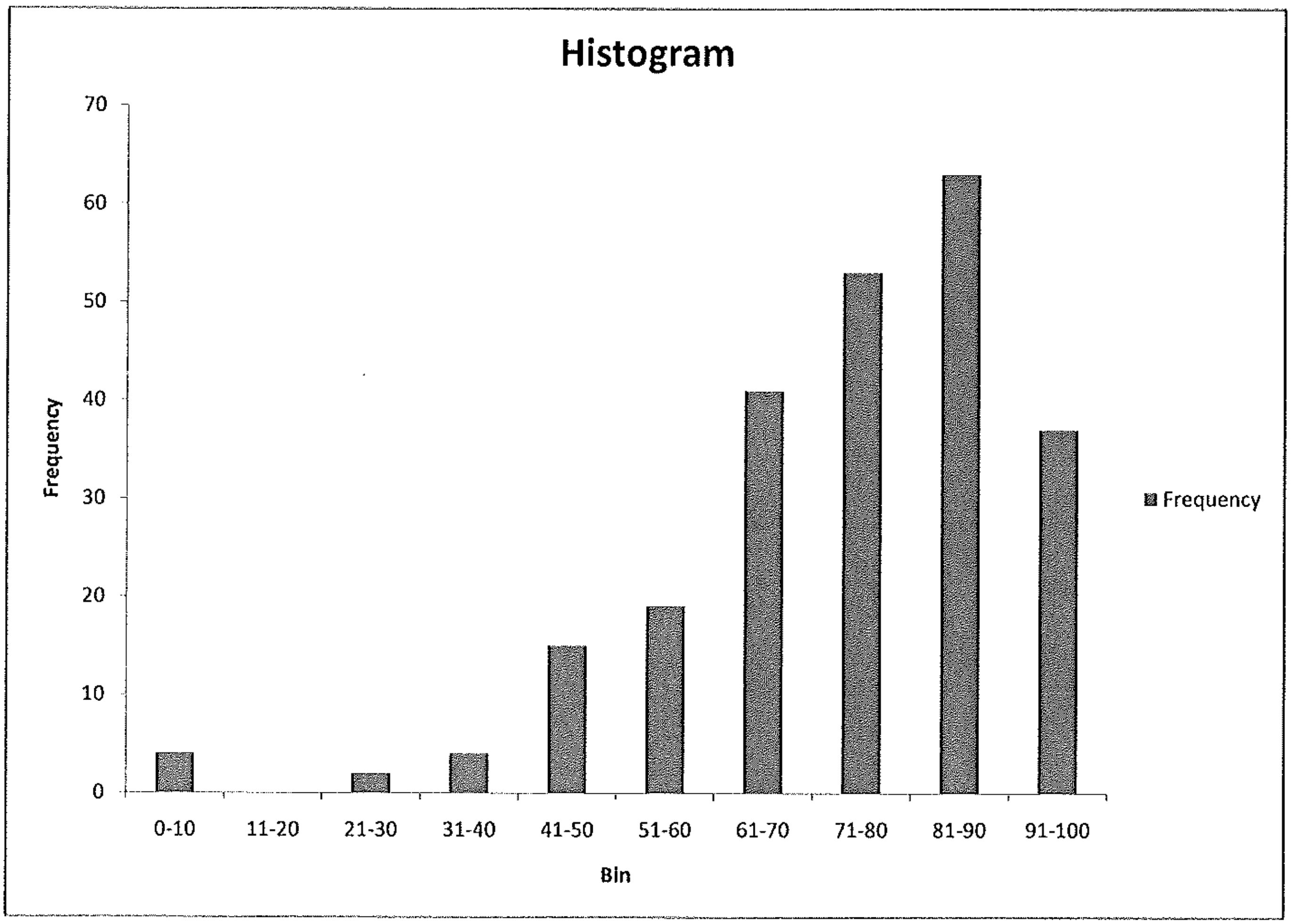
Histogram
70
60
50
40
30
20
10
0
Frequency
0-10
11-20
21-30
31-40
41-50
51-60
61-70
71-80
81-90
91-100
Bin
Frequency

CMPS 130 Computational Models Winter 2017

Midterm 1, April 26, 2017

NAME KEY

ID. NUM.

