TURING MACHINE

MOST POWERFUL AUTOMATA

INVENTED BY ALAN TURING, 1936

CAN COMPUTE ANY FUNCTION NORMALLY CONSIDERED COMPUTABLE.

REASONABLE TO DEFINE COMPUTABLE AS CAN BE ~~PROGRAM~~ DONE BY T.M.

BEFORE COMPUTERS
EFFECTIVE COMPUTATION E.G. EUCLIDS G.C.D.
SEVERAL ALGORITHMS KNOWN BUT NO GOOD DEFINITION FOR EFFECTIVELY COMPUTABLE.

HOW CAN YOU ANSWER QUESTION?
IS SOMETHING COMPUTABLE OR NOT?

OTHER FORMALISMS DEVELOPED.

POST SYSTEMS EMIL POST

μ-RECURSIVE FUNCTIONS GÖDEL, HERBRAND

λ-CALCULUS CHURCH, KLEENE

COMBINATORY LOGIC SCHÖNFINKEL, CURRY

WORK ON DIFFERENT TYPES OF DATA

BUT NATURAL TRANSLATIONS BETWEEN ALL OF THEM.

ALL CAN ~~E~~ SIMULATE EACH OTHER.

MODERN PROGRAMMING LANGUAGES CAN ALSO BE ADDED.

T.M. CLOSEST TO LOOKING LIKE A COMPUTER.

CHURCHES THESIS. (1936)

ALL THESE FORMALISMS CAPTURE PRECISELY OUR INTUITION ABOUT WHAT IT MEANS TO BE EFFECTIVELY COMPUTABLE.

UNIVERSALITY AND SELF-REFERENCE

PROGRAMS AS DATA.

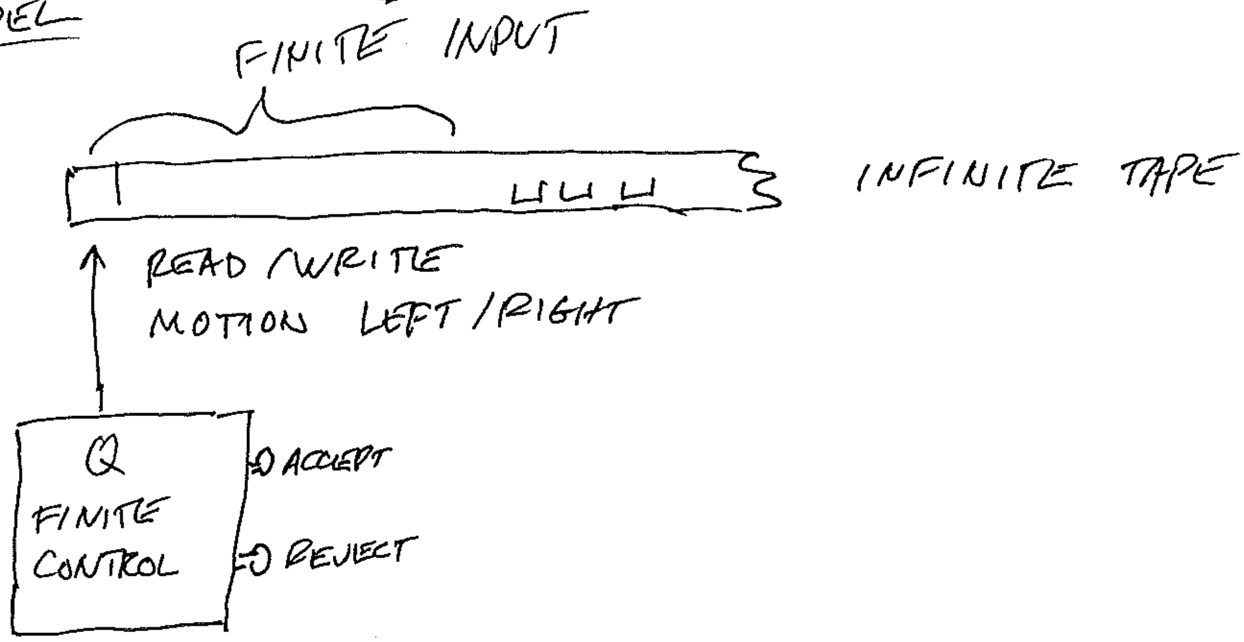
UNIVERSAL SIMULATION

UNIVERSAL PROGRAM OR MACHINE

$$\underbrace{\text{DESCRIPTION OF } M,\ \text{INPUT FOR } M}_{\boxed{\text{SIMULATE } M}}$$

