

# MATH 1729 HOMEWORK 12

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1. The quick brown fox jumps over the lazy dog.

**Theorem 1.1.** *If  $\iota : \mathbb{Z} \hookrightarrow \mathbb{Q}$  is the canonical injection, then  $\iota$  is an epimorphism which is not surjective; nonetheless, writing  $\mathbb{Z} \twoheadrightarrow \mathbb{Q}$  would be confusing.*

Over  $\mathbb{C}$ , all irreducible polynomials have degree 1, and over  $\mathbb{R}$ , they all have degree 1 or 2. However, over  $\mathbb{F}_p$ , they may have any positive degree. This makes life in  $\mathbb{P}\mathbb{F}_p$  more interesting.

**Lemma 1.2.** *There is a bijection  $\mathbb{N} \rightarrow \mathbb{N} \times \mathbb{N}$ .*

*Proof.* Count up each diagonal from  $(0, 0)$ , then  $(1, 0)$  and  $(0, 1)$ , then  $(2, 0)$ ,  $(1, 1)$ , and  $(0, 2)$ , and so on; then, every pair in  $\mathbb{N} \times \mathbb{N}$  will be reached at some finite step.  $\square$

**Claim 1.3.**  *$\mathbb{Q}$  is dense in  $\mathbb{R}$ .*

**Claim.**  *$\overline{\mathbb{Q}}$  is dense in  $\mathbb{C}$ .*

The derivative is defined by

$$\frac{df}{dx} = \lim_{\varepsilon \rightarrow 0} \frac{f(x + \varepsilon) - f(x)}{\varepsilon}, \quad (*)$$

and the partial derivative by

$$\frac{\partial \varphi}{\partial x_i} = \lim_{\varepsilon \rightarrow 0} \frac{\varphi(\mathbf{x} + \varepsilon \mathbf{e}_i) - \varphi(\mathbf{x})}{\varepsilon}. \quad (\dagger)$$

Notice the similarities to  $(*)$ .

2. And now we test delimiters.

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \left( \left( \frac{b-a}{n} \right) \sum_{j=1}^n f(x_j^*) \right).$$

In one dimension,

$$\left| \sum_{i=1}^n x_i \right| \leq \sum_{i=1}^n |x_i|. \quad (\ddagger)$$

In multiple dimensions,

$$\left\| \sum_{i=1}^n \mathbf{x}_i \right\| \leq \sum_{i=1}^n \|\mathbf{x}_i\|. \quad (\S)$$

In an inner product space,  $\|\mathbf{x}\|^2 = \langle \mathbf{x}, \mathbf{x} \rangle$ . Thus, in  $\ell^2$ , if  $\{x^{(n)}\}$  is a sequence converging to  $x$ , then  $\langle x^{(m)} - x^{(n)}, x^{(m)} - x^{(n)} \rangle \rightarrow 0$  as  $m, n \rightarrow \infty$ .

3. This item will test that I've set my `itemize` and `enumerate` environments correctly: one of the biggest reasons I wanted to fix my problem set template was that paragraphs in nested enumeration environments have no indentation or extra space by default, and this makes more textual arguments look bad.

Here, paragraphs are indented by default, but not the one right after the `\item` command. I'd like to replace that with extra whitespace.

- (a) This is the next level of indentation. I expect to use it a lot, e.g. when one problem comes with multiple subparts. As such, I would like it to look good.

In particular, there should be space between these paragraphs, so that it's easier to see where one part of an argument ends and another begins.

There should also be space between items of the same list.

- (b) Here's another item, with some math in it. If  $f$  is a holomorphic function on a disc  $D$  and  $\gamma$  is a  $C^1$  loop whose image is in  $D$ , then

$$\oint_{\gamma} f(z) dz = 0.$$

- (c) We must go deeper.

- This level will also come up a lot, though I don't expect to go much further. I tend to use bulleted lists to organize proofs with a lot of steps that don't depend on each other, e.g. to verify that something is an abelian group, one typically has to check associativity, commutativity, the presence of an identity, and the existence of inverses, but these don't entirely depend on each other.

Here's another paragraph of the same argument.

- And another bullet point, for completeness.