DETERMINISTIC FINITE AUTOMATON (DFA) A STRUCTURE

$M = (Q, \Sigma, \delta, s, F)$, WHERE

Q A FINITE SET OF "STATES";

Σ A FINITE SET, "INPUT ALPHABET", CONSISTS OF "SYMBOLS";

$\delta : Q \times \Sigma \to Q$ "TRANSITION FUNCTION";

($Q \times \Sigma = \{(q, a) \mid q \in Q$ AND $a \in \Sigma\}$
IF M IS IN STATE q AND SEES INPUT a
IT MOVES TO STATE $\delta(q, a)$)

$s \in Q$ THE "START" STATE;

$F \subseteq Q$ "ACCEPT" OR "FINAL" STATES

DEF (INFORMAL) $x \in \Sigma^*$ ACCEPTED BY M IF M STOPS IN F

DEF $L \in 2^{\Sigma^*}$ ACCEPTED BY M, $L(M)$ (NOTE $L \in 2^{\Sigma^*}$)
$= \{x \in \Sigma^* \mid x$ IS ACCEPTED BY $M\}$

DEF $L \subseteq \Sigma^*$ IS REGULAR IF THERE IS A DFA
M SUCH THAT $L = L(M)$

MEMBERSHIP PROBLEM FOR L CAN BE SOLVED WITH A DFA
REGULAR $L \Rightarrow L$ IS SOLVABLE

EXTENDED TRANSITION FUNCTION

$$\hat{\delta} : Q \times \Sigma^* \longrightarrow Q$$

INDUCTIVE DEFINITION

FOR ALL $q \in Q$, $x \in \Sigma^*$, $a \in \Sigma$

$$\hat{\delta}(q, \epsilon) = q$$

$$\hat{\delta}(q, xa) = \delta(\hat{\delta}(q, x), a)$$

NOTE:

$$\hat{\delta}(q, a) = \hat{\delta}(q, \epsilon a) = \delta(\hat{\delta}(q, \epsilon), a) = \delta(q, a)$$

DEFINITION OF <u>ACCEPTANCE OF A STRING</u>

FOR ALL $x \in \Sigma^*$

x IS ACCEPTED BY M iff $\hat{\delta}(s, x) \in F$

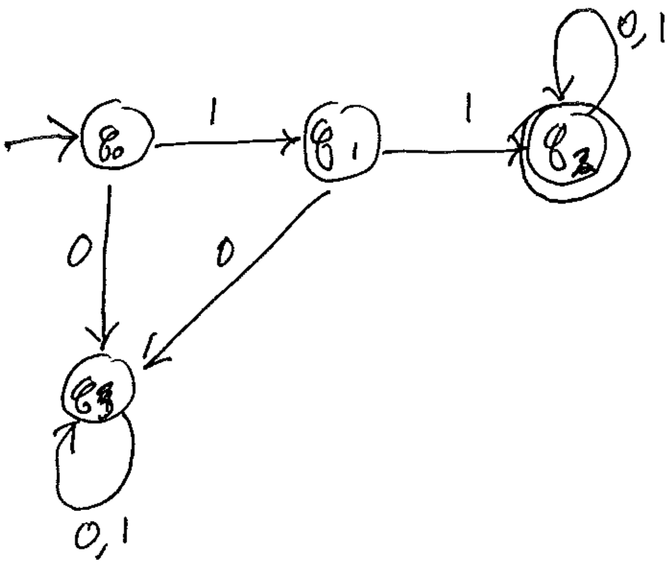
DEFINITION OF <u>LANGUAGE ACCEPTED BY M</u>.

$$L(M) = \{ x \in \Sigma^* \mid x \text{ IS ACCEPTED BY } M \}$$

$$= \{ x \in \Sigma^* \mid \hat{\delta}(s, x) \in F \}$$

A LANGUAGE (SET) $A \subseteq \Sigma^*$, IS <u>REGULAR</u> iff $A = L(M)$ FOR SOME DFA M.

