

Physics 1

Class Notes

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Lynn Public Schools
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<http://www.mrbigler.com/Physics-1>

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This is a set of class notes for Physics 1. This hardcopy is provided so that you can fully participate in class discussions without having to worry about writing everything down.

For Physics 1, we are not using a textbook, though you may refer to *Physics: Principles and Problems*, by Paul W. Zitzewitz, Robert F. Neff and Mark Davids (Glencoe/McGraw-Hill, 1995). In some cases, the notes and the textbook differ in method or presentation, but the physics is the same. There may be errors and/or omissions in the textbook. There are certainly errors and omissions in these notes, despite my best efforts to make them clear, correct, and complete.

As we discuss topics in class, you will almost certainly want to add your own notes to these. If you have purchased this copy, you are encouraged to write directly in it, just as you would write in your own notebook. However, if this copy was issued to you by the school and you intend to return it at the end of the year, you will need to write your supplemental notes on separate paper. If you do this, be sure to write down page numbers in your notes, to make crossreferencing easier.

You should bring these notes to class every day, because lectures and discussions will follow these notes, which will be projected onto the SMART board.

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Cornell Notes

Unit: Introduction

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding:

- how to take advantage of the Cornell note-taking system

Language Objectives:

- Understand the term Cornell Notes and be able to describe how Cornell Notes are different from ordinary note-taking.

Notes:

The Cornell note-taking system was developed about fifty years ago at Cornell University. I think it's a great way to get more out of your notes. I think it's an especially useful system for adding your comments to someone else's notes (such as mine).

The main features of the Cornell Notes system are:

1. The main section of the page is for what actually gets covered in class.
2. The left section (officially 2½ inches, though I have shrunk it to 2 inches for these notes) is for "cues"—questions or comments of yours that will help you find, remember, or effectively use these notes.
3. The bottom section (2 inches) is officially for you to add a 1–2 sentence summary of the page in your own words. This is a good idea. However, because the rest of the page is my notes, not yours, you may also want to use that space for anything else you want to remember that wasn't in the pre-printed notes.

Use this space for summary and/or additional notes:

Reading & Taking Notes from a Textbook

Unit: Introduction

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Skills:

- pre-reading and reading a section of a textbook and taking notes

Language Objectives:

- understand and be able to describe the reading and note-taking strategies presented in this section

Notes:

If you read a textbook the way you would read a novel, you probably won't remember much of what you read. Before you can understand anything, your brain needs enough context to know how to file the information. This is what Albert Einstein was talking about when he said, "It is the theory which decides what we are able to observe."

When you read a section of a textbook, you need to create some context in your brain, and then add a few observations to solidify the context before reading in detail.

René Descartes described this process in 1644 in the preface to his *Principles of Philosophy*:

"I should also have added a word of advice regarding the manner of reading this work, which is, that I should wish the reader at first go over the whole of it, as he would a romance, without greatly straining his attention, or tarrying at the difficulties he may perhaps meet with, and that afterwards, if they seem to him to merit a more careful examination, and he feels a desire to know their causes, he may read it a second time, in order to observe the connection of my reasonings; but that he must not then give it up in despair, although he may not everywhere sufficiently discover the connection of the proof, or understand all the reasonings—it being only necessary to mark with a pen the places where the difficulties occur, and continue reading without interruption to the end; then, if he does not grudge to take up the book a third time, I am confident that he will find in a fresh perusal the solution of most of the difficulties he will have marked before; and that, if any remain, their solution will in the end be found in another reading."

Use this space for summary and/or additional notes:

The following 4-step system takes about the same amount of time you're used to spending on reading and taking notes, but it will probably make a tremendous difference in how much you understand and remember.

