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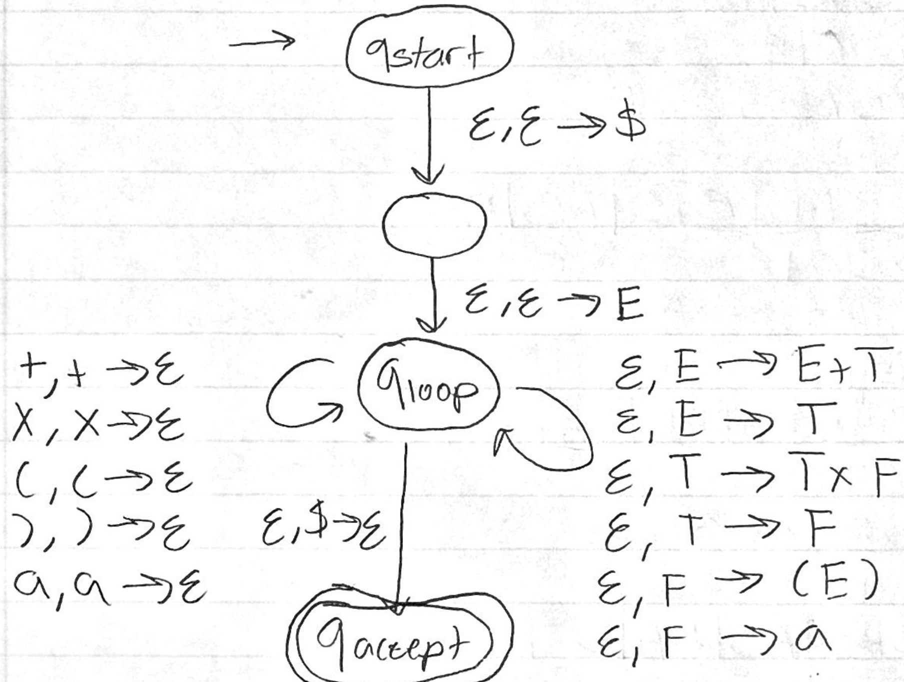
Homework 8

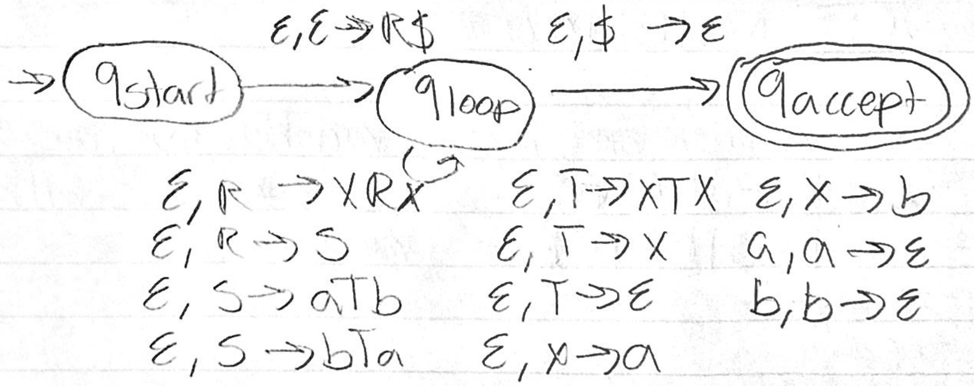
2.11 Convert the CFG given in Exercise 2.1 to an equivalent PDA, using the procedure given in Theorem 2.20.

$$E \to E + T \mid T$$
$$T \to T \times F \mid F$$
$$F \to (E) \mid a$$



2.12 Convert the CFG given in Exercise 2.3 to an equivalent PDA.

$$R \to XRX \mid S$$
$$S \to aTb \mid bTa$$
$$T \to XTX \mid X \mid \varepsilon$$
$$X \to a \mid b$$



2.14 Convert the following CFG into an equivalent CFG in Chomsky normal form, ~~using the procedure given~~

$A \to BAB \mid B \mid \varepsilon$

$B \to 00 \mid \varepsilon$

add a start variable

$S \to A$

$A \to BAB \mid B \mid \varepsilon$

$B \to 00 \mid \varepsilon$

remove epsilons of B

$S \to A$

$A \to \underline{B}A\underline{B} \mid \underline{B} \mid \varepsilon \mid AB \mid BA$

$B \to 00 \mid \cancel{\varepsilon}$

remove epsilon of A

$S \to \underline{A} \mid \varepsilon$

$A \to B\underline{A}B \mid B \mid \underline{A}B \mid B\underline{A} \mid \cancel{\varepsilon}$

