Department of Financial Mathematics

Compare the quality of forecasting models for value at risk

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Contents

Chapter 1

Introduction

1.1 Motivation

Management of risk in financial institutions in order to maintain solvency. One of the most important types of risk is the market risk, which is the measurable uncertainty associated with changes in the interest rates, exchange rates and financial instruments price values.

Value at Risk(VaR) is the most common way of measuring market risk. It determines the greatest possible loss, assuming an α significance level under a normal market condition at a set time period.

Many VaR estimation methods have been developed in order to reduced uncertainty. It is however of interest to compare these method and determine the prevalence of one VaR estimation approach over others.

1.2 Literature review

The first papers involving the comparison of VaR methodologies, such as those by Beder (1995, 1996), Hendricks (1996), and Pritsker (1997), reported that the Historical Simulation performed at least as well as the methodologies developed in the early years, the Parametric approach and the Monte Carlo simulation. These papers conclude that among earlier methods, no approach appeared to perform better than the others. The evaluation and categorization of models carried out in the work by McAleer, Jimenez-Martin and Perez-Amaral(2009) and Shams and Sina (2014), among others, try to determine the conditions under which certain models predict the best. Researchers compared models in periods of varying volatility-before the crisis and after the crisis (When there was no high volatility and when volatility was high, respectively). However, this confirms that some models have good predictions before the start of the crisis, but their quality reduces with increased volatility. Others are more conservative during periods of low volatility, but in the time of the crisis the number of errors made by these models is relatively low.

Bao et al.(2006), Consigli(2002) and Danielson(2002), among others, show that in stable periods, parametric models provide satisfactory results that become less satisfactory during high volatility periods. Additional studies that find evidence in favour of parametric methods are Sarma et al.(2003), who compare Historical simulation and Parametric methods, and Danielson and Vries(2000) in a similar comparison that also includes Extreme value theory methods. Chong(2004), who uses parametric methods to estimate VaR under a Normal distribution and under a Student's t-distribution, finds a better performance under Normality. McAleer et al.(2009) showed that RiskMetrics$^{\mathrm{TM}}$ was the best fitted model during a crisis, while Shams and Sina(2014) recognized GARCH(1,1) and GJR-GARCH as well forecasting models. In contrast to the results obtained by McAleer et al.(2009), the level of quality of forecasts generated by the RiskMetrics$^{\mathrm{TM}}$ model was considered unsatisfactory by them. However, attention needs to be drawn to one difference in the samples, on which the study was conducted, i.e. the first one comes from a developed country (USA, S&P500), and the second one from a developing country (Iran, TSEM). Taylor(2020) evaluate Value at Risk using quantile skill score and the conditional autoregressive model outperformed others.

Attempts have been made to predicts VaR with ANN. VaR estimation on the exchange rate market in the context of ANNs is dealt with in Locarek-Junge and Prinzler (1999), who illustrate how VaR estimates can be obtained by using a USD-portfolio. The empirical outcomes demonstrate an evident superiority of the neural network to other VaR models. Hamid and Iqbal(2004) compared volatility forecasts from neural networks with forecasts of implied volatility from S&P500 index futures options, using the Barone-Adesi and Whaley (BAW) American futures options pricing model. Forecasts from NN outperformed implied volatility forecasts. Similar results are put forth by He et al. (2018), who propose an innovative EMD-DBN type of ANN to estimate VaR on the USD against the AUD, CAD, CHF and the EUR. The authors find positive performance improvement in the risk estimates, and argue that the utilization of an EMD-DBN network can identify more optimal ensemble weights and is less sensitive to noise disruption compared to a FNN. Nevertheless, it is worthwhile to mention that although foreign exchange volatility forecasting through ANNs have gained some attention in the academic field, it still remains a fairly undeveloped area.

All in all, there is no full approval in the evaluation of which models should be used during periods of calm (low volatility), and which ones during crisis (High volatility).

1.3 Thesis Structure

The next chapter of discusses the properties and basic methods to estimate VaR. Subsequent chapters discuss use of Neural Network in Estimating Value at Risk and numerical comparison of the methods with examples. Findings are summarized in the last chapter.