$\{ x \in \{0, 1\}^* \mid x \text{ BEGINS WITH } 11 \}$



$Q = \{ q_0, q_1, q_2, q_3 \}$

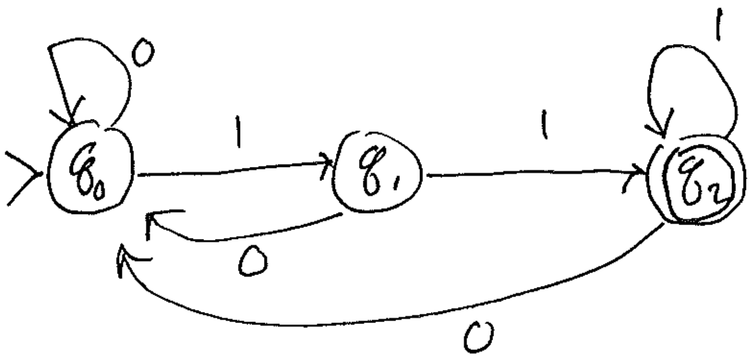
$\Sigma = \{0, 1\}$

$S = q_0$

δ	0	1
q_0	q_3	q_1
q_1	q_3	q_2
q_2	q_2	q_2
q_3	q_3	q_3

$F = \{ q_2 \}$

$\{ x \in \{0,1\}^* \mid x \text{ ENDS WITH 2 CONSECUTIVE 1's} \}$



$Q = \{ q_0, q_1, q_2 \}$

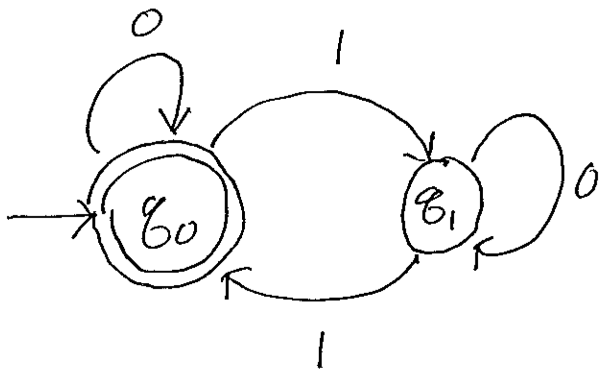
$\Sigma = \{0, 1\}$

δ	0	1
q_0	q_0	q_1
q_1	q_0	q_2
q_2	q_0	q_2

$s = q_0$

$F = \{q_2\}$

~~NUMBER REPRESENTED BY STRING x IN BINARY~~



$$L = \{ x \in \Sigma^* \mid \#1x \text{ IS EVEN} \}$$

NUMBER OF '1's IN STRING x.

