Theory of Computation – Homework 1

Course Instructor: Imran Farid Khan

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Instructions:

1. The deadline is Monday 22nd February, 3:00 PM (before the class).
2. You can work in groups of 2 to 3 people, but only during the brainstorming session, provided that all notes are destroyed after the session. Each member will have to write their own solutions, without looking at anyone else's solutions. You'll have to mention names and roll #s of the students you collaborated with.
3. You are NOT allowed to consult solution manuals or do internet searches. I can assure you that doing so will be more damaging to your grades.

Problem 1:

a) (i) Give a formal description of the Turing machine whose implementation level description is given in Example 4.11 of the language $C = \{a^i b^j c^k \mid i \times j = k \text{ and } i, j, k \geq 1\}$ (by formal description I meant the 7-tuple that defines the TM, i.e. complete set of states, the complete transition function in form of a state diagram or a transition table, etc).
(ii) Give the first twenty configurations of this Turing machine on the input *aabbcccc*.

b) Give a formal description of a Turing machine that takes as input a string w over the alphabet $\Sigma = \{0, 1, 2\}$. And when it halts, the input tape contains the string that follows w in the shortlex order (eg on input 02212 the machine tranforms it into 02220 and halts). You should assume that this TM never rejects any input.

Problem 2:

Let the 7-tuple $(Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$ be the definition of a 2-head Turing machine, where each element of the 7-tuple is defined just as in the definition of the standard Turing machine, except δ, which is defined as:

$$\delta : Q \times \Gamma^2 \to Q \times \Gamma^2 \times \{L, R\}^2$$

Where $\delta(q, a, b) = (p, c, d, L, R)$ says that if the 2-head TM is in state q and the first head is reading a and the second head is reading b, then transition to state p, replace a with c and b with d, move the first head left, and move the right head right. When the 2-head TM begins computation, both of the heads are on the first cell of the tape. Show that a language is Turing-recognizable if and only if a 2-head Turing machine recognizes it.

Problem 3:

Solve Problem # 3.22 of the textbook (on k-PDAs).

Problem 4:

We say a set A is *closed* under an operation $\diamond$ if for all $a, b \in A$, $a \diamond b$ is also in A. Prove that set of Turing-recognizable languages is closed under

1. the intersection operation (you'd need to show if L_1 and L_2 are any two Turing-recognizable languages, then $L_1 \cap L_2 = \{w \mid w \in L_1 \text{ and } w \in L_2\}$ is also a Turing-recognizable language).
2. the concatenation operation (you'd need to show if L_1 and L_2 are any two Turing-recognizable languages, then the concatenation

$$L_1 L_2 = \{w \mid w \text{ can be written as concatenation of two strings xy, where } x \in L_1 \text{ and } y \in L_2\}$$

is also a Turing-recognizable language).