VARIENTS OF TURING MACHINES

MULTITAPE

UNIVERSAL T.M.

HALTING PROBLEM

(COLLATZ)

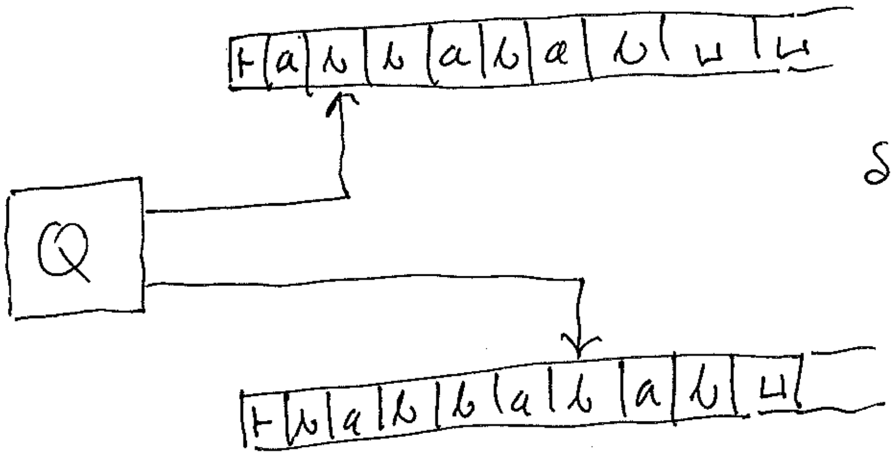
REDUCTIONS

H.P. → M.P.

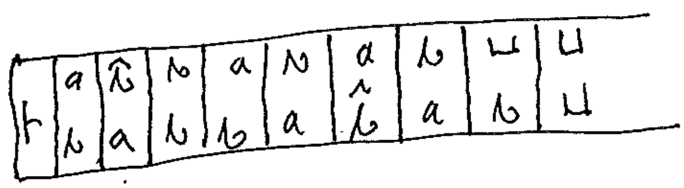
RICE'S THM.

(DEFS OF $\left\{ \begin{array}{l} \text{REC. ENUMERABLE} \\ \text{REC.} \end{array} \right.$)

MULTITAPE MACHINE



$$\delta : Q \times \Gamma^2 \to Q \times \Gamma^2 \times \{L, R\}^2$$



TWO WAY INFINITE TAPE

FOLD.

TWO STACKS INSTEAD OF TAPE

UNIVERSAL TURING MACHINE

UNIVERSAL TURING MACHINE IMITATION ALGORITHM

GIVEN 1. INITIAL TAPE INFORMATION

2. THE FUNCTIONAL MAXTRIX FOR A TM,

SIMULATE THE OPERATION OF TM.

INSTRUCTION 1. SCAN SYMBOL UNDER R/W HEAD

2. LOOK UP ENTRY IN FUNCTION TABLE
FOR CURRENT STATE AND THE SYMBOL READ

WRITE SECOND SYMBOL OF ENTRY
MOVE R/W HEAD ACCORDING TO THIRD SYMBOL OF ENTRY,
SET CURRENT STATE TO FIRST SYMBOL OF ENTRY

3. IF CURRENT STATE ACC OR REJ DO SO.

4. GO TO 1.

SPECIAL CODING

NEED WAY TO DISTINGUISH BETWEEN 3 KINDS
OF SYMBOLS. L R INPUT ALPHABET STATES
TAPE

L	101		
R	1001		
S_1	10001	3 ZEROS	ODD # > 1
S_2	1000001	5 ZEROS	
⋮			
g_1	100001	4 ZEROS	EVEN # > 2
g_2	10000001	6 ZEROS	
⋮			

INTRO TO HALTING PROBLEM

UNIV. MACH. U

TAKES INPUT ENCODING OF M and x

AND SIMULATES OPERATION OF M ON x

HALTS ACCEPTS $\iff$ M HALTS AND ACCEPTS x

HALTS REJECTS $\iff$ M HALTS AND REJECTS x

LOOPS $\iff$ M LOOPS ON x

? MACH U' POSSIBLE ?

HALTS ACCEPTS IF M HALTS AND ACCEPTS x

HALTS REJECTS IIF M HALTS AND REJECTS x

HALTS REJECTS IIF M LOOPS ON x

PROOF THAT HALTING IS NOT DECIDABLE

LET x BE BINARY NUMBER
M_x BE TM WITH ENCODING x
(IF x NOT VALID ENCODING M_x IS Machine that Halts.)

CONSIDER MATRIX

	ϵ	0	1	00	01	10	11	. . .
M_ϵ	H	H	H	H	H	.	-	-
M_0	H	L	H	L	L	H		

x ALL STRINGS

M_x M_x

TOWARD CONTRADICTION
ASSUME MACHINE K EXISTS ← MACHINE THAT DECIDES HALTING PROB.
THAT GIVEN M_x, x CAN DETERMINE H or L.
M # x INPUT

K HALTS AND ACCEPTS IF M HALTS ON x
K HALTS AND REJECTS IF M LOOPS ON x

BUILD Machine N USING K

ON INPUT x

BUILDS M_x AND PUTS M_x, x ON

TAPE

RUNS K

IF K ACCEPTS LOOP

IF K REJECTS ACCEPT

N COMPLEMENTS DIAGONAL

(N HALTS ON $x \iff K$ REJECT $M_x \# x$

$\iff M_x \# x$ LOOPS)

SO N DIFFERENT ON A LEAST ONE STRING

FOR EVERY M_x IN TABLE.

