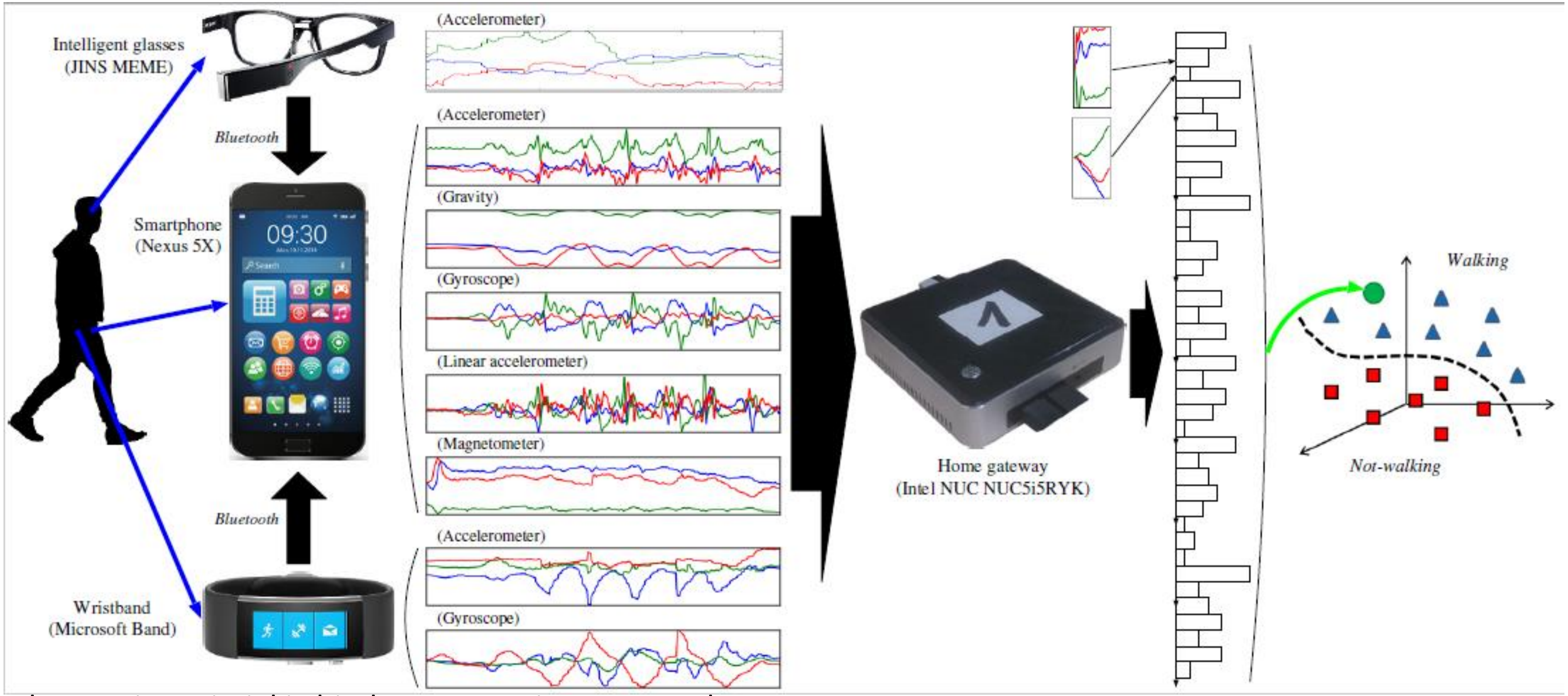
Data Science for Assistive Health Technologies

Dr. Muhammad Adeel Nisar

Assistant Professor – Department of IT,
Faculty of Computing and Information Technology,
University of the Punjab, Lahore

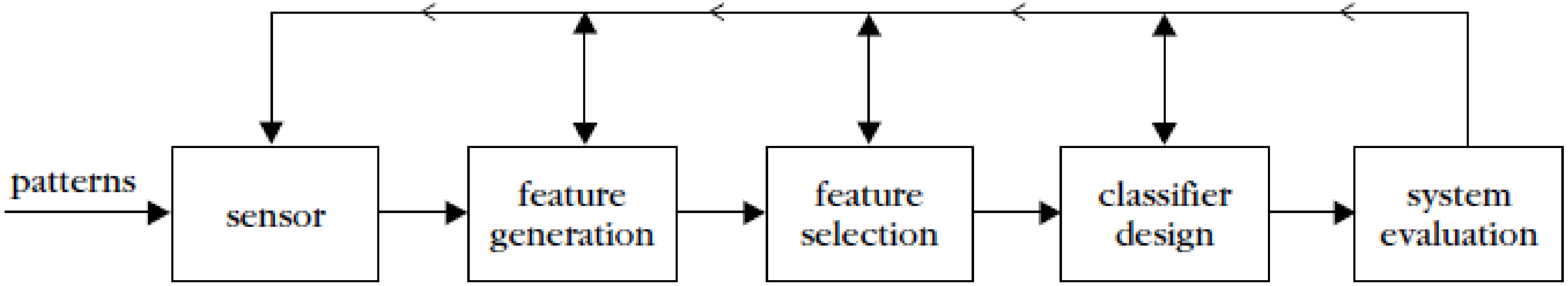
Contents

Recap – Sensor Data Acquisition System



Lukas Köping, Kimiaki Shirahama, Marcin Grzegorzek,
A general framework for sensor-based human activity recognition, Computers in Biology and Medicine, Volume 95, 2018, Pages 248-260, ISSN 0010-4825, https://doi.org/10.1016/j.compbiomed.2017.12.025.

Basic Stages of Pattern Analysis



Feature Generation – Introductory Statements

- Given a set of measurements, the goal is to discover compact and informative representations of the obtained data – features.

- The representations are generated after processing a large amount of sensory data.

- Measurements are transformed to a new set of features.

- In good features, the classification-related information is “squeezed” in a relatively small number of features.

Feature Generation – Introductory Statements

- Appropriately chosen feature transform can exploit and remove information redundancies.
- For example, using image pixels as features would be highly inefficient as pixels have a large degree of correlation.
- However, for instance, the Fourier transform turns out to be much more efficient for feature extraction.
- Fourier transform is just one of the tools from a palette of possible transforms.

Manual Feature Engineering vs. Feature Learning

- **Manual Feature Engineering:** features manually selected by experts from a certain application domain.

- **Feature Learning:** a set of techniques allowing a system to automatically discover the raw data representation needed for classification.

Manual Feature Engineering

Hand-Crafted Features		
Maximum	Percentile 50	First-order mean
Minimum	Percentile 80	Norm of the first-order mean
Average	Interquartile	Second-order mean
Standard-deviation	Skewness	Norm of the second-order mean
Zero-crossing	Kurtosis	Spectral energy
Percentile 20	Auto-correlation	Spectral entropy

Li, F.; Shirahama, K.; Nisar, M.A.; Köping, L.; Grzegorzek, M. Comparison of Feature Learning Methods for Human Activity Recognition Using Wearable Sensors. *Sensors* **2018**, *18*, 679. https://doi.org/10.3390/s18020679

Supervised vs. Unsupervised Feature Learning

- In **supervised feature learning,** features are learned using labelled input data. Examples include supervised neural networks, multilayer perceptron and (supervised) dictionary learning.

- In **unsupervised feature learning,** features are learned with unlabeled input data. Examples include dictionary learning, independent component analysis, autoencoders, matrix factorization and various forms of clustering.

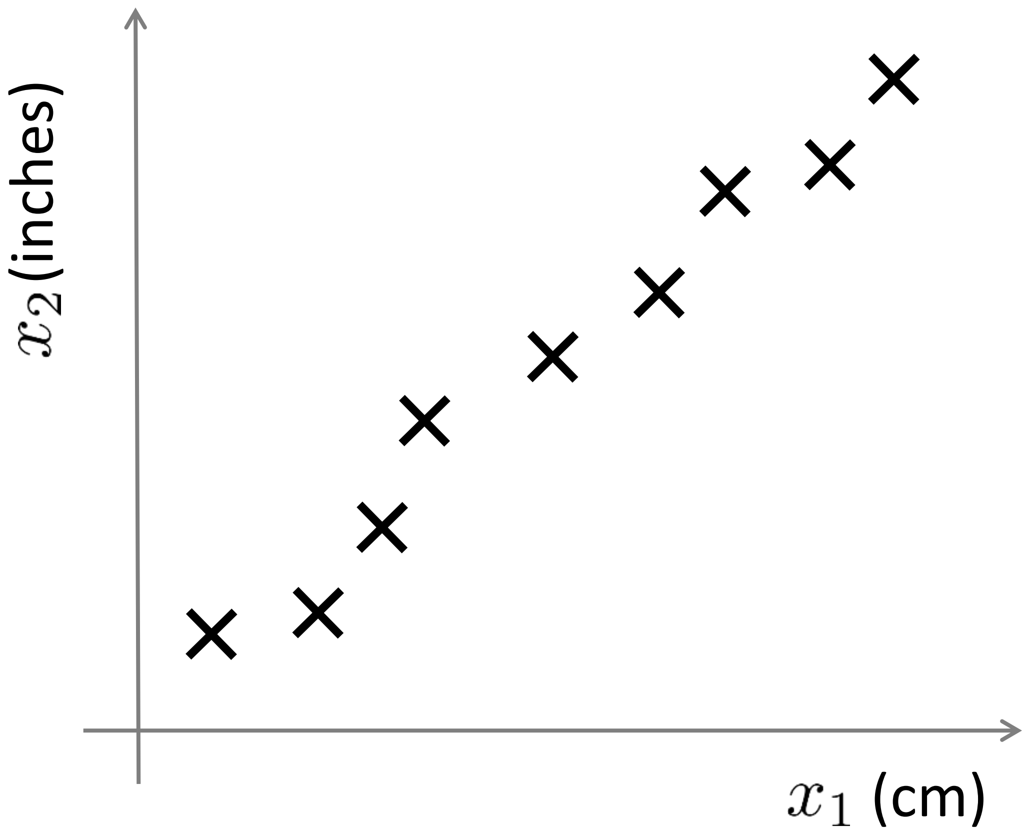
PCA – Introductory Statements

- The Principal Component Analysis (PCA) is one of the most popular methods for feature generation and dimensionality reduction in pattern recognition.

- The computation of the transformation matrix exploits the statistical information describing the data.

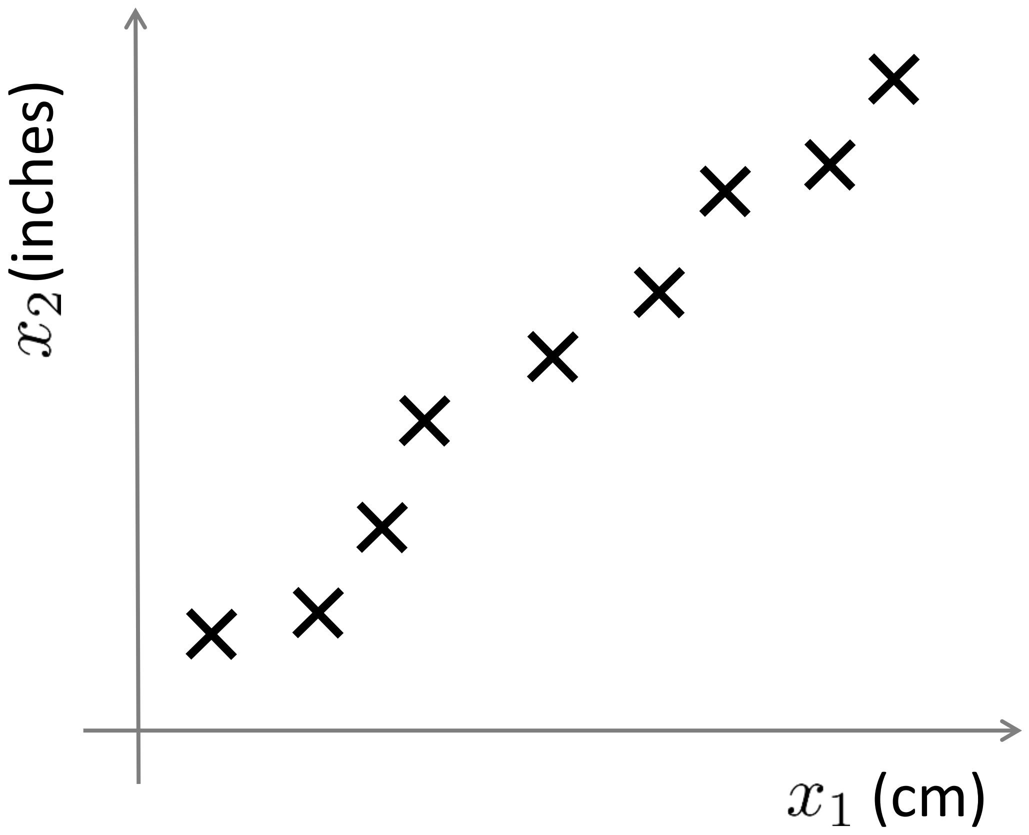
- The labels of the training samples are not used (unsupervised mode).

Data Compression



Reduce data from 2D to 1D

Data Compression



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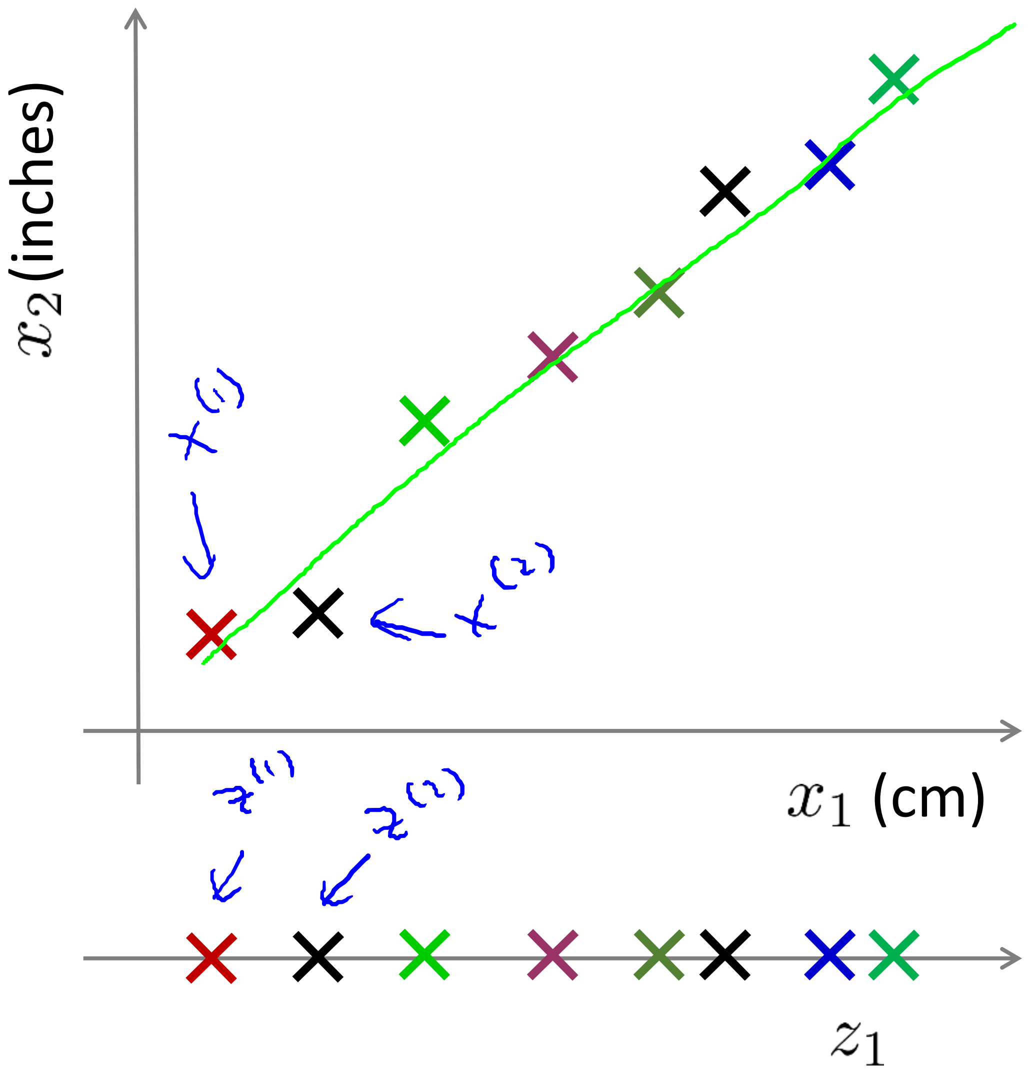


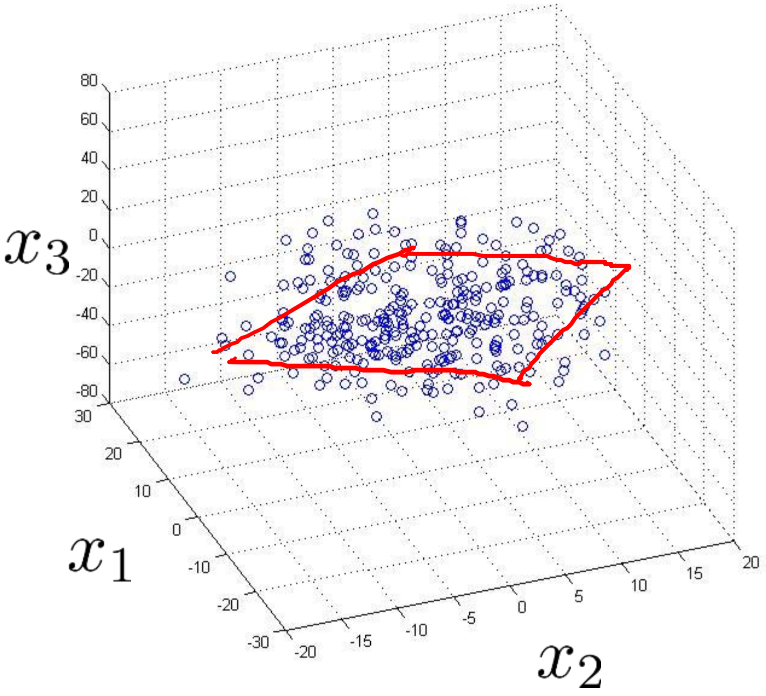
Reduce data from 2D to 1D

$$x^{(1)} \in \mathbb{R}^2 \rightarrow z^{(1)} \in \mathbb{R}$$
$$x^{(2)} \in \mathbb{R}^2 \rightarrow z^{(2)} \in \mathbb{R}$$
$$\vdots$$
$$x^{(m)} \in \mathbb{R}^2 \rightarrow z^{(m)} \in \mathbb{R}$$

