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HomeWork 6

1.51 Let x and y be strings and let L be any language. We say that x and y are distinguishable by L if some string z exists whereby exactly one of the strings xz and yz is a member of L; otherwise, for every string z, we have $xz \in L$ whenever $yz \in L$ and we say that x and y are indistinguishable by L. If x and y are indistinguishable by L, we write $x \equiv_L y$. Show that $\equiv_L$ is an equivalance relation.

Equivalence relation - A binary representation that is reflexive, symmetric, and transitive.

reflexive:

Since x, y are indistinguishable and are strings in any language L, we have $xy \in L$ iff $xy \in L$. So $x \equiv_L x$, and since x and y were any strings in L, $\equiv_L$ is reflexive.

Symmetric:

Since x, y are indistinguishable and are strings in any language L, we have $xy \in L$ iff $yx \in L$. So $x \equiv_L x$ and $y \equiv_L y$, and since x and y were any strings in L, $\equiv_L$ is symmetric.

Transitive:

Since x, y, z are indistinguishable and are strings in any language L, we have $x \equiv_L y$, $y \equiv_L z$. In order to prove transitivity $x \equiv_L z$. For this to occur, $xu \in L$. If this were the case, $x \equiv_L y$ means $yu \in L$, which means $y \equiv_L z$ thus $zu \in L$. Therefore $xu \in L$ implies $zu \in L$ thus $x \equiv_L z$, and since x, y, z were any string in L, $\equiv_L$ is transitive.

∴ Since all three properties were satisfied, $\equiv_L$ is an equivalence relation.

Use the procedure shown in class to minimize the following DFA: In all cases $\Sigma = \{a, b\}$ and the start state is the one on the first row of the table and F indicates accept states.

a)

	a	b
→1	6	3
2	5	6
3F	4	5
4F	3	2
5	2	1
6	1	4

1					
X	2				
✓	✓	3			
✓	✓	X	4		
X	X	✓	✓	5	
✓	X	✓	✓	X	6

(1,2) a (6,5)
(1,2) b (3,6) m
(1,5) a (6,2)
(1,5) b (3,1) m
(1,6) a (6,1)
(1,6) b (3,4)

(2,5) a (5,2)
(2,5) b (6,4)
(2,6) a (5,1) m
(2,6) b (6,4) m
(3,4) a (4,3)
(3,4) b (5,2) m

(5,6) a (2,1) m
(5,6) b (1,4) m

b)

	a	b
1	2	3
2	5	6
3F	1	4
4F	6	3
5	2	1
6	5	4

1					
X	2				
✓	✓	3			
✓	✓	_	4		
X	_	✓	✓	5	
_	✓	✓	✓	✓	6

(1,2) a (2,5)
b (3,6) m
(1,5) a (2,2)
b (3,1) m
(1,6) a (2,5)
b (3,4)
(2,5) a (5,2)
b (6,1)

(2,6) a (5,5)
b (6,4) m
(3,4) a (1,6)
b (4,3)
(5,6) a (2,5)
b (1,4) m

1 ≃ 6
2 ≃ 5
3 ≃ 4

c)

	a	b
0F	3	2
1F	3	5
2	2	6
3	2	1
4	5	4
5	5	3
6	5	0

	0	1	2	3	4	5
1	_					
2	✓	✓				
3	✓	✓	✓			
4	✓	✓	✓	✓		
5	✓	✓	_	✓	✓	
6	✓	✓	✓	_	✓	✓

(0,1) a(3,3)
b(2,5)
(2,3) a(2,2)
b(6,1)m
(2,4) a(2,5)
b(6,4)m_2
(2,5) a(2,5)
b(6,3)
(2,6) a(2,5)
b(6,0)m

(3,4) a(2,5)
b(1,4)m
(3,5) a(2,5)
b(1,3)
(3,6) a(2,5)
b(1,0)
(4,5) a(5,3)
b(4,3)m
(4,6) a(5,5)
b(4,0)m

(5,6) a(5,5)
b(3,0)m

0 ≃ 1
2 ≃ 5
3 ≃ 6

d)

	a	b
0	3	5
1	2	4
2	6	3
3	6	6
4F	0	2
5F	1	6
6	2	6

	0	1	2	3	4	5
1	_					
2	X	X				
3	X	X	_			
4	✓	✓	✓	✓		
5	✓	✓	✓	✓	_	
6	X	X	_	_	✓	✓

0 ≃ 1
2 ≃ 3
2 ≃ 6
3 ≃ 6
4 ≃ 5

(0,1) a(3,2)
b(5,4)
(0,2) a(3,6)
b(5,3)m
(0,3) a(3,6)
b(5,6)m

(0,6) a(3,2)
b(5,6)m
(1,2) a(2,6)
b(4,3)m
(1,3) a(2,6)
b(4,6)m

(1,6) a(2,2)
b(4,6)m
(2,3) a(6,6)
b(3,6)
(2,6) a(6,2)
b(3,6)

(3,6) a(6,2)
b(6,6)
(4,5) a(0,1)
b(2,6)