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Question: Consider an unknown linear transformation T that maps vecto…

2. Consider an unknown linear transformation T that maps vectors from R^4 to vectors in R^4.

The following information is provided:

$$T(e_1) = \begin{bmatrix}1\\2\\0\\1\end{bmatrix},\quad T(e_2) = \begin{bmatrix}1\\0\\0\\4\end{bmatrix}$$

Here, $e_1 = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}$, $e_2 = \begin{bmatrix}0\\1\\1\\2\end{bmatrix}$

a. Find the image of the vector $x = \begin{bmatrix}3\\2\\2\\4\end{bmatrix}$ under this linear transformation.

b. Can we find the image of the vector $x = \begin{bmatrix}1\\2\\3\\4\end{bmatrix}$ under this transformation?

c. Do we have enough information to find the matrix A such that $T(x) = Ax$?

d. Now, we are further given $T(e_3) = \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$, where $e_3 = \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$. Can we determine the transformation matrix A?

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Expert Answer

MANDALA KRISHNA answered this
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$Let\ x = ae_1 + be_2$

$$\begin{bmatrix}3\\2\\2\\4\end{bmatrix} = a\begin{bmatrix}1\\0\\0\\0\end{bmatrix} + b\begin{bmatrix}0\\1\\1\\2\end{bmatrix}$$

$$\begin{bmatrix}3\\2\\2\\4\end{bmatrix} = \begin{bmatrix}a\\b\\b\\2b\end{bmatrix}$$

$a = 3, b = 2$

$Hence\ x = 3e_1 + 2e_2$

$$T(x) = T(3e_1 + 2e_2)$$
$$= 3T(e_1) + 2T(e_2)$$
$$= 3\begin{bmatrix}1\\2\\0\\1\end{bmatrix} + 2\begin{bmatrix}1\\0\\0\\4\end{bmatrix}$$
$$= \begin{bmatrix}5\\6\\0\\11\end{bmatrix}$$

$$T(x) = \begin{bmatrix}5\\6\\0\\11\end{bmatrix}$$

$Which is the image of\ x\ under\ T.$

(b)

$Let\ x = ae_1 + be_2$

$$\begin{bmatrix}1\\2\\3\\4\end{bmatrix} = a\begin{bmatrix}1\\0\\0\\0\end{bmatrix} + b\begin{bmatrix}0\\1\\1\\2\end{bmatrix}$$

$$\begin{bmatrix}1\\2\\3\\4\end{bmatrix} = \begin{bmatrix}a\\b\\b\\2b\end{bmatrix}$$

$a = 1, b = 2, b = 3$

$Therefore\ b\ has\ two\ different\ values\ hence$

$Hence\ we\ can\ not\ find\ the\ image\ of\ x\ under\ T$

$$T\left(\begin{bmatrix}3\\2\\2\\4\end{bmatrix}\right)=A\begin{bmatrix}3\\2\\2\\4\end{bmatrix}$$

$$\begin{bmatrix}5\\6\\0\\11\end{bmatrix}=A_{4\times4}\begin{bmatrix}3\\2\\2\\4\end{bmatrix}$$

$$\begin{bmatrix}5\\6\\0\\11\end{bmatrix}=\begin{bmatrix}a_1&a_2&a_3&a_4\\b_1&b_2&b_3&b_4\\c_1&c_2&c_3&c_4\\d_1&d_2&d_3&d_4\end{bmatrix}\begin{bmatrix}3\\2\\2\\4\end{bmatrix}$$

$From\,this\,we\,can\,not\,\,find\,\,A$

(d)

$Here\{e_1,e_2,e_3\}\;does\,not\;form\,a\,basis\,,\,since\,\dim R^4=4$

$Hence\,we\,can\,'t\,\,find\,transformation\,matrix.$

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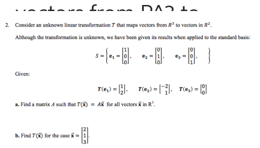

See answer

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A: See answer 100% (4 ratings)

Q: 3. Consider an unknown lincar transformation T that maps vectors from R4 to vectors in R4 Although the transformation is unknown, we have been given its results when applied to this non-standard basis: The following information is provided: nformation 1s 7(b1) = | 0 7(by) = Find the image of the vector e1 = under this linear transformation. Hint: e,--. (b1-b2+ b3-b4) 0 a. b. Find...

A: See answer

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