EX 3

$D = \{ ww \mid w \in \{a,b\}^* \}$ IS NOT A CFL

NOTE $\quad$ CFL $\cap$ REG $\to$ CFL

CONSIDER

$D' = D \cap L(a^*b^*a^*b^*)$

$= \{a^n b^m a^n b^m \mid a, b \geq 0\}$

SO IF D IS CFL THEN D' MUST BE BUT D' IS NOT

p

$s = a^p b^p a^p b^p$ $\qquad \boxed{a}\boxed{b}\boxed{a}\boxed{b}$

4 BLOCKS

$= uvwxy$

$|vwx| \leq p$

$vx \neq \varepsilon$

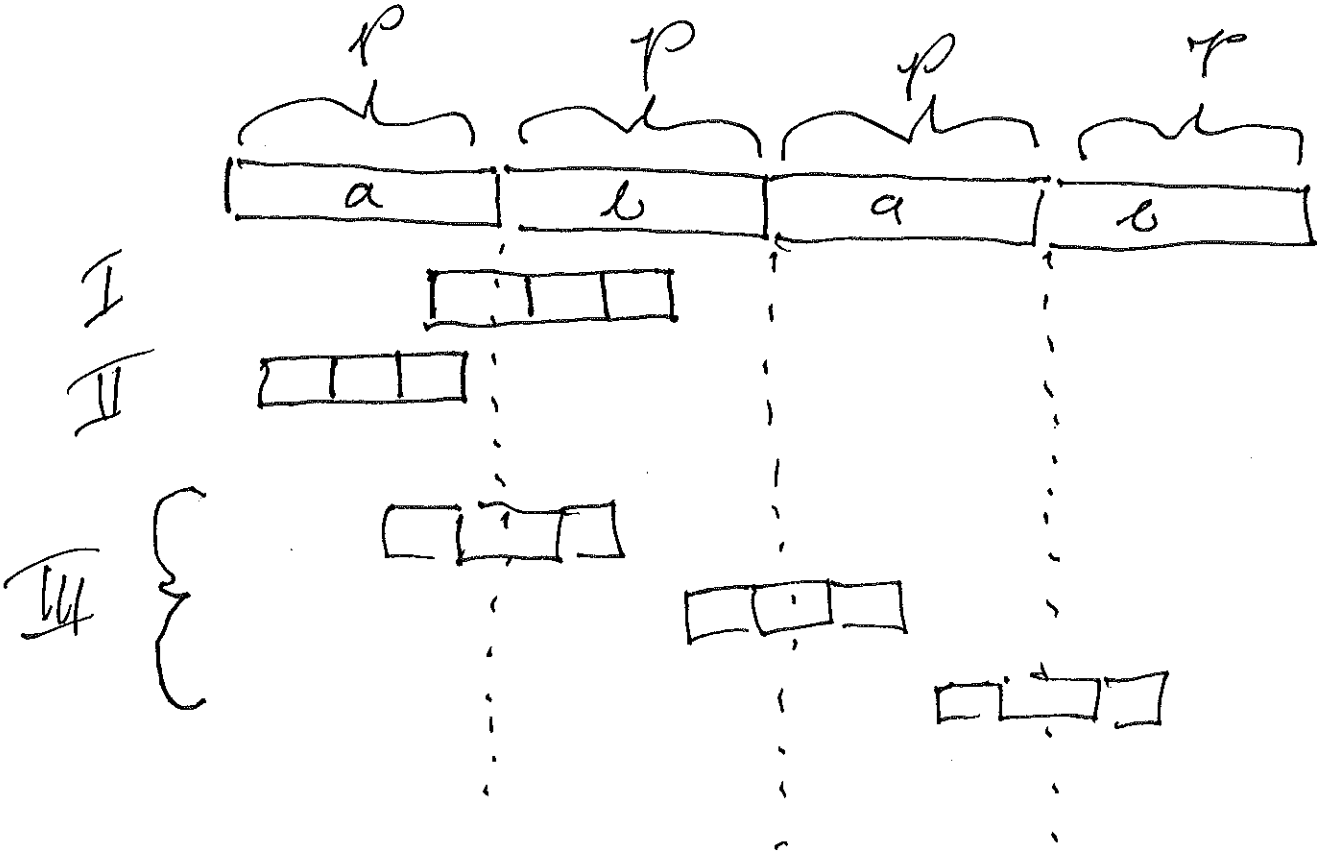
$i = 2$

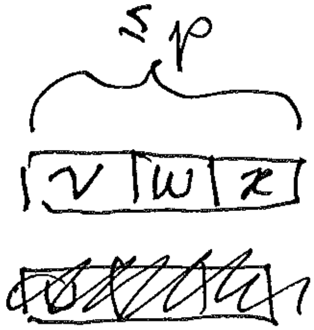
I. ONE OF v OR x CONTAINS BOTH as AND bs. THEN PUMPING DESTROYS FORM $a^*b^*a^*b^*$

II v AND x BOTH FROM SAME BLOCK THEN MORE as ON ONE SIDE THAN THE OTHER OR MORE bs ON ONE SIDE THAN THE OTHER

III v CONTAINS ONE SYMBOL x THE ~~OTHER~~ SYMBOL. SO BLOCKS MUST BE ADJACENT. WILL CREATE IMBALANCE FOR THE # as AND/OR bs FOR THE TWO HALFS.

EX 3





$\{a,b\}^* - \{ww \mid w \in \{a,b\}^*\}$ K 155

