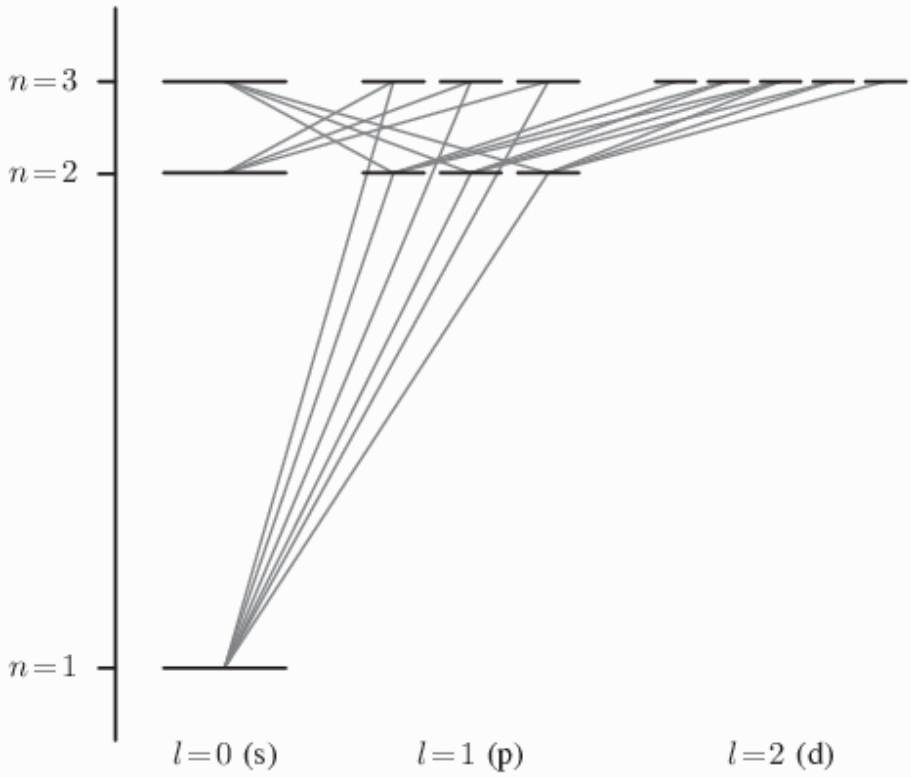
PROBLEM STATEMENT

Selection rules

The hydrogen atom consists of a single electron orbiting a positively charged nucleus. The electron can exist only in discrete orbitals characterized by the radial quantum number $n = 1, 2, 3, \ldots$ and the angular momentum quantum numbers $l = 0, 1, 2, \ldots, n-1$ and $m = -l, \ldots, -1, 0, 1, \ldots, l$. The energy of each orbital, $E_n = -(13.6\,\text{eV})/n^2$, is a function of the radial quantum number alone. Hence, each energy level is g_n-fold degenerate, where $g_n = \sum_{l=0}^{n-1}(2l+1) = n^2$. (In other words, there are $g_n = n^2$ states having the same energy E_n.)



Transitions between orbitals can occur if the electron absorbs or emits a photon. But since a quantum of light has intrinsic angular momentum (1 in units of $\hbar$), conservation laws put a strict limit on which atomic transitions are possible. This leads to the famous electric dipole *selection rules*: $\Delta l = \pm 1$ and $\Delta m = 0, \pm 1$. Allowed transitions between the various states with $n = 1, 2, 3$ are shown in the following diagram.

Complete the program `selection.cpp` so that it computes all possible transitions of the form $n_2 \to n_1$ with $n_1 < n_2 \leq 20$. Have the program output the results in four columns indicating the initial and final radial quantum numbers, the number of allowed pathways, the energy of the emitted photon $\Delta E = E_2 - E_1$, and its wavelength $\lambda = hc/\Delta E$ in nanometers. (Recall that $hc = 1240\,\text{eV} \cdot \text{nm}$.)

```
$ make selection
g++ -o selection selection.cpp -O2 -ansi -pedantic -Wall -lm
$ ./selection | head -n5
 2->1          3            10.2          121.569
 3->1          3         12.0889          102.574
 3->2         15         1.88889          656.471
 4->1          3           12.75          97.2549
 4->2         15            2.55          486.275
```

Modify `selection.cpp` so that the spectral lines in the visible spectrum, $380\,\text{nm} < \lambda < 750\,\text{nm}$, are marked with a lowercase `v`.

```
$ make selection
g++ -o selection selection.cpp -O2 -ansi -pedantic -Wall -lm
$ ./selection | head -n5
 2->1          3            10.2          121.569
 3->1          3         12.0889          102.574
 3->2v        15         1.88889          656.471
 4->1          3           12.75          97.2549
 4->2v        15            2.55          486.275
$ ./selection | grep v
 3->2v        15         1.88889          656.471
 4->2v        15            2.55          486.275
 5->2v        15           2.856          434.174
 6->2v        15         3.02222          410.294
 7->2v        15         3.12245          397.124
 8->2v        15          3.1875           389.02
 9->2v        15          3.2321          383.652
```