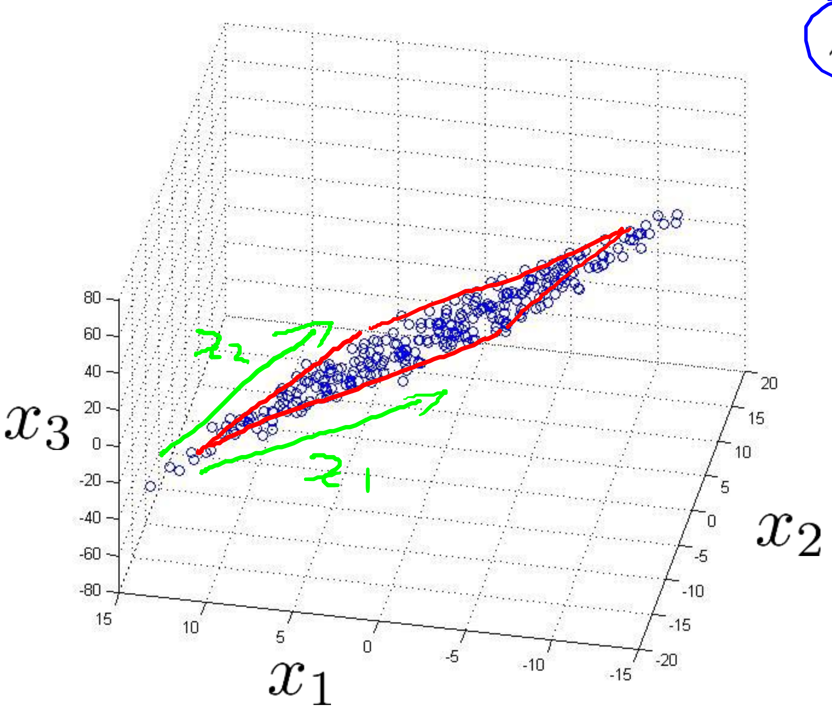
Data Compression

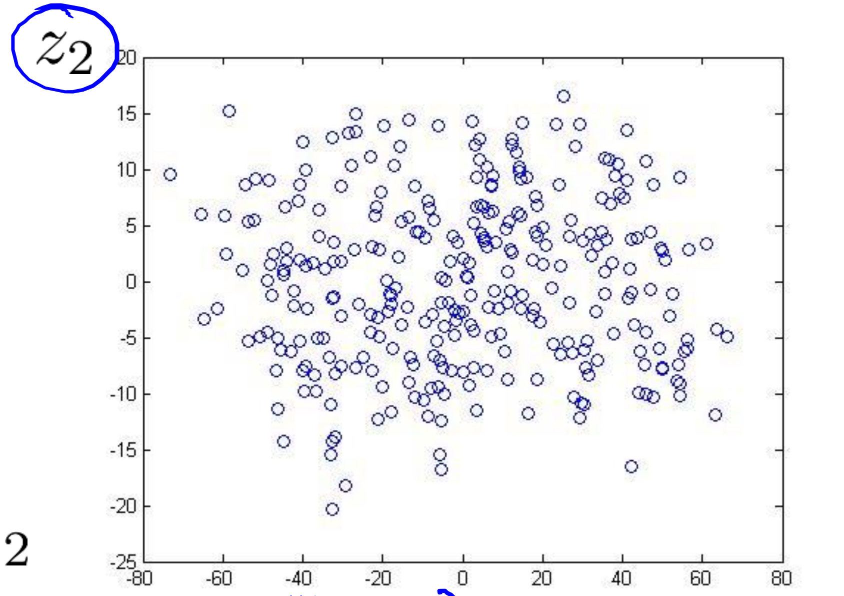
$1000D \rightarrow 100D$

Reduce data from 3D to 2D



$x^{(i)} \in \mathbb{R}^3$





$z^{(i)} \in \mathbb{R}^2$

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \qquad z^{(i)} = \begin{bmatrix} z_1^{(i)} \\ z_2^{(i)} \end{bmatrix}$$

Data Visualization

$x \in \mathbb{R}^{50}$ $\quad x^{(i)} \in \mathbb{R}^{50}$

Country	x_1 GDP (trillions of US$)	x_2 Per capita GDP (thousands of intl. $)	x_3 Human Develop-ment Index	x_4 Life expectancy	x_5 Poverty Index (Gini as percentage)	x_6 Mean household income (thousands of US$)	…
Canada	1.577	39.17	0.908	80.7	32.6	67.293	…
China	5.878	7.54	0.687	73	46.9	10.22	…
India	1.632	3.41	0.547	64.7	36.8	0.735	…
Russia	1.48	19.84	0.755	65.5	39.9	0.72	…
Singapore	0.223	56.69	0.866	80	42.5	67.1	…
USA	14.527	46.86	0.91	78.3	40.8	84.3	…
…	…	…	…	…	…	…	

[resources from en.wikipedia.org]

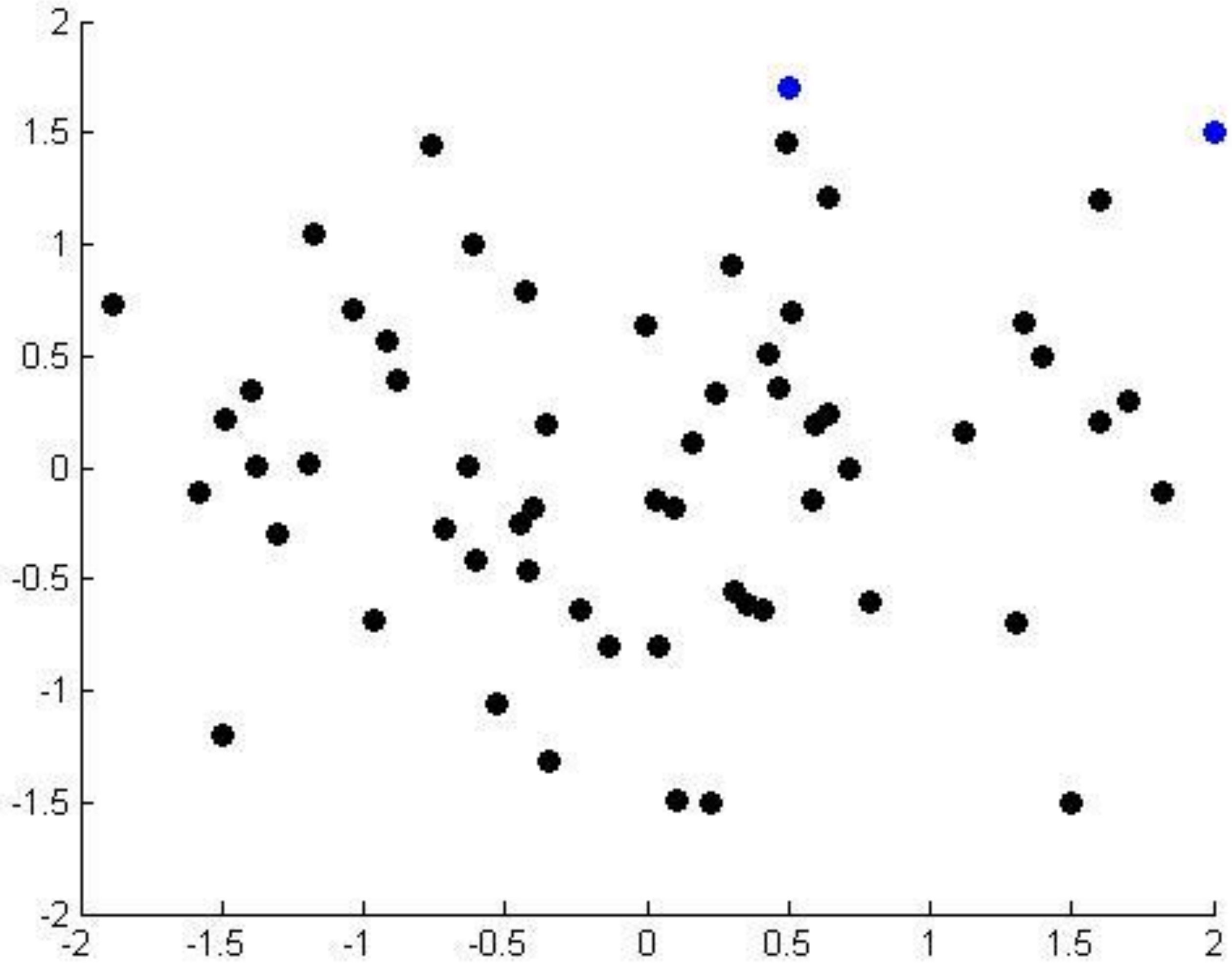
Data Visualization

Country	z_1	z_2
Canada	1.6	1.2
China	1.7	0.3
India	1.6	0.2
Russia	1.4	0.5
Singapore	0.5	1.7
USA	2	1.5
…	…	…

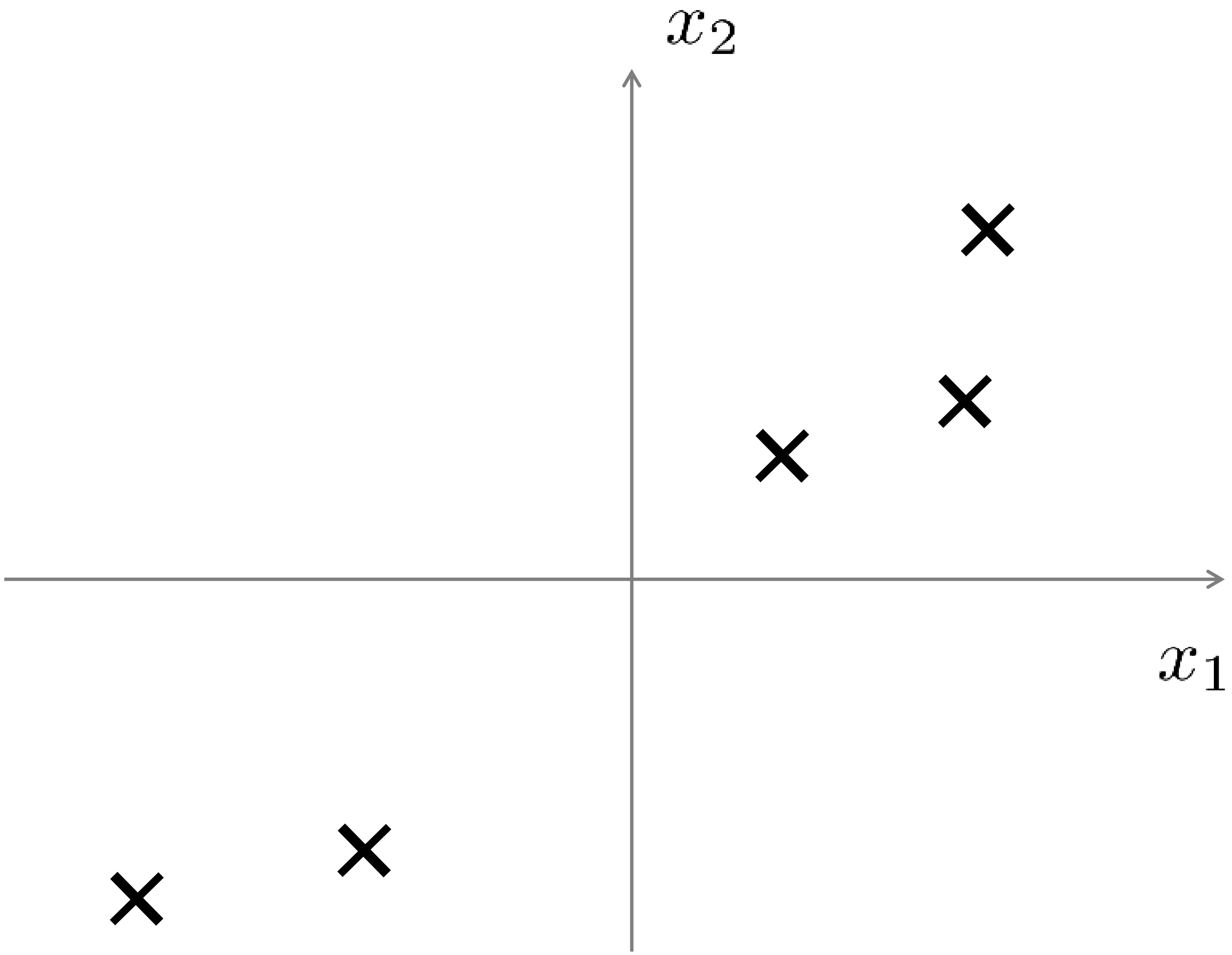
$z^{(i)} \in \mathbb{R}^2$

Reduce data from 50D to 2D

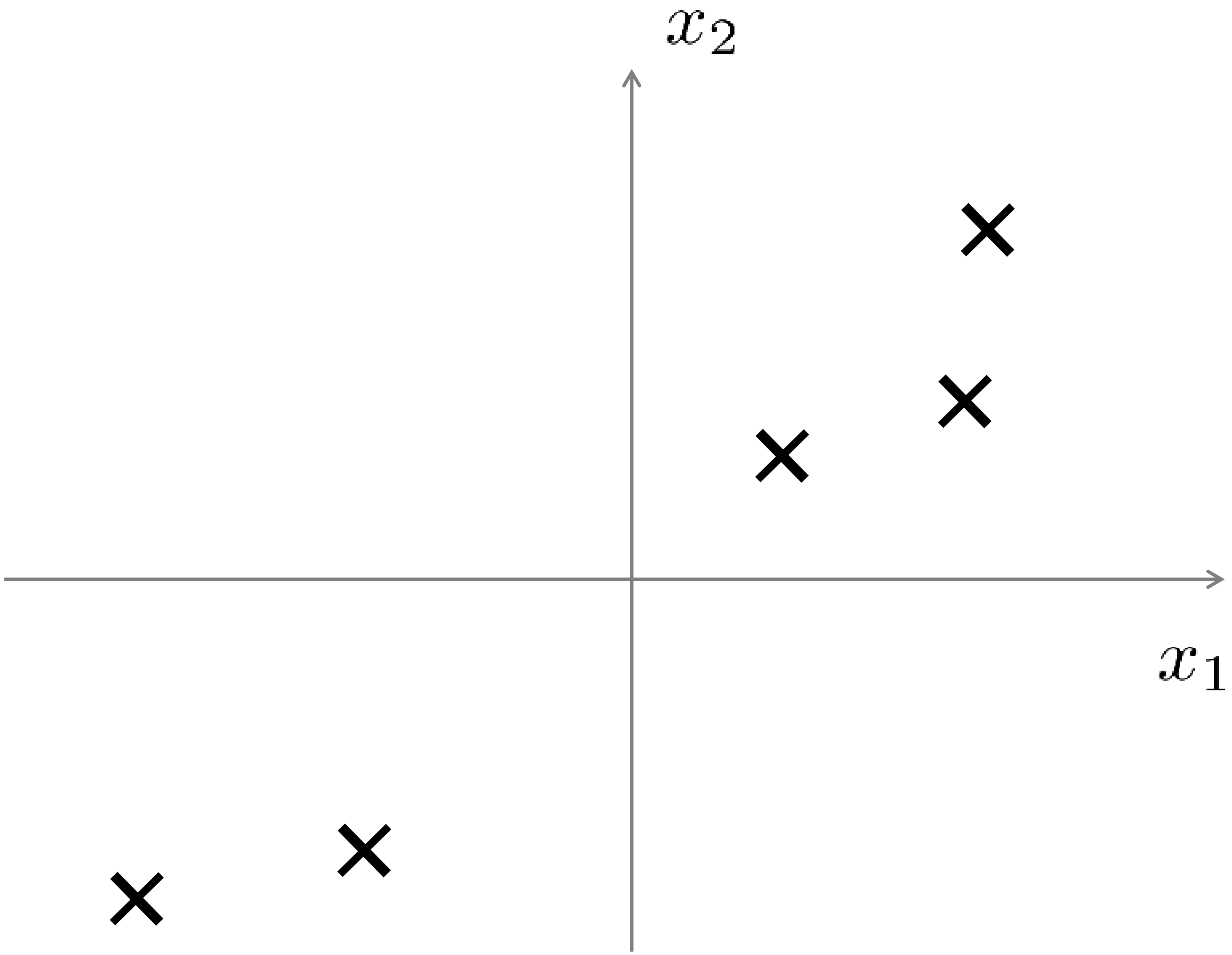
Data Visualization



Principal Component Analysis (PCA) problem formulation



Principal Component Analysis (PCA) problem formulation



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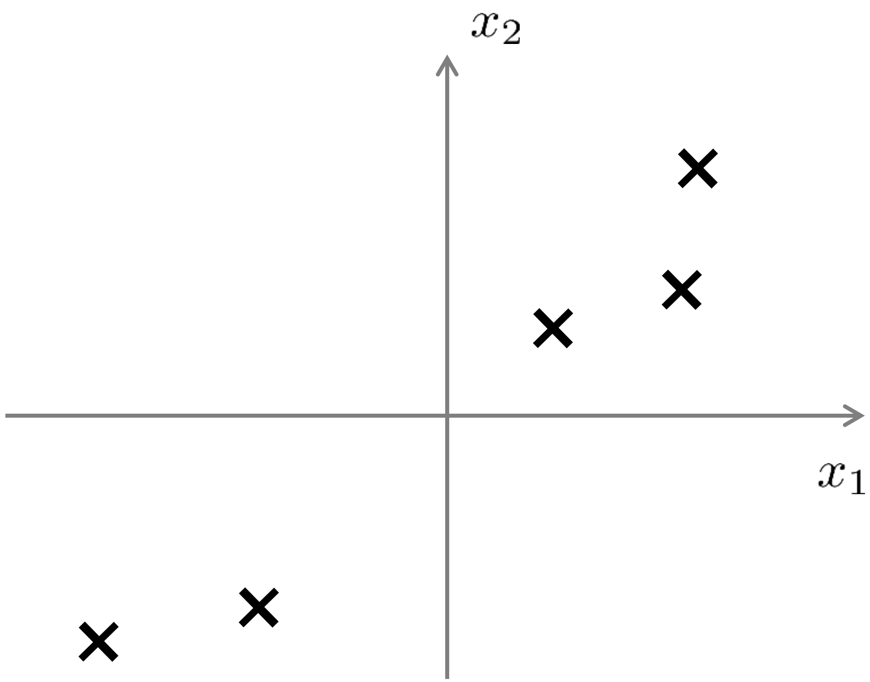
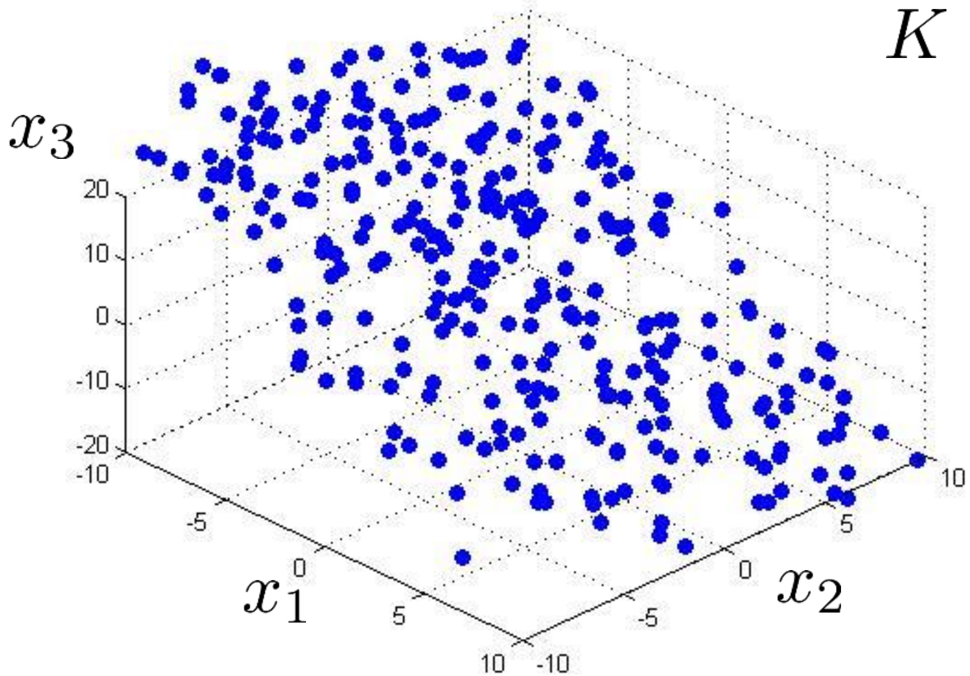
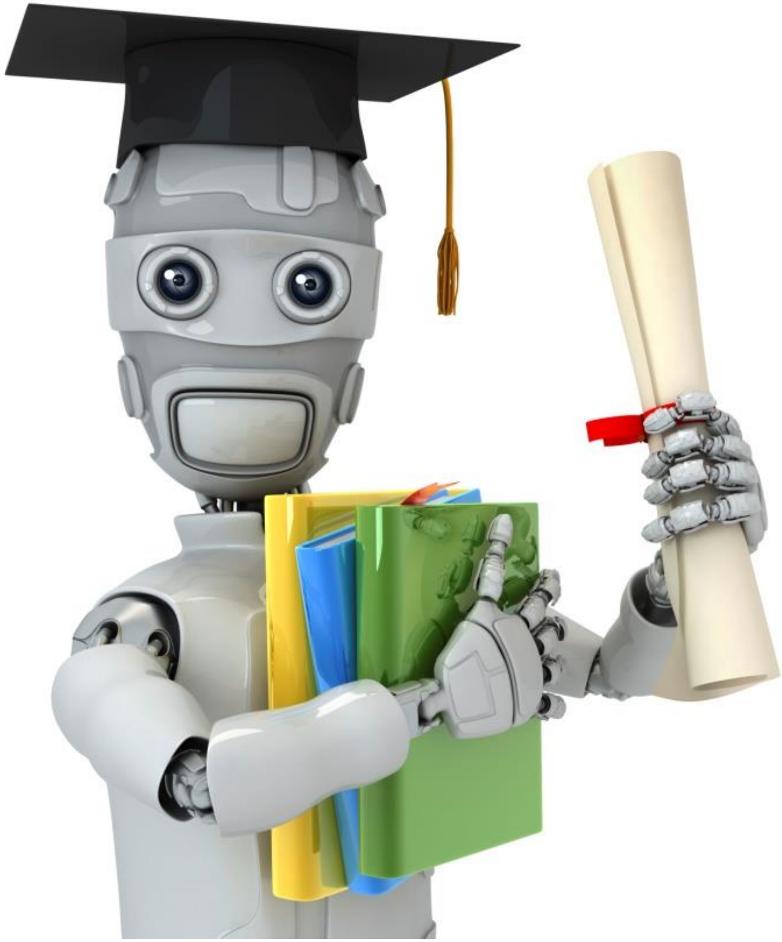


$3D \rightarrow 2D$
$K = 2$



Reduce from 2-dimension to 1-dimension: Find a direction (a vector $u^{(1)} \in \mathbb{R}^n$) onto which to project the data so as to minimize the projection error.

