18.906: Problem Set IV

Due Wednesday, April 12, 2017, in class.

Homework is an important part of this class. I hope you gain from the struggle. Collaboration can be effective, but be sure that you grapple with each problem on your own as well. If you do work with others, you must indicate with whom on your solution sheet. Scores will be posted on the Stellar website.

Extra credit for finding mistakes and telling me about them early!

16. **(a)** Let $p : P \downarrow B$ be a principal G bundle. Construct an isomorphism of fiber bundles over B:

$$\begin{tikzcd} P \times G \arrow[rr] \arrow[dr] & & P \times_B P \arrow[dl] \\ & B & \end{tikzcd}$$

where $P \times G \to B$ sends $(x, g) \mapsto p(x)$.

(b) Give an example of a trivial sub vector bundle of a trivial vector bundle with nontrivial quotient vector bundle.

(c) Let $P \downarrow X$ be a principal G-bundle and F a left G-space. Establish a continuous bijection between the space of sections of the associated bundle with fiber F and the space of G-equivariant maps $P \to F$ (where G acts from the left on P by $gx = xg^{-1}$).

17, 18. $\varnothing$

19. Let X and Y be simplicial sets. Show that the evident natural map

$$|X \times Y| \to |X| \times |Y|$$

is a homeomorphism. Here is an idea of how to do this. Start by showing that it's true if X and Y are both simplicial simplices. (Hint: The Eilenberg-Zilber triangulation.) Then show that any simplical set X is the colimit of a functor to simplicial sets taking values in the subcategory of simplicial simplices. There's a canonical such functor, with source the "category of simplices" of X, i.e. the translation category of X regarded as a functor $\mathbf{\Delta}^{\mathrm{op}} \to \mathbf{Set}$. Further hint: This is only true if we use the product in k-spaces.

20. Suppose the group G acts on a set X. Write XG for the translation category of this action; so $\mathrm{ob}(XG) = X$, and $XG(x, y) = \{g \in G : gx = y\}$. Show that

$$EG \times_G X \cong B(XG)$$

21. Verify the first claim in the Proposition in the notes on Graeme Segal's perspective on classifying spaces, `math.mit.edu/~hrm/18.906/segal-notes.pdf`.