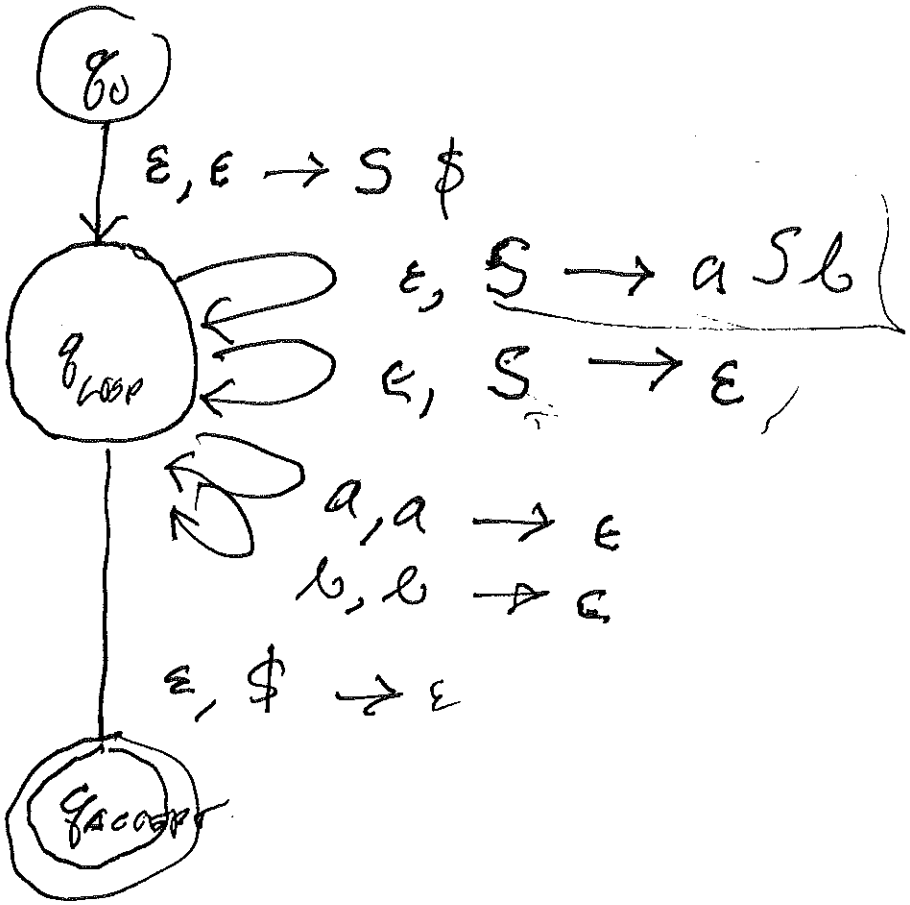
IF TOP OF STALK IS TERMINAL
IF MATCHES SCAN ADV.
IF NOT FAIL.

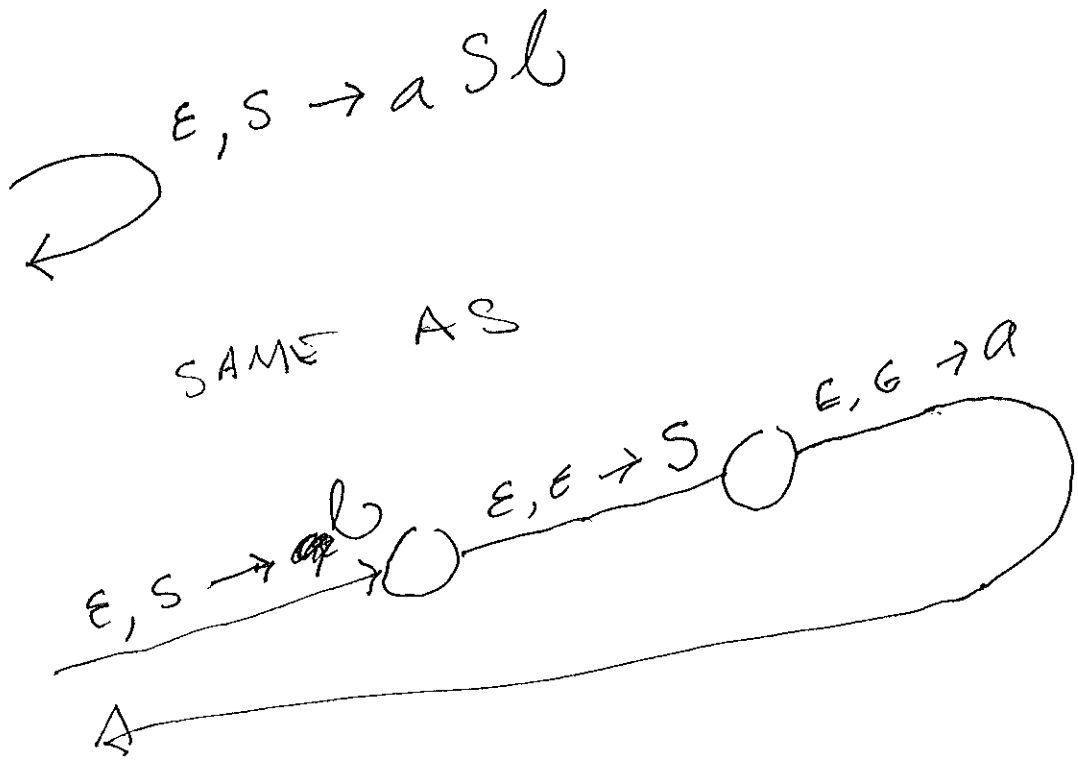
IF TOP $ GO TO ACCEPT STATE.