Reduce from n-dimension to k-dimension: Find k vectors $u^{(1)}, u^{(2)}, \ldots, u^{(k)}$ onto which to project the data, so as to minimize the projection error.



Machine Learning

Dimensionality Reduction

Principal Component Analysis algorithm

Data preprocessing

Training set: $x^{(1)}, x^{(2)}, \ldots, x^{(m)}$

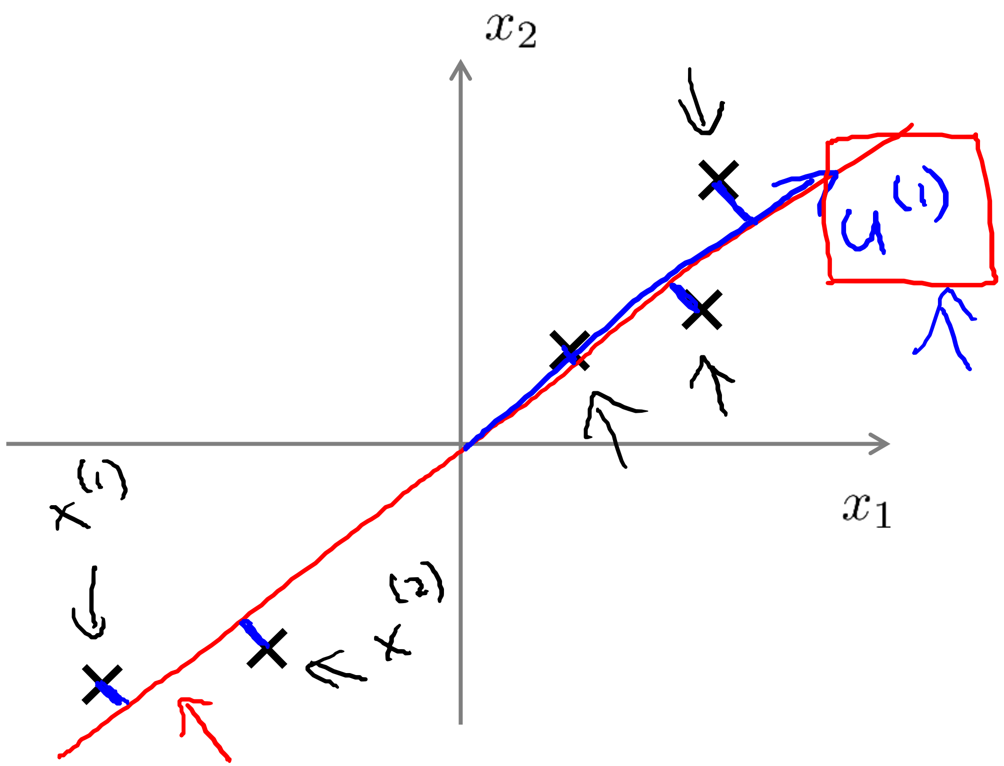
Preprocessing (feature scaling/mean normalization):

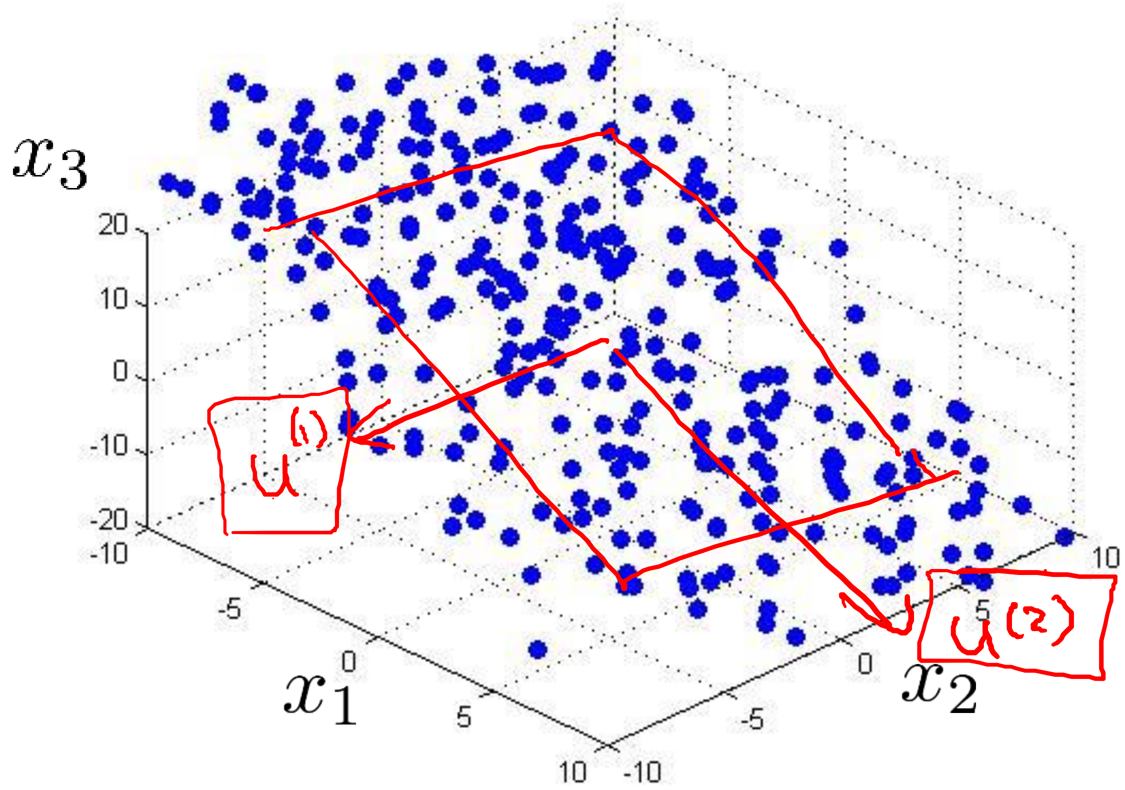
$$\mu_j = \frac{1}{m}\sum_{i=1}^{m} x_j^{(i)}$$

Replace each $x_j^{(i)}$ with $x_j - \mu_j$

If different features on different scales (e.g., $x_1 =$ heart rate, $x_2 =$ skin conductance), scale features to have comparable range of values.

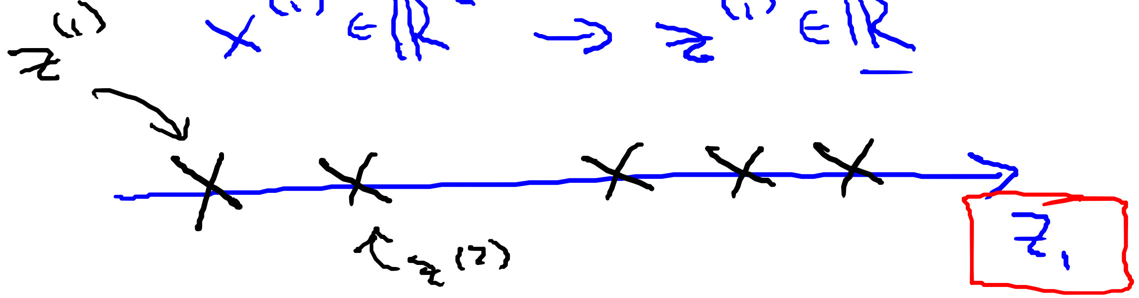
Principal Component Analysis (PCA) algorithm





Reduce data from 2D to 1D

$x^{(i)} \in \mathbb{R}^2 \rightarrow z^{(i)} \in \mathbb{R}$



Reduce data from 3D to 2D

$x^{(i)} \in \mathbb{R}^3 \rightarrow z^{(i)} \in \mathbb{R}^2$

$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$

Principal Component Analysis (PCA) algorithm

Reduce data from n-dimensions to k-dimensions

Compute “covariance matrix”:

$$\Sigma = \frac{1}{m}\sum_{i=1}^{n} \boxed{\underbrace{(x^{(i)})}_{n\times 1}\underbrace{(x^{(i)})^T}_{1\times n}} \quad n \times n$$

Sigma

Compute “eigenvectors” of matrix Σ:

```
[U,S,V] = svd(Sigma);
```

→ Singular value decomposition

eig(Sigma)

$n \times n$ matrix.

$$U = \begin{bmatrix} | & | & | & & | \\ \underbrace{u^{(1)} & u^{(2)} & u^{(3)}}_{k} & \dots & u^{(m)} \\ | & | & | & & | \end{bmatrix}$$

$U \in \mathbb{R}^{n \times n}$

$u^{(1)}, \dots, u^{(k)}$

Principal Component Analysis (PCA) algorithm

From `[U,S,V] = svd(Sigma)` we get:

$$U = \Bigg[\underbrace{\boxed{\begin{matrix} | & | \\ u^{(1)} & u^{(2)} \\ | & | \end{matrix}}}_{k} \quad \ldots \quad \begin{matrix} | \\ u^{(n)} \\ | \end{matrix}\Bigg] \in \mathbb{R}^{n\times n}$$

$$x \in \mathbb{R}^n \longrightarrow z \in \mathbb{R}^k$$

$$z^{(i)} = \underbrace{\begin{bmatrix} | & | & & | \\ u^{(1)} & u^{(2)} & \ldots & u^{(k)} \\ | & | & & | \end{bmatrix}}_{\substack{n\times k \\ U_{reduce}}}^T x^{(i)} = \underbrace{\underbrace{\begin{bmatrix} \text{---}\,(u^{(1)})^T\,\text{---} \\ \vdots \\ \text{---}\,(u^{(k)})^T\,\text{---} \end{bmatrix}}_{k\times n} \underbrace{x^{(i)}}_{n\times 1}}_{k\times 1}$$

$$z \in \mathbb{R}^k$$

Principal Component Analysis (PCA) algorithm summary

- After mean normalization (ensure every feature has zero mean) and optionally feature scaling:

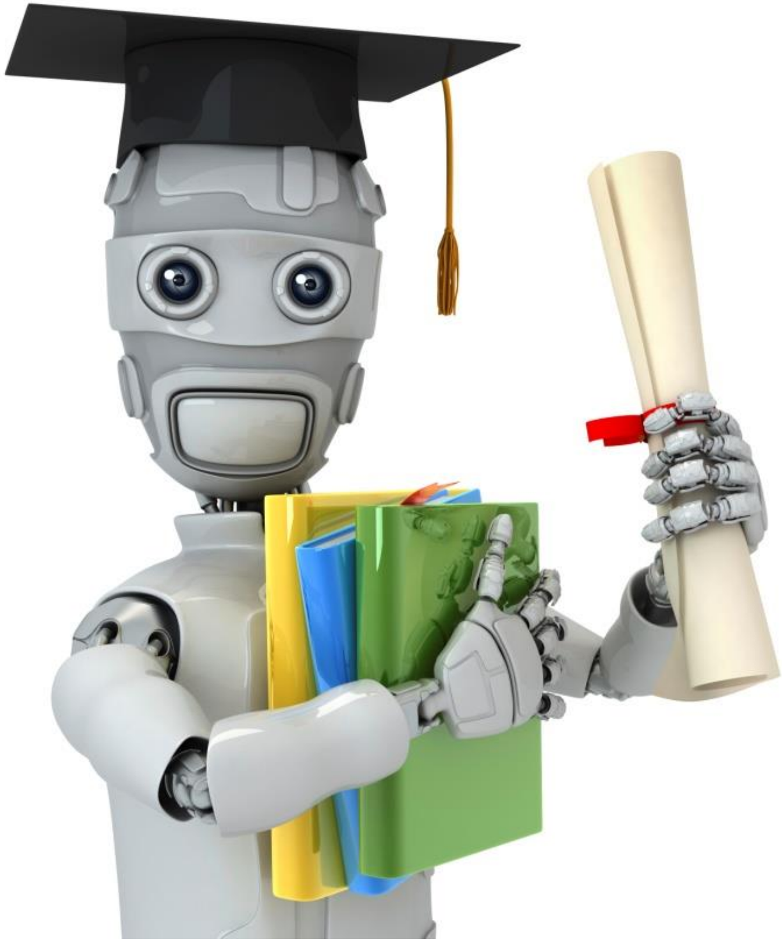
$$\texttt{Sigma} = \frac{1}{m}\sum_{i=1}^{m}(x^{(i)})(x^{(i)})^T$$

$$\underline{X} = \begin{bmatrix} — & x^{(1)T} & — \\ & \vdots & \\ — & x^{(m)T} & — \end{bmatrix}$$

$$\boxed{\texttt{Sigma = (1/m) * X' * X;}}$$

```
[U,S,V] = svd(Sigma);
Ureduce = U(:,1:k);
z = Ureduce'*x;
```

$x \in \mathbb{R}^n$ $\cancel{x_0 = 1}$

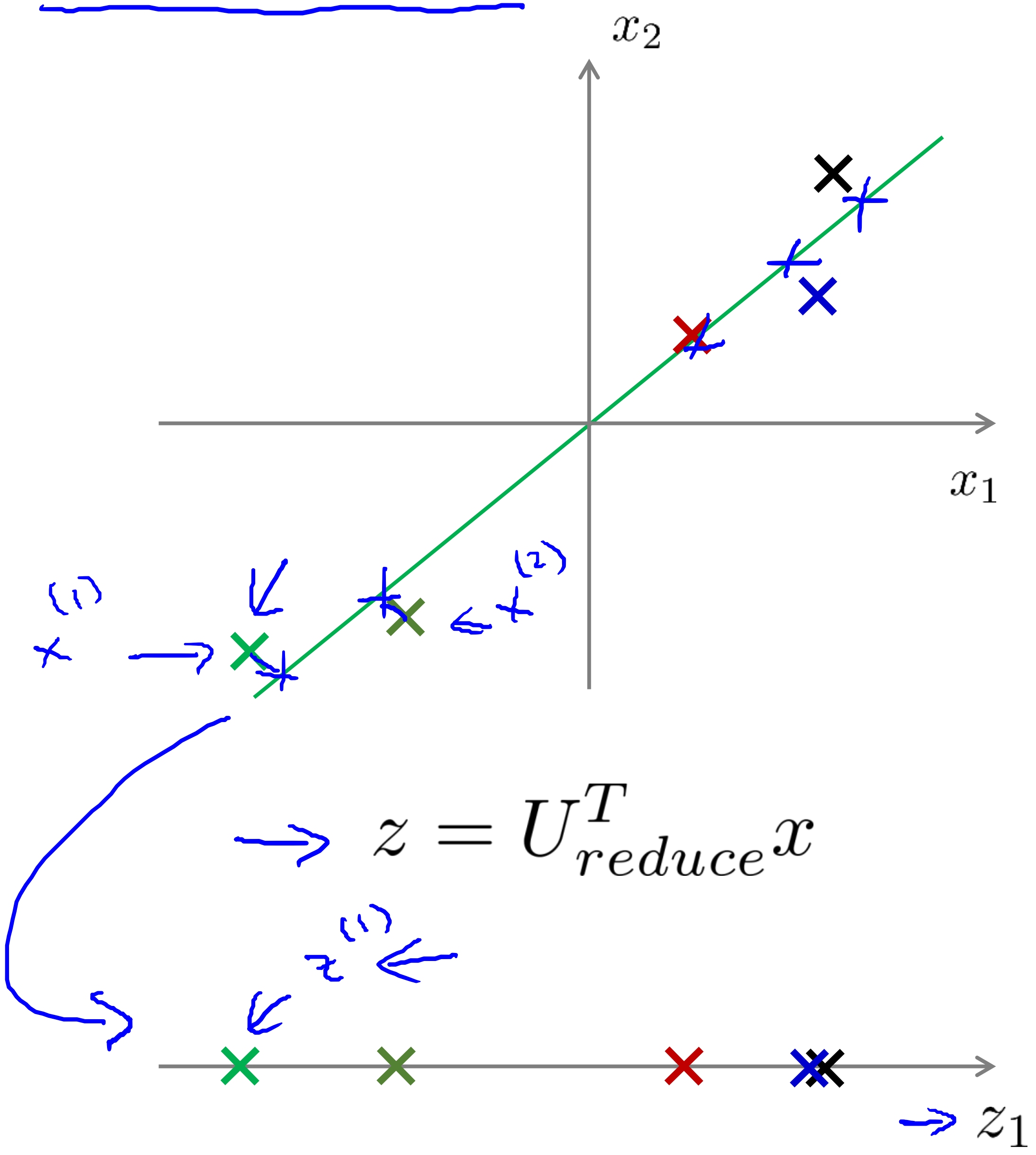


Machine Learning

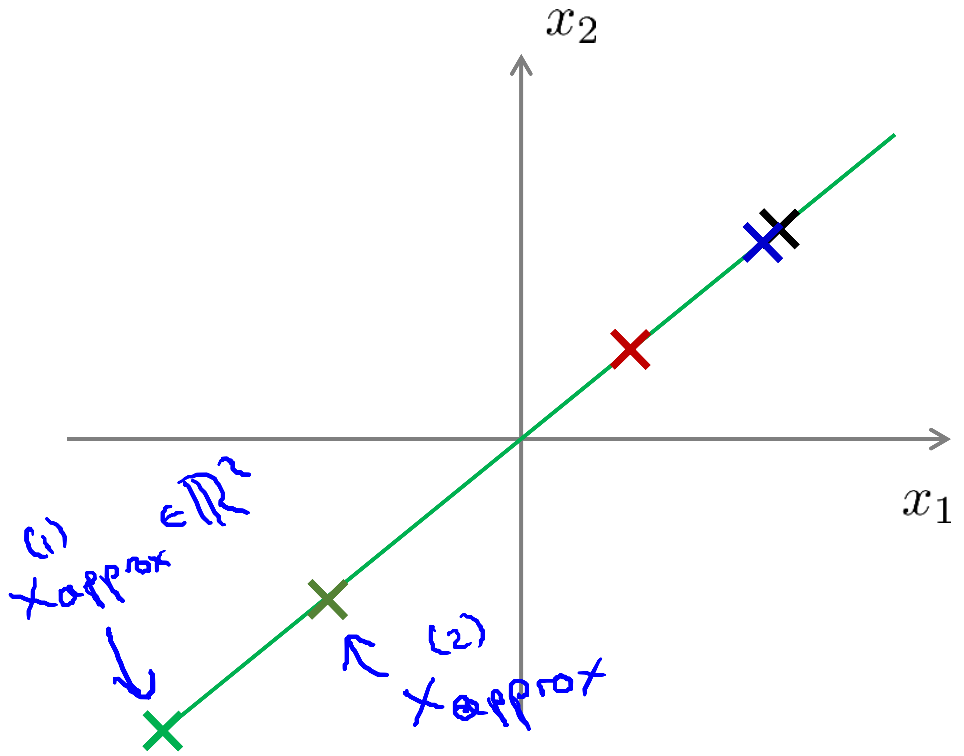
Dimensionality Reduction

Reconstruction from compressed representation

Reconstruction from compressed representation



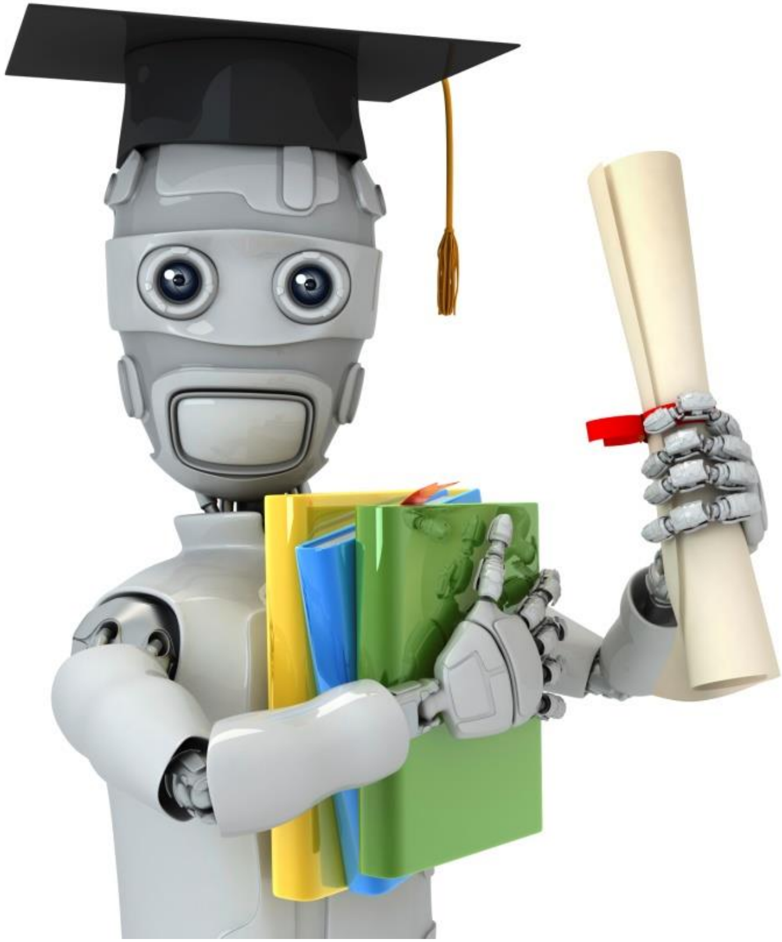
$$z = U_{reduce}^T x$$



$z \in \mathbb{R} \rightarrow x \in \mathbb{R}^2$

$\tilde{x}$

$\underbrace{x_{approx}^{(1)}}_{\mathbb{R}^n} = \underbrace{\underbrace{U_{reduce}}_{n \times k} \cdot \underbrace{z^{(1)}}_{k \times 1}}_{n \times 1}$



