

### DEPARTMENT OF FINANCIAL MATHEMATICS

# Comparison of Forecasting Models for Value at Risk

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A thesis submitted in fulfilment of the requirements for the degree of Master of Science

## Declaration of Authorship

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### Abstract

Value at Risk (VaR) is one of the most popular methods of financial risk measure and a topic of interest in Financial Mathematics. In this thesis, Historical simulation, Garch (1,1) and the long short term memory neural network are used for daily Value at Risk estimation for stock markets of UK, US and Japan. We evaluate our VaR estimates and compare the three methods.

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## **Basics**

#### 1.1 Section types

The basic sections in a book are chapter, section, subsections and so on. These sections will appear in the *tableofcontents* of the book.

#### 1.1.1 Example subsection

This is just one example for a subsection.

#### 1.2 Basic page settings

In order to show how the basic page settings are set up, we will use a long dummy text with the package *blindtext*.

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#### 1.3 Head and foot

The head and foot of the document can be adapted using the packages  $\underline{fancyhdr}$ . The using the commands, e.g.,  $\underline{pagestyle\{fancy\}}$ ,  $\underline{l/c/rhead/foot}$  or with  $\underline{fancyhead/foot[EL,CO]}$  the respective parts can be edited as needed.

#### 1.4 New commands and input

If some long commands for formatting or other utilities are used very often, e.g., **test**, test, test, test, then the definition of new personal commands is very useful. For this the command newcommand is used before the document, e.g., test. Over time the collection of personal commands will grow, so for these it is better to create a separate file and copy this file to the current project folder. The file can be then loaded within the project with the command input before the document.

### 1.5 Language

The default language for the document is English. This can be changed, e.g., in German (with the package <u>ngerman</u>) or whatever you need. This changes the language of automatically generated words like chapter, figure, table, and others. It should be also noted that in most latex editors dictionaries for several languages can be used (see *options*).

# Math stuff

#### 2.1 Equations and math mode

We are able to create automatically enumerated equation as the following one

$$f(x) = A_{ijkl}^{23}(x) \int_{0}^{l} g(y, x) \frac{\partial h(y, x)}{\partial y} dy . \qquad (2.1)$$

Equation can be given a name/label. In order to refer to it later in the text the package amsmath has to be included. After including the package, the command to refer to labeled equation is (2.1).

Equation without a number can be created as follows

$$f(x) = A_{ijkl}^{23}(x) \int_{0}^{l} g(y, x) \frac{\partial h(y, x)}{\partial y} dy ,$$

or alternatively

$$f(x) = A_{ijkl}^{23}(x) \int_{0}^{l} g(y,x) \frac{\partial h(y,x)}{\partial y} dy .$$

You can also create a so called equation array with automatic numbering, e.g.,

$$f(x) = (x+a)^2 (2.2)$$

$$= (x+a)(x+a)$$

$$= x^2 + 2xa + a^2$$
(2.3)

$$= x^2 + 2xa + a^2 (2.4)$$

You can refer to (2.2) and (2.4) separately. The very same can be created without any numbers as

$$f(x) = (x+a)^2$$
$$= (x+a)(x+a)$$
$$= x^2 + 2xa + a^2$$

Sometimes math content will be explained directly within the text. For these cases the math mode using <u>\$\mathscr{S}\$</u> can be used, e.g.,  $f(x) = x_{ijkl}^{234}$ 

#### 2.2Arrays and matrices

Arrays can be used within math environments in order to create a grid with math elements, e.g.,

$$x + y + z \quad m_{1234567} \quad 13425436543634$$

$$A_{ijkl}^{23}(x) \int_{0}^{l} g(y, x) \frac{\partial h(y, x)}{\partial y} dy \qquad n_k \qquad 123$$

$$(2.5)$$

A set of equations can also be arranged as follows

$$f(x) = A_{ijkl}^{23}(x) \int_{0}^{l} g(y,x) \frac{\partial h(y,x)}{\partial y} dy$$

$$= 7x.$$
(2.6)

This is an alternative to <u>equarray</u> with a single centered number, but some symbols may not be displayed as wanted. In order to force a full size display of a chosen element of the array the command <u>displaystyle</u> can be used

$$f(x) = A_{ijkl}^{23}(x) \int_{0}^{l} g(y,x) \frac{\partial h(y,x)}{\partial y} dy$$

$$= 7x.$$
(2.7)

Arrays can also be used in order to represent matrices, e.g.,

$$\begin{pmatrix}
123123 & 324 & 214 & 4 \\
43 & 345345645 & 45353465 & 346
\end{pmatrix}.$$
(2.8)