$B \to 00$

remove epsilon of S

$S \to A \mid \cancel{\varepsilon}$

$A \to BAB \mid AB \mid BA \mid B$

$B \to 00$

remove/replace $S \to A$

$S \to \cancel{A} \mid BAB \mid AB \mid BA \mid B$

$A \to BAB \mid AB \mid BA \mid \underline{B}$

$B \to 00$

replace B with 00

$S \to \underline{BAB} \mid AB \mid BA \mid 00$

$A \to \underline{BAB} \mid AB \mid BA \mid 00$

$B \to 00$

Create a new variable and replace variables that have 72 groupings

$S \to CB \mid AB \mid BA \mid DD$

$A \to CB \mid AB \mid BA \mid DD$

$B \to DD$

$C \to BA$

$D \to 0$

replace terminals with variables

2.30 Use the pumping Lemma to show that the following languages are not context free.

a) $\{0^n 1^n 0^n 1^n \mid n \geq 0\}$

Assume the language is context free. If so, the pumping lemma states there exist p (the pumping length), $\forall s \in A$ such that $|s| \geq p$, $s = uvxyz$ and

1. $uv^i xy^i z \in A \mid i \geq 0$
2. $|vy| > 0$
3. $|vxy| \leq p$

$s = 0^p 1^p 0^p 1^p$, $|s| \geq p$

If vxy is composed of all 0s or all 1s then it is either in the first string of 0s/1s or the second string of 0s/1s. This forces the other string to have a different number of 0s/1s thus not being in the language.
If vxy is composed of 0 and 1 or 1 and 0, then there would be an extra 0 or 1 and also create a string not in the language when pumped.

b) $\{0^n \# 0^{2n} \# 0^{3n} \mid n \geq 0\}$

$s = 0^p \# 0^{2p} \# 0^{3p}$, $|s| \geq p$

If vxy is composed of 0^p, when pumped down, it will decrease the number of 0s without modifying the other 0s thus removing it from the language.
If vx is composed of 0^p and y composed of $\# 0^{2p}$, then when pumped, it would contain multiple #s, removing it from the language.
If v is 0^p, x is #, and y is 0^{2p}, when pumped it wouldn't modify 0^{3p} thus removing it from the language.
Same concept applies if vxy were swapped with any of the values.

c) $\{w\#t \mid w$ is a substring of t, where $w,t \in \{a,b\}^*\}$

$S = a^P b^P \# a^P b^P \quad |S| \geq P$

If vxy is a^P of either side, when it gets pumped, the opposing side would not change so the string is not in the language.

If vx is a^P while y contains some as and some b^P then the pumping would create symbols that were out of order like abababb, thus resulting in the string not being in the language.

If v is a^P, x is b^P, y is $\#$, then clearly the string wouldn't be in the langvge because of the pumped y and a/b don't match.

Thus if we apply this logic to every combination we can see that any string pumped is not in the language.

d) $\{t_1 \# t_2 \# \ldots \# t_k \mid k \geq 2$, each $t_i \in (a,b)^*$, and $t_i = t_j$ for some $i \neq j\}$

$S = a^P b^P \# a^P b^P \quad |S| \geq P$

If vxy is $a^P b^P$ of either side, when pumped, the opposing sides $t_1 \neq t_2$ thus not being in the language

If v is b^P, x is $\#$, and y is a^P, when pumped, the a^P of the left most and b^P of the right most would not match.

2.31 Let B be the language of all palindromes over $\{0,1\}$ containing equal numbers of 0s and 1s. Show B is not context free

$S = 0^P 1^P 1^P 0^P \quad |S| \geq P$

If vxy is $0^P 1^P$ or $1^P 0^P$, pumping either would resut in a string that is not a palindrome. The same applies when pumping each 0^P or 1^P individually.

2.32. Let $\Sigma = \{1,2,3,4\}$ and $C = \{w \in \Sigma^* \mid$ in w, the number of 1s equals the number of 2, and the number of 3s equal the number of 4s$\}$. Show that C is not context free?

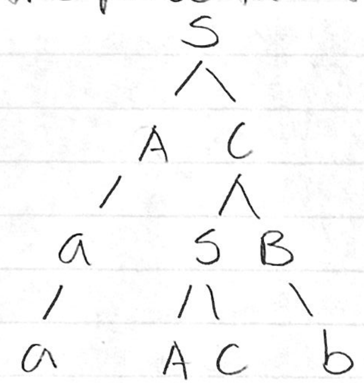
$$S = 1^p 2^p 3^p 4^p \quad |S| \geq p$$

If vxy is the string of only $1^p, 2^p, 3^p$, or 4^p, then when pumped, their corresponding pair would not have the same amount of numbers.

If vxy contained a mixture of numbers than because of the form uv^ixy^iz, when pumped, 1 of the numbers will have an extra value thus also not having the same amount of numbers to its equivalent pair.

2.35 Let G be a CFG in chomsky normal form that contains b variables. Show that if G generates some string with a derivation having at least 2^b steps, $L(G)$ is infinite.

Using the parse tree from class as an example



In this example we can see that the chomsky normal form which requires $A \to BC$, $A \to a$ would result in a repetition of terminals, if the derivation had at least 2^b steps. Other wise each variable would be able to end at a terminal.