Dane Truong
CMPS 130
dtruong8@ucsc.edu

HomeWork 5

1.29 Use the pumping lemma to show that the following languages are not regular.

a) $A_1 = \{0^n 1^n 2^n \mid n \geq 0\}$

Assume that A_1 is regular. A_1 must have a pumping length. Let "P" be the pumping length.
The string we will use to get the contradiction is

$$S = 0^P 1^P 2^P$$

lets divide S into pieces xyz, to be regular it must follow the three conditions.

(a) $|y| > 0$
(b) $|xy| \leq p$
(c) $\forall i > 0, xy^iz \in L$

When dividing S into three pieces, we have multiples cases in which the string of y can be. In the case in which y is 0,1,or 2, when pumped, the number of 0s, 1s, or 2s would no longer match. In another case, if the string contained all three values or two of each, then when pumped it would no long be in order, removing it from the language. Therefore, no matter how we break the language into XYZ, it cannot be pumped thus the language is not regular.

b) $A_2 = \{www \mid w \in \{a,b\}^*\}$

Assume that A_2 is regular. A_2 must have a pumping length. Let "P" be the pumping length.
The string we will use to get the contradiction is

$S = a^P b a^P b a^P b$, now split S into three pieces $S = XYZ$

using condition (a) $|y| > 0$
(b) $|xy| \leq p$, we can see that no matter what y we choose for any value $p > 0$ the string length of xy would not be less than or equal to P. Therefore a contradiction meaning A_2 is not regular.

c) $A_3 = \{a^{2^n} \mid n \geq 0\}$ (Here, a^{2^n} means a string of 2^n a's)
Assume A_3 is regular. A_3 must have a pumping length.
Let "P" be the pumping length. the string used to get the contradiction is
$S = a^{2^P}$, now split S into three pieces $S = xyz$
Since a is a string of 2^P a's, that would mean we are unable to split S into three pieces since all the values are the same in the string, there would only be a y^i state and no x and z, thus making a contradiction.

1.30 Describe the error in the following "proof" that 0^*1^* is not a regular language. The proof is by contradiction. Assume that 0^*1^* is regular. Let p be the pumping length for 0^*1^* given by the pumping lemma. Choose S to be the string 0^P1^P. You know that s is a member of 0^*1^*, but example 1.73 shows that s cannot be pumped. Thus you have a contradiction. So 0^*1^* is not regular.
The error in the proof is that the string choosen does not match the languange. 0^*1^* means any number of 0s or any number of 1s. In example 1.73, the proof is for the language in which 0's and 1's must be of the same length.

1.42 For languages A and B, let the shuffle of A and B be the language $\{w \mid w = a_1b_1 \cdots a_kb_k$, where $a_1 \cdots a_k \in A$ and $b_1 \cdots b_k \in B$, each $a_i, b_i \in \Sigma^*\}$. Show that the class of regular languages is closed under shuffle.
For the langues to be closed and regulor, a DFA for A and B must be created such that a third NFA is used to keep track of both DFA(A, B). After the whole string is processed and both DFA(A, B) are in the final state then the string is accepted.

1.46 Prove that the following languages are not regular. You may use the pumping lemma and the closure of the class of regular languages under union, intersect, and complement.

a) $\{0^m 1^m 0^n \mid m,n \geq 0\}$

Using the pumping lemma to get a contradiction.

$S = 0^p 1 0^p$

we can see that xy contains only 0s and when we pump it, the languages no longer match because the left 0s > right 0s.

b) $\{0^m 1^n \mid m \neq n\}$

using the pumping lemma to get a contradiction.

$S = 0^p 1^n$

If we pump 0 or 1 then the condition $|xy| \leq p$ would fail, If we pumped both 0,1, then the string would repeat in an incorrect manner that wouldn't match the languge. Therefor a contradiction, making the language not regular.

c) $\{w \mid w \in \{0,1\}^*$ is not a palindrome$\}$

To simplify the problem, we will use the compliment in which the language is a palindrome. By showing that this compliment is not regular, the languyge is also not regular.

Using pumping lemma to get a contradiction we choose

$S = 0^p 1 0^p$

we can see that xy contans only 0s and when we pump it, the right 0's no longer match the left 0s therefore losing its palindrome.

d) $\{wtw \mid w,t \in \{0,1\}^+\}$

Using the pumping lemma to get a contradicting we choose

$S = 0^p 1 0^p$

we can see that xy contains only 0s and when we pump it, there will be more zeros in the left string.

1.47 let $\Sigma = \{1, \#\}$ and let
$Y = \{w \mid w = x_1 \# x_2 \# \ldots \# x_k$ for $k \geq 0$, each $x_i \in 1^*$, and $x_i \neq x_j$ for $i \neq j\}$. Prove Y is not regular.
using the pumping lemma to get a contradiction.
$S = 1^p \# 1^{p+1} \#, \ldots \# 1^{2p}$
we can observe that if we pump any value, we get a contradiction to the condition $|xy| \leq p$.

1.55. The pumping lemma says that every regular language has a pumping length p, such that every string in the language can be pumped if it has length p or more. If p is a pumping length of a language A, so is any length $p' \geq p$. The minimum pumping length for A is the smallest p that is a pumping length for A. For example if $A = 01^*$, the minimum pumping length is 2. The reason is that the string s=0 is in A and has length 1 yet s cannot be pumped, but any string in A of length 2 or more contains a 1 and hence can be pumped by dividing it so that x = 0, y = 1, and z is the rest. For each of the following languages, give the minimum pumping length and justify your answer.

e) $(01)^*$ 0101

The minimum pumping length is 2, because this allows string 01 to repeat which fits the language restriction.

f) ε

The minimum pumping length is 0, because the string is empty and there is nothing to pump

i) 1011

The minimum pumping length is 2, because it can be pumped by dividing it x=1, y=0, and z the rest.

j) Σ^*

The minium pumping length is ∞ because the languge cardinality is ∞