Alternatively matrices can be created with the following environments

$$\begin{pmatrix} 123123 & 324 & 214 & 4 \\ 43 & 345345645 & 45353465 & 346 \end{pmatrix} \quad \begin{bmatrix} 123123 & 324 & 214 & 4 \\ 43 & 345345645 & 45353465 & 346 \end{bmatrix} \qquad (2.9)$$

#### 2.3 Math fonts

Depending on what is to be presented or discussed in the work, several math fonts might be useful for different concepts. For extended fonts the package  $\underline{amssymb}$  is needed. The basic fonts are then

The commands <u>mathbb</u> and others can be changed using different packages, e.g., <u>euscript</u> and <u>lucida</u> (look for latex math fonts in stackexchange). It is very useful to define the most used fonts as new commands within your personal macros, e.g., A.

#### 2.4 Math symbols

The amount of math symbols offered in latex is immense. Some of them are, e.g.,

$$\sum, \int, \iiint, \nabla, \cdot, \times, \otimes, \to, \Rightarrow, \bigcup, \in, \subset.$$
 (2.10)

You will have to look for those you might need.

# Hyperlinks and references

#### 3.1 The package hyperref

The package <u>hyperref</u> is the package for referring to labeled elements of a document and hyperlinks. Now, chapters, sections, equations, figures, tables and other elements can be labeled and referred to, e.g., Equation 2.1, section 2.4 and chapter 3. These are clickable links which in the pdf redirects the reader to the referred element (with ALT+LEFT you can then go back to where you were reading). Here, different alternatives can be used, e.g., 3, chapter 3 or Chapter 3. Depending on which language you have to write something, you may need language options (e.g., ngerman for German hyperlinks).

#### 3.2 Hyperlinks to internet sites, email and attached files

Hyperlinks can be added as, e.g., http://miktex.org/ or click me. Sending an email to a prescribed address can be done by name.lastname@address.org. If the pdf is delivered within a folder with useful files, these files can be linked in the pdf, e.g., manipulate or video.

#### 3.3 Literature references

Bibtex files with literature information can be created either manually or using literature manager programs like Mendeley or Citavi. The bibtex file must be included in the project with <u>bibliography</u> pointing to the file, together with <u>bibliographystyle</u> and a packages for citing commands. With the commands <u>cite/p</u> elements of the included file are then cited, e.g., Hill (1952) and (Kröner, 1977). Make sure that while compiling you have chosen a procedure including bibtex (see compiling options). Sometimes it may be necessary to delete all files but not the main.tex file in order to be able to compile again the project, if bibliography styles have been changed.

# Figures, tables, enumerate and itemize

#### 4.1 Figures

In almost every document figures will be needed in order to explain a concept or just present something. The package *graphicx* is needed for embedding figures.

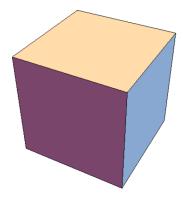


Figure 4.1: A figure caption beneath the figure for description of the depicted concept which sometimes can be very long

In Figure 4.1, for example, a PNG image is depicted (compiled with pdflatex). Alternatively, EPS figures can be embedded if dvips and ps2pdf compilation is used. All figures are listed in the list of figures with the command *listoffigures*.

#### 4.2 Tables

Data can be presented in tables, e.g., as shown in Table 4.1.

	Property 1	Property 2
Criterion 1	764	23546
Criterion 2	3	34

Table 4.1: Exemplary table

Sometimes very long tables must be presented which may also go over pages. For this cases the packages *longtable* is useful, as used in