1. Copy the titles/headings of each section. Leave about $\frac{1}{4}$ page of space after each one. (Don't do anything else yet.) This should take about 2–3 minutes.
2. Do not write anything yet! Look through the section for pictures, graphs, and tables. Take a minute to look at these—the author must have thought they were important. Also read over (but don't try to answer) the homework questions/problems at the end of the section. (For the visuals, the author must think these things illustrate something that is important enough to dedicate a significant amount of page real estate to it. For the homework problems, these illustrate what the author thinks you should be able to do once you know the content.) This process should take about 10–15 minutes.
3. Actually read the text, one section at a time. For each section, jot down keywords and sentence fragments that remind you of the key ideas. You are not allowed to write more than the $\frac{1}{4}$ page allotted. (You don't need to write out the details—those are in the book, which you already have!) This process is time consuming, but shorter than what you're probably used to doing for your other teachers.
4. Read the summary at the end of the chapter or section—this is what the author thinks you should know now that you've finished the reading. If there's anything you don't recognize, go back and look it up. This process should take about 5–10 minutes.

You shouldn't need to use more than about one sheet of paper (both sides) per 10 pages of reading!

Use this space for summary and/or additional notes.

Taking Notes on Math Problems

Unit: Introduction

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Skills:

- taking notes on a mathematical problem

Language Objectives:

- understand and be able to describe the strategies presented in this section

Notes:

If you were to copy down a math problem and look at it a few days or weeks later, chances are you'll recognize the problem, but you won't remember how you solved it.

Solving a math problem is a process. For notes to be useful, they need to describe the process as it happens, not just the final result.

If you want to take good notes on how to solve a problem, you need your notes to show what you did at each step.

For example, consider the following physics problem:

A 25 kg cart is accelerated from rest to a velocity of $3.5 \frac{\text{m}}{\text{s}}$ over an interval of 1.5 s. Find the net force applied to the cart.

The process of solving this problem involves applying two equations:

$$v = v_0 + at \quad \text{and} \quad F = ma.$$

Use this space for summary and/or adional notes:

A good way to document the process is to use a two-column format, in which you show the steps of the solution in the left column, and you write an explanation of what you did and why for each step in the right column.

For this problem, a two-column description might look like the following:

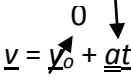
Step	Description/Explanation
$m = 25 \text{ kg}$ $v_o = 0$ $v = 3.5 \frac{\text{m}}{\text{s}}$ $t = 1.5 \text{ s}$ $F = \text{quantity desired}$	Define variables.
$\underline{F} = \underline{ma}$	Choose a formula that contains F . See if we have the other variables.  → No. Therefore, we need to find a before we can solve the problem.
$\underline{v} = \underline{v}_o + \underline{at}$	Choose a formula that contains a . See if we have the other variables. → Yes.
$3.5 = 0 + a(1.5)$ $3.5 = 1.5a$ $\frac{3.5}{1.5} = \frac{1.5}{1.5}a$ $2.33 = a$	Solve the 2 nd equation for a by substituting the values of v , v_o and t into it.
$F = ma$ $F = (25)(2.33)$ $F = 58.33$	Substitute a from the 2 nd equation and m from the problem statement into the 1 st equation, and solve for F .
$F = \boxed{58.33 \text{ N}}$	Include the units and box the final answer.

Use this space for summary and/or additional notes.

Sometimes you might need to solve problems symbolically rather than numerically. A symbolic version of the same problem might look like this:

A cart with mass m is accelerated from rest to velocity v over time interval t . Derive an expression for the net force applied to the cart in terms of m , v and t .

For this problem, a two-column description might look like the following:

Step	Description/Explanation
$F_{net} = ma$	Choose a formula that contains F . See if the other variables are all given in the problem statement. → No. Acceleration a is not given, so we need to use additional equations to eliminate it from our answer.
$v = v_0 + at$ 	Choose a formula that contains a . See if the other variables are given in the problem statement. → Yes. (The cart starts from rest, which means $v_0 = 0$.)
$a = \frac{v}{t}$	Rearrange the 2 nd equation to solve for a .
$F_{net} = ma$ $F_{net} = \frac{mv}{t}$	Substitute the expression for a into the 1 st equation and solve for F .

Of course, AP problems are more complex than this, but the same methodology applies.

Use this space for summary and/or additional notes.

