QUESTIONS

WHAT IS COMPUTER SCIENCE?

→ STUDY OF ALGORITHMS

WHAT ARE ALGORITHMS GOOD FOR?

→ SOLVING ALG. PROBLEMS

WHAT ARE ALGORITHMIC PROBLEMS?

→ next page

WHAT ARE ALGORITHMS GOOD FOR?
SOLVING ALGORITHMIC PROBLEMS!

WHAT IS AN ALGORITHMIC PROBLEM?

I. Given graph G, is G connected?

II. " " " " " 3-colorable?

III Given a Boolean formula ϕ, is ϕ satisfiable? (CIRCUIT DESIGN QUESTION?)

IV Given natural number n, is n prime?

DECISION

V Given 2 integers m & n, find greatest common divisor. FUNCTION

VI Given a natural number n, find all its prime factors.

VII Given a Boolean Formula ϕ, find all its satisfying assignment. ENUMERATION

VIII " " " " ", How many sat assignments. FUNCTION

Given a network of cities + distance between them

IX find tour of minimal length. OPTIMIZATION

X find length of shortest tour. FUNCTION

OVERVIEW (REVIEW)

ALGORITHMIC PROBLEMS → EXPAND ON TYPES OF PROBLEMS.

FOCUS ON DECISION PROBLEMS

SIMPLIST

AS DIFFICULT AS ANY PROBLEMS

COMPUTATION / SOLUTION

INSTANCE OF PROBLEM → $\boxed{\text{QUESTION DOES INSTANCE HAVE PROPERTY?}}$ → YES / NO

$x \in \Sigma^* \rightarrow \boxed{\text{IS } x \in L \text{?}} \rightarrow$ YES / NO

$L \subseteq \Sigma^*$

instances that have property
can be considered to make up a language.

BIG QUESTIONS

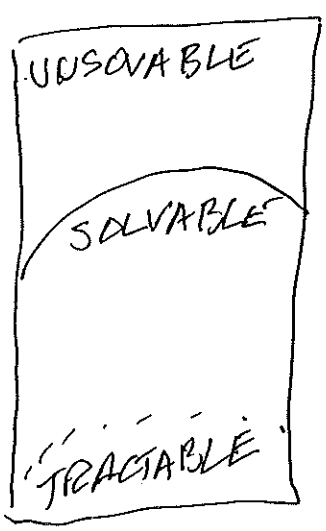
SOLUTION POSSIBLE

EFFICIENT SOLUTION POSSIBLE

FINITE AUTOMATA

PUSH DOWN AUTOMATA

TURING MACHINES



TAXONOMY OF PROBLEMS
RE RESOURCES REQUIRED.

SET - COLLECTION OF DISTINGUISHABLE OBJECTS

$x \in S$ ELEMENT MEMBER

SPECIFY / DESCRIBED

COMPLETE DENOTATION $S = \{1, 5, 9\}$

ELEMENTS NOT REPEATED AND NOT ORDERED.

EQUALITY - CONTAIN SAME ELEMENTS

FREQUENT SETS

$\emptyset$ empty set null set

$\mathbb{Z}$ integers $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

$\mathbb{R}$ real numbers

$\mathbb{N}$ natural numbers $\{1, 2, 3, \ldots\}$

$\mathbb{Q}$ rational numbers

SUBSET

$A \subseteq B$ if all $x \in A$ are $\in B$

$\forall x \quad x \in A \Rightarrow x \in B$

SET OPERATIONS

INTERSECTION

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

UNION

$A \cup B = \{x : x \in A \text{ OR } x \in B\}$

EXAMPLES

$\square$

DIFFERENCE

$A - B = \{x : x \in A \text{ AND } x \notin B\}$

SYMETRIC DIFFERENCE

$A \triangle B = \{x : x \in (A - B) \cup (B - A)\}$

"LAWS" FOR OPPERATIONS

$A \cap \emptyset = \emptyset$ EMPTY SET LAWS

$A \cup \emptyset = A$

$A \cap A = A$ IDEMPOTENT

$A \cup A = A$

$A \cap B = B \cap A$ COMMUTATIVE

$A \cup B = B \cup A$

$A \cap (B \cap C) = (A \cap B) \cap C$ ASSOCIATIVE

$A \cup (B \cup C) = (A \cup B) \cup C$

$A \cap (A \cup B) = A$

ABSORPTION LAWS

$A \cup (A \cap B) = A$

$$\left.\begin{aligned} A - (B \cap C) &= (A - B) \cup (A - C) \\ A - (B \cup C) &= (A - B) \cap (A - C) \end{aligned}\right. \quad \text{DE MORGAN}$$