$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
8       16       24         27       54       81         64       128       192         125       250       375         216       432       648         343       686       1029         512       1024       1536         729       1458       2187         1000       2000       3000         1331       2662       3993         1728       3456       5184         2197       4394       6591         2744       5488       8232         3375       6750       10125         4096       8192       12288         4913       9826       14739         5832       11664       17496         6859       13718       20577         8000       16000       24000         9261       18522       27783         10648       21296       31944         12167       24334       36501         13824       27648       41472         15625       31250       46875         17576       35152       52728         19683       39366 <td< td=""></td<>
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64       128       192         125       250       375         216       432       648         343       686       1029         512       1024       1536         729       1458       2187         1000       2000       3000         1331       2662       3993         1728       3456       5184         2197       4394       6591         2744       5488       8232         3375       6750       10125         4096       8192       12288         4913       9826       14739         5832       11664       17496         6859       13718       20577         8000       16000       24000         9261       18522       27783         10648       21296       31944         12167       24334       36501         13824       27648       41472         15625       31250       46875         17576       35152       52728         19683       39366       59049         21952       43904       65856         24389       4
125       250       375         216       432       648         343       686       1029         512       1024       1536         729       1458       2187         1000       2000       3000         1331       2662       3993         1728       3456       5184         2197       4394       6591         2744       5488       8232         3375       6750       10125         4096       8192       12288         4913       9826       14739         5832       11664       17496         6859       13718       20577         8000       16000       24000         9261       18522       27783         10648       21296       31944         12167       24334       36501         13824       27648       41472         15625       31250       46875         17576       35152       52728         19683       39366       59049         21952       43904       65856         24389       48778       73167         27000
216       432       648         343       686       1029         512       1024       1536         729       1458       2187         1000       2000       3000         1331       2662       3993         1728       3456       5184         2197       4394       6591         2744       5488       8232         3375       6750       10125         4096       8192       12288         4913       9826       14739         5832       11664       17496         6859       13718       20577         8000       16000       24000         9261       18522       27783         10648       21296       31944         12167       24334       36501         13824       27648       41472         15625       31250       46875         17576       35152       52728         19683       39366       59049         21952       43904       65856         24389       48778       73167         27000       54000       81000         29791
343       686       1029         512       1024       1536         729       1458       2187         1000       2000       3000         1331       2662       3993         1728       3456       5184         2197       4394       6591         2744       5488       8232         3375       6750       10125         4096       8192       12288         4913       9826       14739         5832       11664       17496         6859       13718       20577         8000       16000       24000         9261       18522       27783         10648       21296       31944         12167       24334       36501         13824       27648       41472         15625       31250       46875         17576       35152       52728         19683       39366       59049         21952       43904       65856         24389       48778       73167         27000       54000       81000         29791       59582       89373         327
512         1024         1536           729         1458         2187           1000         2000         3000           1331         2662         3993           1728         3456         5184           2197         4394         6591           2744         5488         8232           3375         6750         10125           4096         8192         12288           4913         9826         14739           5832         11664         17496           6859         13718         20577           8000         16000         24000           9261         18522         27783           10648         21296         31944           12167         24334         36501           13824         27648         41472           15625         31250         46875           17576         35152         52728           19683         39366         59049           21952         43904         65856           24389         48778         73167           27000         54000         81000           29791         59582
729         1458         2187           1000         2000         3000           1331         2662         3993           1728         3456         5184           2197         4394         6591           2744         5488         8232           3375         6750         10125           4096         8192         12288           4913         9826         14739           5832         11664         17496           6859         13718         20577           8000         16000         24000           9261         18522         27783           10648         21296         31944           12167         24334         36501           13824         27648         41472           15625         31250         46875           17576         35152         52728           19683         39366         59049           21952         43904         65856           24389         48778         73167           27000         54000         81000           29791         59582         89373           32768         65536
1000         2000         3000           1331         2662         3993           1728         3456         5184           2197         4394         6591           2744         5488         8232           3375         6750         10125           4096         8192         12288           4913         9826         14739           5832         11664         17496           6859         13718         20577           8000         16000         24000           9261         18522         27783           10648         21296         31944           12167         24334         36501           13824         27648         41472           15625         31250         46875           17576         35152         52728           19683         39366         59049           21952         43904         65856           24389         48778         73167           27000         54000         81000           29791         59582         89373           32768         65536         98304           35937         7
1331       2662       3993         1728       3456       5184         2197       4394       6591         2744       5488       8232         3375       6750       10125         4096       8192       12288         4913       9826       14739         5832       11664       17496         6859       13718       20577         8000       16000       24000         9261       18522       27783         10648       21296       31944         12167       24334       36501         13824       27648       41472         15625       31250       46875         17576       35152       52728         19683       39366       59049         21952       43904       65856         24389       48778       73167         27000       54000       81000         29791       59582       89373         32768       65536       98304         35937       71874       107811         39304       78608       117912         42875       85750       128625
1728     3456     5184       2197     4394     6591       2744     5488     8232       3375     6750     10125       4096     8192     12288       4913     9826     14739       5832     11664     17496       6859     13718     20577       8000     16000     24000       9261     18522     27783       10648     21296     31944       12167     24334     36501       13824     27648     41472       15625     31250     46875       17576     35152     52728       19683     39366     59049       21952     43904     65856       24389     48778     73167       27000     54000     81000       29791     59582     89373       32768     65536     98304       35937     71874     107811       39304     78608     117912       42875     85750     128625       46656     93312     139968
2197       4394       6591         2744       5488       8232         3375       6750       10125         4096       8192       12288         4913       9826       14739         5832       11664       17496         6859       13718       20577         8000       16000       24000         9261       18522       27783         10648       21296       31944         12167       24334       36501         13824       27648       41472         15625       31250       46875         17576       35152       52728         19683       39366       59049         21952       43904       65856         24389       48778       73167         27000       54000       81000         29791       59582       89373         32768       65536       98304         35937       71874       107811         39304       78608       117912         42875       85750       128625         46656       93312       139968