TURING MACHINE

MODEL



TAPE ALPHABET

INPUT ALPHABET

TRANSITION FUNCTION

CURRENT STATE

SYMBOL AT READ HEAD

↓

ACTION WRITE SYMBOL

MOVE LEFT/RIGHT

CHANGE STATE

STOP IF ACCEPT STATE

STOP IF REJECT STATE

TM $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$

Q IS A FINITE SET OF STATES

Σ IS A FINITE SET OF SYMBOLS THE INPUT ALPHABET (DOES NOT INCLUDE $\sqcup$ BLANK)

Γ IS A FINITE SET OF SYMBOLS THE TAPE ALPHABET $\sqcup \in \Gamma$ AND $\Gamma \supset \Sigma$.

$\delta: Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$ IS THE TRANSITION FUNCTION.

$q_0 \in Q$ IS THE START STATE

$q_{acc} \in Q$ IS THE ACCEPT STATE

$q_{REJ} \in Q$ IS THE REJECT STATE

CONFIGURATION

$\alpha q_j a \beta$

HEAD IS SCANING THE a IN THE STRING $\alpha a \beta$ AND MACHINE IS IN STATE q_j.

SIMPLE EXAMPLES

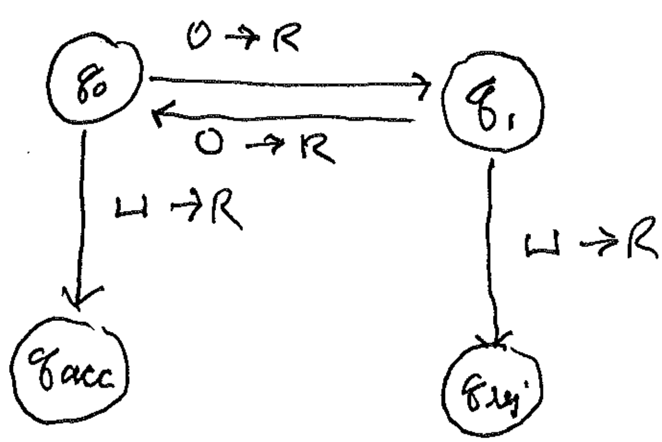
STRINGS OF 0s

	0	$\sqcup$
q_0	q_1 R	acc R
q_1	q_0 R	rej R

EVEN # 0

	0	1	$\sqcup$
q_0	q_1 R	R	acc R
q_1	q_0 R	R	rej R

EVEN # 0
IGNORE 1s.



AOD 1

$\triangleright\ 00101\ \sqcup$

$\Sigma = \{0, 1\}^*$

$\Gamma = \Sigma \cup \{\triangleright, \sqcup\}$

	$\triangleright$	0	1	$\sqcup$
q_0	R	R	R	q_1 L
q_1		q_{acc} 1 L	0 L	

	0	1	␣
q_0	R	q_1 R	accept.
q_1	q_0 R	R	reject

assume string begins with 0
number of transitions from 0 to 1 equals
" " " " 1 to 0

	0	1
q_0	q_0 R	q_1 R

$\{ ww \mid w \in \{a,b\}^* \}$

SCAN LEFT TO RT COUNTING SYMBOLS MOD 2
IF NOT EVEN REJECT.
WHEN REACH ⊔ PUT DOWN RT END MARKER ⊣
THEN REPEATABLY SCAN LEFT & RIGHT OVER TAPE
WHEN SCANNING RT TO LEFT
MARK FIRST UNMARKED ~~symbol~~ a/b with ´
WHEN SCANNING LEFT TO RT.
MARK FIRST UNMARKED (a/b) WITH ˋ
CONTINUE UNTIL ALL SYMBOLS OF INPUT MARKED
(THIS DETERMINES MIDDLE OF INPUT STRING
THEN REPEATEDLY SCAN LEFT TO RIGHT
REMEMBER AND ERASE FIRST ˋ SYMBOL
CHECK FIRST ´ MATCHES AND ERASE.
(NO MATCH REJECT)
WHEN ALL SYMBOLS ERASED ACCEPT.