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Page 8	60 points	
Page 9	40 points	

1. (20pts) What are decision problems and why is it reasonable to focus on them in the study of computational models?

PROBLEMS WHOSE ANSWER IS YES OR NO.
EASIER TO ANALYZE
CAN BE AS DIFFICULT AS ANY PROBLEM
MANY OTHER PROBLEMS CAN BE SOLVED
WITH DECISION PROBLEMS

2. (20pts) Let Σ equal the alphabet $\{a, b, c\}$. For each of the following sets indicate if its cardinality is finite or infinite. If it is finite what is its cardinality and if it is not finite is it countable or uncountable?

a.) Σ FINITE 3
b.) 2^{Σ^*} INFINITE UNCOUNTABLE
c.) Σ^* INFINITE COUNTABLE
d.) 2^{Σ} FINITE 8
e.) $\emptyset$ FINITE 0

3. (40pts) Complete the following definitions:

A **Deterministic Finite Automaton**, DFA, is a structure M such that:

The **extended transition function** for M is recursively defined i n terms of the transition function as follows:

A string $x \in \Sigma^*$ is **accepted** by M if

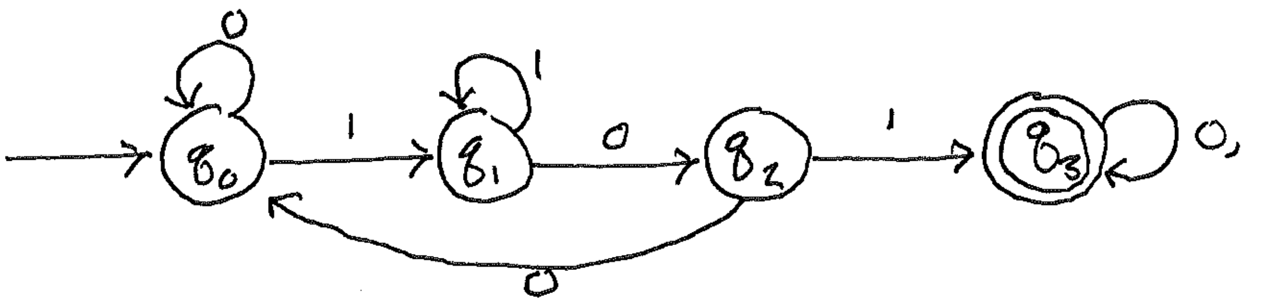
The language **accepted or recognized** by M is

The formal definition that a language is **regular** is

SEE DFA HANDOUT IN RESOURCES

4. (40pts) Consider the language $L = \{x \in \{0,1\}^* \mid x \text{ contains the substring } 101\ \}$.

Give the state diagram for a DFA that recognizes L.



Give the formal description for the DFA above that recognizes L.