OF COURSE IF NO INPUT LEFT - ACCEPT
INPUT LEFT FAIL.

$S \rightarrow a S b$

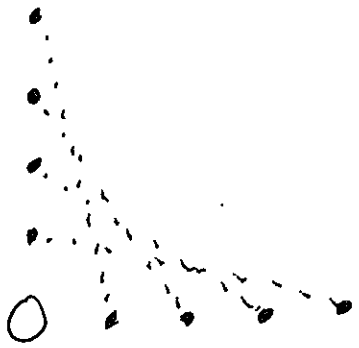
$S \rightarrow \varepsilon$

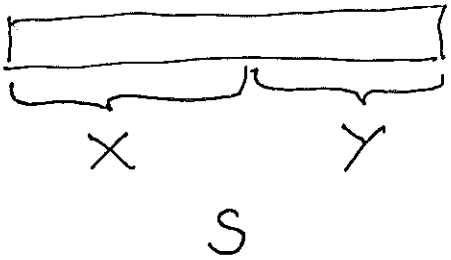



ε, S → a S b
SAME AS
ε, S → ab
ε, ε → S
ε, G → a

CKY ALGORITHM

COCKE
KASAMI
YOUNGER





COCKE-KASAMI-YOUNGER

CFG G

GIVEN AND A STRING $x \in \Sigma^*$

HOW CAN YOU TELL IF $x \in L(G)$?

CASE I

$L(G)$ IS DETERMINISTIC CFL AND WE ARE GIVEN A DPDA FOR IT.

WRITE PROGRAM TO SIMULATE THE DPDA.

CASE II

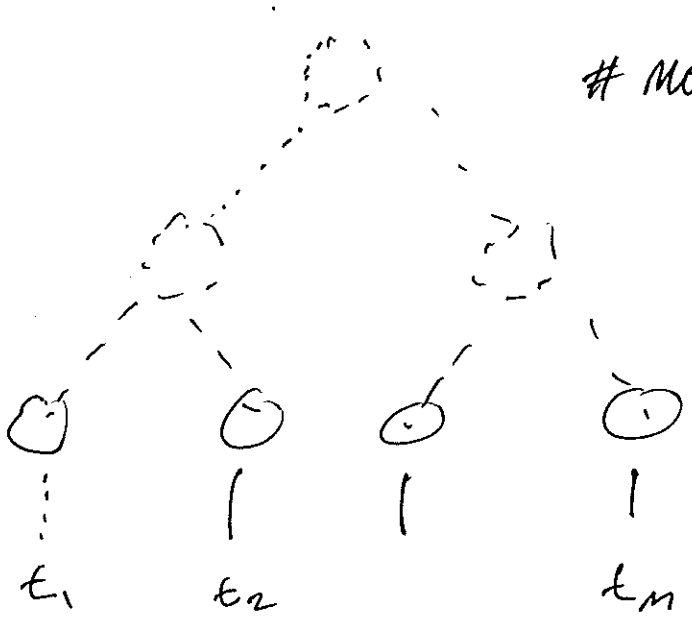
A IS NOT A DETERMINISTIC CFL OR EVEN IF IT IS, WE DO NOT HAVE A DPDA FOR IT.

USE CKY ALGORITHM

CONVERT G TO CNF.

CONSIDER ALL POSIBLE DERIVATION TREES FOR x . OF LENGTH $2|x|-1$

THE UPPER BOUND FOR NUMBER OF THESE IS $\#VARS^{2|x|-1}$, I.E., EXPONENTIAL.