Machine Learning

Dimensionality Reduction

Choosing the number of principal components

Choosing k (number of principal components)

Average squared projection error: $\frac{1}{m}\sum_{i=1}^{m}\|x^{(i)} - x_{approx}^{(i)}\|^2$

Total variation in the data: $\frac{1}{m}\sum_{i=1}^{m}\|x^{(i)}\|^2$

Typically, choose k to be smallest value so that

$$\frac{\frac{1}{m}\sum_{i=1}^{m}\|x^{(i)} - x_{approx}^{(i)}\|^2}{\frac{1}{m}\sum_{i=1}^{m}\|x^{(i)}\|^2} \leq \cancel{0.01} \qquad \cancel{(1\%)}$$

~~0.05~~ ~~5%~~

0.10 (10%)

"~~99%~~ of variance is retained"

~~95%~~ 90%

Choosing k (number of principal components)

Algorithm:

Try PCA with $\cancel{k=1}$ $\cancel{k=2}$ $\cancel{k=3}$ $k=4$ $\vdots$

Compute $U_{reduce}, z^{(1)}, z^{(2)}, \ldots, z^{(m)}, x^{(1)}_{approx}, \ldots, x^{(m)}_{approx}$

Check if

$$\frac{\frac{1}{m}\sum_{i=1}^{m}\|x^{(i)} - x^{(i)}_{approx}\|^2}{\frac{1}{m}\sum_{i=1}^{m}\|x^{(i)}\|^2} \leq 0.01?$$

$k = 17$

```
[U,S,V] = svd(Sigma)
```

$$S = \begin{bmatrix} S_{11} & & & & 0 \\ & S_{22} & & & \\ & & S_{33} & & \\ & & & \ddots & \\ 0 & & & & S_{nn} \end{bmatrix}$$

$k=3$

For given k

$$1 - \frac{\sum_{i=1}^{k} S_{ii}}{\sum_{i=1}^{n} S_{ii}} \leq 0.01$$

$$\boxed{\frac{\sum_{i=1}^{k} S_{ii}}{\sum_{i=1}^{n} S_{ii}} \geq 0.99}$$

Choosing k (number of principal components)

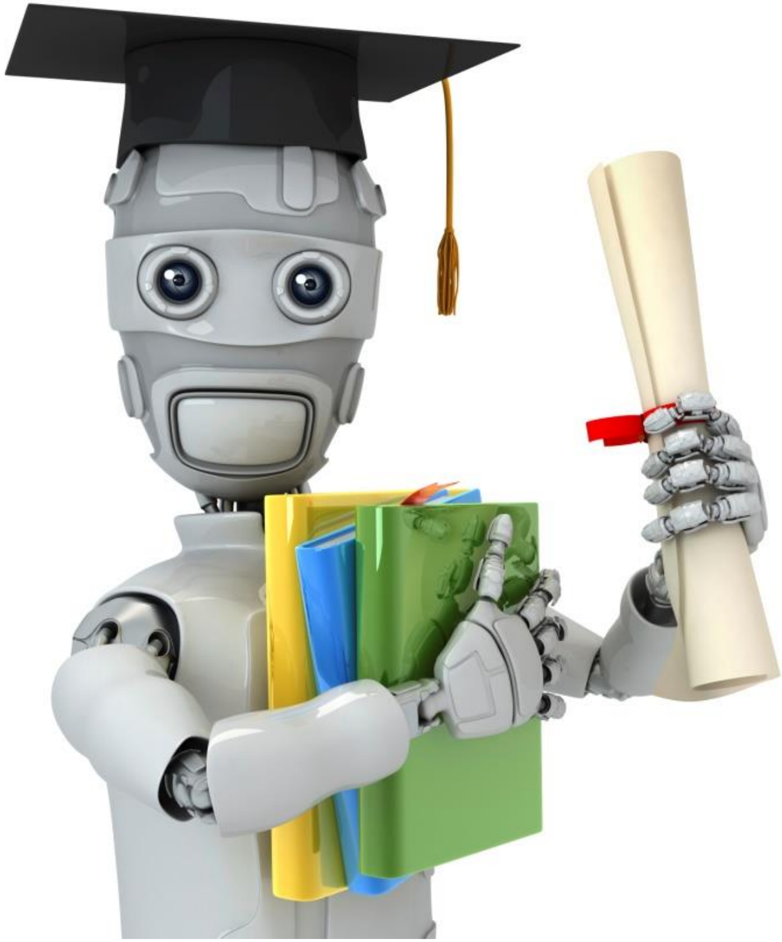
```
[U,S,V] = svd(Sigma)
```

Pick smallest value of k for which

$$\boxed{\frac{\sum_{i=1}^{k} S_{ii}}{\sum_{i=1}^{m} S_{ii}}} \geq 0.99$$

$k=100$

(99% of variance retained)



Machine Learning

Dimensionality Reduction

Advice for applying PCA

Supervised learning speedup

$x^{(i)} \in \mathbb{R}^{10,000}$

100

100

$\boxed{(x^{(1)}, y^{(1)})}, (x^{(2)}, y^{(2)}), \ldots, (x^{(m)}, y^{(m)})$

Extract inputs:

Unlabeled dataset: $x^{(1)}, x^{(2)}, \ldots, x^{(m)} \in \mathbb{R}^{10000}$

$\downarrow PCA$

x

$\downarrow$

z

U_{reduce}

$z^{(1)}, z^{(2)}, \ldots, z^{(m)} \in \mathbb{R}^{1000}$

New training set:

$\boxed{(z^{(1)}, y^{(1)})}, (z^{(2)}, y^{(2)}), \ldots, (z^{(m)}, y^{(m)})$

$h_\theta(z) = \frac{1}{1+e^{-\theta^T z}}$

Note: Mapping $x^{(i)} \rightarrow z^{(i)}$ should be defined by running PCA only on the training set. This mapping can be applied as well to the examples $x_{cv}^{(i)}$ and $x_{test}^{(i)}$ in the cross validation and test sets.

$x \rightarrow z$

Application of PCA

- Compression
  - Reduce memory/disk needed to store data
  - Speed up learning algorithm ←

Choose k by % of variance retain

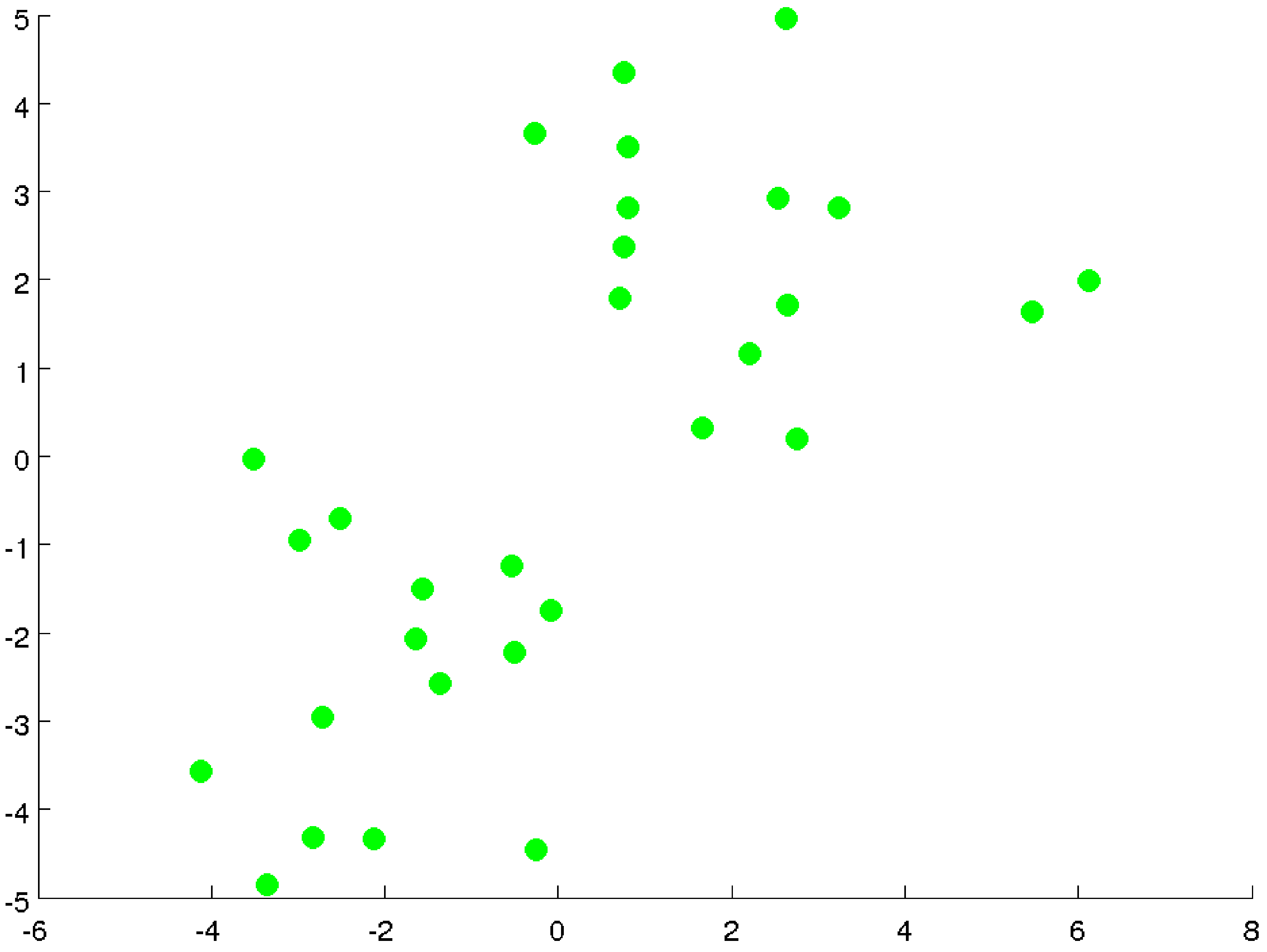
- Visualization

k = 2 or k = 3

Codebook Approach

Clustering

K-means algorithm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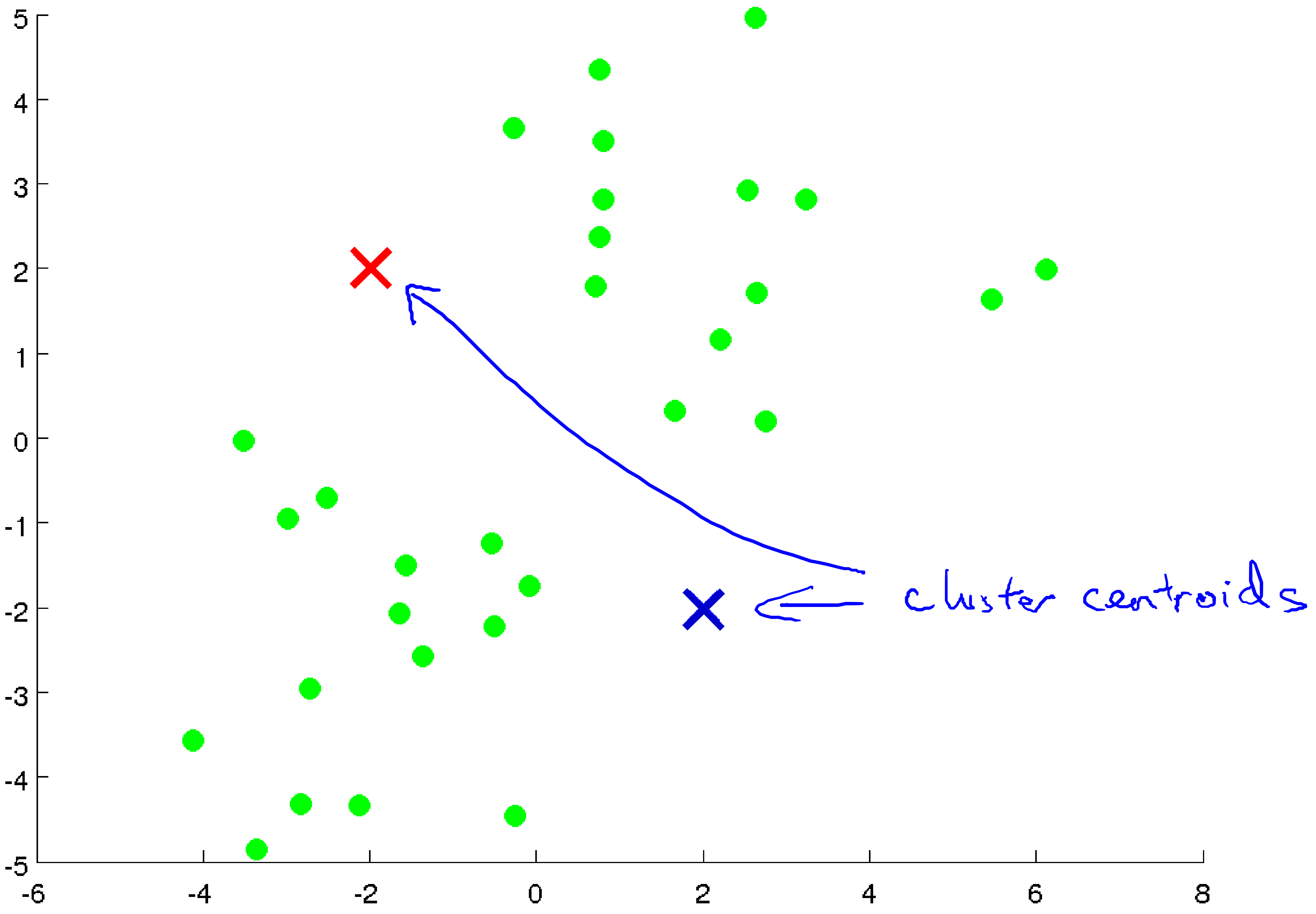
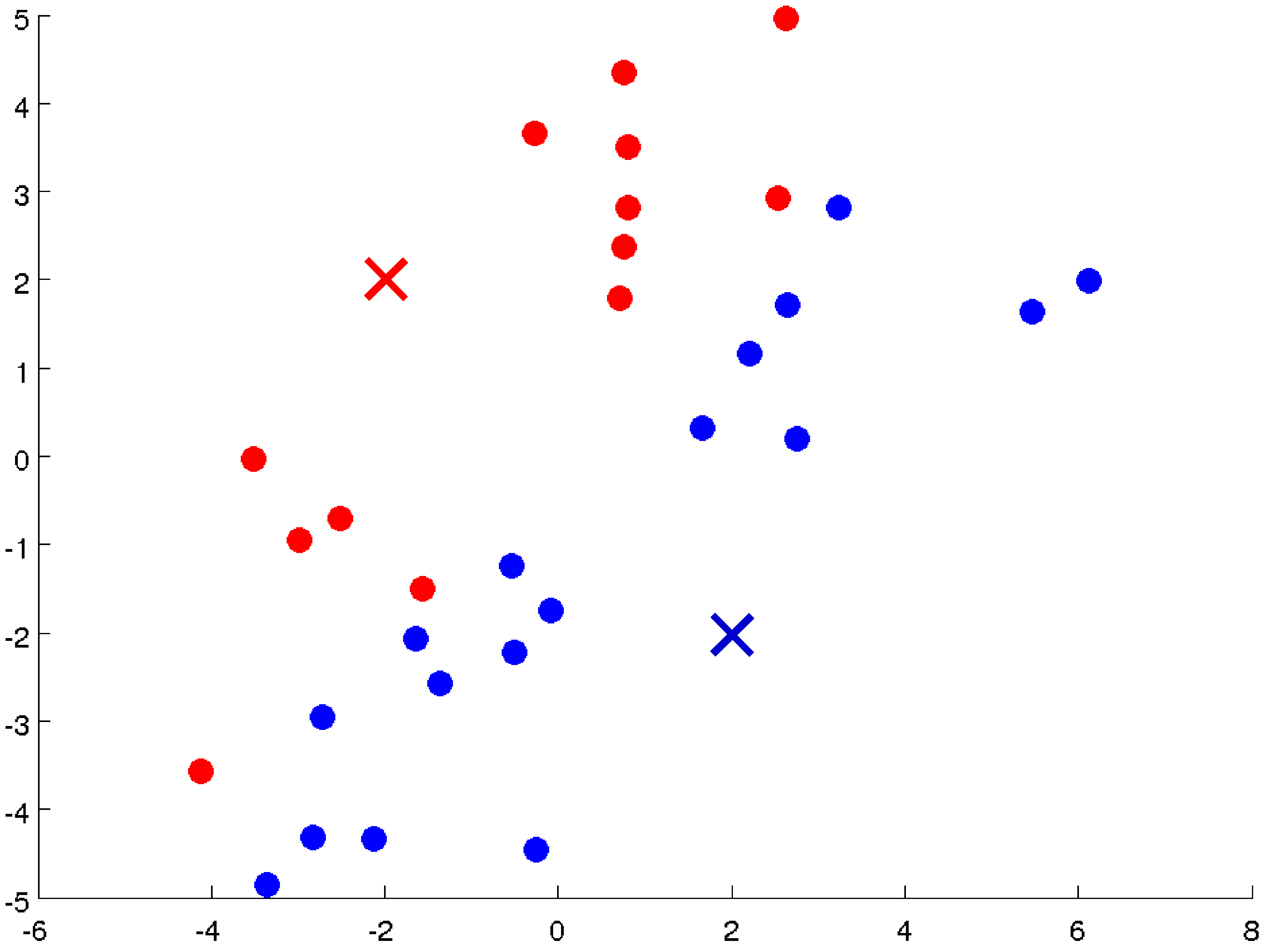