IS A CFL

$S \to A \mid B$

$S \to AB \mid BA$

$A \to LAL \mid a$ ODD LENGTH WITH a IN MIDDLE

$B \to LBL \mid b$ ODD LENGTH WITH b IN MIDDLE

$L \to a \mid b$

$$L^n a L^m L^n b L^m$$

|||

$$L^n a L^n L^m b L^m$$

NOTE $\{ww \mid w \in \{a,b\}^*\}$ IS NOT A CFL

SO CFLs NOT CLOSED UNDER COMPLEMENT

CFLs NOT CLOSED UNDER INTERSECTION

$$\{a^m b^m c^n \mid m,n \geq 0\} \cap \{a^m b^n c^n \mid m,n \geq 0\}$$

$$= \{a^n b^n c^n \mid n \geq 0\} \leftarrow \text{NOT A CFL}$$

$S \to AC$
$A \to aAb \mid \varepsilon$
$C \to cC \mid \varepsilon$

$S \to AB$
$A \to aA \mid \varepsilon$
$B \to bBc \mid \varepsilon$

BUT NOTE

CFL $\cap$ REG $\to$ CFL

A PRODUCT CONSTRUCTION.

CSCHUSTE
MPDURGA

PUSH DOWN AUTOMATA

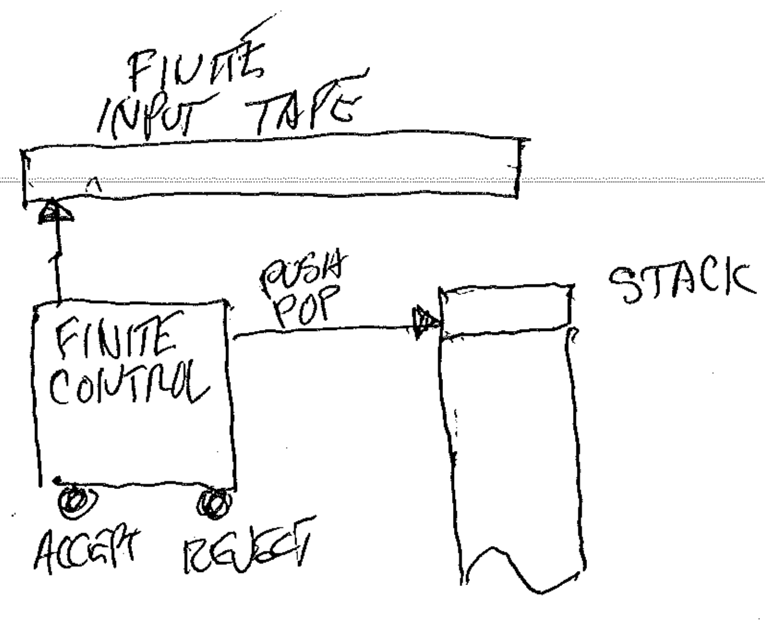
EXTRA COMPONENT - STACK

CAN RECOGNIZE SOME NON-REGULAR LANGS.

