TECHNISCHE UNIVERSITÄT
KAISERSLAUTERN

Department of Financial Mathematics

Comparison of Forecasting Models for Value at Risk

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October 28, 2021

A thesis submitted in fulfilment of the requirements for the degree of Master of Science at the Technische Universität Kaiserslautern

Declaration of Authorship

I, Hafees Adebayo YUSUFF, hereby declare the following thesis titled "Comparison of Forecasting Models for Value at Risk" to be my own work and I confirm that:

- The thesis I am submitting is entirely my own work except where otherwise indicated.
- It has not been submitted, either partially or in full, for a qualification at this or any other University.
- I have clearly signalled the presence of all material I have quoted from other sources, including any diagrams, charts, tables or graphs.
- I have acknowledged appropriately any assistance I have received.

Signature	Date

Abstract

Financial institutions need to have enough capacity to meet their responsibilities and sop up unexpected losses. Since they are exposed to risks, managing them is very important. Several methods have been provided for managing risk, of which Value at Risk (VaR) is the most common for market risk. VaR is a statistical method used to measure the amount of potential loss that could happen in an investment portfolio over a specified period of time under normal market conditions.

This study compares some VaR estimation methods: Historical simulation, Garch (1,1) model and Long short term memory (LSTM) neural network using the Japan, UK and US stock markets. Each stock market contains 8476 daily log-returns from 05/01/1988 to 30/06/2020. For the Historical simulation and Garch (1,1) model, we use the first 7090 days as our rolling window. As for the LSTM VaR model, the first 7000 daily log-returns (83% of data) of each series are used for training, 1400 (20%of data for training) are used for validation while the remaining 1386 are used for testing. From the kupiec test, Historical simulation outperforms other models as it is accepted for both the 95% and 99% confidence level. Only the Garch (1,1) model with 95% confidence level is accepted for all considered stock markets, while that of 99% is rejected. The LSTM VaR model with 95% confidence interval is accepted for S&P 500 (US) and FTSE 100 (UK), but rejected for NIKKEI 225 (Japan). However, the Kupiec test disapproves the LSTM VaR model with 99% confidence level for all the three stocks markets.

Contents

Chapter 1

Basics

1.1 Section types

The basic sections in a book are chapter, section, subsections and so on. These sections will appear in the *tableofcontents* of the book.

1.1.1 Example subsection

This is just one example for a subsection.

1.2 Basic page settings

In order to show how the basic page settings are set up, we will use a long dummy text with the package *blindtext*.
Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede. Vivamus nunc nunc, molestie ut, ultricies vel, semper in, velit. Ut porttitor. Praesent in sapien. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Duis fringilla tristique neque. Sed interdum libero ut metus. Pellentesque placerat. Nam rutrum augue a leo. Morbi sed elit sit amet ante lobortis sollicitudin. Praesent blandit blandit mauris. Praesent lectus tellus, aliquet aliquam, luctus a, egestas a, turpis. Mauris lacinia lorem sit amet ipsum. Nunc quis urna dictum turpis accumsan semper. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Etiam lobortis facilisis sem. Nullam nec mi et neque pharetra sollicitudin. Praesent imperdiet mi nec ante. Donec ullamcorper, felis non sodales commodo, lectus velit ultrices augue, a dignissim nibh lectus placerat pede.

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1.3 Head and foot

The head and foot of the document can be adapted using the packages *fancyhdr*. The using the commands, e.g., *pagestyle{fancy}*, *l/c/rhead/foot* or with *fancyhead/foot[EL,CO]* the respective parts can be edited as needed.

1.4 New commands and input

If some long commands for formatting or other utilities are used very often, e.g., **test**, *test*, ***test***, then the definition of new personal commands is very useful. For this the command *newcommand* is used before the document, e.g., ***test***. Over time the collection of personal commands will grow, so for these it is better to create a separate file and copy this file to the current project folder. The file can be then loaded within the project with the command *input* before the document.

1.5 Language

The default language for the document is English. This can be changed, e.g., in German (with the package *ngerman*) or whatever you need. This changes the language of automatically generated words like chapter, figure, table, and others. It should be also noted that in most latex editors dictionaries for several languages can be used (see *options*).

Chapter 2

Math stuff

2.1 Equations and math mode

We are able to create automatically enumerated equation as the following one

$$f(x) = A^{23}_{ijkl}(x) \int_0^l g(y,x) \frac{\partial h(y,x)}{\partial y} dy \ . \tag{2.1}$$

Equation can be given a name/label. In order to refer to it later in the text the package *amsmath* has to be included. After including the package, the command to refer to labeled equation is (2.1).
Equation without a number can be created as follows

$$f(x) = A^{23}_{ijkl}(x) \int_0^l g(y,x) \frac{\partial h(y,x)}{\partial y} dy \ ,$$

or alternatively

$$f(x) = A^{23}_{ijkl}(x) \int_0^l g(y,x) \frac{\partial h(y,x)}{\partial y} dy \ .$$

You can also create a so called equation array with automatic numbering, e.g.,

$$\begin{aligned} f(x) &= (x+a)^2 && (2.2)\\ &= (x+a)(x+a) && (2.3)\\ &= x^2 + 2xa + a^2 && (2.4) \end{aligned}$$

You can refer to (2.2) and (2.4) separately. The very same can be created without any numbers as

$$\begin{aligned} f(x) &= (x+a)^2 \\ &= (x+a)(x+a) \\ &= x^2 + 2xa + a^2 \end{aligned}$$

Sometimes math content will be explained directly within the text. For these cases the math mode using *$$* can be used, e.g., $f(x) = x^{234}_{ijkl}$.