GIVEN UNIVERSE OF DISCOURSE SAY U

$\overline{A} = U - A$

$\overline{\overline{A}} = A$

$A \cap \overline{A} = \emptyset$

$A \cup \overline{A} = U$

$$\left.\begin{aligned} \overline{A \cap B} &= \overline{A} \cup \overline{B} \\ \overline{A \cup B} &= \overline{A} \cap \overline{B} \end{aligned}\right\} \text{DE MORGAN}$$

CAN PROVE FROM DEFINITIONS

$$A \subseteq A$$

$$\left.\begin{array}{l} A \subseteq B \\ B \subseteq C \end{array}\right\} \Rightarrow A \subseteq C$$

$$\left.\begin{array}{l} A \subseteq B \\ B \subseteq A \end{array}\right\} \Rightarrow A = B$$

for all A $\quad \emptyset \subseteq A$

MORE WAYS TO SPECIFY NATURAL NUMBERS

For example ~~ODD LISTING~~ EVEN ~~[illegible]~~

$$\{x \mid x \in N \text{ and } \frac{x}{2} \in N\}$$

OR $\{x \in N \mid \frac{x}{2} \in N\}$

<RUSSELL PARADOX?>

Russels Paradox

set of all tea cups

EXTRAORDINARY SETS
CONTAIN THEM SELFS

- [ ] set of all non-teacups.
- [ ] set of all sets that can be defined (specified with less than 1000 characters.

Stanford Encyc. of Philosophy

RÜSSEL PARADOX.

$x \in x \Rightarrow x \in E$ EXTRAORDINARY

$\{ y \mid y \notin E \}$?

SET OF ALL ORDINARY SETS

IF ORDINARY THEN MUST BE EXTRA ORDINARY

IF EXTRA ORDINARY THEN MUST CONTAIN ITSELF
THAT IS IT IS ~~NO~~ A MEMBER OF ORDINARY SETS.

RUSSEL

$p \to \neg p$

$\neg(p \wedge p)$

$\neg p$

$\neg p \vee q$

$p \to q$

---

p

$\neg p$

$\neg p \vee q$

$p \to q$

q

$a \to b$

$\neg a \vee b$

$\neg(a \wedge \neg b)$

<u>ordinary set</u> set does not have self as a member (a set that is not a member of itself.)

extraordinary set set that is a member of itself.

consider set of all ordinary sets

$? = \{x : x \text{ is ordinary}\}$

X is ordinary

$X \in X$ BECAUSE X IS SET OF ALL ordinary sets.

X is extraordinary BECAUSE $X \in X$

X is extraordinary

$X \in X$ def. of extraordinary

X IS ORDINARY

RELATIONS

Ex. $<$

SET OF ALL PAIRS SUCH THAT $a < b$

ORDERED PAIR

(a, b) — DIFFERENT THAN $\{a, b\}$

TUPLE

$(a, b) \neq (b, a)$ in gen.

(a, a) valid

CARTESIAN PRODUCT

OF TWO SETS A AND B

$\{(a, b) \mid a \in A$ AND $b \in B\}$

EX.