$Q = \{ q_0, q_1 \}$

$\Sigma = \{ 0, 1 \}$

$\delta(q_0, 0) = q_0$

$\delta(q_0, 1) = q_1$

$\delta(q_1, 0) = q_1$

$\delta(q_1, 1) = q_0$

$s = q_0$

$F = \{ q_0 \}$

$\{x \in \{a, b\}^* \mid x$ CONTAINS AT LEAST 3 a'S $\}$

$M = \{Q, \Sigma, \delta, s, F\}$ WHERE

$Q = \{0, 1, 2, 3\}$

$\Sigma = \{a, b\}$

$s = 0$

$F = \{3\}$

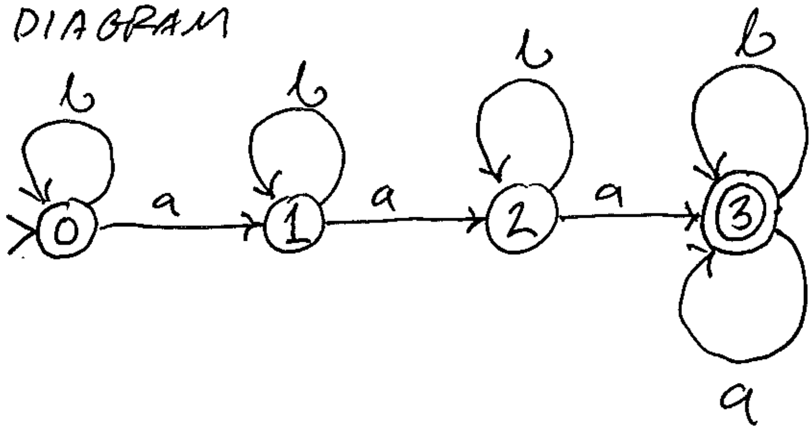
$\delta(0, a) = 1$
$\delta(1, a) = 2$
$\delta(2, a) = 3$
$\delta(3, a) = 3$

