Regular Expression Definitions

A **Regular Expression**, re, over an alphabet Σ, is a string over the symbols in the set, $\Sigma \cup \{\epsilon, \emptyset, (,), +, \cdot, *\}$, that can be formed according to the following rules:

1. ϵ is a re.
2. $\emptyset$ is a re.
3. $\forall a \in \Sigma$, a is a re.
4. If r_1, r_2 are re's then $(r_1 + r_2)$ and $(r_1 \cdot r_2)$ are also.
5. If r is a re then r^* is also.
6. Nothing else is a re.

Semantics of a Regular Expression: A regular expression is a syntactic object that denotes a language. The language denoted by the re r is $L(r)$ where

1. $L(\epsilon) = \{\epsilon\}$.
2. $L(\emptyset) = \emptyset$.
3. $\forall a \in \Sigma, L(a) = \{a\}$.
4. $L(r_1 + r_2) = L(r_1) \cup L(r_2)$.
5. $L(r_1 \cdot r_2) = L(r_1) \cdot L(r_2)$.
6. $L(r^*) = (L(r))^*$.