Physics 540: Class 16

Thursday, June 22, 2021

We want to simulate two balls with masses m_1, m_2 and radii R_1, R_2 under freefall conditions:

$$x_i(t) = x_i(0) + v_{i,x}(0)t, \quad y_i(t) = y_i(0) + v_{i,y}(0)t - \frac{1}{2}gt^2,$$

$$v_{x,i}(t) = v_{x,i}(0), \qquad v_{y,i}(t) = v_{y,i}(0) - gt.$$

We make the approximation that motion is confined to a common plane. Alternatively, you might imagine that we're actually simulating discs on an air hockey table that's not quite level.

The positions and velocities are updated in the function `advance` according to the formulae above. Those data are organized in a `Ball` structure that is passed by reference. At each time step $\Delta t \leftrightarrow$ `dt`, we solve the quadratic equation $(x_2 - x_1)^2 + (y_2 - y_1)^2 = (R_1 + R_2)^2$ for the possible collision times t_{c1} and t_{c2}. (A complex-conjugated pair lying off the real axis indicates that there is no collision). We take the actual collision time to be the smallest such time that lies in the future ($t_c > 0$).
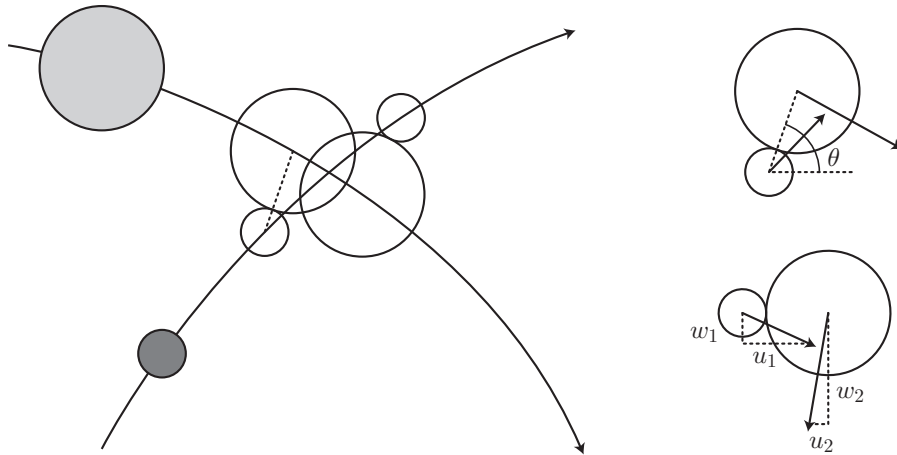
If the collision lies more than Δt in the future, then we simply advance the particles to their $t + \Delta t$ positions. If $0 < t_c < \Delta t$, however, we evolve by t_c, recompute the velocities after collision (by calling `elastic_scatter`) and then evolve by $\Delta t - t_c$. The new velocities can easy be determined by rotating to a reference frame in which the particles collide head-on along one of the orthogonal axes:

$$\begin{pmatrix} u_i \\ w_i \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} v_{i,x} \\ v_{i,y} \end{pmatrix}.$$

The velocities u_1 and u_2 take new values

$$u_1' = \frac{u_1(m_1 - m_2) + 2m_2 u_2}{m_1 + m_2}$$

$$u_2' = \frac{u_2(m_2 - m_1) + 2m_1 u_1}{m_1 + m_2}$$

and are rotated back to the conventional frame:

$$\begin{pmatrix} v_{i,x}' \\ v_{i,y}' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} u_i' \\ w_i' \end{pmatrix}.$$

On the other hand, if the balls are perfectly sticky and weld to one another on collision, the final velocities of both balls must be identical. Conservation of momentum tells us that

$$v'_{1,x} = v'_{2,x} = \frac{m_1 v_{1,x} + m_2 v_{2,x}}{m_1 + m_2},$$
$$v'_{1,y} = v'_{2,y} = \frac{m_1 v_{1,y} + m_2 v_{2,y}}{m_1 + m_2}.$$

We can implement this in the function `inelastic_scatter` and just comment out the `elastic_scatter` function we were using previously:

```
inelastic_scatter();
//elastic_scatter();
```

Here, it's good to check what happens to the conservation of energy.