CONTRADICTION.

IMP . . . . EACH ROW DIFFERENT FROM IM AT AT LEAST ONE x.

x

ALL POSSIBLE T.Ms

	ϵ	0	1	00	01	10	11	.
M_ϵ	H	H	H	H	H	H	H	
M_0								
M_1								
M_{00}								
M_{01}								
M_{10}								
M_{11}								
M_x	H	L	H	H	L			

$H(\langle M \rangle x)$ = YES IF $\langle M \rangle x$ HALTS

= NO IF $\langle M \rangle x$ LOOPS

IM (x)

FIND $H(\langle M_x \rangle x)$

IF YES LOOP

IF NO H

SO IM(x) IS OPPOSITE OF $\langle M_x \rangle x$

* EACH ROW DIFFERENT FROM IMP IN AT LEAST ONE SPOT

IM IS DIFFERENT FROM EACH M_x ROW

IN TABLE FOR AT LEAST ONE INPUT NAMELY x

HALTING PROBLEM

H

INPUT

$\langle M \rangle\, x$

OUTPUT

YES $\quad M(x)$ HALTS

NO $\quad M(x)$ DOES NOT HALT

ASSUME H EXISTS
TOWARD CONTRADICTION

N

INPUT

$\langle M \rangle\, x \qquad H(\langle M \rangle, x)$

$H(\langle N \rangle\, x)$ YES $\qquad$ LOOP

$H(\langle N \rangle\, x)$ NO $\qquad$ HALT

BUILD HYPOTHETICAL MACHINE
BASED ON EXISTANCE OF H

N

INPUT

$\langle N \rangle \quad \langle N \rangle$

$H(\langle N \rangle \langle N \rangle)$ YES $\qquad$ LOOP

$H(\langle N \rangle \langle N \rangle)$ NO $\qquad$ HALT

HYPOTHETICAL MACHINE
IS NOT POSSIBLE
ON INPUT $\langle N \rangle$ IT
LOOPS IF IT HALTS
HALTS IF IT LOOPS

<u>CONTRADICTION</u>

REDUCTION

IF A CAN BE REDUCED TO B

B MUST BE HARDER OR EQUAL.

$$A \leq_p B$$

HALTING $\leq_p$ MEMBERSHIP.

IF HALTING PROB CAN BE REDUCED TO MEM. PROB,

THEN MEM PROB MUST BE AS HARD AS HALTING PROB.

HP $<$ MP

$\langle M \rangle \# x$ DOES M HALT ON x

$\langle M' \rangle \# x$ IS $x \in L(M')$

M ACC $\rightarrow$ M' ACC

M REJ $\rightarrow$ M' ACC

M LOOPS $\rightarrow$ M' LOOPS

$x \in L(M')$

IFF M HALTS ON x

I.E. ACC OR REJ

MP $<$ HP

$\langle M \rangle$ IS $x \in L(M)$

$\langle M' \rangle$ DOES M' HALT ON x

M ACC $\rightarrow$ M' ACC

M REJ $\rightarrow$ M' LOOPS

M LOOPS $\rightarrow$ M' LOOPS

M' HALTS IFF $x \in L(M)$

META QUESTIONS ABOUT F.A.s

DECISION PROBLEM WITH F.A. AS INPUT

1. GIVEN F.A M IS $L(M) \neq \emptyset$?
2. GIVEN F.A. M IS $L(M) = \Sigma^*$?
3. GIVEN F.A. M IS $|L(M)|$ infinite ?
4. GIVEN F.A.s M_1 & M_2 IS $L(M_1) = L(M_2)$

THM
EACH OF THESE IS SOLVABLE
THERE EXISTS AN ALGORITHM.

TRADE OFF
LIMITED EXPRESSIVE POWER
GOOD ALGs FOR DECISIONS ABOUT THEM

FOR TURING MACHS ALL OF THESE ARE NOT SOLVABLE.

RICE'S THEOREM

1. $L(M) \neq \emptyset$ IFF $\exists w \in \Sigma^*$ s.t. $|w| \leq n$

AND $\hat{\delta}(q_0, w) \in F$

EXHAUSTIVE SEARCH.

2. $L(M) = \Sigma^*$ IFF $\overline{L(M)} = \emptyset$ IFF $L(\overline{M}) = \emptyset$

WHERE $\overline{M} = (Q, \Sigma, q_0, Q - F, \delta)$

3. $L(M)$ IS INFINITE IFF

M ACCEPTS STRING w s.t. $|Q| \leq |w| < 2|Q|$

4. $L(M_1) = L(M_2)$

IFF $L(M_1) \subseteq L(M_2)$ AND $L(M_2) \subseteq L(M_1)$

IFF $L(M_1) - L(M_2) = \emptyset$ AND $L(M_2) - L(M_1) = \emptyset$

NOTE GIVEN M_1, M_2

$$L(M) = L(M_1) - L(M_2)$$
$$= L(M_1) \cap \overline{L(M_2)}$$

SET DEFINITIONS

recursively enumerable
if S = L(M) FOR SOME T.M.

recursive if S = L(M) FOR SOME total T.M.
i.e. never loops
halts on all input

PROPERTY DECIDABLE
IF SET OF STRINGS $\bar{c}$ PROPERTY IS RECURSIVE
I.E. THERE EXISTS total T.M.
ACCEPTS STRINGS $\bar{c}$ PROP. P
REJECTS STRINGS $\bar{c}$-OUT PROP P.