$M = (Q, \Sigma, \delta, s, F)$ WHERE

$Q = \{q_0, q_1, q_2, q_3\}$

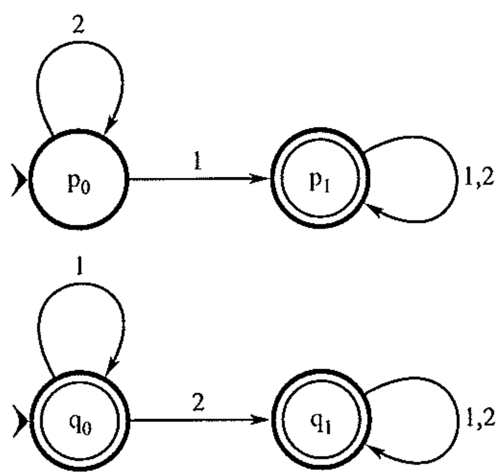
$\Sigma = \{0, 1\}$

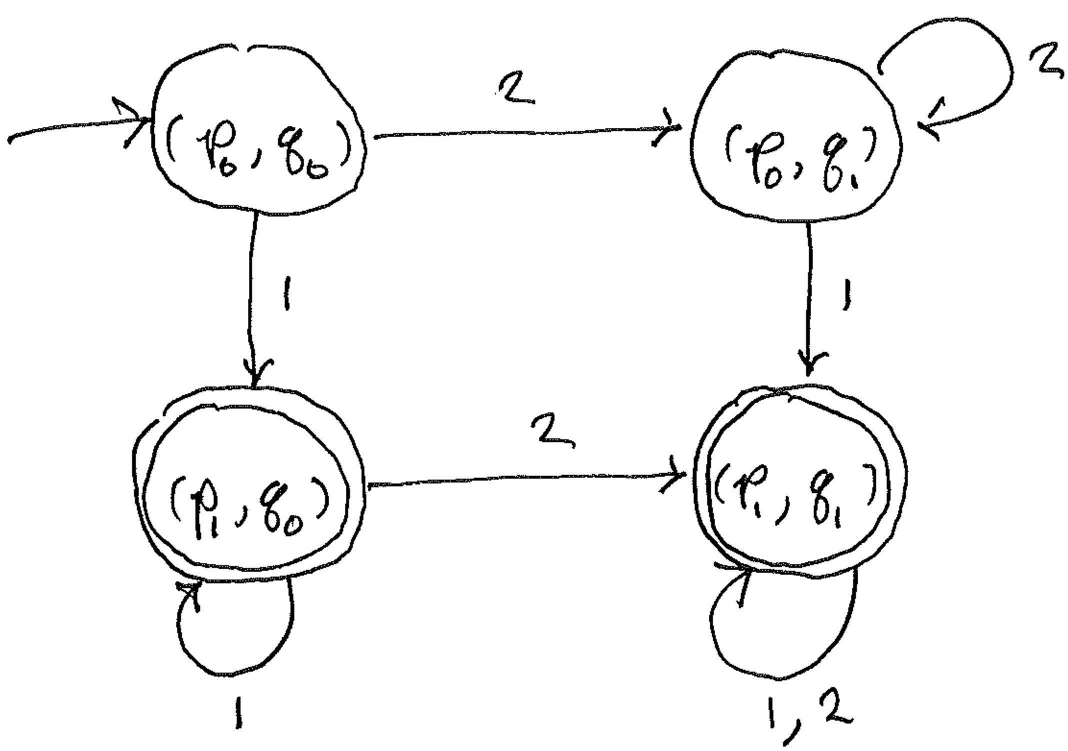
δ	0	1
q_0	q_0	q_1
q_1	q_2	q_1
q_2	q_0	q_3
q_3	q_3	q_3

$s = q_0$

$F = \{q_3\}$

5. (20pts) Use the product construction to build a DFA that recognizes the intersection of the languages recognized by the following two DFAs. Clearly show your steps so it is clear that you used the product construction.





6. (20pts) Prove by induction, that the finite union of regular languages is regular. In other words show that for all $n \geq 2$

$$L_1 \cup L_2 \cup L_3 \cup \cdots \cup L_n$$

where all the L_i are regular languages must be regular. You may use the fact that the union of two regular languages is a regular language.

BASIS

$\bigcup_{i=1}^{2} L_i = L_1 \cup L_2$ GIVEN SO BASE CASE TRUE

INDUCTIVE STEP

ASSUME $\bigcup_{i=1}^{M} L_i$ REGULAR PROVE $\bigcup_{i=1}^{M+1} L_i$ REGULAR

$$\bigcup_{i=1}^{M+1} L_i = \left(\bigcup_{i=1}^{M} L_i\right) \cup L_{M+1}$$

$\bigcup_{i=1}^{M} L_i$ REGULAR I.H.

L_{M+1} REGULAR GIVEN

SO $\bigcup_{i=1}^{M+1} L_i$ = REGULAR $\cup$ REGULAR

THEREFORE IS REGULAR

Q.E.D.

7. (20pts) Prove that the complement of a non-regular language must be non-regular.