DERIVE EQUIVALENT NPDA AND TRY ALL POSSIBILITIES, STILL EXPONENTIAL

CKY CUBIC $O(pm^3)$

DYNAMIC PROGRAMMING.

p # OF PRODUCTIONS

$M = |x|$ LENGTH OF STRING

FOR EACH SUBSTRING OF x

WHAT SET OF ~~TERMINALS~~ VARS COULD GENERATE IT.

GIVEN CNF GRAMMAR G ← KOZEN EXAMPLE p. 192

$S \to AB \mid BA \mid SS \mid AC \mid BD$

$A \to a$

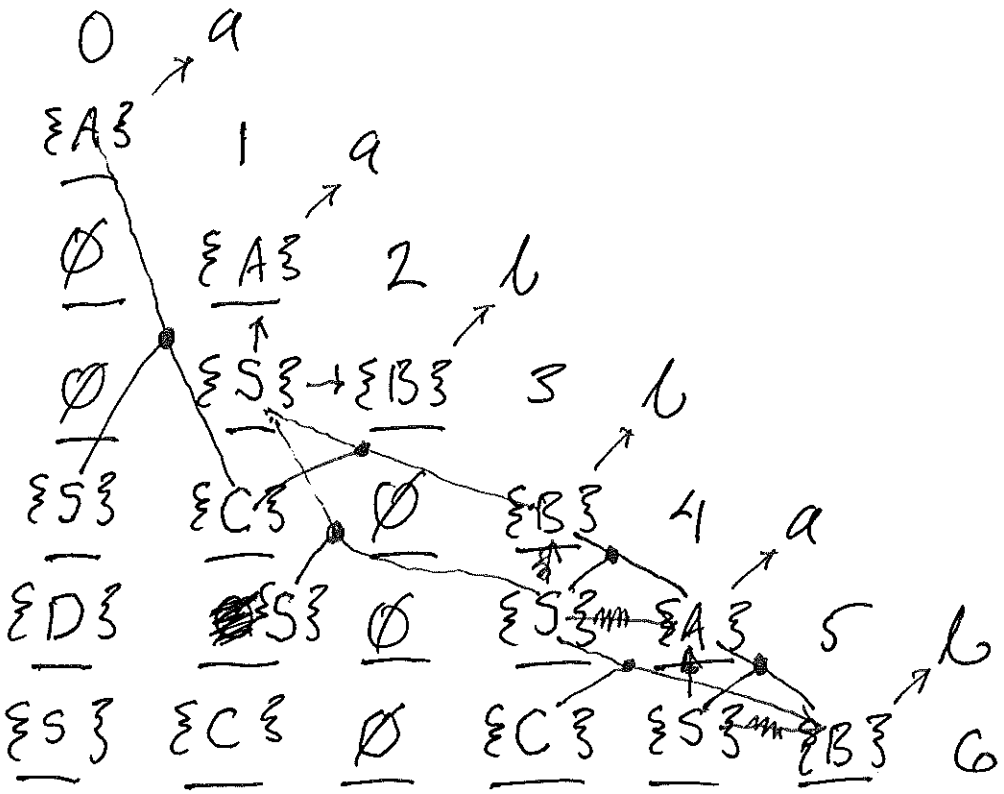
$B \to b$

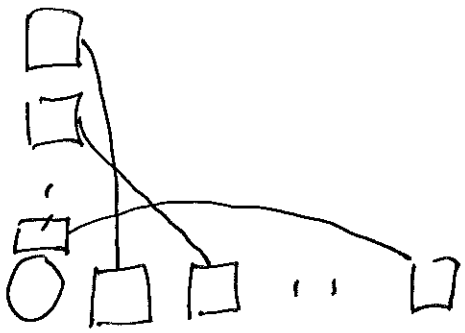
$C \to SB$

$D \to SA$

IS STRING $aabbab \in L(G)$?

| a | a | b | b | a | b |
0 1 2 3 4 5 6





FIRST DIAGONAL

t_{01}	? → a	{A}
t_{12}	? → a	{A}
t_{23}	? → b	{B}
t_{34}	? → b	{B}
t_{45}	? → a	{A}
t_{56}	? → b	{B}

2ND	DIAGONAL	QUESTION	ANSWER
t_{02}	t_{01} t_{12}	? → {A} {A}	∅
t_{13}		? → {A} {B}	S
t_{24}		? → {B} {B}	∅
t_{35}		? → {B} {A}	S
t_{46}		? → {A} {B}	S