Chapter 2

Value-at-Risk: Concept, fundamental properties and popular estimation methods

2.1 Concept

Higher volatility in exchange markets, credit defaults, even endangering countries, and the call for more regulation drastically changed the circumstances in which banks operate. These situations of uncertainty are called risks and managing them is of great importance to financial institutions (e.g Banks) in order to keep them afloat. A possible method of measurement is the evaluation of losses likely to be incurred when the price of the portfolio falls. Value at Risk (VaR) does this.

According to Jorion (2001), "VaR measure is defined as the worst expected loss over a given horizon under normal market conditions at a given level of confidence. For instance, a bank might say that the daily VaR of its trading portfolio is $2 million at the 99% confidence level. In other words, under normal market conditions, only 1% of the time, the daily loss will exceed $2 million (99% of the time, their loss will not be more than $2 million)."

Mathematically,

Let $r_1, r_2, ..., r_n$ be independently and identically distributed(iid) random variables representing financial returns. Use $F(r)$ to denote the cumulative distribution function, $F(r) = Pr(r_t < r|\Omega_{t-1})$ conditional on the information set Ω_{t-1} available at time t-1. Assume that $\{r_t\}$ follows the stochastic process;

$$\begin{aligned} r_t &= \mu_t + \varepsilon_t = \mu_t \sigma_t + z_t \\ \varepsilon_t &= \sigma_t z_t \qquad z_i \sim (0,1) \end{aligned} \tag{2.1}$$

where $\sigma_t^2 = E[z_t^2|\Omega_{t-1}]$ and z_t has a conditional distribution function $G(z)$, $G(z) = Pr(z_t < z|\Omega_{t-1})$. The VaR with a given probability $\alpha \; \epsilon (0,1)$, denoted by VaR(α), is defined as the α quantile of the probability distribution of financial returns:
$F(\text{VaR}(\alpha)) = Pr(r_t < \text{VaR}(\alpha)) = \alpha$ or $\text{VaR}(\alpha) = \inf\{v|P(r_t \leq v) = \alpha\}$

One can estimate this quantile in two different ways: (1) inverting the distribution function of financial returns, F(r), and (2) inverting the distribution function of innovations, with regard to $G(z)$ the latter, it is also necessary to estimate σ_t^2.

$$\text{VaR}(\alpha) = F^{-1}(\alpha) = \mu + \sigma_t G^{-1}(\alpha) \tag{2.2}$$

Hence, a VaR model involves the specification of $F(r)$ or $G(r)$. There are several method for these estimations which will be discussed later in this chapter.

2.2 Value at Risk methods

The estimation of these functions can be carried out using the following methods: (1) non-parametric methods; (2) parametric methods and (3) semi-parametric methods. Below we will describe the methodologies, which have been developed in each of these three cases to estimate VaR

2.2.1 Non-parametric methods

The Non-parametric approaches seek to measure a portfolio VaR without making strong assumptions about returns distribution. The essence of these approaches is to let data speak for themselves as much as possible and to use recent returns empirical distribution – not some assumed theoretical distribution – to estimate VaR. All Non-parametric approaches are based on the underlying assumption that the near future will be sufficiently similar to the recent past for us to be able to use the data from the recent past to forecast the risk in the near future. The Non-parametric approaches include (a) Historical Simulation and (b) Delta-Gamma approximation

Historical simulation

The historical simulation involves using past data to predict future. First of all, we have to identify the market variables that will affect the portfolio. Then, the data will be collected on the movements in these market variables over a certain time period. This provides us the alternative scenarios for what can happen between today and tomorrow. For each scenario, we calculate the changes in the dollar value of portfolio between today and tomorrow. This defines a probability distribution for changes in the value of portfolio. For instance, VaR for a portfolio using 1-day time horizon with 99% confidence level for 500 days data is nothing but an estimation of the loss when we are at the fifth-worst daily change.

Basically, historical simulation is extremely different from other type of simulation in that estimation of a covariance matrix is avoided. Therefore, this approach has simplified the computations especially for the cases of complicated portfolio.

The core of this approach is the time series of the aggregate portfolio return. More importantly, this approach can account for fat tails and is not prone to the accuracy of the model due to being independent of model risk. As this method is very powerful and intuitive, it is then become the most widely used methods to compute VaR.

