This print-out should have 6 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

CalC3e06a
001 10.0 points

Find the derivative of f when

$$f(x) = 1\tan(x) - 4\cot(x).$$

1. $f'(x) = \dfrac{1 - 3\sin^2(x)}{\sin^2(x)\cos^2(x)}$

2. $f'(x) = \dfrac{1 - 3\cos^2(x)}{\sin^2(x)\cos^2(x)}$

3. $f'(x) = \dfrac{1 - 3\cos(x)}{\sin(x)\cos(x)}$

4. $f'(x) = \dfrac{1 + 3\sin^2(x)}{\sin^2(x)\cos^2(x)}$

5. $f'(x) = \dfrac{1 + 3\cos^2(x)}{\sin^2(x)\cos^2(x)}$ **correct**

6. $f'(x) = \dfrac{1 + 3\cos(x)}{\sin(x)\cos(x)}$

Explanation:
After differentiation

$$\begin{aligned} f'(x) &= 1\sec^2(x) + 4\csc^2(x) \\ &= \frac{1}{\cos^2(x)} + \frac{4}{\sin^2(x)} \\ &= \frac{1\sin^2(x) + 4\cos^2(x)}{\sin^2(x)\cos^2(x)}. \end{aligned}$$

Now

$$\begin{aligned} &1\sin^2(x) + 4\cos^2(x) \\ &\quad = 1\left(1 - \cos^2(x)\right) + 4\cos^2(x) \\ &\quad = 1 + 3\cos^2(x). \end{aligned}$$

Consequently,

$$\boxed{f'(x) = \frac{1 + 3\cos^2(x)}{\sin^2(x)\cos^2(x)}}.$$

CalC3e05a
002 10.0 points

Find the derivative of

$$f(x) = 2x\sin(x) - x^2\cos(x).$$

1. $f'(x) = (x^2 + 2)\cos(x)$

2. $f'(x) = (2 - x^2)\cos(x)$

3. $f'(x) = (x^2 - 2)\sin(x)$

4. $f'(x) = (2 + x^2)\sin(x)$ **correct**

5. $f'(x) = (2 - x^2)\sin(x)$

6. $f'(x) = (x^2 - 2)\cos(x)$

Explanation:
By the Product Rule

$$\frac{d}{dx}(2x\sin(x)) = 2\sin(x) + 2x\cos(x),$$

while

$$\frac{d}{dx}(x^2\cos(x)) = 2x\cos(x) - x^2\cos(x).$$

Consquently,

$$\boxed{f'(x) = (2 + x^2)\sin(x)}.$$

keywords: DerivTrig, DerivTrigExam,

CalC3e33s
003 10.0 points

A ladder 16 feet long rests against a vertical wall. Let θ be the angle between the top of the ladder and the wall and let x be the distance from the bottom of the ladder to the wall.

If the bottom of the ladder slides away from the wall, how fast does x change with respect to θ when $\theta = \pi/3$?

1. 5 ft/rad

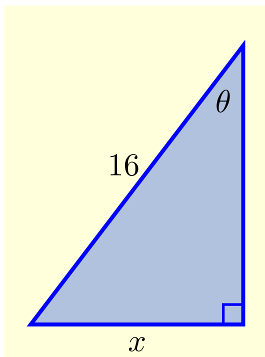
2. 6 ft/rad

3. 8 ft/rad **correct**

4. 7 ft/rad

5. 4 ft/rad

Explanation:
When the ladder is 16 feet long, the variables x and θ are shown in



From this we see that

$$\sin(\theta) = \frac{x}{16}, \quad i.e., \quad x = 16\sin(\theta).$$

The rate of change of x with respect to θ is thus the derivative

$$\frac{dx}{d\theta} = 16\cos(\theta).$$

When $\theta = \pi/3$, therefore,

$$\boxed{\frac{dx}{d\theta} = 16\cos\left(\frac{\pi}{3}\right) = 8 \text{ ft/rad}}.$$

CalC3e09a
004 10.0 points

Find the derivative of f when

$$f(x) = \frac{\sin x}{\cos x - 1}.$$

1. $f'(x) = -\dfrac{1}{1+\cos x}$

2. $f'(x) = \dfrac{1}{1-\sin x}$

3. $f'(x) = \dfrac{1}{\sin x - 1}$

4. $f'(x) = \dfrac{1}{\cos x + 1}$

5. $f'(x) = -\dfrac{1}{\sin x + 1}$

6. $f'(x) = \dfrac{1}{1+\sin x}$

7. $f'(x) = \dfrac{1}{\cos x - 1}$

8. $f'(x) = \dfrac{1}{1-\cos x}$ **correct**

Explanation:
By the quotient rule,

$$\begin{aligned} f'(x) &= \frac{\cos x\,(\cos x - 1) + \sin^2 x}{(\cos x - 1)^2} \\ &= \frac{\left(\cos^2 x + \sin^2 x\right) - \cos x}{(\cos x - 1)^2}. \end{aligned}$$

But $\sin^2 x + \cos^2 x = 1$, so

$$\begin{aligned} f'(x) &= \frac{1-\cos x}{(\cos x - 1)^2} \\ &= \frac{1-\cos x}{(1-\cos x)^2}. \end{aligned}$$

Consequently,

$$\boxed{f'(x) = \frac{1}{1-\cos x}}.$$

TrigDeriv12b
005 10.0 points

Find the derivative of

$$f(x) = \frac{\tan x - \sec x}{x^2}.$$

1. $f'(x) = \dfrac{(x\sec x - 2)(\tan x + \sec x)}{x^3}$

2. $f'(x) = -\dfrac{(x\sec x - 2)(\tan x + \sec x)}{x^3}$

3. $f'(x) = -\dfrac{(x\sec x + 2)(\tan x + \sec x)}{x^3}$

4. $f'(x) = -\dfrac{(x\sec x + 2)(\tan x - \sec x)}{x^3}$ **correct**

5. $f'(x) = \dfrac{(x\sec x + 2)(\tan x - \sec x)}{x^3}$

6. $f'(x) = \dfrac{(x\sec x - 2)(\tan x - \sec x)}{x^3}$

Explanation:

By the Quotient Rule,

$$f'(x) = \frac{x^2(\sec^2 x - \sec x\tan x) - 2x(\tan x - \sec x)}{x^4}.$$

But then

$$f'(x) = \frac{x\sec x(\sec x - \tan x) - 2(\tan x - \sec x)}{x^3}.$$

Consequently,

$$\boxed{f'(x) = -\frac{(x\sec x + 2)(\tan x - \sec x)}{x^3}}.$$

keywords:

CalC3c64exam
006 10.0 points

If f is a differentiable function, express the value of

$$\lim_{x\to 1} \frac{xf(x) - f(1)}{x-1}$$

in terms of f and f'.

1. limit $= f'(1) - 2f(1)$

2. limit $= 2f'(1)$

3. limit $= f'(1) - f(1)$

4. limit $= f'(1) + f(1)$ **correct**

5. limit $= f'(1) + 2f(1)$

6. limit does not exist

Explanation:

The value, $F'(a)$, at $x = a$ of the derivative of a function F is given by

$$F'(a) = \lim_{x\to a} \frac{F(x) - F(a)}{x-a}.$$

Thus the given limit is simply $F'(a)$ with

$$F(x) = xf(x), \qquad a = 1.$$

But by the Product Rule,

$$F'(x) = xf'(x) + f(x).$$

Consequently,

$$\boxed{\lim_{x\to 1} \frac{xf(x) - f(1)}{x-1} = f'(1) + f(1)}.$$