PROPOSITIONAL LOGIC

A proposition ~~is~~ IS A STATEMENT THAT IS TRUE OR FALSE.

propositional connective

not	$\neg$
and	$\wedge$
or	$\vee$
implies	$\rightarrow$
iff	$\leftrightarrow$

constants $0, 1$

variables $X = \{P, Q, R, \ldots\}$ FINITE

BOOLEAN LOGIC

P, Q, ... VARIABLES VALUES 0 1
PROPOSITIONAL FALSE TRUE

OPERATIONS

P	$\neg P$
0	1
1	0

P Q	$P \wedge Q$	$P \vee Q$
0 0	0	0
0 1	0	1
1 0	0	1
1 1	1	1

P Q	$P \rightarrow Q$	$P \leftrightarrow Q$
0 0	1	1
0 1	1	0
1 0	0	0
1 1	1	1

if $x < 10$ then $x < 100$

x	P	Q
1 – 9	T	T
10 | 99	F	T
100 ↓	F	F

VALID ARGUMENT

$$P \to Q$$

$$\underline{P}$$

$$\therefore Q$$

P	Q	$P \to Q$	P	$\neg Q$	$(P \to Q) \wedge (P) \wedge (\neg Q)$
0	0	1	0	1	0
0	1	1	0	0	0
1	0	0	1	1	0
1	1	1	1	0	0

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array} \qquad \begin{array}{l} p \to q \\ \neg q \\ \hline \therefore \neg p \end{array} \qquad \begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array} \qquad \begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

$$\begin{array}{l} p \\ \hline \therefore p \vee q \end{array} \qquad \begin{array}{l} p \wedge q \\ \hline \therefore q \end{array} \qquad \begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array} \qquad \begin{array}{l} \neg p \vee r \\ p \vee q \\ \hline \therefore q \vee r \end{array}$$

CARDINALITY

$|A|$ = # OF ELEMENT IN A WHEN A IS FINITE

$|\emptyset| = 0$

SAME CARDINALITY

CAN PUT ELEMENTS IN 1-TO-1 CORRESPONDENCE

(HOTTENTOTS :)

FINITE

INFINITE

COUNTABLY INFINITE

UNCOUNTABLY INFINITE

CANTOR (1845-1918)