cluster centroids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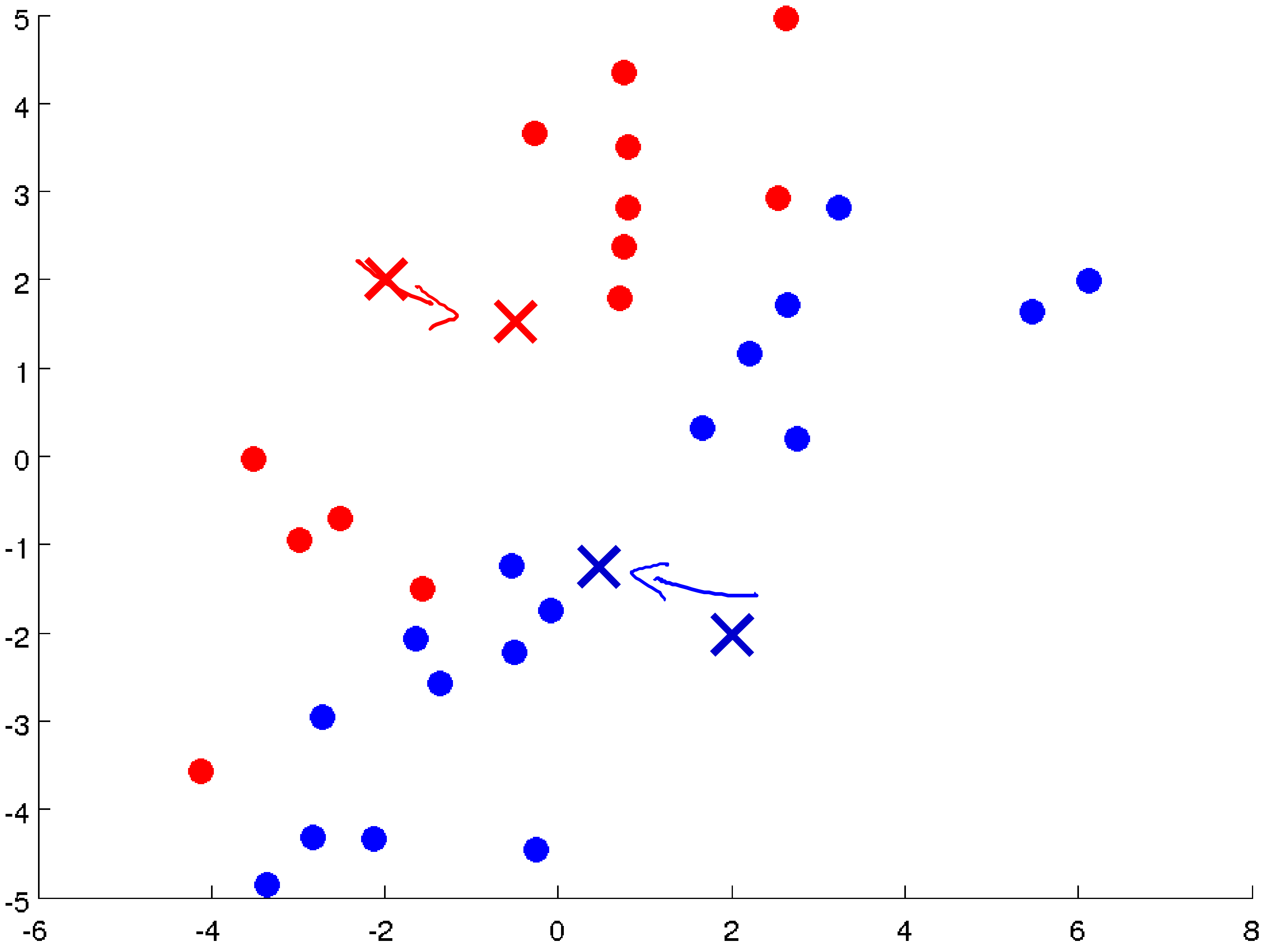
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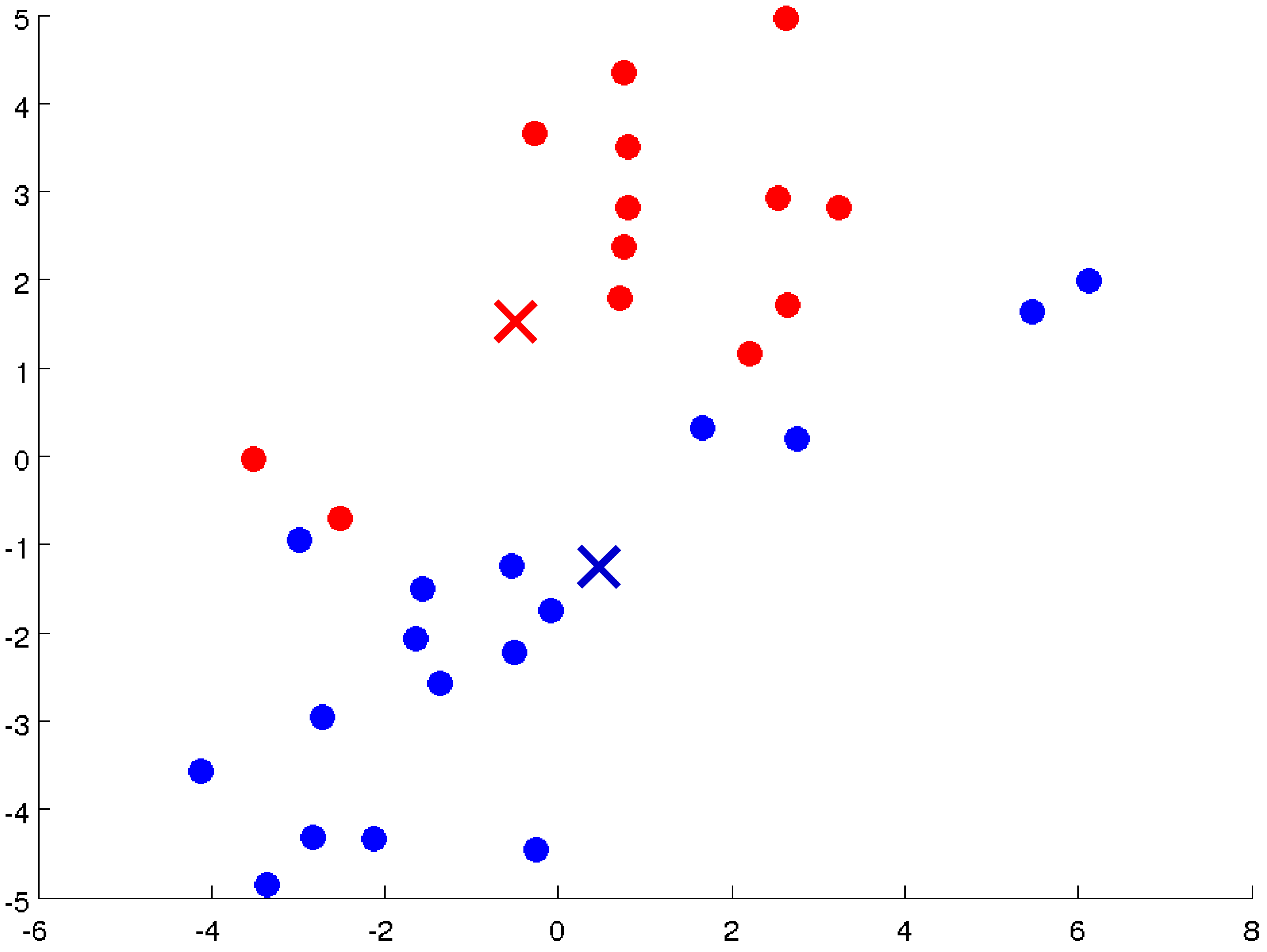
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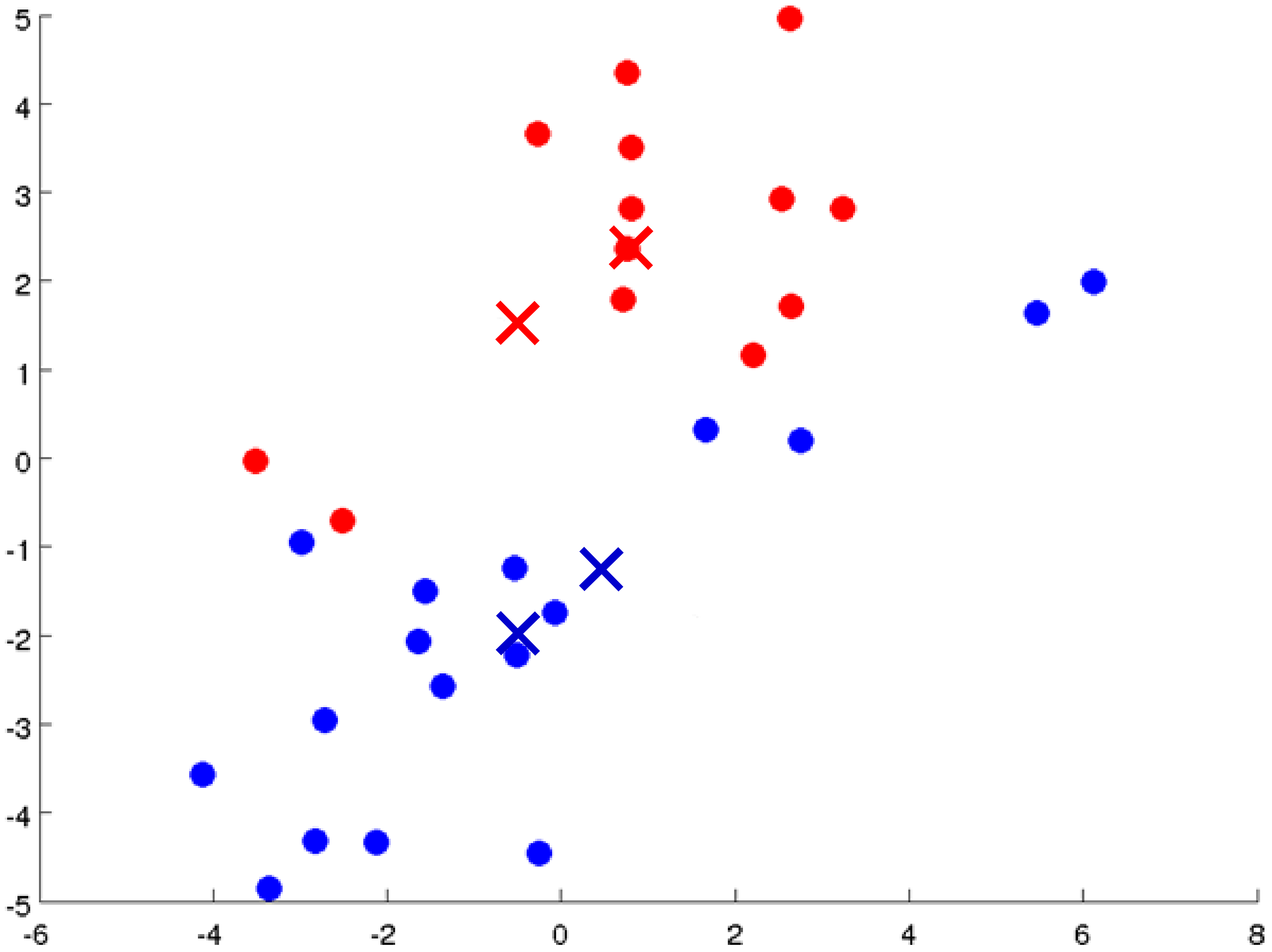
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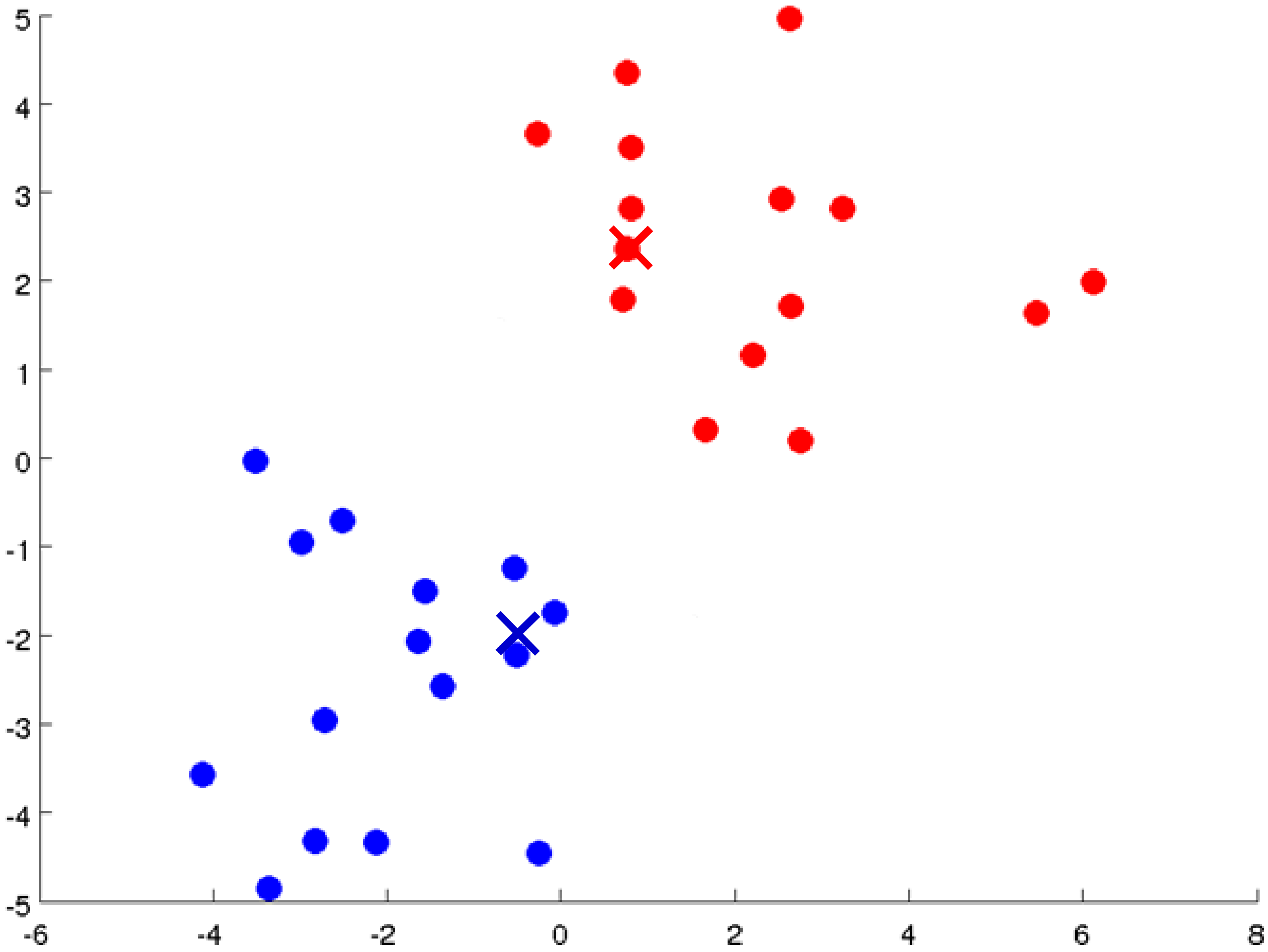
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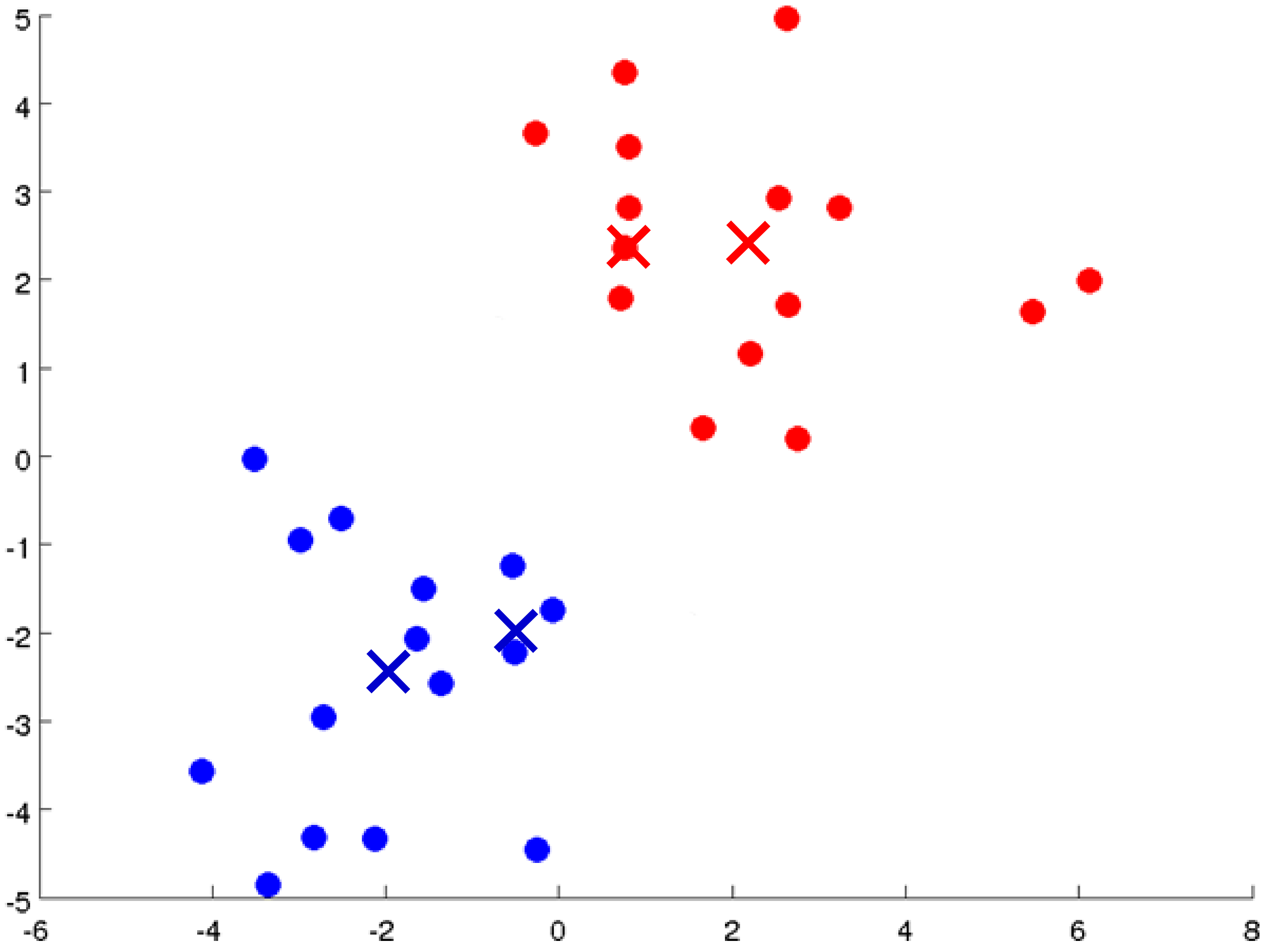

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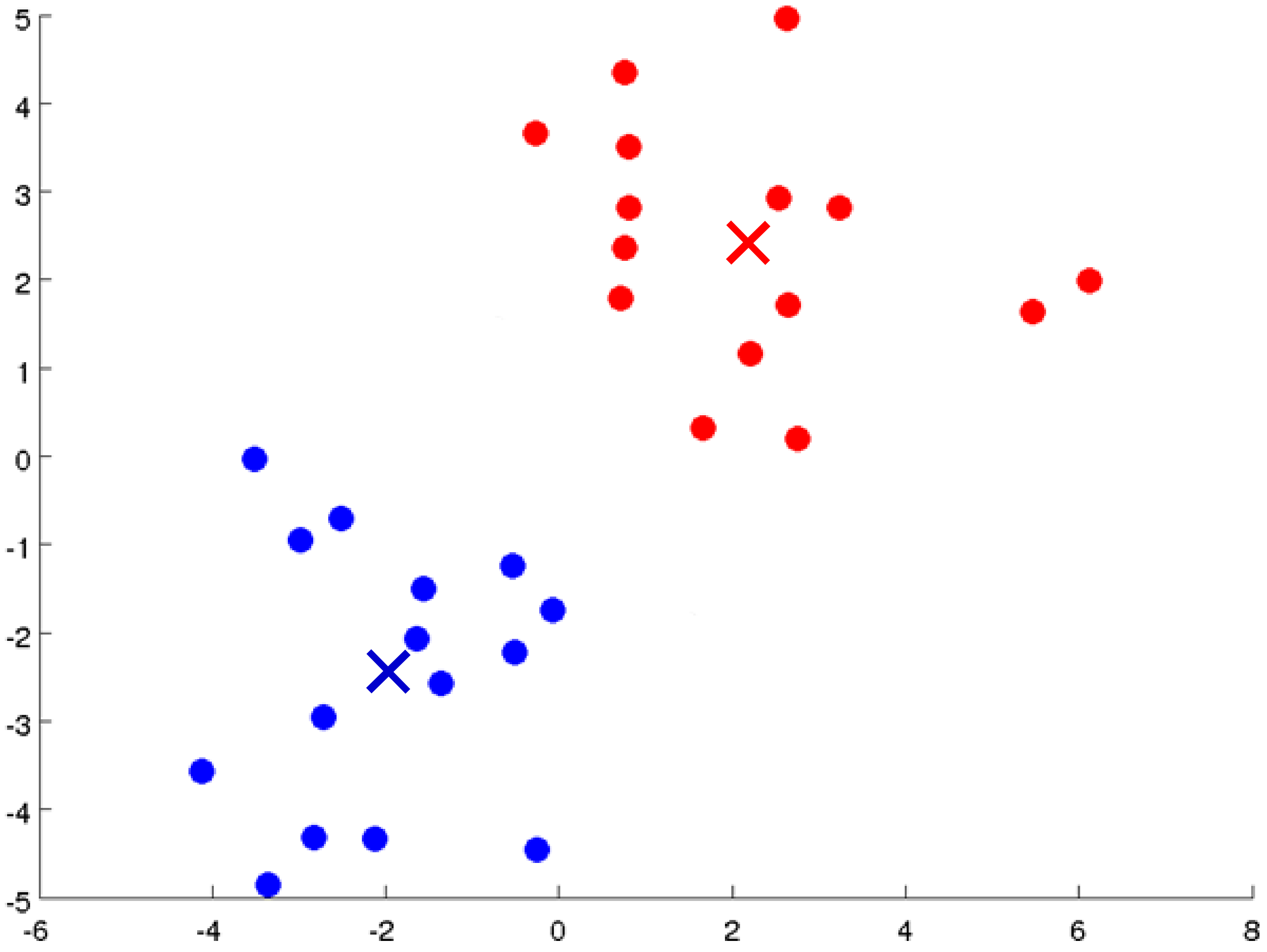
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K-means algorithm

