CMPS 130 **Computational Models** **Winter 2017**

Homework Assignment 6

- Read through page 98 the book. However, the material on pages 70 through 76 is optional.
- Problem 1.51 page 90.
- Use the procedure shown in class to Minimize the following Deterministic Finite Automata: In all cases $\Sigma = \{a, b\}$ and the start state is the one on the first row of the table and F indicates accept state. (from Kozen)

A.

	a	b
1	6	3
2	5	6
3F	4	5
4F	3	2
5	2	1
6	1	4

B.

	a	b
1	2	3
2	5	6
3F	1	4
4F	6	3
5	2	1
6	5	4

C.

	a	b
0F	3	2
1F	3	5
2	2	6
3	2	1
4	5	4
5	5	3
6	5	0

D.

	a	b
0	3	5
1	2	4
2	6	3
3	6	6
4F	0	2
5F	1	6
6	2	6

1 1.51

To prove that $\equiv_L$ is an equivalency relation, we need to show that it is reflexive, symmetric and transitive.

Proof that $\equiv_L$ is reflexive: Assuming that x is a string and L a language, then for any string z, the string xz will either be in the language L or not, so $x \equiv_L x$, so $\equiv_L$ is reflexive.

Proof that $\equiv_L$ is symmetric: Assuming that x and y are strings and L a language such that $x \equiv y$, then for all strings z, $xz \in L$ whenever $yz \in L$. Therefore $yz \in L$ whenever $xz \in L$, which implies $y \equiv_L x$, so $\equiv_L$ is symmetric.

Proof that $\equiv_L$ is transitive: Assuming that x_1, x_2 and x_3 are strings and L a language such that $x_1 \equiv x_2$ and $x_2 \equiv x_3$. For a string z such that $x_1 z \in L$, it follows that $x_2 z \in L$ which further implies that $x_3 z \in L$. Similarly, if $x_1 z \notin L$ then $x_2 z \notin L$ and thereby $x_3 z \notin L$. Combined, this implies that $x_1 \equiv_L x_3$, so $\equiv_L$ is transitive.

2 Minimization

2.1 A.

δ	a	b
1	6	3
2	5	6
3F	4	5
4F	3	2
5	2	1
6	1	4

eq	1	2	3	4	5	6
1	=					
2	$\neq$	=				
3	$\neq$	$\neq$	=			
4	$\neq$	$\neq$	**=**	=		
5	$\neq$	**=**	$\neq$	$\neq$	=	
6	**=**	$\neq$	$\neq$	$\neq$	$\neq$	=

δ	a	b
1,6	1,6	3,4
2,5	2,5	1,6
3,4F	3,4	2,5

2.2 B.

δ	a	b
1	2	3
2	5	6
3F	1	4
4F	6	3
5	2	1
6	5	4

eq	1	2	3	4	5	6
1	=					
2	$\neq$	=				
3	$\neq$	$\neq$	=			
4	$\neq$	$\neq$	**=**	=		
5	$\neq$	**=**	$\neq$	$\neq$	=	
6	**=**	$\neq$	$\neq$	$\neq$	$\neq$	=

δ	a	b
1,6	2,5	3,4
2,5	2,5	1,6
3,4F	1,6	3,4

2.3 C.

δ	a	b
0F	3	2
1F	3	5
2	2	6
3	2	1
4	5	4
5	5	3
6	5	0

eq	0	1	2	3	4	5	6
0	=						
1	**=**	=					
2	$\neq$	$\neq$	=				
3	$\neq$	$\neq$	$\neq$	=			
4	$\neq$	$\neq$	$\neq$	$\neq$	=		
5	$\neq$	$\neq$	**=**	$\neq$	$\neq$	=	
6	$\neq$	$\neq$	$\neq$	**=**	$\neq$	$\neq$	=

δ	a	b
0,1F	3,6	2,5
2,5	2,5	3,6
3,6	2,5	0,1

2.4 D.

δ	a	b
0	3	5
1	2	4
2	6	3
3	6	6
4F	0	2
5F	1	6
6	2	6

eq	0	1	2	3	4	5	6
0	=						
1	**=**	=					
2	$\neq$	$\neq$	=				
3	$\neq$	$\neq$	**=**	=			
4	$\neq$	$\neq$	$\neq$	$\neq$	=		
5	$\neq$	$\neq$	$\neq$	$\neq$	**=**	=	
6	$\neq$	$\neq$	**=**	**=**	$\neq$	$\neq$	=

δ	a	b
0,1	2,3,6	4,5
2,3,6	2,3,6	2,3,6
4,5F	0,1	3,3,6