Delta-Gamma Approximation

The delta-gamma approximation (DGA) method is similar to the delta approximation approach, but with a higher order of sensitivity. As portfolios in real life are composed of instruments that are nonlinearly related to the underlying risk factors, the delta approximation fares poorly due to the linear assumption involved (as the first order derivative is nothing but the slope of the curve at a given point). The natural remedy to this is to incorporate the next order derivative. The delta-gamma approximation requires the calculation of the second order derivative, gamma, in addition to the delta and the returns of each instrument are calculated as below (derived from the Taylor's series):

$$\Delta(V) = \text{Delta} * \Delta(X) + 0.5 * \text{Gamma} * \Delta(X)^2$$

- $\Delta(V)$ is the change in the value of the instrument

- $\Delta(X)$ is the change in the value of the underlying
- Delta is the first order derivative of the instrument with respect to the underlying.
- Gamma is the first order derivative of the instrument with respect to the underlying.

From the returns computed using the above formula, the portfolio returns are determined and the required quantile was chosen as the VaR. The delta-gamma method works reasonably well for the simple nonlinear instruments (such as bonds without put/call options) as the curvature of the relationship with the underlying risk factor can be approximated by the convexity measure.

2.2.2 Parametric methods

Parametric approaches assume that portfolio losses follow a pre-determined distribution, such as a Normal distribution or a Student t-distribution, whereas non-parametric approaches do not depend on any distributional assumption. Instead, they approaches focus directly on the underlying loss distribution of a portfolio, by relying on a sample of observed losses. As a result, these approaches are applicable to any instrument, while parametric approaches, such as the Normal distribution, might not be. For example, under normality, the probability loss distribution is assumed to have no excess kurtosis, however, it has been empirically observed that financial data indeed exhibits excess kurtosis. Thus, assuming normally distributed losses when using financial data may lead to incorrect VaR estimates.

2.2.3 Semi-parametric methods

The Semi-parametric methods combine the Non-parametric approach with the Parametric approach. The most important Semi-parametric methods are Volatility-weight Historical Simulation, Filtered Historical Simulation (FHS),the approach based on Extreme Value Theory and those that directly model the conditional quantile for a chosen probability level using quantile regression, such as conditional autoregressive VaR and . Directly modeling a quantile avoids the need for a distributional assumption, and allows the dynamics of the quantiles to differ for each probability level.

2.3 Properties

Fix α $\epsilon(0,1)$, then the Value at Risk of a portfolio where the net payoff is modelled by X at a level α is given as:
$\mathrm{VaR}_\alpha(X) = \inf\{x\epsilon\mathbb{R}|P(X \leq x) = \alpha\}$ has the following properties

- Monotonicity
  if $X \leq Y$ then $\mathrm{VaR}_\alpha(X) \leq \mathrm{VaR}_\alpha(Y)$
- Positive Homogeneity
  $\mathrm{VaR}_\alpha(cX) = c\mathrm{VaR}_\alpha(X)$
- Translation invariance
  $\mathrm{VaR}_\alpha(X + c) = \mathrm{VaR}_\alpha(X) + c$

Chapter 3

Hyperlinks and references

3.1 The package hyperref

The package *hyperref* is the package for referring to labeled elements of a document and hyperlinks. Now, chapters, sections, equations, figures, tables and other elements can be labeled and referred to, e.g., **??**, **??** and chapter 3. These are clickable links which in the pdf redirects the reader to the referred element (with ALT+LEFT you can then go back to where you were reading). Here, different alternatives can be used, e.g., 3, chapter 3 or Chapter 3. Depending on which language you have to write something, you may need language options (e.g., ngerman for German hyperlinks).

3.2 Hyperlinks to internet sites, email and attached files

Hyperlinks can be added as, e.g., `http://miktex.org/` or click me. Sending an email to a prescribed address can be done by name.lastname@address.org. If the pdf is delivered within a folder with useful files, these files can be linked in the pdf, e.g., manipulate or video.

3.3 Literature references

Bibtex files with literature information can be created either manually or using literature manager programs like Mendeley or Citavi. The bibtex file must be included in the project with *bibliography* pointing to the file, together with *bibliographystyle* and a packages for citing commands. With the commands *cite/p* elements of the included file are then cited, e.g., Hill (1952) and (Kröner, 1977). Make sure that while compiling you have chosen a procedure including bibtex (see compiling options). Sometimes it may be necessary to delete all files but not the main.tex file in order to be able to compile again the project, if bibliography styles have been changed.