Input:

- K (number of clusters)
- Training set $\{x^{(1)}, x^{(2)}, \ldots, x^{(m)}\}$

$x^{(i)} \in \mathbb{R}^n$

K-means algorithm

μ_1 $\times$ μ_2 $\times$

Randomly initialize K cluster centroids $\mu_1, \mu_2, \dots, \mu_K \in \mathbb{R}^n$

Repeat {

Cluster assignment step

for i = 1 to m

$c^{(i)}$:= index (from 1 to K) of cluster centroid closest to $x^{(i)}$

$\min_k \|x^{(i)} - \mu_k\|^2 \rightarrow c^{(i)}$

Move centroid

for k = 1 to K

μ_k := average (mean) of points assigned to cluster k

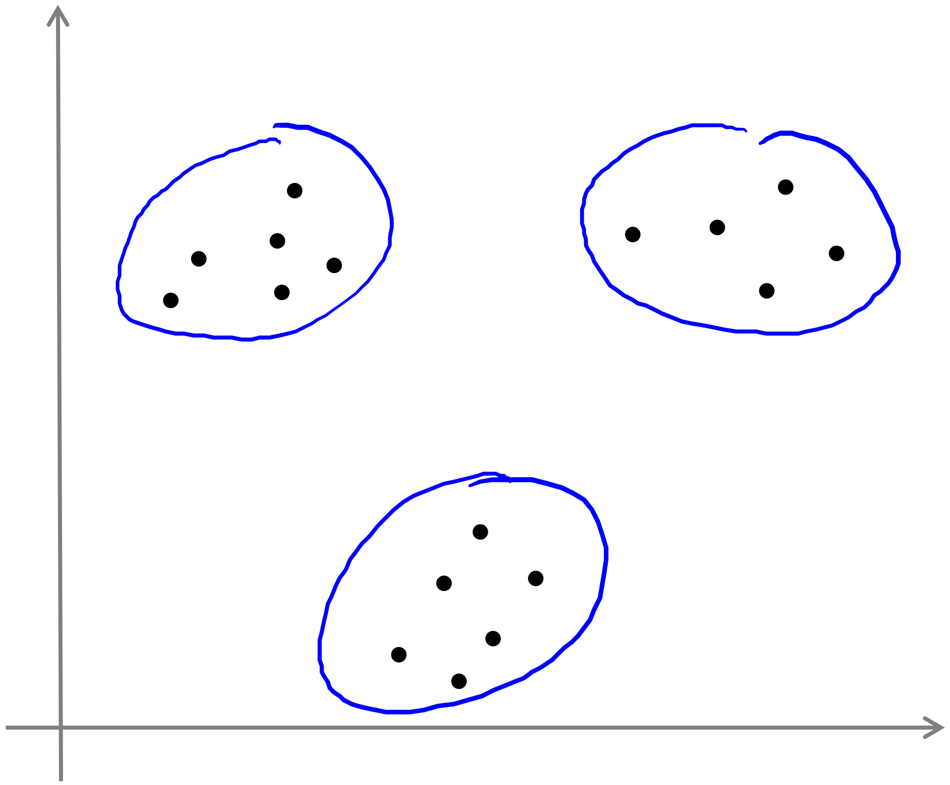
$x^{(1)}, x^{(5)}, x^{(6)}, x^{(10)} \rightarrow c^{(1)}=2, c^{(5)}=2, c^{(6)}=2, c^{(10)}=2$

$$\mu_2 = \frac{1}{4}\left[x^{(1)} + x^{(5)} + x^{(6)} + x^{(10)}\right] \in \mathbb{R}^n$$

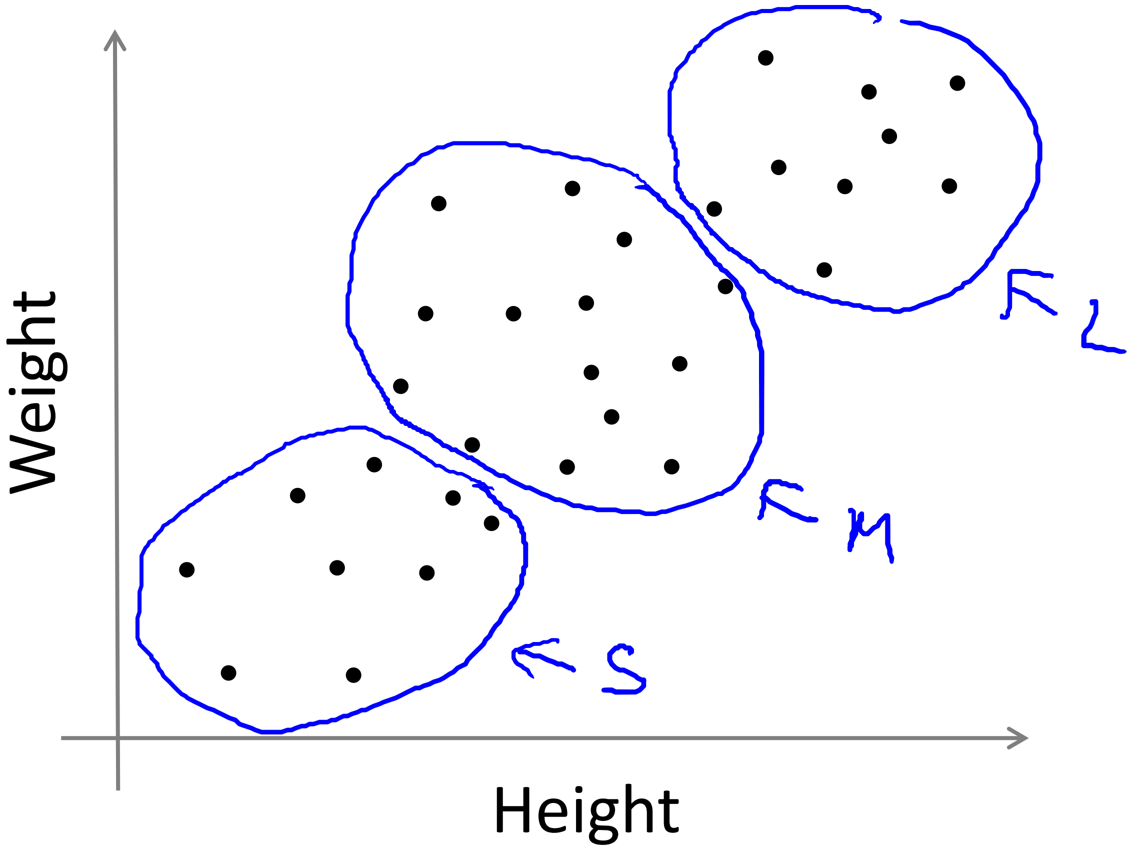
}

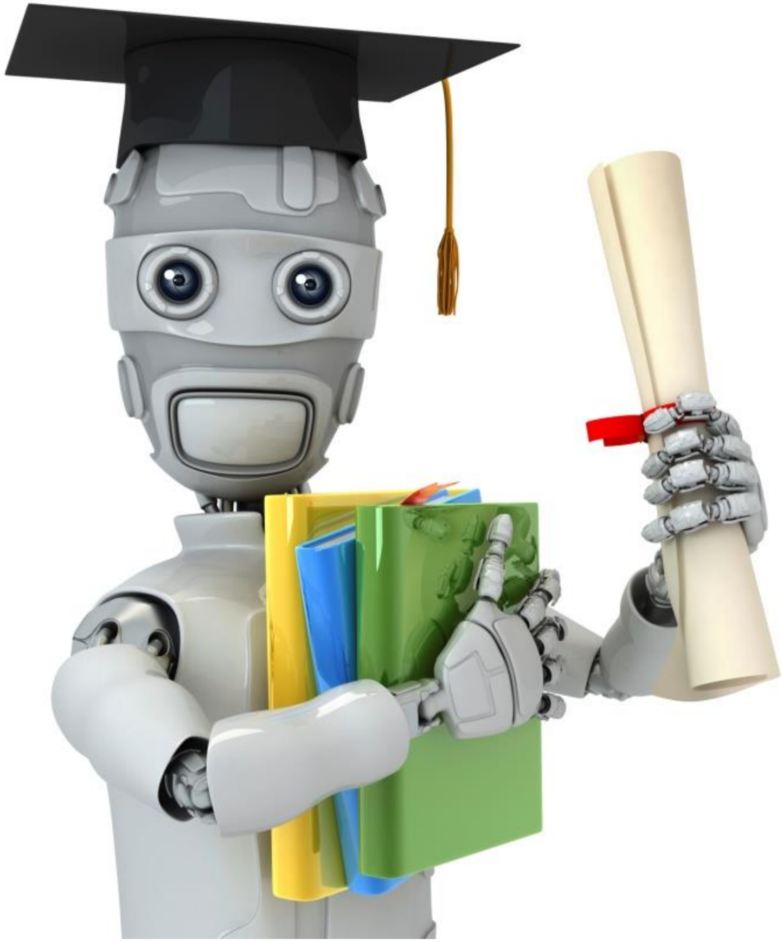
K-means for non-separated clusters

S, M, L



T-shirt sizing





Machine Learning

Clustering

Optimization objective

K-means optimization objective

$c^{(i)}$ = index of cluster (1,2,...,K) to which example $x^{(i)}$ is currently assigned

μ_k = cluster centroid $\underline{k}$ ($\mu_k \in \mathbb{R}^n$) K $k \in \{1,2,\ldots,K\}$

$\mu_{c^{(i)}}$ = cluster centroid of cluster to which example $x^{(i)}$ has been assigned

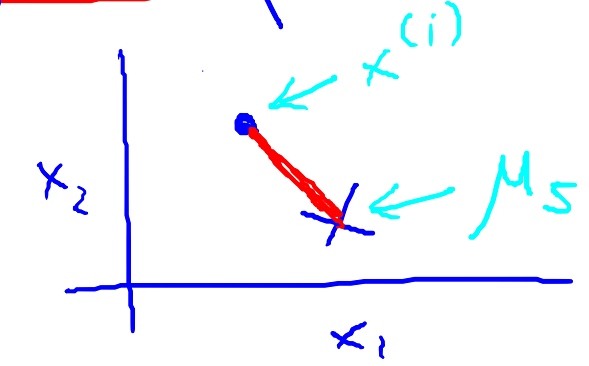
$x^{(i)} \rightarrow \underline{5}$ $\underline{c^{(i)} = 5}$ $\underline{\mu_{c^{(i)}}} = \mu_5$

Optimization objective:

$$J(\underline{c^{(1)},\ldots,c^{(m)},\mu_1,\ldots,\mu_K}) = \frac{1}{m}\sum_{i=1}^{m} \boxed{\|x^{(i)} - \mu_{c^{(i)}}\|^2}$$

$$\min_{\substack{c^{(1)},\ldots,c^{(m)},\\ \mu_1,\ldots,\mu_K}} J(c^{(1)},\ldots,c^{(m)},\mu_1,\ldots,\mu_K)$$

Distortion



K-means algorithm

Randomly initialize K cluster centroids $\mu_1, \mu_2, \ldots, \mu_K \in \mathbb{R}^n$

Cluster assignment step

Minimize $J(\ldots)$ wrt $\boxed{c^{(1)}, c^{(2)}, \ldots, c^{(m)}}$ ←

(holding $\mu_1, \ldots, \mu_K$ fixed)

Repeat {

for i = 1 to m

$c^{(i)}$:= index (from 1 to K) of cluster centroid closest to $x^{(i)}$

move centroid

for k = 1 to K

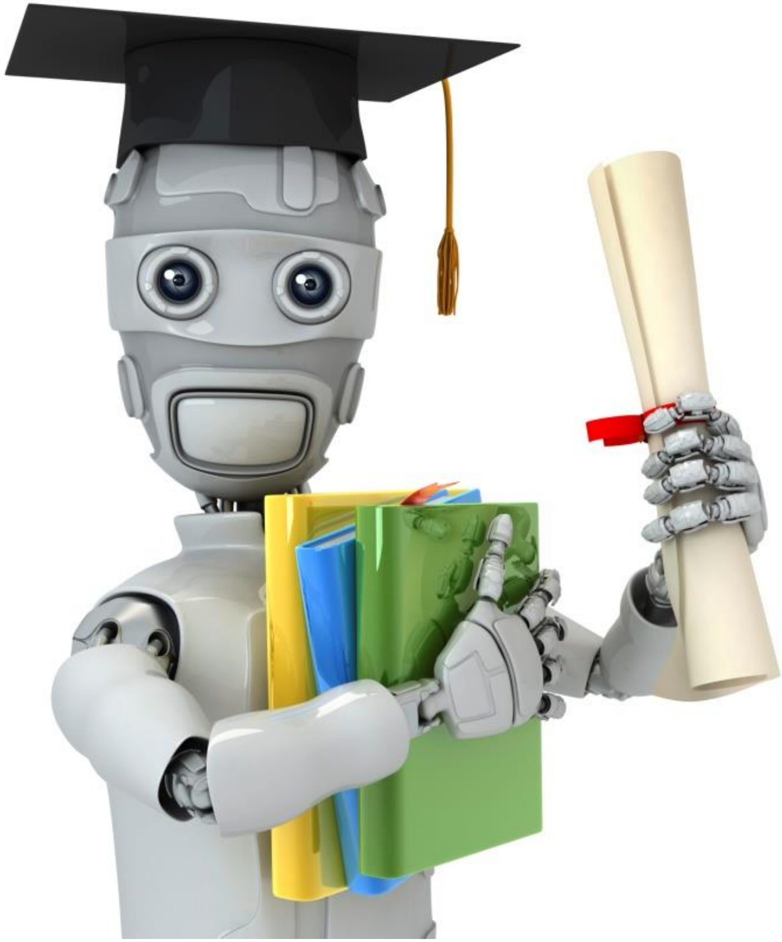
μ_k := average (mean) of points assigned to cluster k

}

minimize $J(\ldots)$ wrt $\boxed{\mu_1, \ldots, \mu_K}$

Clustering

Random initialization



Machine Learning

K-means algorithm

Randomly initialize K cluster centroids $\mu_1, \mu_2, \ldots, \mu_K \in \mathbb{R}^n$

Repeat {

for i = 1 to m

$c^{(i)}$:= index (from 1 to K) of cluster centroid closest to $x^{(i)}$

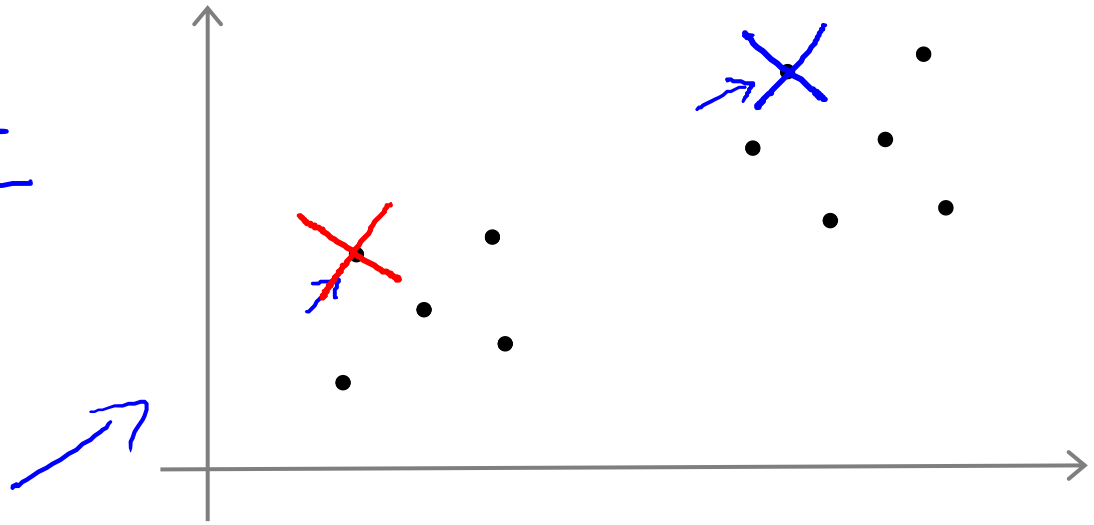
for k = 1 to K

μ_k := average (mean) of points assigned to cluster k

}

Random initialization

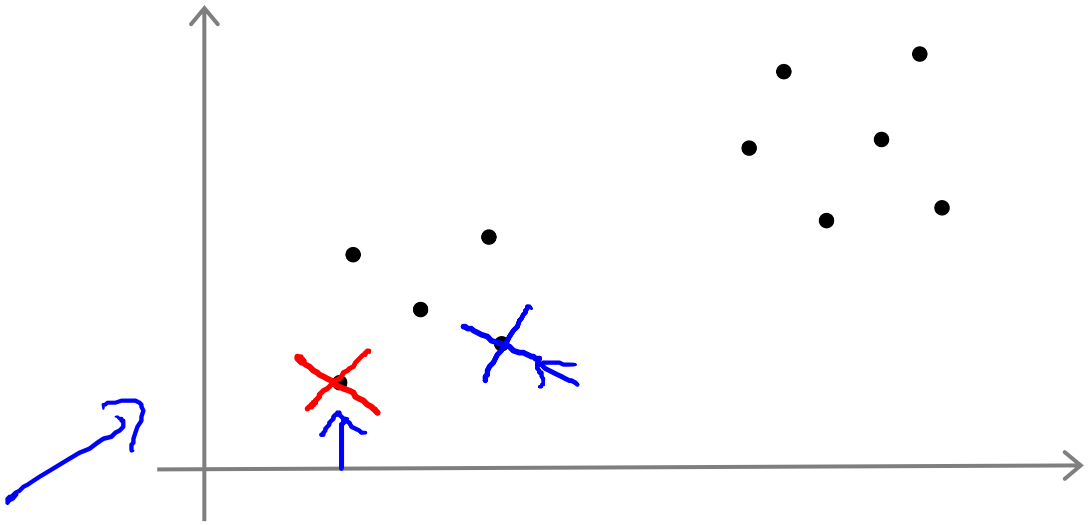
$K=2$



Should have $K < m$

Randomly pick K training examples.

Set $\mu_1, \ldots, \mu_K$ equal to these K examples.

