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HomeWork 7

Use MyHill-Nerode Theorem to prove that the languages are not regular.

1.29 a) $A_1 = \{0^n 1^n 2^n \mid n \geq 0\}$
Consider the set $X = \{0^n \mid n \geq 1\}$. It is clearly infinite. Any two members of X have the form $0^i, 0^j$, where $i \neq j$ and the string $z = 1^i 2^i$ distinguishes them. Since $0^i \underline{1}^i 2^i \in L$ and $0^j \underline{1}^i 2^i \notin L$, the language is not regular.
c) $A_3 = \{a^{2^n} \mid n \geq 0\}$ (here a^{2^n} means a string of 2^n a's)
Consider the set $X = \{a^{2^n+1} \mid n \geq 1\}$ where the string a^{2^n+1} is in the language by appending a^{2^n-1}. It is clearly infinite. Ant two members of X have the form a^{2^i+1}, a^{2^j+1}, where $i \neq j$. Since i and j are any number is the set of natural numbers, we have an infinite number of equivalent classes. Since this contradicts the theorem telling us regular languages have a finite number of subsets, this language is not regular.

1.46 b) $\{0^m 1^n \mid m \neq n\}$
Consider the set $\{0^n \mid n \geq 1\}$. It is clearly infinite. Any two members of X have the form $0^i, 0^j$, where $i \neq j$ and the string $z = 0^i \underline{1}^i$ distinguishes them. Since $0^i 1^i \notin L$ and $0^j 1^j \in L$, the language is not regular
c) $\{w \mid w \in \{0,1\}^*$ is not a palindrome$\}$

Prove any language of your choice is regular using My-Hill Nerode theorem. Could you have done the proof with pumping lemma
$A_2 = \{w \mid w$ begins with 0 and ends with a 0$\}$

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Consider the following finite list of subsets of Σ^*: $A_2 = \{w | w$ begins with a 0 and ends with a 0$\}$, $A_1 = \{w | w$ begins with a 1 and ends with a 1$\}$. Claim the members of A_2 are equivalent to each other, because for any two members, say w^i, w^j where $i \neq j$ and both end and start with a 0, it is the case that $w^i R_L w^j$, since for any $z \in \Sigma^*$, say, w^m, both $w^i w^m$ and $w^j w^m$ are in A_2 if w^m ends with a 0 and both are not in A_2 if w^m ends with a 1. The members of A_1 are all equivalent by a similar argument with the roles of start and end switched. The same logic can be applied to $A_3 = \{w | w$ begins with 0 and ends with 1$\}$ or $A_4 = \{w | w$ begins with 1 and ends with 0$\}$. Now clearly every string is either one of the 4 proposed, so $A_1 \cup A_2 \cup A_3 \cup A_4 = \Sigma^*$. The proof could have been done with the pumping lemma.

2.3 Answer each part for the following context-free grammar G.

$$R \rightarrow XRX \mid S$$
$$S \rightarrow aTb \mid bTa$$
$$T \rightarrow XTX \mid X \mid \varepsilon$$
$$X \rightarrow a \mid b$$

a) What are the variables of G?

R, S, T, X

b) What are the terminals of G?

a, b

c) Which is the start variable of G?

R

d) Give three string in $L(G)$?

ab, ba, abbbb

e) Give three strings not in $L(G)$?

aa, aba, bbb

f) T/F: $T \Rightarrow aba$

False

g) T/F : $T \overset{*}{\Rightarrow} aba$
true

h) T/F : $T \Rightarrow T$
False

i) T/F : $T \Rightarrow^* T$
true

j) T/F : $XXX \Rightarrow^* aba$
true

k) T/F : $X \Rightarrow^* aba$
False

l) T/F : $T \Rightarrow^* XX$
true

m) T/F : $T \Rightarrow^* XXX$
False

n) T/F : $S \Rightarrow^* \varepsilon$
False

o) Give a description in Eglish of $L(G)$.
The language of strings a,b such that it is not a palindrome

2.4 Give a CFG that generates the following languages In all parts Σ is $\{0,1\}$.

a) $\{w \mid w$ contains at least 3 1s$\}$
$S \to X1X1X1X$
$X \to 0X \mid 1X \mid \varepsilon$

b) $\{w \mid w$ starts and ends with the same symbol$\}$
$S \to 0X0 \mid 1X1$
$X \to 0X \mid 1X \mid \varepsilon$

c) $\{w \mid w$ the length of w is odd$\}$
$S \to 0S0 \mid 0S1 \mid 1S0 \mid 1S1 \mid 0 \mid 1$

d) $\{w \mid w$ the length of w is odd and its middle symbol is $0\}$
$S \to 0S0 \mid 0S1 \mid 1S0 \mid 1S1 \mid 0$

e) $\{w \mid w = w^R$, that is, w is a palindrome$\}$

$S \rightarrow 0S0 \mid 1S1 \mid 0 \mid 1 \mid \varepsilon$

f) The empty set

$S \rightarrow \varepsilon$

Give a CFG for the language

$\{x \in \{a,b\}^* \mid x \neq ww$ for some $w \in \{a,b\}^*\}$

$S \rightarrow abS \mid Sba \mid a \mid b \mid \varepsilon$