PROVE RATIONALS COUNTABLE

PROVE REALS UNCOUNTABLE

MORE CARDINALITY

$A \subseteq B \quad |A| \leq |B|$

$|A \cup B| = |A| + |B| - |A \cap B|$ FOR FINITE A & B

SO $|A \cup B| \leq |A| + |B|$

IF $A \cap B = \emptyset$

$|A \cap B| = 0$

$|A \cup B| = |A| + |B|$ FINITE SETS

<u>POWERSET</u>

$\mathcal{P}(S)$ OR 2^S SET OF ALL SUBSETS

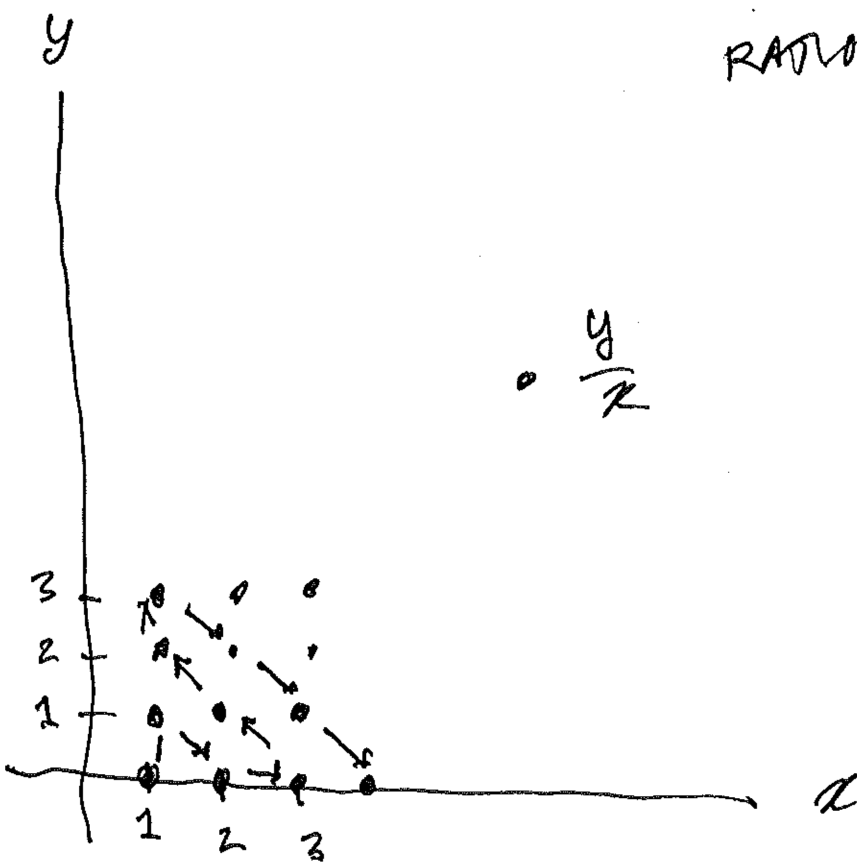
EX: $S = \{1, 2, 3\}$

$2^S = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

NOTE $|2^S| = 2^{|S|}$

Strings of {0,1}		Z^+
ϵ	1	1
0	10	2
1	11	3
00	100	4
01	101	
10	110	
11	111	
000	1000	
001	1001	
010	1010	
011	1011	

RATIONALS



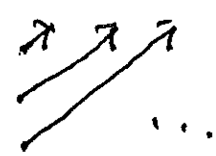
UNION OF A COUNTABLE NUMBER OF COUNTABLE SETS.

	LIST 1	LIST 2	LIST 3	...
ITEM 1	(1, 1)	(2, 1)	(3, 1)	
2	(1, 2)	(2, 2)		
3	(1, 3)			