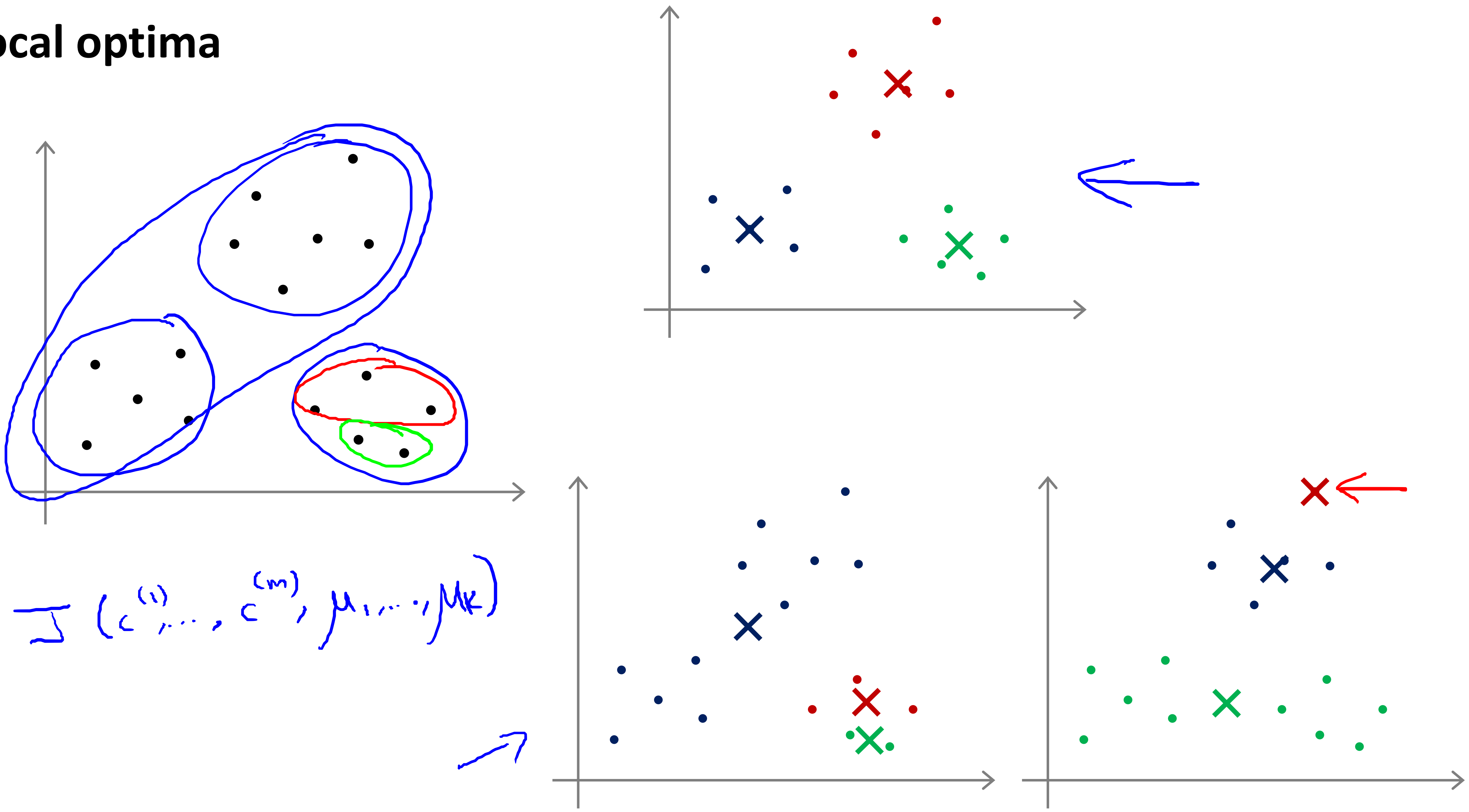


$\mu_1 = x^{(i)}$

$\mu_2 = x^{(j)}$

$\vdots$

Local optima



$$J(c^{(1)}, \ldots, c^{(m)}, \mu_1, \ldots, \mu_K)$$

Random initialization

For i = 1 to 100 {

Randomly initialize K-means.

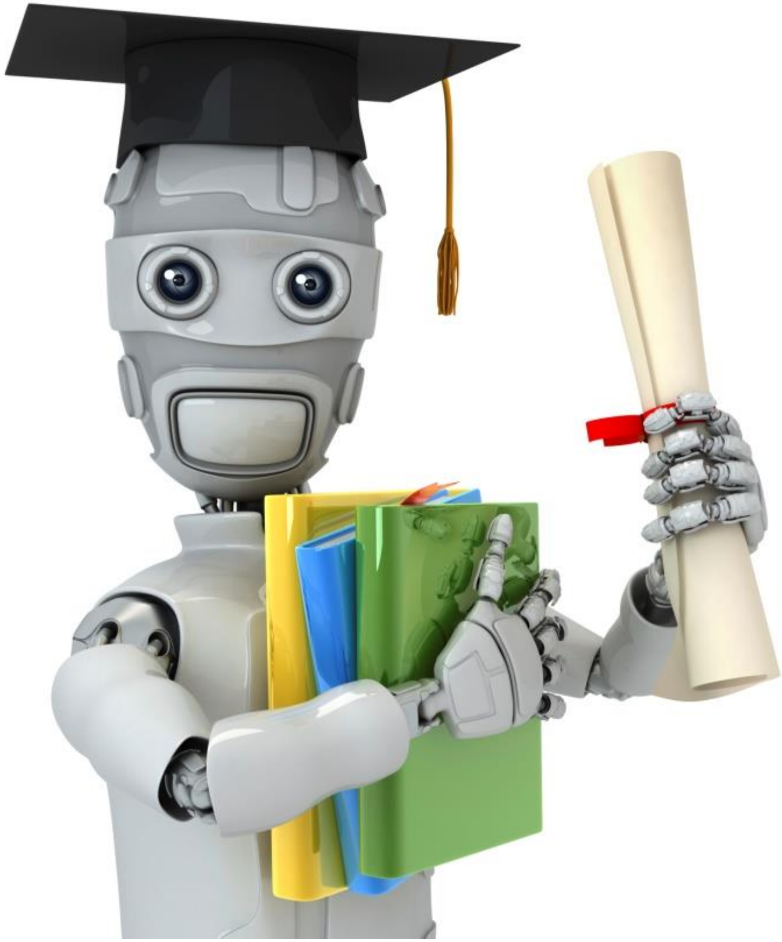
Run K-means. Get $c^{(1)}, \ldots, c^{(m)}, \mu_1, \ldots, \mu_K$

Compute cost function (distortion)

$$J(c^{(1)}, \ldots, c^{(m)}, \mu_1, \ldots, \mu_K)$$

}

Pick clustering that gave lowest cost $J(c^{(1)}, \ldots, c^{(m)}, \mu_1, \ldots, \mu_K)$

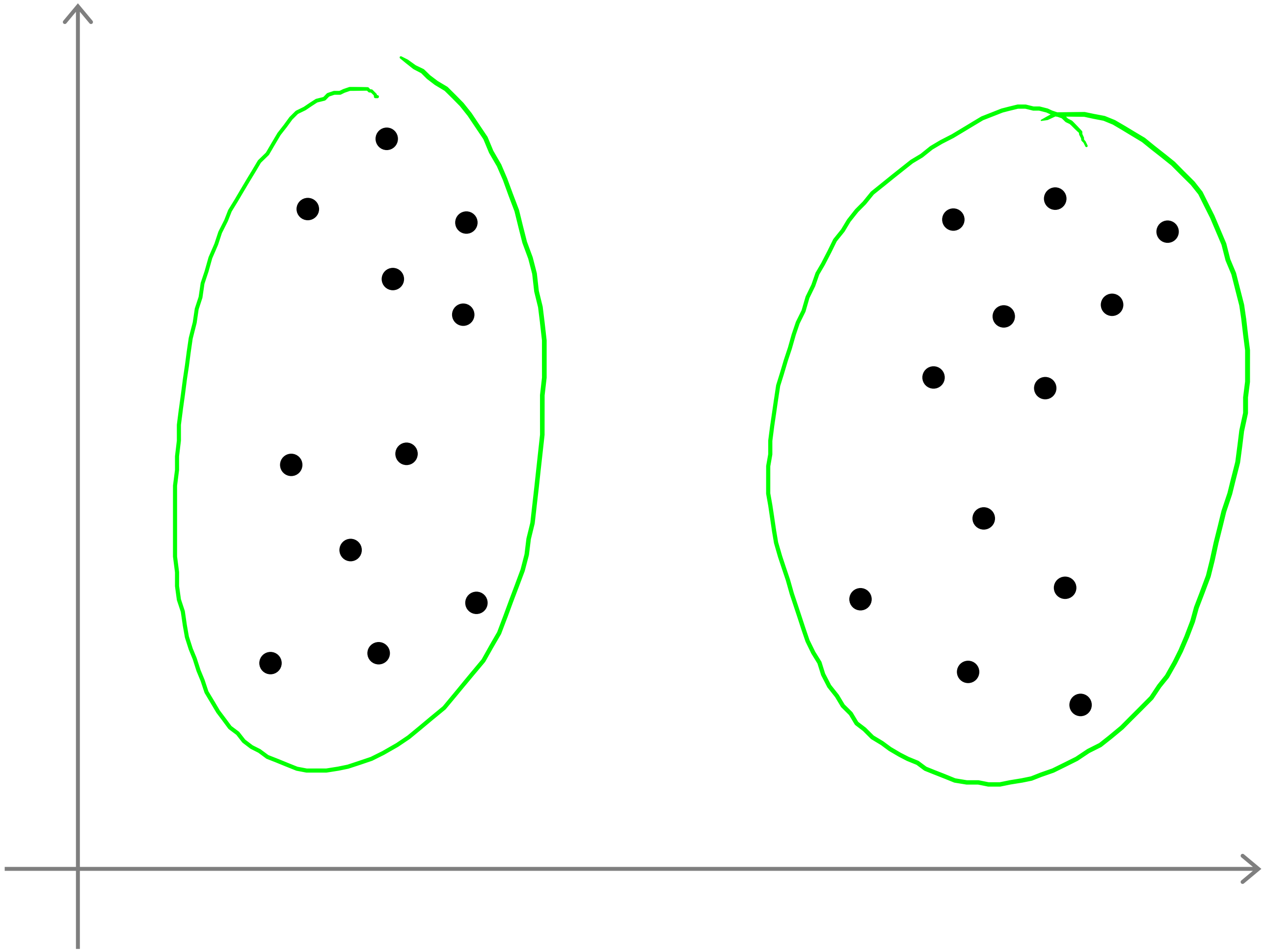


Machine Learning

Clustering

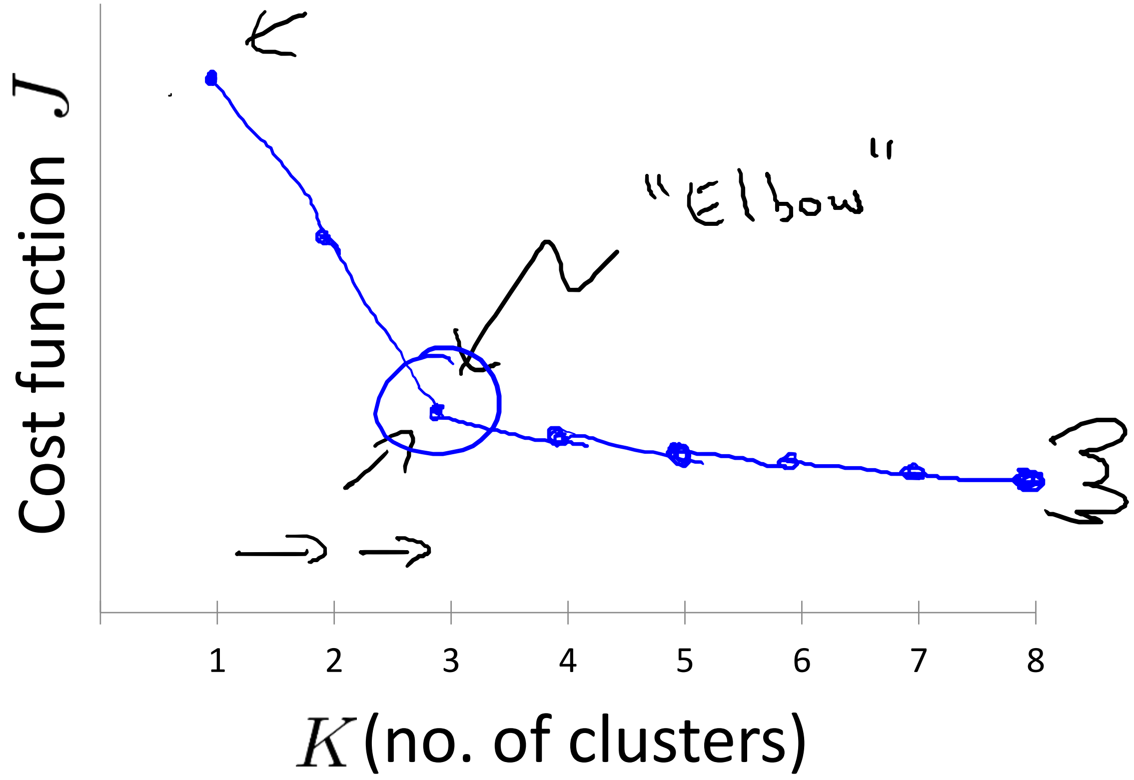
Choosing the number of clusters

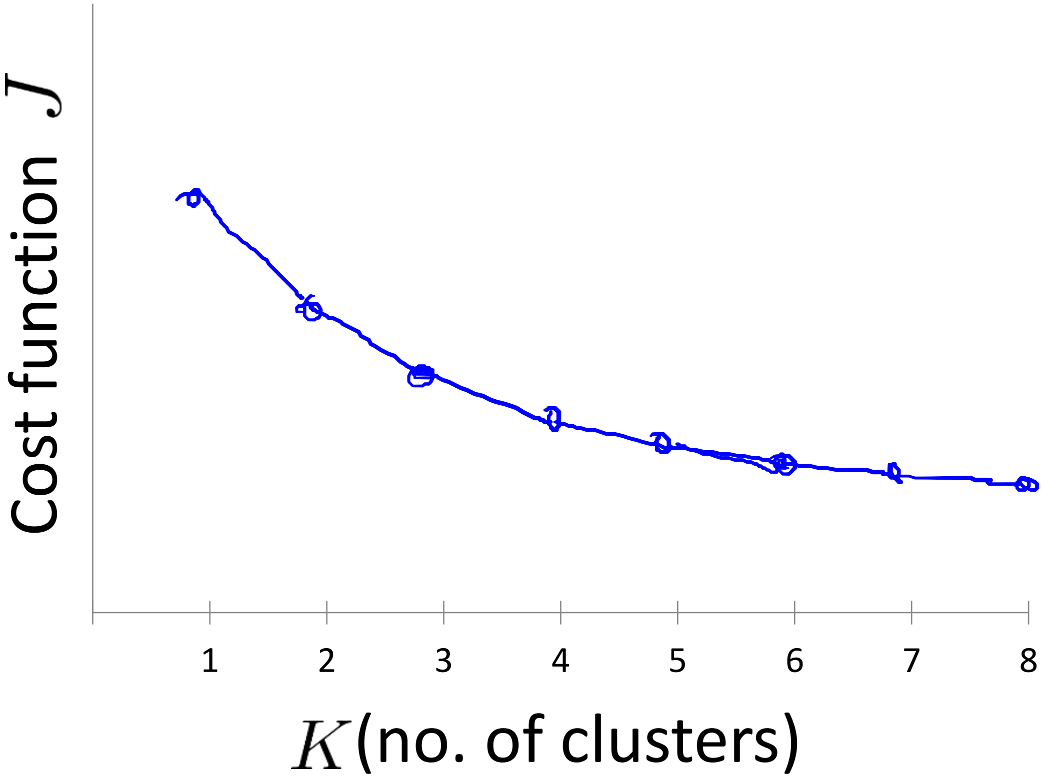
What is the right value of K?



Choosing the value of K

Elbow method:





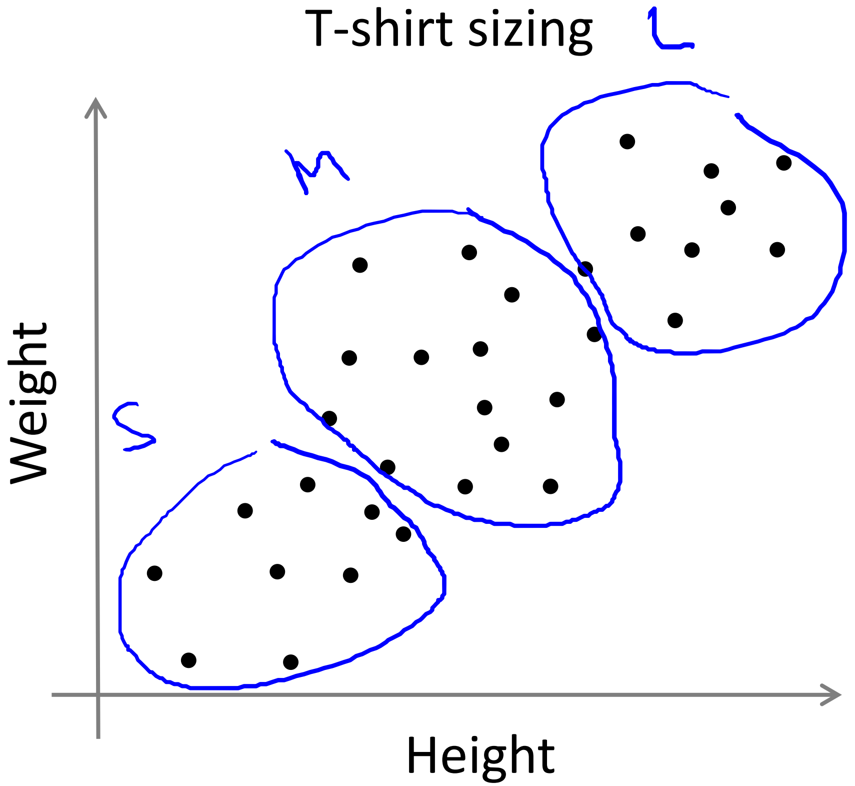
Choosing the value of K

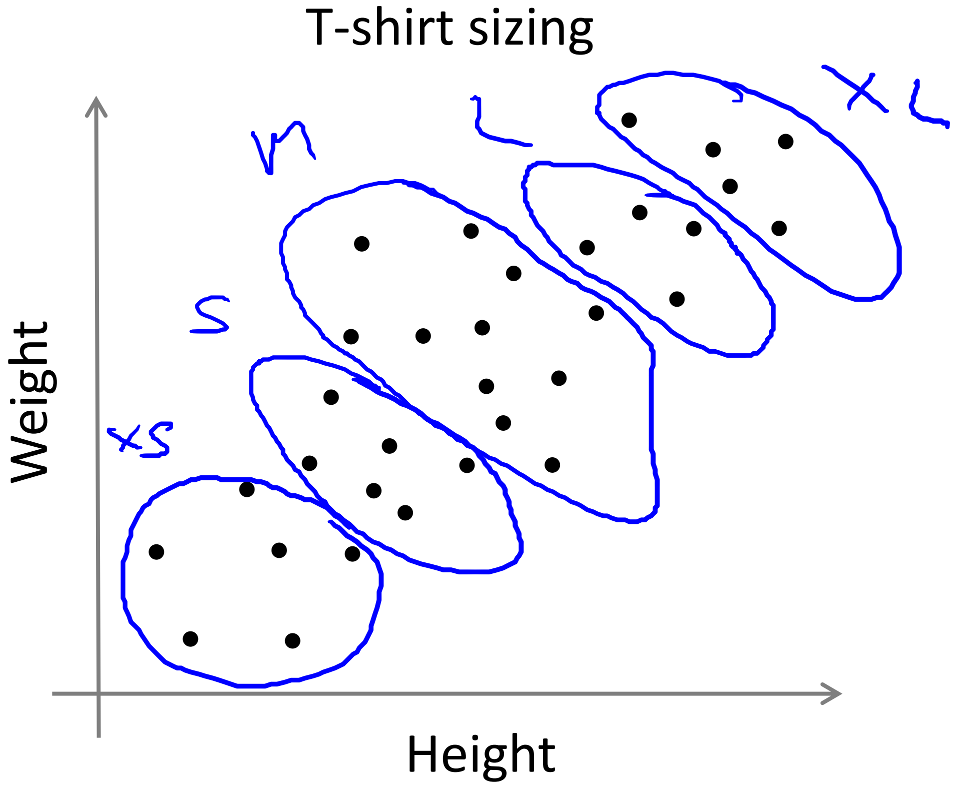
Sometimes, you're running K-means to get clusters to use for some later/downstream purpose. Evaluate K-means based on a metric for how well it performs for that later purpose.