2744     5488     8232       3375     6750     10125       4096     8192     12288       4913     9826     14739       5832     11664     17496       6859     13718     20577       8000     16000     24000       9261     18522     27783       10648     21296     31944       12167     24334     36501       13824     27648     41472       15625     31250     46875       17576     35152     52728       19683     39366     59049       21952     43904     65856       24389     48778     73167       27000     54000     81000       29791     59582     89373       32768     65536     98304       35937     71874     107811       39304     78608     117912       42875     85750     128625       46656     93312     139968
3375       6750       10125         4096       8192       12288         4913       9826       14739         5832       11664       17496         6859       13718       20577         8000       16000       24000         9261       18522       27783         10648       21296       31944         12167       24334       36501         13824       27648       41472         15625       31250       46875         17576       35152       52728         19683       39366       59049         21952       43904       65856         24389       48778       73167         27000       54000       81000         29791       59582       89373         32768       65536       98304         35937       71874       107811         39304       78608       117912         42875       85750       128625         46656       93312       139968
4096       8192       12288         4913       9826       14739         5832       11664       17496         6859       13718       20577         8000       16000       24000         9261       18522       27783         10648       21296       31944         12167       24334       36501         13824       27648       41472         15625       31250       46875         17576       35152       52728         19683       39366       59049         21952       43904       65856         24389       48778       73167         27000       54000       81000         29791       59582       89373         32768       65536       98304         35937       71874       107811         39304       78608       117912         42875       85750       128625         46656       93312       139968
4096       8192       12288         4913       9826       14739         5832       11664       17496         6859       13718       20577         8000       16000       24000         9261       18522       27783         10648       21296       31944         12167       24334       36501         13824       27648       41472         15625       31250       46875         17576       35152       52728         19683       39366       59049         21952       43904       65856         24389       48778       73167         27000       54000       81000         29791       59582       89373         32768       65536       98304         35937       71874       107811         39304       78608       117912         42875       85750       128625         46656       93312       139968
4913       9826       14739         5832       11664       17496         6859       13718       20577         8000       16000       24000         9261       18522       27783         10648       21296       31944         12167       24334       36501         13824       27648       41472         15625       31250       46875         17576       35152       52728         19683       39366       59049         21952       43904       65856         24389       48778       73167         27000       54000       81000         29791       59582       89373         32768       65536       98304         35937       71874       107811         39304       78608       117912         42875       85750       128625         46656       93312       139968
5832       11664       17496         6859       13718       20577         8000       16000       24000         9261       18522       27783         10648       21296       31944         12167       24334       36501         13824       27648       41472         15625       31250       46875         17576       35152       52728         19683       39366       59049         21952       43904       65856         24389       48778       73167         27000       54000       81000         29791       59582       89373         32768       65536       98304         35937       71874       107811         39304       78608       117912         42875       85750       128625         46656       93312       139968
6859         13718         20577           8000         16000         24000           9261         18522         27783           10648         21296         31944           12167         24334         36501           13824         27648         41472           15625         31250         46875           17576         35152         52728           19683         39366         59049           21952         43904         65856           24389         48778         73167           27000         54000         81000           29791         59582         89373           32768         65536         98304           35937         71874         107811           39304         78608         117912           42875         85750         128625           46656         93312         139968
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9261     18522     27783       10648     21296     31944       12167     24334     36501       13824     27648     41472       15625     31250     46875       17576     35152     52728       19683     39366     59049       21952     43904     65856       24389     48778     73167       27000     54000     81000       29791     59582     89373       32768     65536     98304       35937     71874     107811       39304     78608     117912       42875     85750     128625       46656     93312     139968
10648         21296         31944           12167         24334         36501           13824         27648         41472           15625         31250         46875           17576         35152         52728           19683         39366         59049           21952         43904         65856           24389         48778         73167           27000         54000         81000           29791         59582         89373           32768         65536         98304           35937         71874         107811           39304         78608         117912           42875         85750         128625           46656         93312         139968
12167     24334     36501       13824     27648     41472       15625     31250     46875       17576     35152     52728       19683     39366     59049       21952     43904     65856       24389     48778     73167       27000     54000     81000       29791     59582     89373       32768     65536     98304       35937     71874     107811       39304     78608     117912       42875     85750     128625       46656     93312     139968
13824     27648     41472       15625     31250     46875       17576     35152     52728       19683     39366     59049       21952     43904     65856       24389     48778     73167       27000     54000     81000       29791     59582     89373       32768     65536     98304       35937     71874     107811       39304     78608     117912       42875     85750     128625       46656     93312     139968
15625     31250     46875       17576     35152     52728       19683     39366     59049       21952     43904     65856       24389     48778     73167       27000     54000     81000       29791     59582     89373       32768     65536     98304       35937     71874     107811       39304     78608     117912       42875     85750     128625       46656     93312     139968
17576     35152     52728       19683     39366     59049       21952     43904     65856       24389     48778     73167       27000     54000     81000       29791     59582     89373       32768     65536     98304       35937     71874     107811       39304     78608     117912       42875     85750     128625       46656     93312     139968
19683     39366     59049       21952     43904     65856       24389     48778     73167       27000     54000     81000       29791     59582     89373       32768     65536     98304       35937     71874     107811       39304     78608     117912       42875     85750     128625       46656     93312     139968
21952     43904     65856       24389     48778     73167       27000     54000     81000       29791     59582     89373       32768     65536     98304       35937     71874     107811       39304     78608     117912       42875     85750     128625       46656     93312     139968
24389     48778     73167       27000     54000     81000       29791     59582     89373       32768     65536     98304       35937     71874     107811       39304     78608     117912       42875     85750     128625       46656     93312     139968
27000     54000     81000       29791     59582     89373       32768     65536     98304       35937     71874     107811       39304     78608     117912       42875     85750     128625       46656     93312     139968
29791     59582     89373       32768     65536     98304       35937     71874     107811       39304     78608     117912       42875     85750     128625       46656     93312     139968
32768     65536     98304       35937     71874     107811       39304     78608     117912       42875     85750     128625       46656     93312     139968
35937     71874     107811       39304     78608     117912       42875     85750     128625       46656     93312     139968
39304 78608 117912 42875 85750 128625 46656 93312 139968
42875 85750 128625 46656 93312 139968
46656 93312 139968
- aunaa -   101a0h   1a19a9
54872 109744 164616
59319   118638   177957
64000   128000   192000
68921   137842   206763
74088   148176   222264
79507   159014   238521
85184   170368   255552
97336   194672   292008 103823   207646   311469
117649   235298   352947 125000   250000   375000
Table 4.2: Long Table

