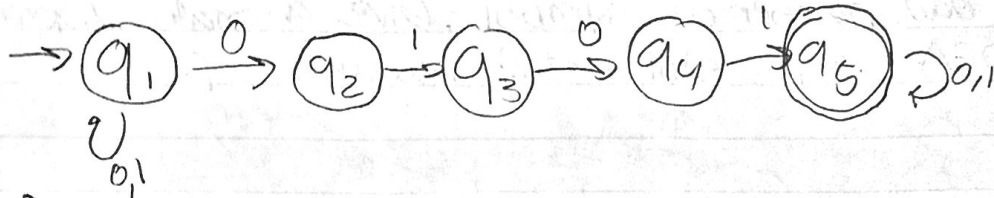
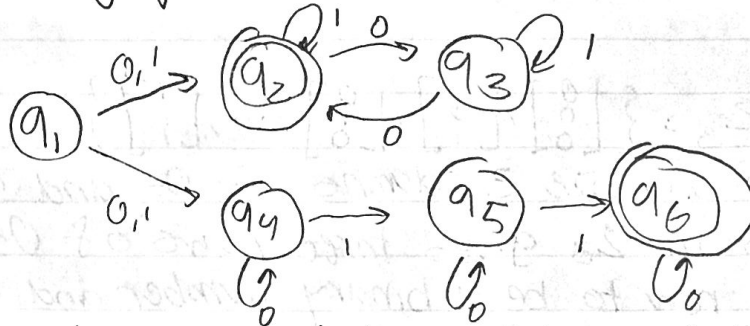


Homework 3

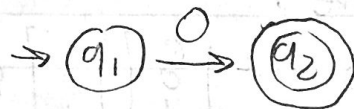
1.7 b) 5 states $\{w \mid w \text{ contains the substring } 0101\}$



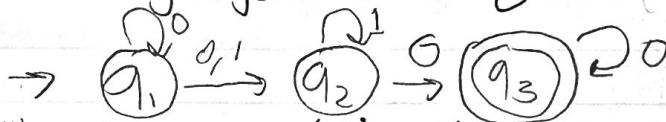
c) The language of exercise 1.61 with six states



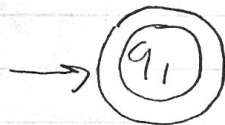
d) The language $\{0\}$ with two states



e) The language $0^*1^*0^+$ with three states



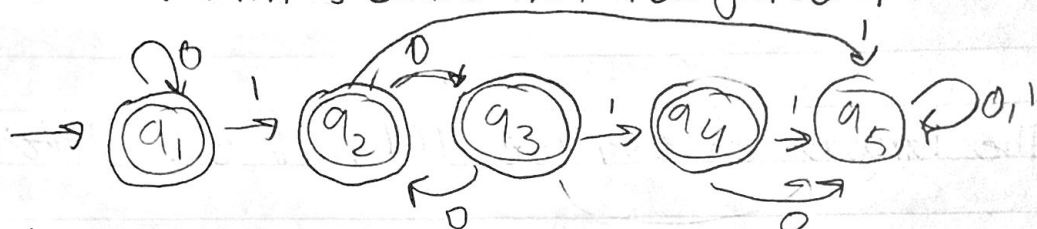
g) The language $\{\epsilon\}$ with one state



h) The language 0^* with one state



1.13 Let F be the language of all strings over $\{0,1\}$ that do not contain a pair of 1s that are separated by an odd number of symbols. Give a state diagram of a DFA with 5 states that recognize F .



1.32 Let

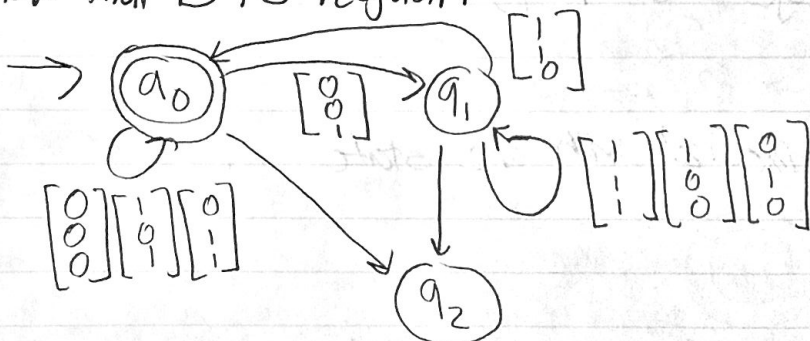
$$\Sigma_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \dots, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

Σ_3 contains all size 3 columns of 0s and 1s. A string of symbols in Σ_3 gives three rows of 0s and 1s. Consider each row to be a binary number and let

$$B = \{ w \in \Sigma_3^* \mid \text{the bottom row of } w \text{ is the sum of the top two rows} \}$$

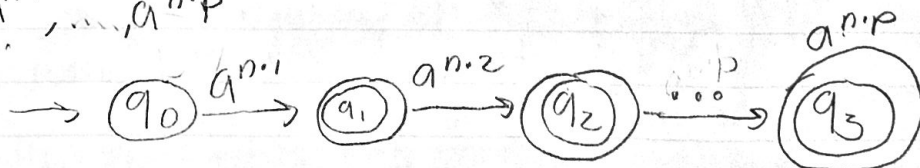
For example $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \in B$ but $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \notin B$

Show that B is regular.



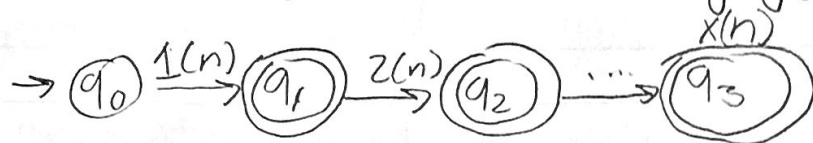
1.36 Let $B_n = \{ a^k \mid k \text{ is a multiple of } n \}$. Show that for each $n \geq 1$ the language B_n is regular.

Since k is a multiple of $n \forall n \geq 1$, a^k can be written as $a^{n \cdot 1}, \dots, a^{n \cdot p}$



1.37 Let $C_n = \{x \mid x \text{ is a binary number that is a multiple of } n\}$

Show that for each $n \geq 1$, the language C_n is regular



1.41 For Languages A and B , let the perfect shuffle of A and B be the language

$\{w \mid w = a_1 b_1 \dots a_k b_k, \text{ where } a_1 \dots a_k \in A \text{ and } b_1 \dots b_k \in B, \text{ each } a_i, b_i \in \Sigma\}$.

Show that the class of regular languages is closed under perfect shuffle.