Introduction: Laboratory & Measurement

Unit: Laboratory & Measurement

Topics covered in this chapter:

The Scientific Method	15
Designing & Performing Experiments.....	19
Graphical Solutions	25
Accuracy & Precision.....	28
Uncertainty & Error Analysis.....	30
Significant Figures	48
Keeping a Laboratory Notebook.....	55
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Formal Laboratory Reports	63

The purpose of this chapter is to teach skills necessary for designing and carrying out laboratory experiments, recording data, and writing summaries of the experiment in different formats.

- *Designing & Performing Experiments* discusses strategies for coming up with your own experiments and carrying them out.
- *Accuracy & Precision, Uncertainty & Error Analysis*, and *Recording and Analyzing Data* discuss techniques for working with the measurements taken during laboratory experiments.
- *Keeping a Laboratory Notebook* and *Formal Laboratory Reports* discuss ways in which you might communicate (write up) your laboratory experiments.

Calculating uncertainty (instead of relying on significant figures) is a new and challenging skill that will be used in lab write-ups throughout the year.

Standards addressed in this chapter:

Next Generation Science Standards (NGSS):

No NGSS standards are addressed in this chapter.

Use this space for summary and/or adional notes:

Massachusetts Curriculum Frameworks (2006):

No MA curriculum frameworks are specifically addressed in this chapter.

Skills learned & applied in this chapter:

- Designing laboratory experiments
- Error analysis (calculation & propagation of uncertainty)
- Formats for writing up lab experiments

Use this space for summary and/or additional notes.

The Scientific Method

Unit: Laboratory & Measurement

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding Goals:

- Understand the scientific method

Language Objectives:

- Understand and correctly use terms relating to the scientific method, such as “peer review”

Notes:

The scientific method is a fancy name for “figure out what happens by trying it.”

In the middle ages, “scientists” were called “philosophers.” These were church scholars who decided what was “correct” by a combination of observing the world around them and then arguing and debating with each other about the mechanisms and causes.

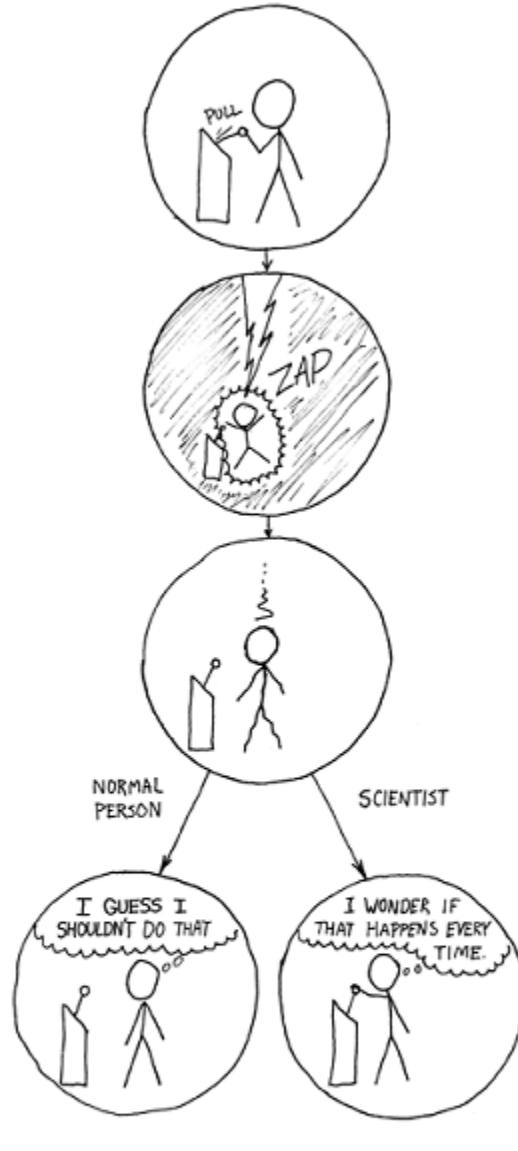
During the Renaissance, scientists like Galileo Galilei and Leonardo da Vinci started using experiments instead of argument to decide what really happens in the world.