Table 4.2: Long Table

All tables are listed with *listoftables*.

#### 4.3 Enumerate and itemize

If important sequential points are to presented the environment  $\underline{enumerate}$  can be used as follows:

- 1. Some important stuff
- 2. More stuff

With the package *enumerate* some options can be used, e.g.,

- a) Some important stuff
- b) More stuff

or

- 1) Some important stuff
- 2) More stuff

Alternatively, point can be just presented without any enumeration with the environment *itemize* 

- Some important stuff
- More stuff

# Appendix, footnotes, todos and index

#### **Appendix** 5.1

For many reasons some concept may be important for the document but too long for the main text. In this kind of cases these concept can be presented with the environment appendix in appendices, e.g., as in Appendix A and Appendix B.

#### 5.2Footnotes

You may want to give additional information to some points<sup>1</sup> in the text<sup>2</sup>.

#### 5.3 Todos

With the package <u>todonotes</u> comments pointing to their place can be embedded into the text. like this These comments are veeeery useful if you are writing something for the first time or are working on a draft. The todos can be listed with *listoftodos* where you want it to appear in order to see what is unfinished or needs some more work.

#### 5.4 Index

If the document is very long, it may be very useful for a lot of readers to have an index for searching key words and certain concepts (Crtl+F is usually very helpful in PDFs but not always the best solution). For this, the package <u>makeidx</u>, the commands <u>makeindex</u> and <u>printindex</u> and the compiling option make index are needed. You may want to index different words like heterogeneous materials, effective properties and homogenization.

<sup>&</sup>lt;sup>1</sup>Bla bla

<sup>&</sup>lt;sup>2</sup>Blu blup

# Appendix A

# Just an example appendix

### A.1 Bla blup

Sme stuff

$$f(x) = \int_{\Omega} g(x)dx . \tag{A.1}$$

# Appendix B

# Another example

B.1 More stuff

Bla bla.

# **Bibliography**

- R. Hill. The elastic behaviour of a crystalline aggregate. *Proceedings of the Physical Society*. Section A, 65:349–354, 1952.
- E. Kröner. Bounds for effective elastic moduli of disordered materials. *Journal of the Mechanics and Physics of Solids*, 25(3):137–155, 1977.

# List of Figures

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	sometimes can be very long	

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