Definition. *A **Context-Free Grammar**, CFG, is a structure, G such that:*

$G = (N, \Sigma, P, S)$, *where*
N *is a finite set of "non-terminal symbols or variables".*
Σ *is a finite set of "terminal symbols".*
P *is a finite subset of* $N \times (N \cup \Sigma)^*$, *the "productions or rules".*
$S \in N$ *is the "start symbol or variable".*

We will use:

$A, B, C, \ldots$	*for nonterminals.*
$a, b, c, \ldots$	*for terminals.*
$\alpha, \beta, \gamma, \ldots$	*for strings in* $(N \cup \Sigma)^*$.
$A \to \alpha$	*for* (A, α).
$A \to \alpha_1 \mid \alpha_2 \mid \alpha_3$	*for* $A \to \alpha_1, A \to \alpha_2, A \to \alpha_3$.

We say β *is **derivable in one step** from* α, *if* β *can be obtained from* α *by replacing some occurrence of a non-terminal, say* A *in* α *with* γ, *where* $A \to \gamma \in P$. *That is, if there exists* $\alpha_1, \alpha_2 \in (N \cup \Sigma)^*$ *and production* $A \to \gamma \in P$, *such that* $\alpha = \alpha_1 A \alpha_2$ *and* $\beta = \alpha_1 \gamma \alpha_2$. *We can write this as:* $\alpha \overset{1}{\underset{G}{\to}} \beta$.

*We define **derivable in n steps**,* $\overset{n}{\underset{G}{\to}}$, *inductively as*

$$\alpha \overset{0}{\underset{G}{\to}} \alpha \quad \text{for all } \alpha$$
$$\alpha \overset{n+1}{\underset{G}{\to}} \beta \quad \text{if there exists } \gamma \text{, such that, } \alpha \overset{n}{\underset{G}{\to}} \gamma \text{ and } \gamma \overset{1}{\underset{G}{\to}} \beta.$$

We say β *is **derivable** from* α, $\alpha \overset{*}{\underset{G}{\to}} \beta$, *if* $\alpha \overset{n}{\underset{G}{\to}} \beta$ *for some* $n \geq 0$. *Note* $\overset{1}{\underset{G}{\to}}$ *is a relation on* $(N \cup \Sigma)^*$ *and* $\overset{*}{\underset{G}{\to}}$ *is its reflexive transitive closure.*

If $S \overset{*}{\underset{G}{\to}} x$, *that is* x *is derivable from the start symbol, we say* x *is a **sentential form**. If* x *consists only of terminal symbols, that is, if* $x \in \Sigma^*$, *we say* x *is a **sentence**.*

*The **language generated by a grammar G**,* $L(G)$, *is the set of strings* $\{x \in \Sigma^* \mid S \overset{*}{\underset{G}{\to}} x\}$. *A subset* $S \subseteq \Sigma^*$ *is a **Context-Free Language**, CFL, if* $S = L(G)$ *for some CFG, G.*