K=3 S, M, L

K=5 XS, S, M, L, XL

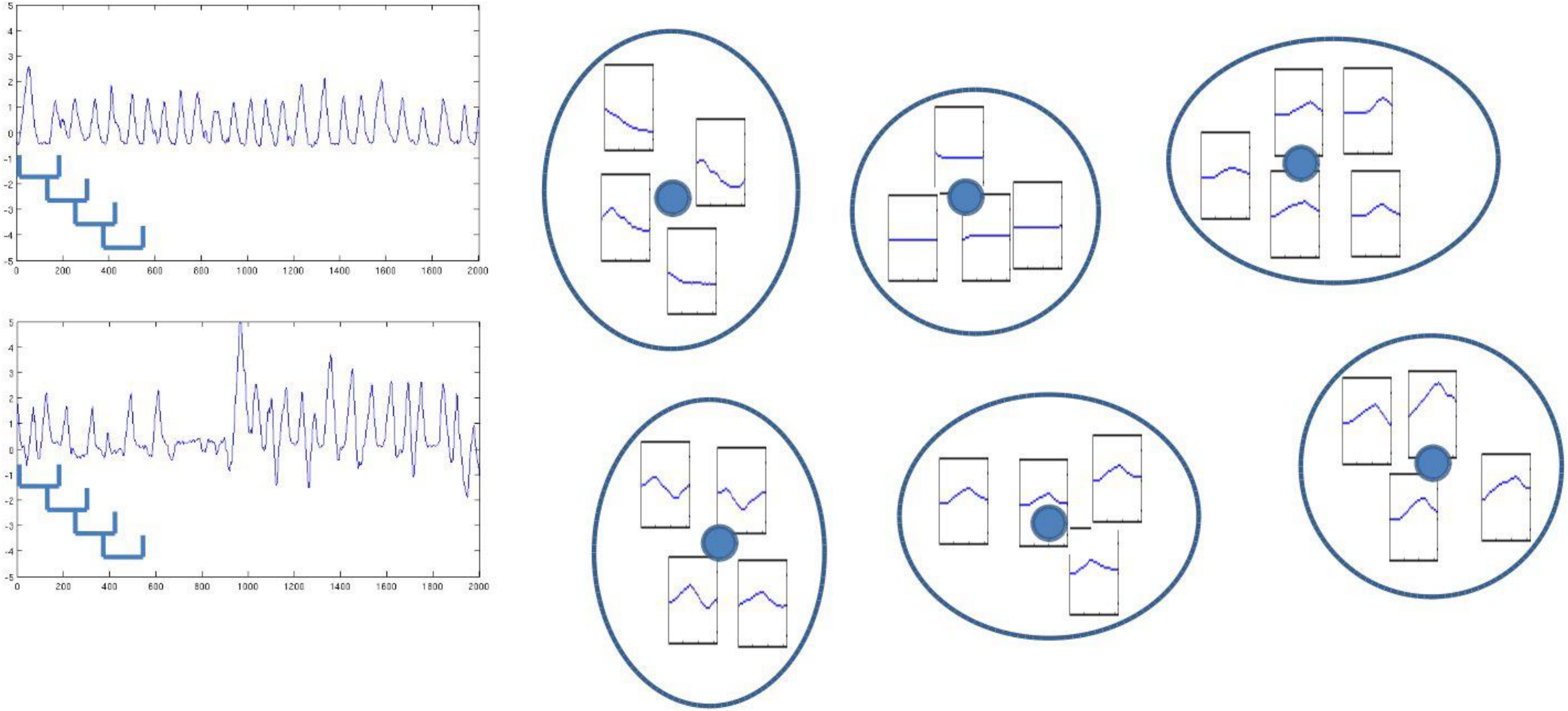
E.g.





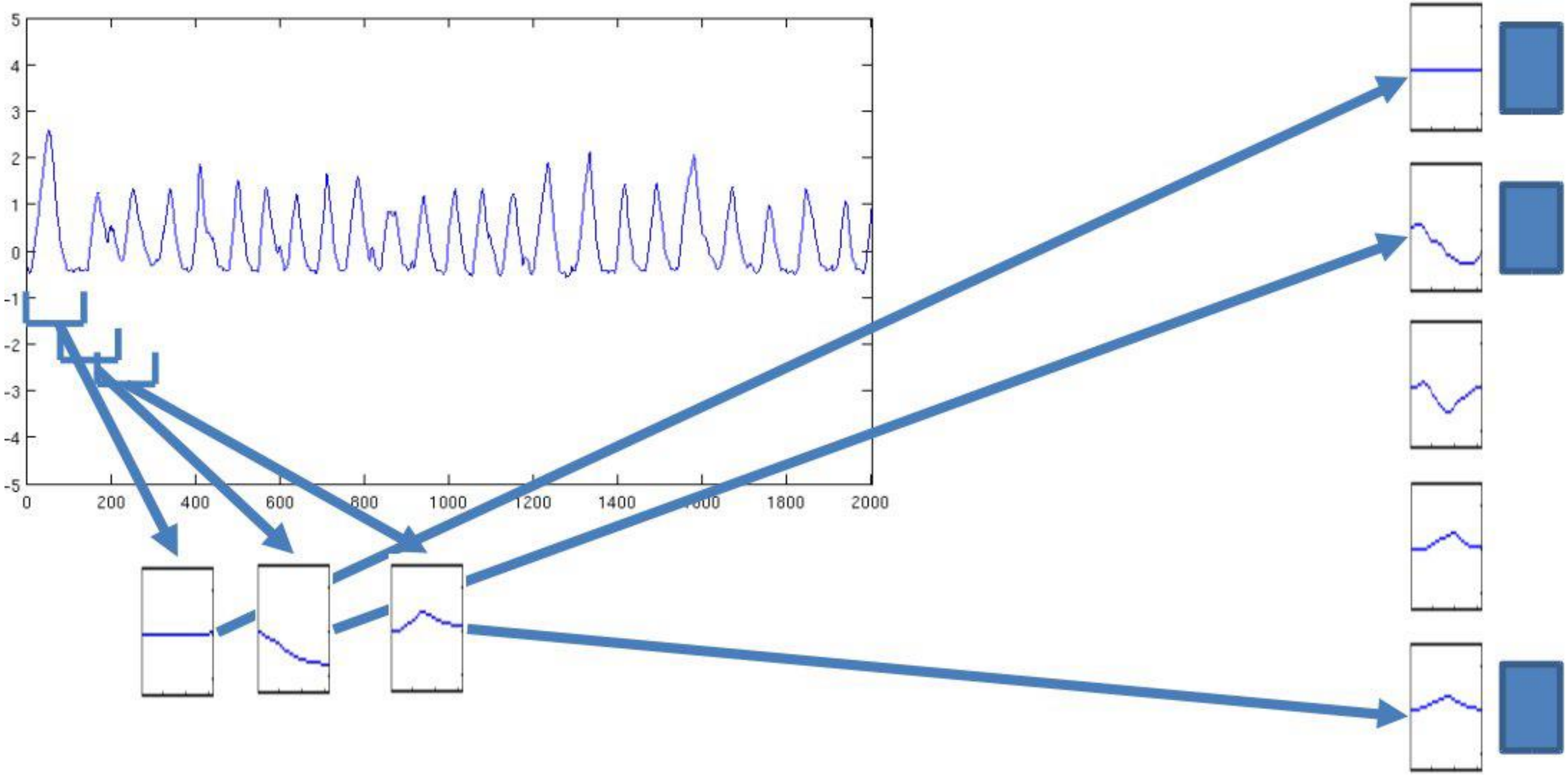
Codebook - Construction

- Clustering of Time Series subsequences



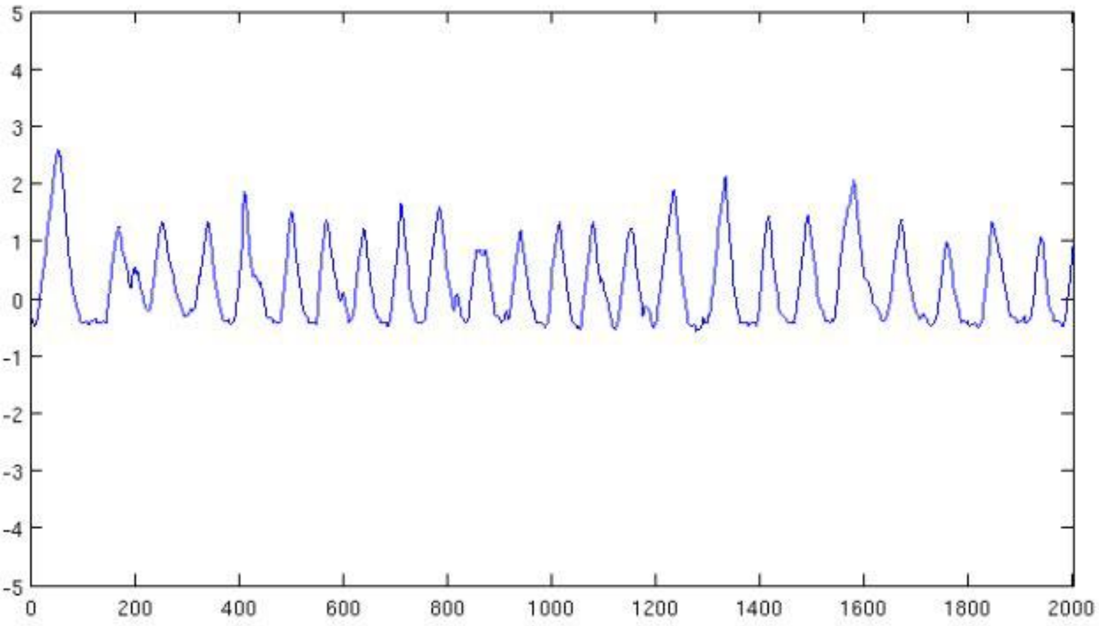
Codeword-Assignment

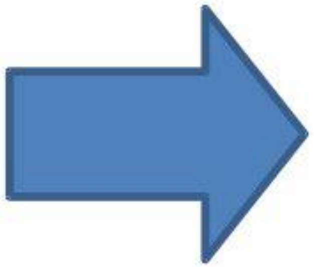
- Assignment of Codewords to the Constructed Clusters

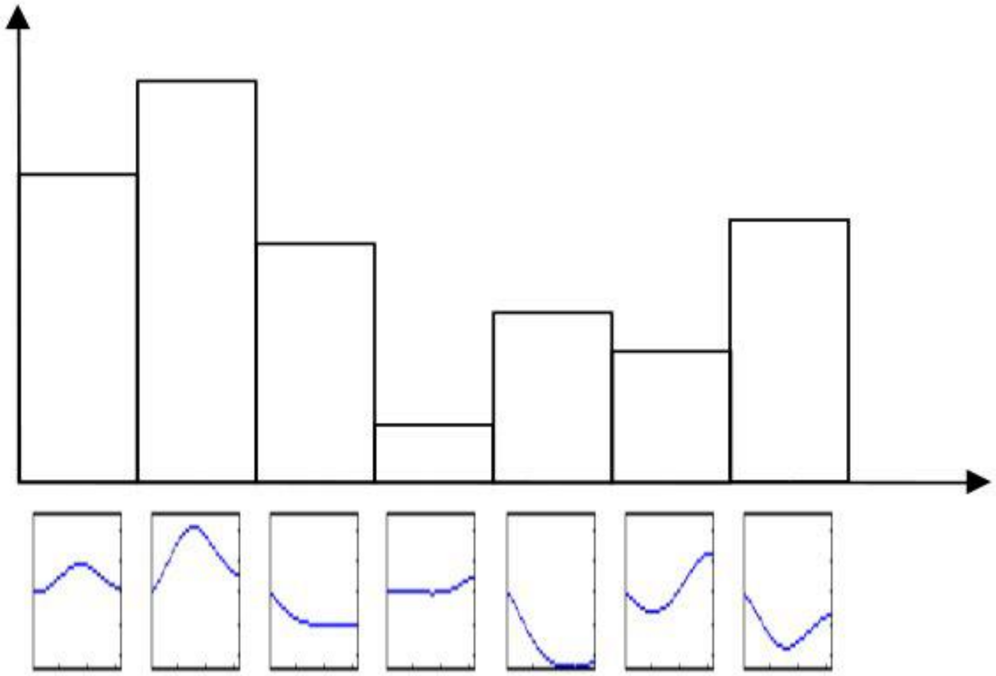


Histogram Generation

- Time Series Sequence Representation by a Histogram

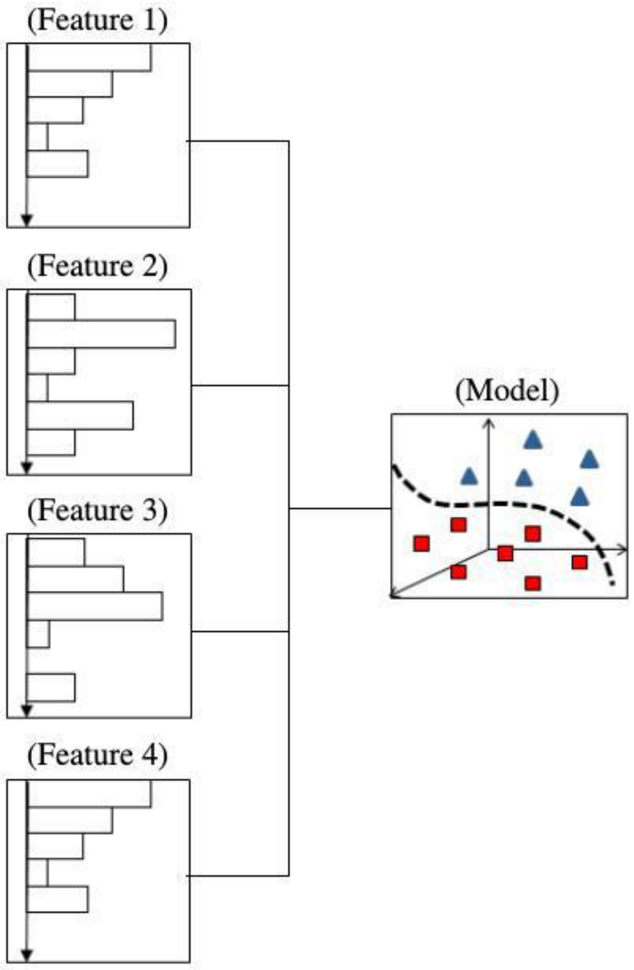




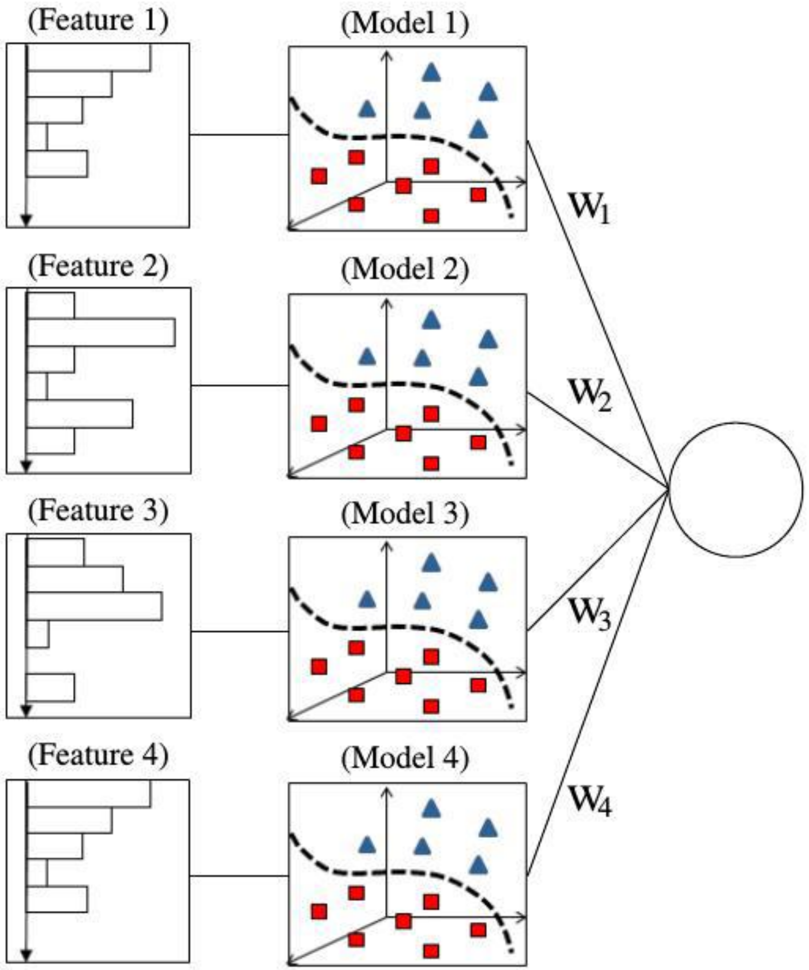


Fusion and Classification

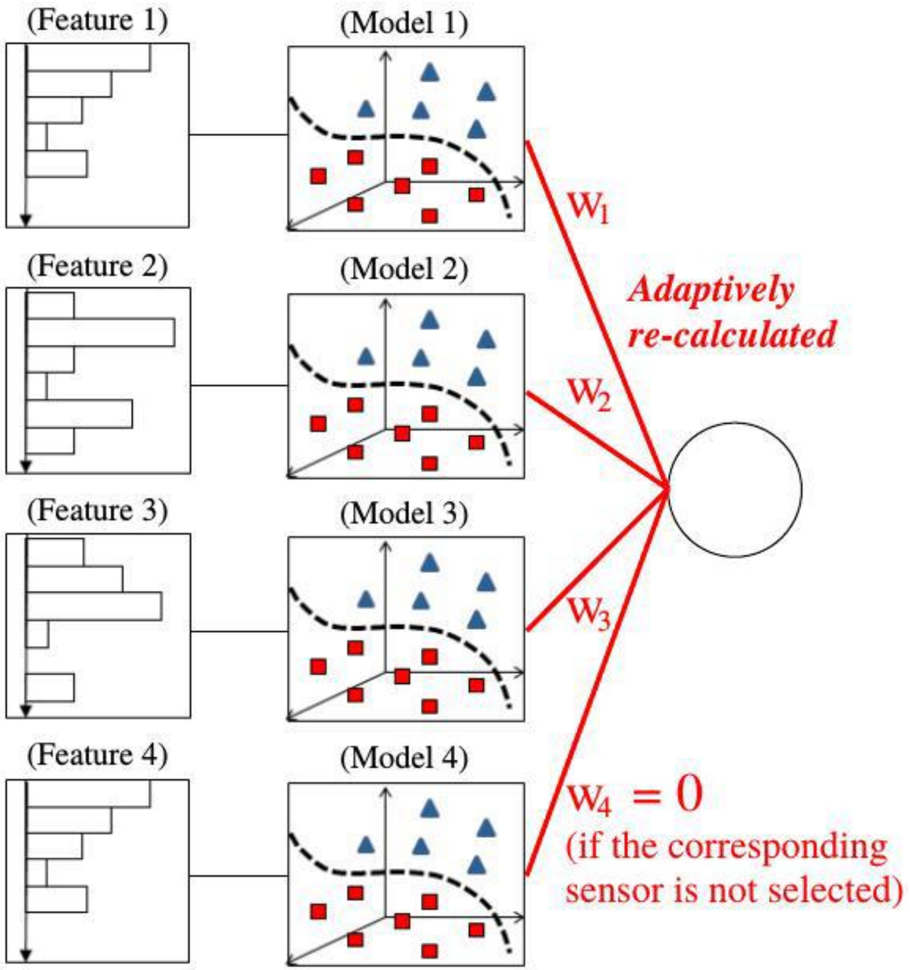
- Early Fusion



Late Fusion



Dynamic Late Fusion



Final Statements

- Feature learning delivers an optimum representation of the data, however, the semantic meaning of single feature dimensions gets often lost.

- Although feature learning algorithms seem to take over, manual feature engineering will remain important in the future, especially in the medical domain.

- Deep Neural Networks are a very powerful technique to automatically learn discriminative data representation in a supervised scenario.

Reading Homework

Article

Comparison of Feature Learning Methods for Human Activity Recognition Using Wearable Sensors