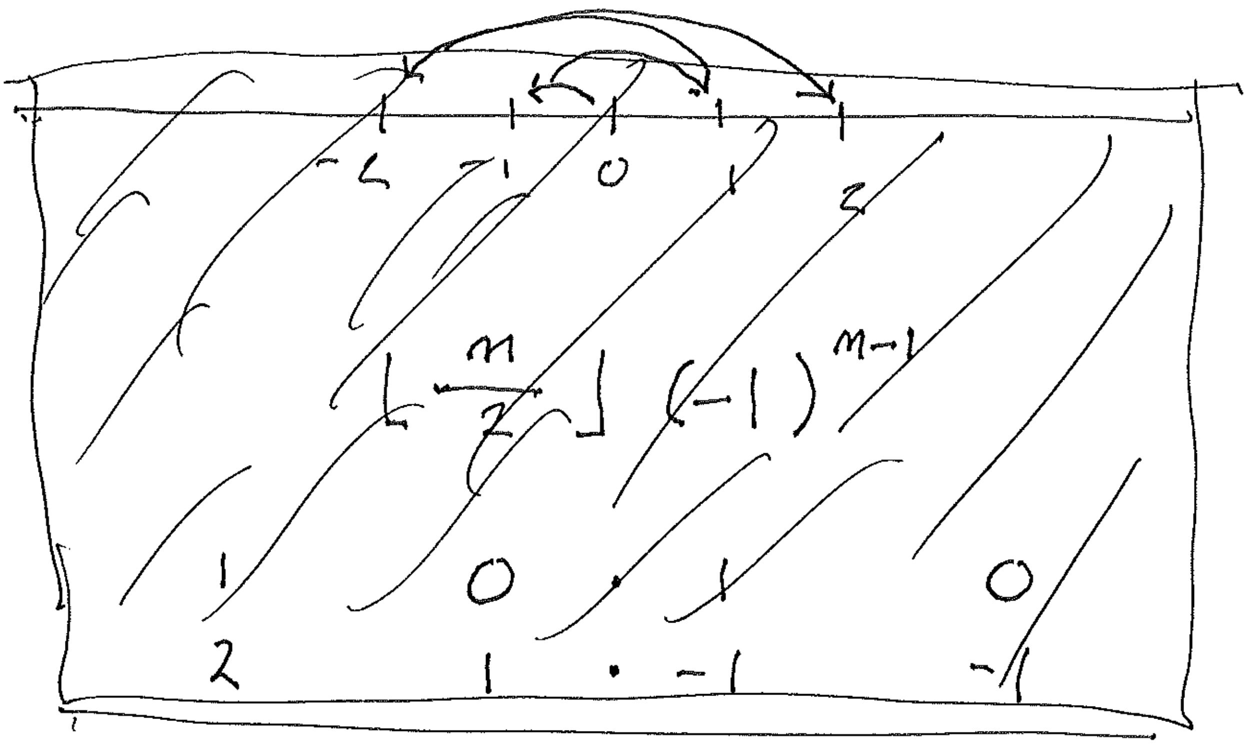
ITEM (n, i) ITEM i OF LIST n

(1, 1)	2
(1, 2)	3
(2, 1)	
(1, 3)	4
(2, 2)	
(3, 1)	