$\delta(0, b) = 0$
$\delta(1, b) = 1$
$\delta(2, b) = 2$
$\delta(3, b) = 3$

TABLE

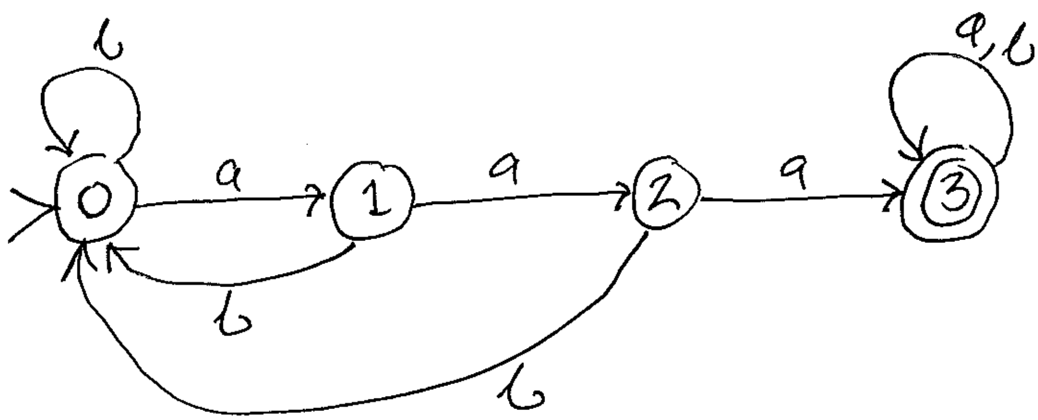
	a	b
>0	1	0
1	2	1
2	3	2
3F	3	3

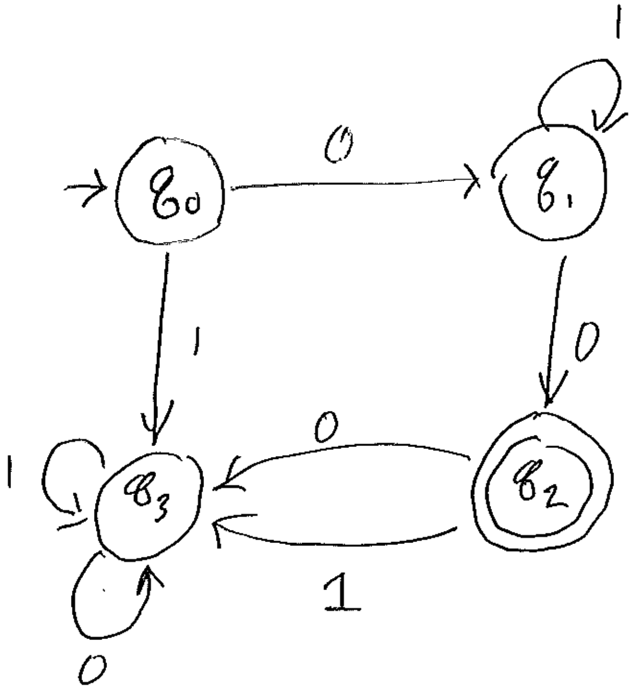
TRANSITION DIAGRAM



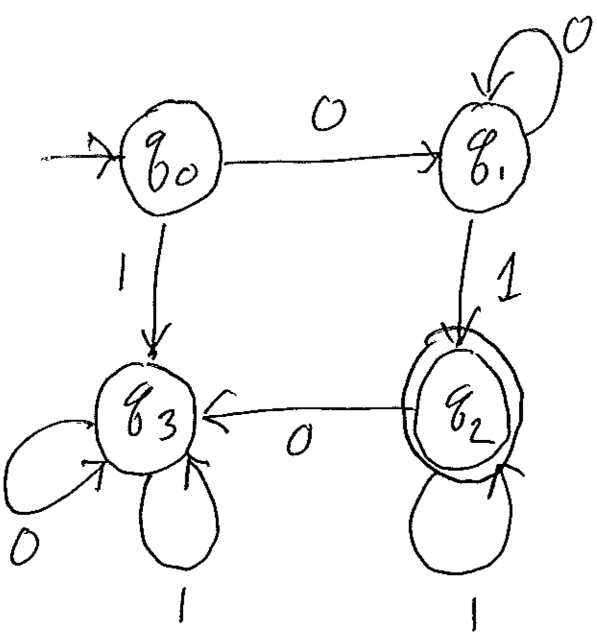
$\{ x \in \{a, b\}^* \mid x \text{ CONTAINS 3 CONSECUTIVE } a\text{'S} \}$

	a	b
> 0	1	0
1	2	0
2	3	0
3F	3	3





$L(M) = \{ 0 1^n 0 \mid n \geq 0 \}$

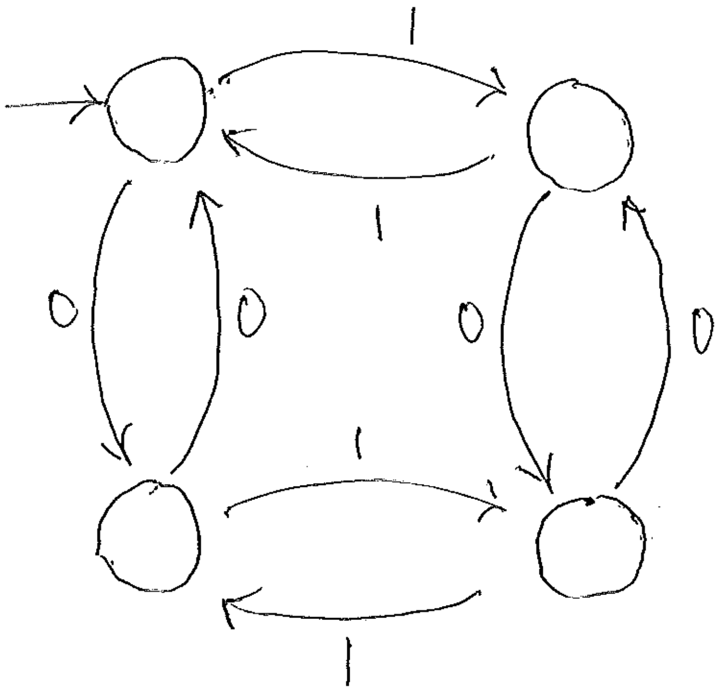


$L(M) = \{ 0^n 1^m \mid n \geq 1,\ m \geq 1 \}$

NOTE $\{ 0^n 1^n \mid n \geq 1 \}$ NOT REGULAR

EVEN # 1s

ODD # 1s



EVEN # 0s

ODD # 0s

16 DIFFERENT LANGUAGES CAN BE DEFINED WITH CHOICE OF ACCEPT STATES

A FINITE IS ALWAYS REG.

