MATH 1729 HOMEWORK 12

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(1) The quick brown fox jumps over the lazy dog.

Theorem 1.1. *If $\iota : \mathbb{Z} \hookrightarrow \mathbb{Q}$ is the canonical injection, then ι is an epimorphism which is not surjective.*

Over $\mathbb{C}$, all irreducible polynomials have degree 1, and over $\mathbb{R}$, they all have degree 1 or 2. However, over $\mathbb{F}_p$, they may have any positive degree. This makes life in $\mathbb{PF}_p$ more interesting.

Lemma 1.2. *There is a bijection $\mathbb{N} \to \mathbb{N} \times \mathbb{N}$.*

Claim 1.3. *$\mathbb{Q}$ is dense in $\mathbb{R}$.*

Claim. *$\overline{\mathbb{Q}}$ is dense in $\mathbb{C}$.*

The derivative is defined by

$$\frac{\mathrm{d}f}{\mathrm{d}x} = \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon) - f(x)}{\varepsilon}, \tag{$*$}$$

and the partial derivative by

$$\frac{\partial \varphi}{\partial x_i} = \lim_{\varepsilon \to 0} \frac{\varphi(\mathbf{x} + \varepsilon \boldsymbol{\alpha}_i) - \varphi(\mathbf{x})}{\varepsilon}. \tag{$\dagger$}$$

Notice the similarities to $(*)$.

(2) And now we test delimiters.

$$\int_a^b f(x)\,\mathrm{d}x = \lim_{n \to \infty} \left(\left(\frac{b-a}{n} \right) \sum_{j=1}^n f(x_j^*) \right).$$

In one dimension,

$$\left| \sum_{i=1}^n x_i \right| \le \sum_{i=1}^n |x_i|. \tag{$\ddagger$}$$

In multiple dimensions,

$$\left\| \sum_{i=1}^n \mathbf{x}_i \right\| \le \sum_{i=1}^n \|\mathbf{x}_i\|. \tag{§}$$