2.2 Arrays and matrices

Arrays can be used within math environments in order to create a grid with math elements, e.g.,

$$A^{23}_{ijkl}(x) \int_0^l g(y,x) \frac{\partial h(y,x)}{\partial y} dy \quad \begin{array}{ccc} x+y+z & m_{1234567} & 13425436543634 \\ n_k & & 123 \end{array} \tag{2.5}$$

A set of equations can also be arranged as follows

$$\begin{aligned} f(x) &= A^{23}_{ijkl}(x) \int_0^l g(y,x) \tfrac{\partial h(y,x)}{\partial y} dy \\ &= 7x \ . \end{aligned} \tag{2.6}$$

This is an alternative to *eqnarray* with a single centered number, but some symbols may not be displayed as wanted. In order to force a full size display of a chosen element of the array the command *displaystyle* can be used

$$\begin{aligned} f(x) &= A^{23}_{ijkl}(x) \int_0^l g(y,x) \frac{\partial h(y,x)}{\partial y} dy \\ &= 7x \ . \end{aligned} \tag{2.7}$$

Arrays can also be used in order to represent matrices, e.g.,

$$\begin{pmatrix} 123123 & 324 & 214 & 4 \\ 43 & 345345645 & 45353465 & 346 \end{pmatrix} \ . \tag{2.8}$$

Alternatively matrices can be created with the following environments

$$\begin{pmatrix} 123123 & 324 & 214 & 4 \\ 43 & 345345645 & 45353465 & 346 \end{pmatrix} \quad \begin{bmatrix} 123123 & 324 & 214 & 4 \\ 43 & 345345645 & 45353465 & 346 \end{bmatrix} \tag{2.9}$$

2.3 Math fonts

Depending on what is to be presented or discussed in the work, several math fonts might be useful for different concepts. For extended fonts the package *amssymb* is needed. The basic fonts are then

default	r	R	Sym^+	γ	Γ
bb	$\mathbb{r}$	$\mathbb{R}$	$\mathbb{Sym}^+$	γ	$\mathbb{\Gamma}$
bf	$\mathbf{r}$	$\mathbf{R}$	$\mathbf{Sym}^+$	γ	$\mathbf{\Gamma}$
cal	$\mathcal{r}$	$\mathcal{R}$	$\mathcal{Sym}^+$	γ	$\mathcal{\Gamma}$
frak	$\mathfrak{r}$	$\mathfrak{R}$	$\mathfrak{Sym}^+$	γ	$\mathfrak{\Gamma}$
it	r	R	Sym^+	γ	Γ
rm	r	R	Sym^+	γ	Γ
sf	r	R	Sym^+	γ	$\mathsf{\Gamma}$
tt	$\mathtt{r}$	$\mathtt{R}$	$\mathtt{Sym}^+$	γ	$\mathtt{\Gamma}$
boldsymbol	$\boldsymbol{r}$	$\boldsymbol{R}$	$\boldsymbol{Sym}^+$	$\boldsymbol{\gamma}$	$\boldsymbol{\Gamma}$

The commands *mathbb* and others can be changed using different packages, e.g., *euscript* and *lucida* (look for latex math fonts in stackexchange). It is very useful to define the most used fonts as new commands within your personal macros, e.g., $\mathbb{A}$.

2.4 Math symbols

The amount of math symbols offered in latex is immense. Some of them are, e.g.,

$$\sum, \int, \iiint, \nabla, \cdot, \times, \otimes, \rightarrow, \Rightarrow, \bigcup, \in, \subset \ . \tag{2.10}$$

You will have to look for those you might need.

Chapter 3

Hyperlinks and references

3.1 The package hyperref

The package *hyperref* is the package for referring to labeled elements of a document and hyperlinks. Now, chapters, sections, equations, figures, tables and other elements can be labeled and referred to, e.g., Equation 2.1, section 2.4 and chapter 3. These are clickable links which in the pdf redirects the reader to the referred element (with ALT+LEFT you can then go back to where you were reading). Here, different alternatives can be used, e.g., 3, chapter 3 or Chapter 3. Depending on which language you have to write something, you may need language options (e.g., ngerman for German hyperlinks).

3.2 Hyperlinks to internet sites, email and attached files

Hyperlinks can be added as, e.g., `http://miktex.org/` or click me. Sending an email to a prescribed address can be done by name.lastname@address.org. If the pdf is delivered within a folder with useful files, these files can be linked in the pdf, e.g., manipulate or video.