NON-DETERMINISTIC PDAS EQ. POWER TO CFGs.
(SO TWO WAYS TO PROVE LANG. CFL)

~~[illegible]~~

Langs accepted by NPDA are exactly
the CFLs



NPDA $M = (Q, \Sigma, \Gamma, \delta, s, F)$ WHERE

Q FINITE SET OF STATES

Σ FINITE SET OF SYMBOLS INPUT ALPHABET

Γ FINITE SET OF SYMBOLS STACK ALPHABET

$\delta : Q \times (\Sigma \cup \{\varepsilon\}) \times (\Gamma \cup \{\varepsilon\}) \rightarrow 2^{Q \times (\Gamma \cup \{\varepsilon\})}$ NON-DETERMINISTIC

DON'T READ

DON'T WRITE

$s \in Q$ START STATE

$F \subseteq Q$ ACCEPT STATES

TRANSITION FUNCTION FOR PDA:

$a, b \to c$

IF READING a AND b IS ON STACK

POP b
PUSH c
ADV

$a, \varepsilon \to c$

PUSH c
ADV

$a, b \to \varepsilon$

POP b
ADV

$a, \varepsilon \to \varepsilon$

ADV

IF $\varepsilon, x \to s$

DO ABOVE WITH OUT ADV.

ADV TAPE   POP STACK   PUSH THIS STRING

$$\underset{\text{ADV TAPE}}{a}\ ,\ \overset{\text{POP STACK}}{\underset{\uparrow}{b}} \rightarrow \overset{\text{PUSH THIS STRING}}{\underset{\downarrow}{x}}$$