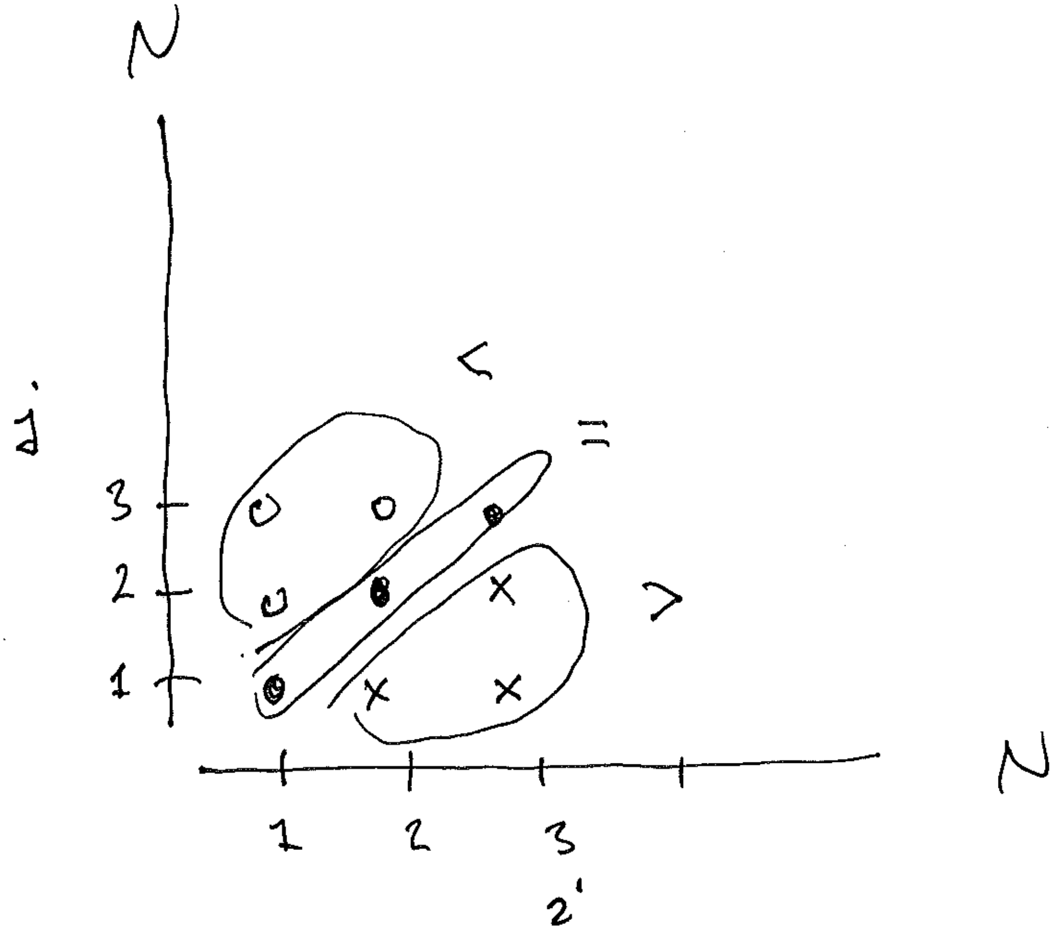
$\{c, d\} \times \{1, 2, 3\}$

$= \{(c,1)(c,2)(c,3)(d,1)(d,2)(d,3)\}$

BINARY RELATION ON A AND B

IS SUBSET OF $A \times B$

EX. $\{(i, j) \mid i, j \in N$ and $i < j\}$



N
j
3
2
1
<
=
>
1
2
3
2'
N

MORE ON BINARY RELATIONS ON A SET A

REFLEXIVE

~~[illegible]~~

$\forall x \in A$ $=$ and $\leq$ but not $<$

$x R x$

SYMMETRIC

$x R y$ implies $y R x$ $=$ but not $<$ or $\leq$

TRANSITIVE

$x R y$ and $y R z$ implies $x R z$

$=$ $<$ $\leq$

SYMMETRIC & TRANSITIVE $\not\Rightarrow$ REFLEXIVE WHY?

EXAMPLE NOT TRANSITIVE

$R = \{ (x, y) \mid x, y \in \mathbb{N} \text{ and } x = y - 1 \}$

$1 R 2$ and $2 R 3$ do not imply $1 R 3$

$R =$ "one less than"

N-FOLD CARTESIAN PRODUCT

$A_1 \times A_2 \times \dots \times A_n$

$\{ (a_1, a_2, \dots, a_n) \mid a_i \in A_i$ FOR EACH $i = 1, 2, \dots, n \}$

N-TUPLE

$A \times A \times A \times \dots \times A$ n times A^n

EX, N^2 PAIRS OF NATURAL NUMBERS

N-ARY RELATION

$A_1, A_2, A_3, \dots A_n$

SUBSET OF $A_1 \times A_2 \times A_3 \times \dots \times A_n$

PARTITIONS

DISJOINT

$$A \cap B = \emptyset$$

PARTITION

$$C = \{ \overbrace{P_1, P_2, \ldots, P_n}^{\text{SETS}} \}$$ PARTITION OF S

$$S = \bigcup_{x \in C} x$$

$$\forall\, i, j \quad P_i, P_j \in C \text{ and } i \neq j \Rightarrow P_i \cap P_j = \emptyset$$

EXAMPLE

LET

$$E = \{ x \in N \mid \tfrac{x}{2} \in N \}$$ EVEN NAT. NUMS

$$O = \{ x \in N \mid \tfrac{x}{2} \notin N \}$$ ODD NAT. NUMS

THEN

$$N = E \cup O$$

AND

$$E \cap O = \emptyset$$

FUNCTION

GIVEN TWO SETS ~~B~~ A and ~~A~~ B

a FUNCTION f IS

BINARY RELATION ON ~~B X A~~ $A \times B$

SUCH THAT

FOR ALL $x \in A$, there exists exactly one $y \in B$

SUCH THAT $(a, b) \in f$

~~f: A → B~~ $f : A \to B$ DEFINE VALUES WITH RULE OR TABLE

$b = f(a)$

f ASSIGNS ONE ELEMENT OF B TO EACH ELEMENT OF A

A IS DOMAIN

B IS ~~RANGE~~ CO-DOMAIN

RANGE IS $f(A)$

IF $A' \subset A$

$f(A') = \{ b \in B \mid b = f(a) \text{ for some } a \in A' \}$

~~image~~ IMAGE

GRAPH

a tuple $G = (V, E)$ where

V is a set

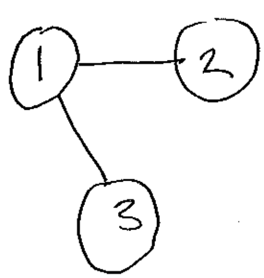
$E \subseteq \{\{x, y\} \mid x, y \in V \text{ and } x \neq y\}$