TOWARD A CONTRADICTION ASSUME L NON-REG AND $\bar{L}$ REG. THEN

$\bar{L}$ REG $\Rightarrow$ $\bar{\bar{L}}$ REG $\Rightarrow$ L REG A CONTRADICTION

Q.E.D.

8. (20pts) Given that the complement of any regular language is regular and that the intersection of any two regular languages is regular, prove that the union of any two regular languages is regular.

A REG GIVEN

B REG GIVEN

$\bar{A}$ REG COMPLEMENT OF REG IS REG

$\bar{B}$ REG COMPLEMENT OF REG IS REG

$\bar{A} \cap \bar{B}$ REG INTERSECTION OF REG IS REG

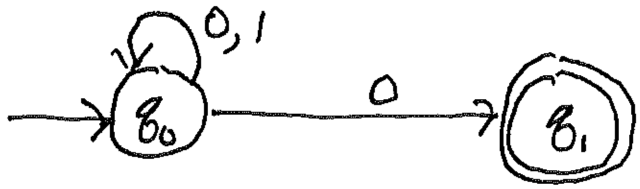
$\overline{\bar{A} \cap \bar{B}}$ REG COMPLEMENT OF REG IS REG

$A \cup B$ REG DE MORGAN

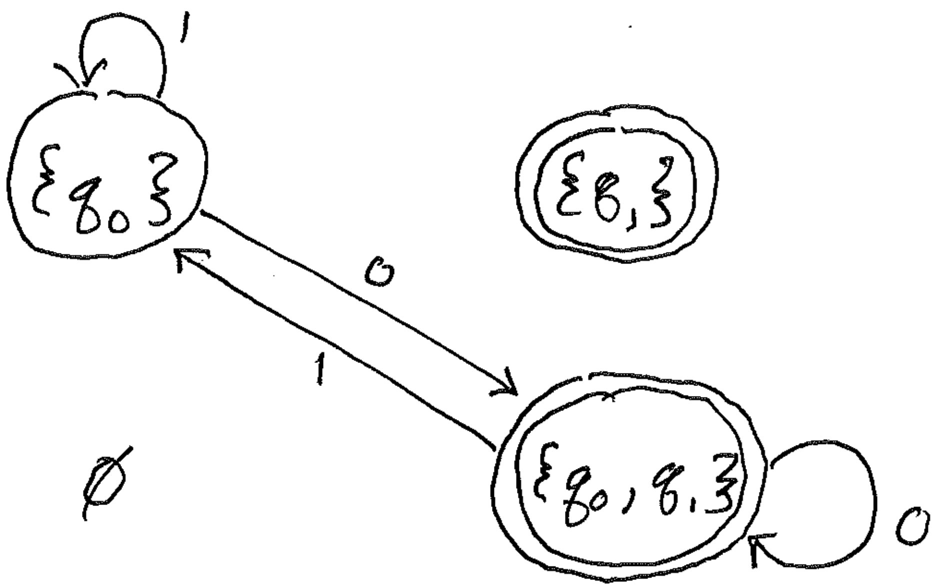
Q.E.D.

9. (40pts) Consider the language $L = \{x \in \{0,1\}^* \mid \text{ the last symbol in } x \text{ is a } 0.\}$.

Show the state diagram for an NFA with no more than two states that recognizes L.



Use the subset construction to build a DFA that recognizes the language recognized by the NFA for the language L. Be sure to label the states in such a way that it is clear that you used the subset construction.



10. (20pts) The subset construction can produce an exponential blow-up in the numbe r of states. Is it possible that some other construction could be found that could a void the exponential blow-up in all cases. Justify your answer. (This does not have to be a formal proof.)

NO

WAS PROVED IN CLASS

11. (20pts) What are the different ways to prove a language is regular?

DESIGN DFA
DESIGN NFA
DESIGN ϵ - NFA
USE CLOSURE PROPERTIES

12. (20pts) For what set operations are regular languages closed?

COMPLEMENT
UNION
INTERSECTION
DIFFERENCE
KLEENE STAR