$M_A = (Q, \Sigma, \delta, s, F)$

$Q = \{q_i \mid i \in \Sigma^* \text{ AND } |i| \le \max|x| \text{ FOR } x \in A\} \cup \{s_0\}$

$\Sigma = \{0, 1\}$

$\delta : Q \times \Sigma \to Q$

$\delta(q_i, a) = q_{ia}$ $\quad |i| \le \max|x| - 1$ FOR $x \in A$

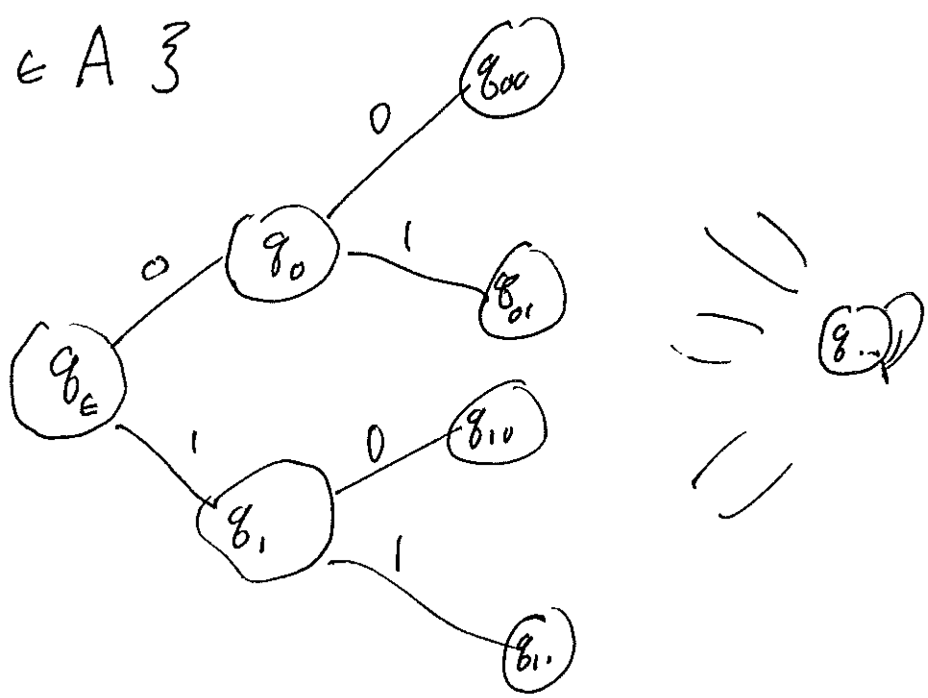
OR

$\delta(q_i, a) = q_R$

$\delta(q_R, a) = q_R$

$s = q_\epsilon$

$F = \{q_i \mid i \in A\}$



COMPLEMENT

ASSUME A IS REGULAR

THEN THERE EXISTS

$$M = (Q, \Sigma, \delta, s, F)$$

AND $L(M) = A$.

SHOW $\Sigma^* - A$ IS REGULAR. $\sim A$

PROOF

LET $M' = (Q, \Sigma, \delta, s, Q-F)$

CLEARLY ANY STRING ACCEPTED BY M' WILL BE REJECTED BY M ~~AND~~ AND ANY STRING REJECTED BY M' WILL BE ACCEPTED BY M. AND VISA VERSA.

$$\hat{\delta}(s, x) \in F \iff \hat{\delta}(s, x) \notin Q - F$$

$$\hat{\delta}(s, x) \in \Sigma^* - F \iff \hat{\delta}(s, x) \notin F$$

COMP REG. L IS REG.

$$M = (Q, \Sigma, \delta, s, F)$$

$$M_c = (Q_c, \Sigma, \delta_c, s_c, F_c)$$

$$Q_c = Q$$

$$\delta_c = \delta$$

$$s_c = s$$

$$F_c = Q - F$$

SHOW $\overline{L(M)} = L(M_c)$

$$x \in \overline{L(M)} \Leftrightarrow \hat{\delta}(s, x) \notin F$$

$$\Leftrightarrow \hat{\delta}(s, x) \in Q - F = F_c$$

$$\Leftrightarrow \hat{\delta}_c(s_c, x) \in F_c$$

$$\Leftrightarrow x \in L(M_c)$$