SO IS A BINARY RELATION

UNDIRECTED

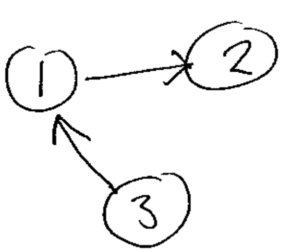
DIRECTED

$E \subseteq \{(x, y) \mid (x, y) \in V \times V\} = V \times V$



$V = \{1, 2, 3\}$

$E = \{\{1, 2\}, \{1, 3\}\}$



$V =$ same

$E = \{(1, 2), (3, 1)\}$

COMPLETE GRAPH

K_n

V can be SPECIFIED like any set

E can be SPECIFIED BY like any set ALSO

ADJACENCY LIST

MATRIX.

PATH - SEQUENCE OF VERTICIES THAT ARE PAIRWISE CONNECTED

SIMPLE - IF ALL VERTICIES DISTINCT

LENGTH NUMBER OF EDGES

CYCLE $(V_0, \ldots, V_n)$ and $V_0 = V_n$

SIMPLE if all others VERTICIES distinct.

CONNECTED $\exists$ paths between any two vertices.

TREE (FREE)

CONNECTED
ACYCLIC
UNDIRECTED

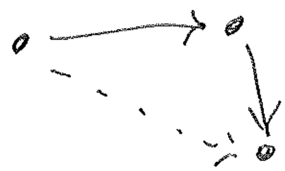
$\Bigg\}$ GRAPH

NOT CONNECTED
FOREST

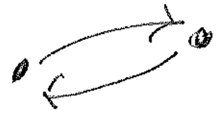
ROOTED ONE VERTEX DISTINGUISHED

GRAPH CAN BE USED TO REPRESENT BINARY RELATION ON A FINITE SET

REFLEXIVE



TRANSITIVE



REFLEXIVE

LET A BE A SET AND

$R \subseteq A \times A$ I.E. R IS A BINARY RELATION ON A.

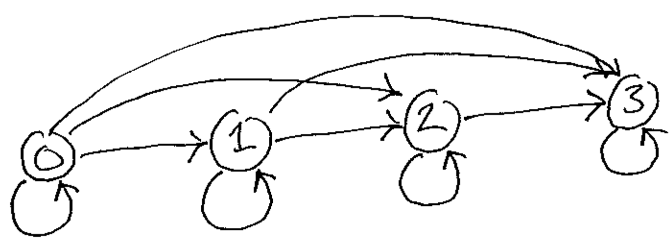
R CAN BE REPRESENTED BY A DIRECTED GRAPH

EACH ELEMENT OF A IS A VERTEX

THE EDGES ARE DEFINED BY

$a, b \in A$ AND aRb iff $a \longrightarrow b$

EXAMPLE: $\leq$ ON NATURAL NUMBERS



STRINGS

<u>ALPHABET</u> ANY FINITE SET
(NOTATION USUALLY CAPITAL GREEK LETTERS, e.g. Σ)

ELEMENTS OF ALPHABET CALLED <u>LETTERS</u> OR <u>SYMBOLS</u>
(NOTATION LOWER CASE LATIN. a, b, c

<u>STRING</u> OVER Σ
FINITE LENGTH SEQUENCE OF ELEMENTS FROM Σ
(NOTATION x, y, z)

E.G. $\Sigma = \{a, b\}$
THEN $aabab$ IS A STRING OVER Σ
OF LENGTH 5.

<u>LENGTH</u> OF A STRING x IS NUMBER OF SYMBOLS IN x.
$|x|$

EMPTY STRING UNIQUE STRING OF LENGTH 0
ϵ DON'T CONFUSE WITH $\in$

a^n INDUCTIVE DEF
$a^0 \stackrel{df}{=} \epsilon$
$a^{n+1} \stackrel{df}{=} a^n a$

Σ^* SET OF ALL STRINGS OVER ALPHABET Σ

NOTE CONVENTION $\emptyset^* \stackrel{df}{=} \{\epsilon\}$