3.3 Literature references

Bibtex files with literature information can be created either manually or using literature manager programs like Mendeley or Citavi. The bibtex file must be included in the project with *bibliography* pointing to the file, together with *bibliographystyle* and a packages for citing commands. With the commands *cite/p* elements of the included file are then cited, e.g., Hill (1952) and (Kröner, 1977). Make sure that while compiling you have chosen a procedure including bibtex (see compiling options). Sometimes it may be necessary to delete all files but not the main.tex file in order to be able to compile again the project, if bibliography styles have been changed.

Chapter 4

Figures, tables, enumerate and itemize

4.1 Figures

In almost every document figures will be needed in order to explain a concept or just present something. The package *graphicx* is needed for embedding figures.

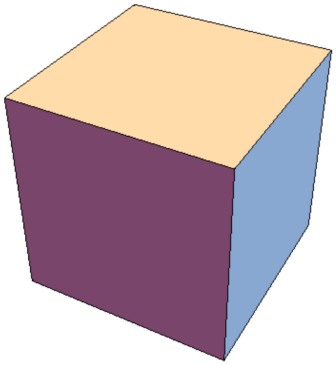

Figure 4.1: A figure caption beneath the figure for description of the depicted concept which sometimes can be very long

In Figure 4.1, for example, a PNG image is depicted (compiled with pdflatex). Alternatively, EPS figures can be embedded if dvips and ps2pdf compilation is used. All figures are listed in the list of figures with the command *listoffigures*.

4.2 Tables

Data can be presented in tables, e.g., as shown in Table 4.1.

	Property 1	Property 2
Criterion 1	764	23546
Criterion 2	3	34

Table 4.1: Exemplary table

Sometimes very long tables must be presented which may also go over pages. For this cases the packages *longtable* is useful, as used in

i^3	$2i^3$	$3i^3$
1	2	3
8	16	24
27	54	81
64	128	192
125	250	375
216	432	648
343	686	1029
512	1024	1536
729	1458	2187
1000	2000	3000
1331	2662	3993
1728	3456	5184
2197	4394	6591
2744	5488	8232
3375	6750	10125
4096	8192	12288
4913	9826	14739
5832	11664	17496
6859	13718	20577
8000	16000	24000
9261	18522	27783
10648	21296	31944
12167	24334	36501
13824	27648	41472
15625	31250	46875
17576	35152	52728
19683	39366	59049
21952	43904	65856
24389	48778	73167
27000	54000	81000
29791	59582	89373
32768	65536	98304
35937	71874	107811
39304	78608	117912
42875	85750	128625
46656	93312	139968
50653	101306	151959
54872	109744	164616
59319	118638	177957
64000	128000	192000
68921	137842	206763
74088	148176	222264
79507	159014	238521
85184	170368	255552
91125	182250	273375
97336	194672	292008
103823	207646	311469
110592	221184	331776
117649	235298	352947
125000	250000	375000

Table 4.2: Long Table

All tables are listed with *listoftables*.

4.3 Enumerate and itemize

If important sequential points are to presented the environment *enumerate* can be used as follows:

1. Some important stuff
2. More stuff

With the package *enumerate* some options can be used, e.g.,

a) Some important stuff

b) More stuff

or

1) Some important stuff
2) More stuff

Alternatively, point can be just presented without any enumeration with the environment *itemize*

- Some important stuff
- More stuff

Chapter 5

Appendix, footnotes, todos and index

5.1 Appendix

For many reasons some concept may be important for the document but too long for the main text. In this kind of cases these concept can be presented with the environment *appendix* in appendices, e.g., as in Appendix A and Appendix B.

5.2 Footnotes

You may want to give additional information to some points[1] in the text[2].

5.3 Todos

With the package *todonotes* comments pointing to their place can be embedded into the text. These comments are veeeery useful if you are writing something for the first time or are working on a draft. The todos can be listed with *listoftodos* where you want it to appear in order to see what is unfinished or needs some more work.

like this one

5.4 Index

If the document is very long, it may be very useful for a lot of readers to have an index for searching key words and certain concepts (Crtl+F is usually very helpful in PDFs but not always the best solution). For this, the package *makeidx*, the commands *makeindex* and *printindex* and the compiling option *make index* are needed. You may want to index different words like heterogeneous materials, effective properties and homogenization.

[1] Bla bla
[2] Blu blup

Appendix A

Just an example appendix

A.1 Bla blup

Sme stuff

$$f(x) = \int_{\Omega} g(x)dx \ . \tag{A.1}$$

Appendix B

Another example

B.1 More stuff

Bla bla.

Bibliography

R. Hill. The elastic behaviour of a crystalline aggregate. *Proceedings of the Physical Society. Section A*, 65:349–354, 1952.

E. Kröner. Bounds for effective elastic moduli of disordered materials. *Journal of the Mechanics and Physics of Solids*, 25(3):137–155, 1977.

List of Figures

List of Tables