ε ε ε

IGNORE TAPE SYMBOL

DO NOT ADV

DON'T READ TAPE

IGNORE TOP OF STACK SYMBOL

DO NOT POP

DON'T READ STACK

DO NOT PUSH ON STACK

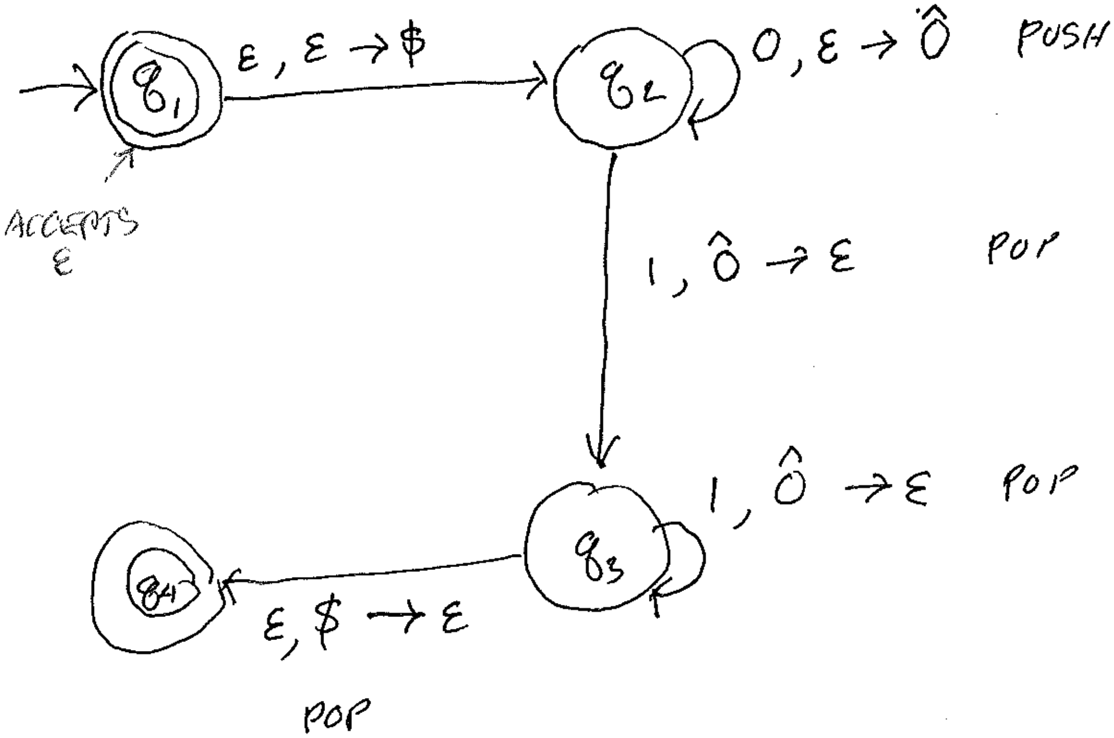
$\{0^m 1^m \mid m \geq 0\}$

$Q = \{q_1, q_2, q_3, q_4\}$

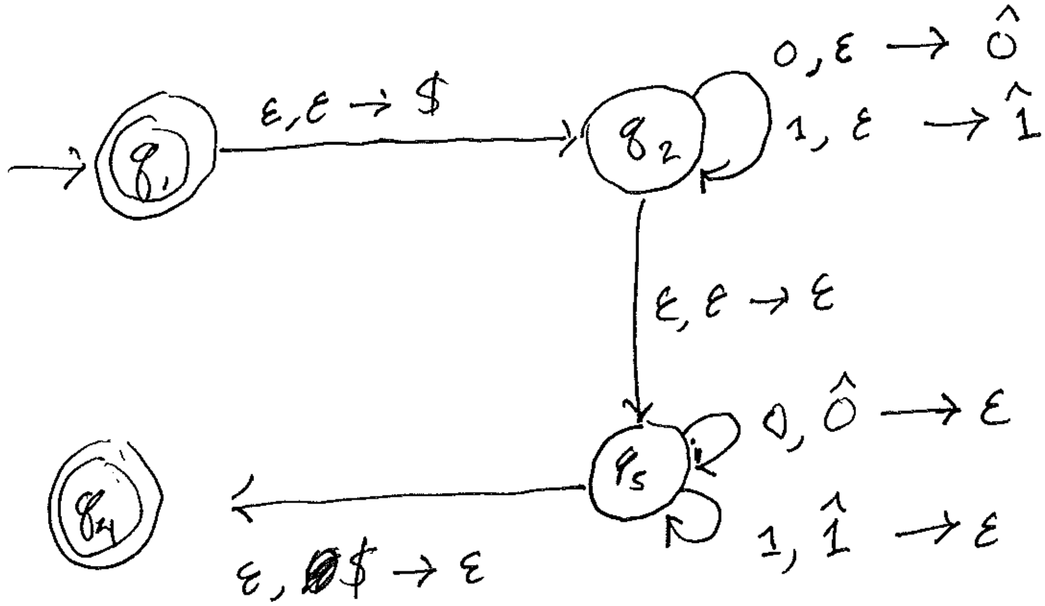
$\Sigma = \{0, 1\}$

$\Gamma = \{\hat{0}, \$\}$

$F = \{q_1, q_4\}$



$\{ ww^R \mid w \in \{0, 1\}^* \}$



THEOREM

<u>IF L IS A CFL THEN THERE EXISTS A NON-DETERMINISTIC PDA</u>
<u>THAT RECOGNIZES IT.</u>

L CFL IMPLIES THERE EXISTS A CFG G_L THAT
WILL GENERATE IT.

WE WILL DESIGN A NON-DETERMINISTIC PDA
THAT WILL CHECK IF ITS INPUT STRING CAN
BE GENERATED BY G_L.

IT NON-DETERMINISM ALLOWS US TO GUESS THE
A DERIVATION IF ONE EXISTS!

I BRANCHES CORRESPOND TO APPLICATION OF
RULES.

HAVE TO KEEP TRACK OF INTERMEDIATE - STRING
AS PROCESS GOES ON. CAN NOT ALL
BE IN STACK, (SINCE CAN ONLY ACCESS TOP OF STACK.)