3RD DIAGONAL

t_{03} $\{A\}\{S\} \cup \emptyset\ \{B\}$
$\emptyset \qquad \emptyset$

t_{14} $\{A\}\emptyset \cup \{S\}\{B\}$
$\emptyset \cup \{C\}$

t_{25} $\{B\}\{S\} \cup \emptyset\ \{A\}$
$\emptyset \cup \emptyset$

t_{36} $\{B\}\{S\} \cup \{S\}\{B\}$
$\emptyset \qquad \{C\}$

4 TH DIAGONAL

t_{04} $\{A\}\{C\} \cup \emptyset\emptyset \cup \emptyset \{B\}$

$S \cup \emptyset \cup \emptyset$

t_{15} $\{A\} \emptyset \cup \{S\}\{S\} \cup \{C\}\{A\}$

$\emptyset \cup S \cup \emptyset$

t_{26} $\{B\}\{C\} \cup \emptyset \{S\} \cup \emptyset \{B\}$

$\emptyset \cup \emptyset \cup \emptyset$

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5 TH DIAGONAL

t_{05} $\{A\}\{S\}\emptyset \cup \emptyset\emptyset \cup \emptyset \{S\} \cup \{S\}\{A\}$

$\emptyset \quad \emptyset \quad \emptyset \quad D$

$\{S\}$

t_{16} $\{A\} \emptyset \cup \{S\}\{C\} \cup \{C\}\{S\} \cup \{B\}$

$\emptyset \cup \emptyset \cup \emptyset \cup C$

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6 TH DIAGONAL

t_{06} $\{A\}\{C\} \cup \emptyset\emptyset \cup \emptyset \{C\} \cup \{S\}\{S\} \cup \{D\}\{B\}$

$\{S\} \cup \emptyset \cup \emptyset \cup \{S\} \cup \emptyset$

$\{S\}$

EXAMPLE WITH SOME MULTIVARIABLE SETS

GIVEN CNF GRAMMAR G

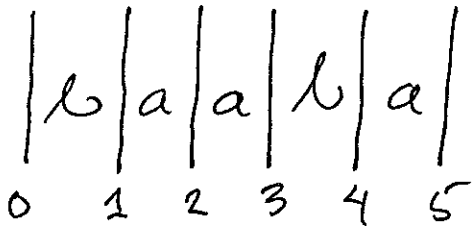
$S \rightarrow AB \mid BC$

$A \rightarrow BA \mid a$

$B \rightarrow CC \mid b$

$C \rightarrow AB \mid a$

IS STRING $baaba \in L(G)$?



0	b		
{B}	1	a	
{S,A}	{A,C}	2	a
∅	{B}	{A,C}	3	b
∅	{B}	{S,C}	{B}	4	a
{S,A,C}	{S,A,C}	{B}	{S,A}	{A,C}	5

2 DIAGONAL

(3)

0,2 ? → {B} {A,C} {S, A}

1,3 ? → {A,C} {A,C} {B}

2,4 ? → {A,C} {B} {S, C}

3,5 ? → {B} {A,C} {S, A}

0,0

3RD DIAGONAL

t_{14} t_{12} t_{24}

$\cup$

t_{13} t_{34}

$\{A,C\}$ $\{S,C\}$ $\cup$ $\{B\}$ $\{B\}$

$\{B\}$ $\emptyset$

$\{B\}$

t_{03} $\{B\}\{B\}$ $\cup$ $\{S,A\}$ $\{A,C\}$

$\emptyset$ $\cup$ $\emptyset$

$\emptyset$

t_{25} $\{A,C\}$ $\{S,A\}$ $\cup$ $\{S,C\}$ $\{A,C\}$

$\emptyset$ $\cup$ $\{B\}$

$\{B\}$

4TH DIAGONAL

t_{04} $t_{01}\,t_{14}$ $t_{02}\,t_{24}$ $t_{03}\,t_{34}$

$\{B\}\{B\} \cup \{S,A\}\{S,C\} \cup \emptyset\,\{B\}$

$\emptyset$ $\emptyset$ $\emptyset$

$\emptyset$

t_{15} $t_{12}\,t_{25}$ $t_{13}\,t_{35}$ $t_{14}\,t_{45}$

$\{A,C\}\{B\} \cup \{B\}\{S,A\} \cup \{B\}\{A,C\}$

$\{S,E\} \cup \{A\} \cup \{S,A\}$

$\{S,A,C\}$

5TH DIAGONAL

t_{05} $t_{01}\,t_{15}$ $t_{02}\,t_{25}$ $t_{03}\,t_{35}$ $t_{04}\,t_{45}$

$\{B\}\{S,A,C\} \cup \{S,A\}\{B\} \cup \emptyset\{S,A\} \cup \emptyset\{A,C\}$

$\emptyset$ $\emptyset$

$\{S,A\} \cup \{S,C\}$

$\{S,A,C\}$