

NFA Definitions

A **Non-deterministic Finite Automaton**, NFA, is a structure M such that:

$M = (Q, \Sigma, \delta, s, F)$, where

Q is a finite set of “states”.

Σ is a finite set of “symbols”, an “alphabet”.

$\delta : Q \times \Sigma \rightarrow 2^Q$ is the “transition function”.

For $p \in Q$, $A \subseteq 2^Q$ and $a \in \Sigma$, $\delta(p, a) = A$ means

when in state p scanning symbol a , a non-deterministic transition is made to one of the states in the set A

$s \in Q$ is the “start state”.

$F \subseteq Q$ is the set of “final states”.

The **extended transition function** for M is the function:

$$\begin{aligned}\hat{\delta} : Q \times \Sigma^* &\rightarrow 2^Q, \quad \text{where} \\ \hat{\delta}(q, \epsilon) &= \{q\}, \quad \text{and} \\ \hat{\delta}(q, xa) &= \bigcup_{p \in \hat{\delta}(q, x)} \delta(p, a)\end{aligned}$$

A string $x \in \Sigma^*$ is **accepted** by M if $\hat{\delta}(s, x) \cap F \neq \emptyset$.

The language **accepted or recognized** by M is

$$L(M) = \{x \in \Sigma^* \mid x \text{ is accepted by } M\}.$$