DIAGONALS



$\mathbb{Z}$ countable

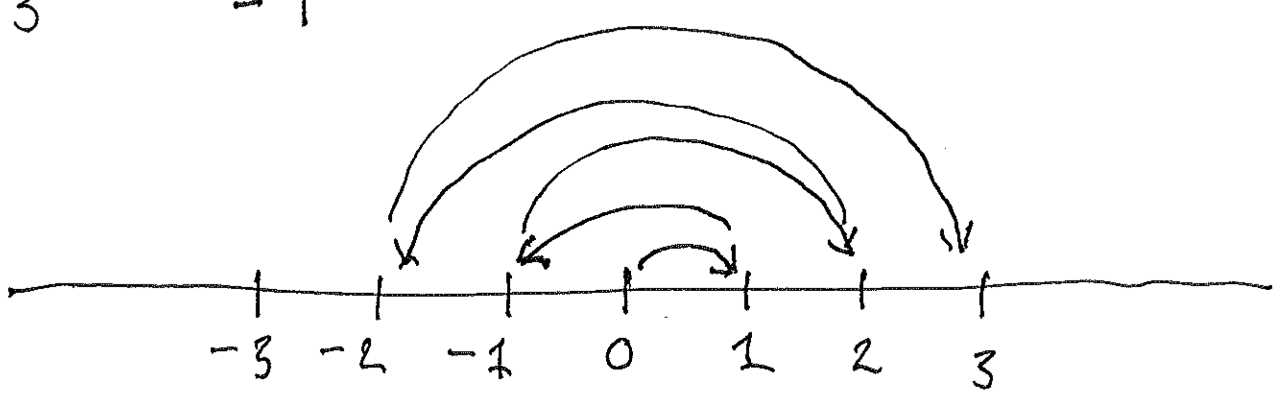


$f(n) = (-1)^n \lfloor \frac{n}{2} \rfloor$

$f : \mathbb{N} \leftrightarrow \mathbb{Z}$

$f^{-1} : \mathbb{Z} \to \mathbb{N}$

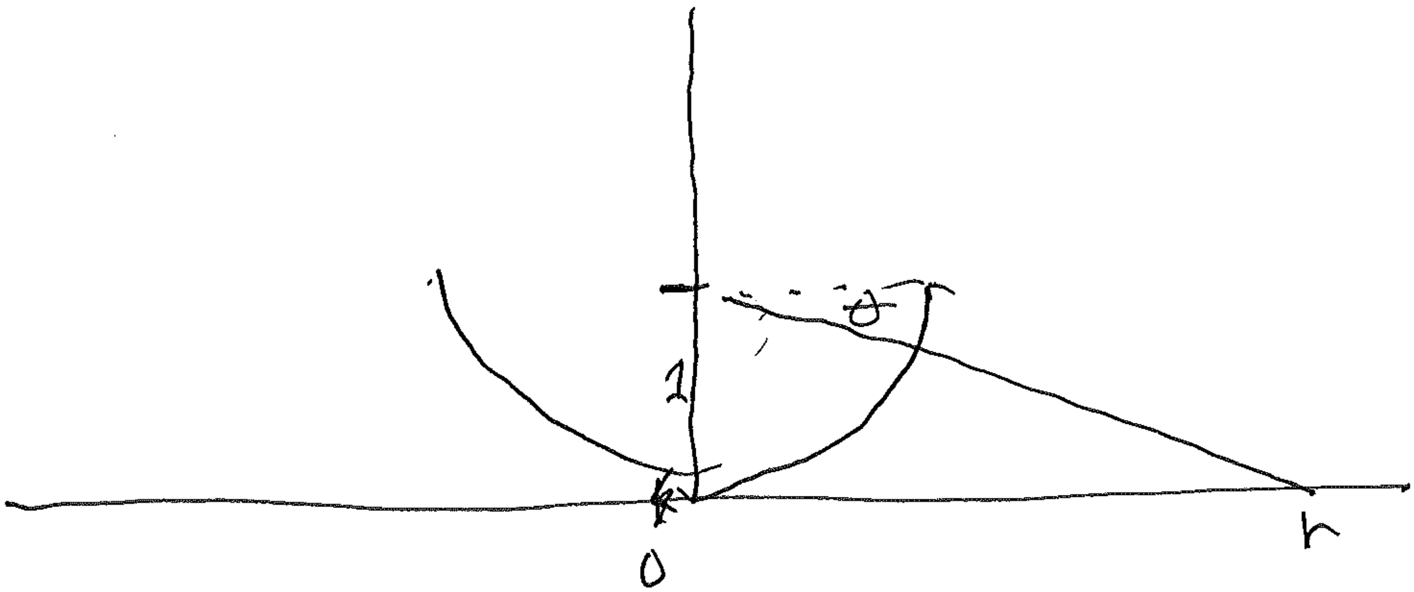
1	0
2	1
3	-1



REALS

1 to 1 CORRESPONDANCE

$(0, 1) \longleftrightarrow (-\infty, +\infty)$



$$\tan\left(\frac{\pi}{2} - \theta\right) = r$$

$$\tan\left(\frac{\pi}{2} - f\pi\right) = r$$

0

$\updownarrow$

1

$$\tan \pi\left(\frac{1}{2} - f\right) = r$$

$-\infty, +\infty$

(0

,

1)

REALS NOT COUNTABLE

LIST OF REALS ≥ 0 AND < 1

	bit 1	2	3	4	5	$\cdots i$
R_1	0	1	1	0	0	
R_2	1	0	1	1	1	
R_3	1	1	0	0	1	
R_m						

i-TH BIT
OF m-TH REAL

$b(m, i)$

CONSTRUCTION OF REAL x THAT
CAN NOT BE IN LIST

$$b(x, i) = b(i, i)$$

SO x DIFFERS IN ONE BIT LOCATION
FROM EVERY NUMBER IN THE LIST
SO NOT IN LIST OF ALL REALS

MORE CARDINALITY

Σ^* COUNTABLY INFINITE

2^{Σ^*} NOT COUNTABLE SET OF ALL LANGUAGES OVER Σ

ε 0 1 00 01 10 11 000 001 010 011 100 101 110 111 0000 , , , , , ,

1

2

3

4

PROOF FINITE STRINGS COUNTABLE

1	1	ϵ
2	10	0
3	11	1
4	100	00
5	101	01
6	110	10
7	111	11
8	1000	000
9	1001	001
10	1010	010
11	1011	011

drop leading 1 →

← add 1 on left
(1os)

1 TO 1 CORRESPONDANCE BETWEEN finite strings of $\{0, 1\}$ and POS BINARY INTEGERS > 0.