NOTE CAN DISCARD ANY TERMINALS ON LEFT
THAT HAVE BEEN MATCHED.
WORK WITH <u>FIRST</u> <u>NON TERMINAL</u> I.E.
LEFT MOST.

PLACE $ AND START VAR ON STACK

DO FOREVER

IF TOP OF STACK A VAR
POP IT AND PUSH A RULE FOR IT.
(NON-DETERMINISTICALLY!)

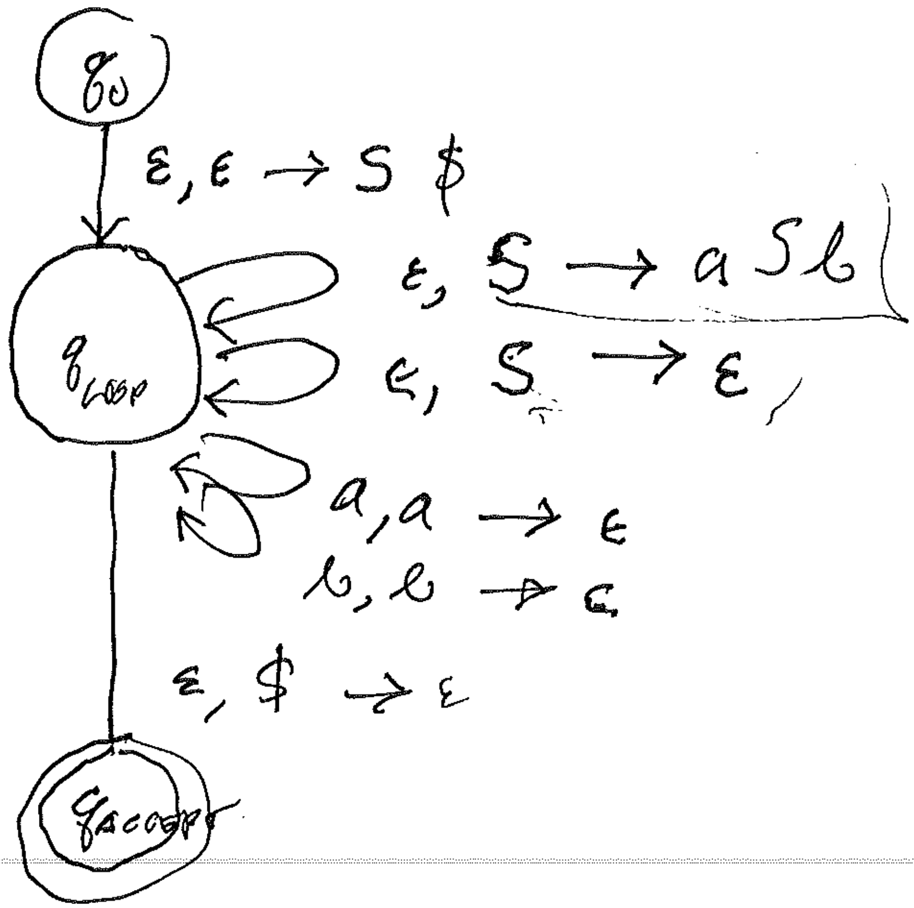
IF TOP OF STACK IS TERMINAL
IF MATCHES SCAN ADV.
IF NOT FAIL.

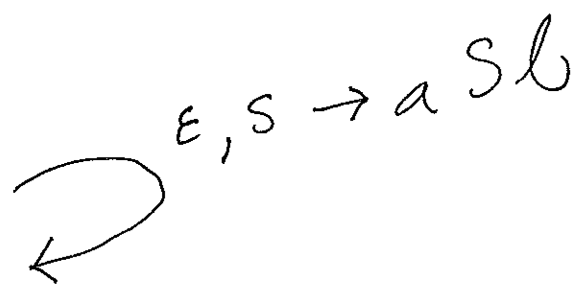
IF TOP $ GO TO ACCEPT STATE.

OF COURSE IF NO INPUT LEFT - ACCEPT
INPUT LEFT FAIL.

$S \to aSb$

$S \to \varepsilon$





SAME AS