Homework 7

CMPS130 Computational Models, Spring 2015

2.3

a. $V = R, S, T, X$

b. $\Sigma = \{a, b\}$

c. R

d. ab, ba, abb

e. a, b, aaa

f. False

g. True

h. False

i. True

j. True

k. False

l. True

m. True

n. False

o. $L(G)$ is the language of all words that are not palindromes $L(G) = \{w | w \neq w^R\}$.

2.4

For all these grammars, the grammar definition is $G = (\{A, B, C, ..., S\}, \{0, 1\}, R, S)$ with the following production rules R:

a.

$$S \to A\ 1\ A\ 1\ A\ 1\ A$$
$$A \to A\ 0\ |\ A\ 1\ |\ \epsilon$$

b.

$$S \to 0\ |\ 1\ |\ 0\ A\ 0\ |\ 1\ A\ 1$$
$$A \to A\ 0\ |\ A\ 1\ |\ \epsilon$$

c.

$$S \to S\ A\ A\ |\ A$$
$$A \to 0\ |\ 1$$

d.

$$S \to A\ S\ A\ |\ 0$$
$$A \to 0\ |\ 1$$

e.

$$S \to 0\ S\ 0\ |\ 1\ S\ 1\ |\ 0\ |\ 1\ |\ \epsilon$$

f.

$$S \to S$$

CFG

The language $L = \{x \in \{\text{a}, \text{b}\}^* \mid x \neq ww \text{ for some } w \in \{\text{a}, \text{b}\}^*\}$ can be generated by the following context-free grammar:

$$S \to X \mid Y \mid X\ Y \mid Y\ X$$
$$X \to Z\ X\ Z \mid \text{a}$$
$$Y \to Z\ Y\ Z \mid \text{b}$$
$$Z \to \text{a} \mid \text{b}$$

Explanation: First, note that X generates all odd-length strings with an a as middle symbol and Y all the odd-length strings with a b in the center position. No odd-length string can ever be divided into ww, so the rules $S \to X \mid Y$ simply generates all possible odd-length strings. For even length strings, the rules $S \to X\ Y \mid Y\ X$ ensure that the first string w has a different symbol at its i'th position than the second string w. Here, the position i is denoted by the number the recursive productions with the left of the two non-terminals (X or Y) while recursive productions with the right non-terminal generate additional symbols beyond i to increase the overall length of a string w. These two parameters can change independently of each other, but in all cases the i'th position will always have a different terminal and therefore these strings cannot be the same and $x \neq ww$