Chapter 4

Figures, tables, enumerate and itemize

4.1 Figures

In almost every document figures will be needed in order to explain a concept or just present something. The package *graphicx* is needed for embedding figures.

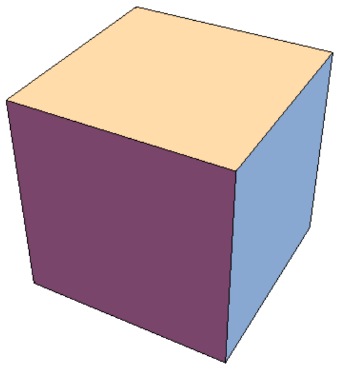

Figure 4.1: A figure caption beneath the figure for description of the depicted concept which sometimes can be very long

In Figure 4.1, for example, a PNG image is depicted (compiled with pdflatex). Alternatively, EPS figures can be embedded if dvips and ps2pdf compilation is used. All figures are listed in the list of figures with the command *listoffigures*.

4.2 Tables

Data can be presented in tables, e.g., as shown in Table 4.1.

	Property 1	Property 2
Criterion 1	764	23546
Criterion 2	3	34

Table 4.1: Exemplary table

Sometimes very long tables must be presented which may also go over pages. For this cases the packages *longtable* is useful, as used in

i^3	$2i^3$	$3i^3$
1	2	3
8	16	24
27	54	81
64	128	192
125	250	375
216	432	648
343	686	1029
512	1024	1536
729	1458	2187
1000	2000	3000
1331	2662	3993
1728	3456	5184
2197	4394	6591
2744	5488	8232
3375	6750	10125
4096	8192	12288
4913	9826	14739
5832	11664	17496
6859	13718	20577
8000	16000	24000
9261	18522	27783
10648	21296	31944
12167	24334	36501
13824	27648	41472
15625	31250	46875
17576	35152	52728
19683	39366	59049
21952	43904	65856
24389	48778	73167
27000	54000	81000
29791	59582	89373
32768	65536	98304
35937	71874	107811
39304	78608	117912
42875	85750	128625
46656	93312	139968
50653	101306	151959
54872	109744	164616
59319	118638	177957
64000	128000	192000
68921	137842	206763
74088	148176	222264
79507	159014	238521
85184	170368	255552
91125	182250	273375
97336	194672	292008
103823	207646	311469
110592	221184	331776
117649	235298	352947
125000	250000	375000

Table 4.2: Long Table

All tables are listed with *listoftables*.

4.3 Enumerate and itemize

If important sequential points are to presented the environment *enumerate* can be used as follows:

1. Some important stuff
2. More stuff

With the package *enumerate* some options can be used, e.g.,

a) Some important stuff

b) More stuff

or

1) Some important stuff
2) More stuff

Alternatively, point can be just presented without any enumeration with the environment *itemize*

- Some important stuff
- More stuff

Chapter 5

Appendix, footnotes, todos and index

5.1 Appendix

For many reasons some concept may be important for the document but too long for the main text. In this kind of cases these concept can be presented with the environment *appendix* in appendices, e.g., as in Appendix A and Appendix B.

5.2 Footnotes

You may want to give additional information to some points[1] in the text[2].

5.3 Todos

With the package *todonotes* comments pointing to their place can be embedded into the text. These comments are veeeery useful if you are writing something for the first time or are working on a draft. The todos can be listed with *listoftodos* where you want it to appear in order to see what is unfinished or needs some more work.

like this one

5.4 Index

If the document is very long, it may be very useful for a lot of readers to have an index for searching key words and certain concepts (Crtl+F is usually very helpful in PDFs but not always the best solution). For this, the package *makeidx*, the commands *makeindex* and *printindex* and the compiling option *make index* are needed. You may want to index different words like heterogeneous materials, effective properties and homogenization.

[1] Bla bla

[2] Blu blup

Appendix A

Just an example appendix

A.1 Bla blup

Sme stuff

$$f(x) = \int_{\Omega} g(x)dx \ . \tag{A.1}$$

Appendix B

Another example

B.1 More stuff

Bla bla.

Bibliography

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