Black-Scholes call price

Question:

Yara Inc is listed on the NYSE with a stock price of $40 - the company is not known to pay dividends. We need to price a call option with a strike of $45 maturing in 4 months. The continuously-compounded risk-free rate is 3%/year, the mean return on the stock is 7%/year, and the standard deviation of the stock return is 40%/year. What is the Black-Scholes call price?

Solution:

Formula: Black-Scholes model

$$c = P_a N(d_1) - P_e N(d_2) e^{-rt}$$

Where:

$$d_1 = \frac{\ln(P_a / P_e) + \left(r + 0.5s^2\right)t}{s\sqrt{t}}$$

$$d_2 = d_1 - s\sqrt{t}$$

$$c = Price\ of\ the\ call\ option$$

$$P_e = The\ exercise\ price$$

$$e = Euler's\ number = 2.71828$$

$$r = The\ risk - free\ interest\ rate$$

$$t = Time\ to\ expiry\ of\ the\ option$$

$$N(d) = normal\ distribution\ probability\ density\ function$$

$$P_a = The\ price\ of\ the\ underlying\ item,\ such\ as\ current\ shore\ price$$

$$s = standard\ devistion\ of\ the\ returns\ of\ the\ underlying\ item.$$

Step 0: Parameters

$$P_a = \$40,\ P_e = \$45,\ t = 0.33,\ r = 7\% = 0.07,\ s = 40\% = 0.4$$

Step 1: *Calculating* $d_1 = \dfrac{\ln(40/45) + \left(0.07 + \left(0.5 \times 0.4^2\right)\right)0.33}{0.4\sqrt{0.33}}$

$d_1 = \dfrac{-0.0214}{0.2298}$

$= \mathbf{-0.0931\ (approx.\ -0.1)}$

Step 2: *Calculating* $d_2 = d_1 - s\sqrt{t}$

$-0.0931 - 0.4\sqrt{0.33}$

$-0.0931 - 0.2298$

$\mathbf{= -0.3229 (approx.\ -0.32)}$

Step 3: $N(d_1),\ N(d_2)$ **From Standard normal distribution table (z)**

$N((d_1) = 0.46017\ (using\ d_1)$

$N((d_2) = 0.37448\ (using\ d_2)$

Step 4: *Calculating* $c = P_a N(d_1) - P_e N(d_2) e^{-rt}$

$40(0.46017) - 45(0.37448)e^{-(0.07 \times 0.33)}$

$18.4068 - 16.4674$

$= \mathbf{1.9394}$