Use this space for summary and/or additional notes.

Steps:

Note that although the steps are numbered, there is no “order” to them. I.e., depending on the circumstances, a scientist could end up jumping from just about any step to just about any other step in the process

1. Observe something interesting.
2. Figure out and perform experiments that will have different outcomes depending on the parameter(s) being tested. You can make a **claim** that describes what you expect will happen (sometimes called a hypothesis), or you can just perform the experiment to see what happens.
3. Repeat the experiment, varying your conditions as many ways as you can.
4. If you are testing a claim, **assume that your claim is wrong**. Try every experiment and make every observation you can think of that might refute your claim.
5. If your claim holds, try to come up with a model that explains and predicts the behavior you observed. This model is called a **theory**. If your claim holds but you cannot come up with a model, try to completely and accurately describe the conditions under which your claim successfully predicts the outcomes. This description is called a **law**.



xkcd.com
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Use this space for summary and/or additi

6. Share your theory (or law), your experimental procedures, and your data with other scientists. Some of these scientists may:
 - a. Look at your experiments to see whether or not the experiments really can distinguish between the different outcomes.
 - b. Look at your data to see whether or not the data really do support your theory.
 - c. Try your experiments or other related experiments themselves and see if the new results are consistent with your theory.
 - d. Add to, modify, limit, refute (disprove), or suggest an alternative to your theory.
 - e. Completely ignore your theory and your experiments. (The vast majority of scientists will do this; scientists are busy people who are in no way obligated to spend their time testing other people's theories.)

This process is called “peer review.” If a significant number of scientists have reviewed your claims and agree with them, and no one has refuted your theory, your theory may gain acceptance within the scientific community.

Note that the word “theory” in science has a different meaning from the word “theory” in everyday language. In science, a theory is a model that:

- has never failed to explain a collection of related observations
- has never failed to successfully predict the outcomes of related experiments

For example, the theory of evolution *has never failed* to explain the process of changes in organisms caused by factors that affect the survivability of the species.

If a repeatable experiment contradicts a theory, and the experiment passes the peer review process, the theory is deemed to be wrong. If the theory is wrong, it must either be modified to explain the new results, or discarded completely.

Use this space for summary and/or additional notes.

Note that, despite what your ninth-grade science teacher may have taught you, it is possible (and often useful) to have a hypothesis or claim before performing an experiment, but an experiment is just as valid and just as useful whether or not an hypothesis was involved.

Theories vs. Natural Laws

The terms “theory” and “law” developed organically, so any definition of either term must acknowledge that common usage, both within and outside of the scientific community, will not always be consistent with the definitions.

A theory is a model that attempts to explain why or how something happens. A law simply describes what happens without attempting to provide an explanation. Theories and laws can both be used to predict the outcomes of related experiments.

For example, the Law of Gravity states that objects attract other objects based on their masses and distances from each other. It is a law and not a theory because the Law of Gravity does not explain *why* masses attract each other.

Atomic Theory states that matter is made of atoms, and that those atoms are themselves made up of smaller particles. The interactions between the particles that make up the atoms (particularly the electrons) are used to explain certain properties of the substances. This is a theory because it gives an explanation for *why* the substances have the properties that they do.

Note that a theory cannot become a law any more than a definition can become a measurement or a postulate can become a theorem.

Use this space for summary and/or additional notes.

Designing & Performing Experiments

Unit: Laboratory & Measurement

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Skills:

- determine what you are testing
- figure out how to get from what you can measure to what you want to determine

Language Objectives:

- Understand and correctly use the terms “dependent variable” and “independent variable.”
- Understand and be able to describe the strategies presented in this section.

Notes:

Most high school physics experiments are relatively simple to understand, set up and execute—much more so than in chemistry or biology. This makes physics well-suited for teaching you how to design experiments.

The education “buzzword” for this is ***inquiry-based experiments***, which means you (or your lab group) will need to figure out what to do to perform an experiment that answers a question about some aspect of physics. In this course, you will usually be given only an objective or goal and a general idea of how to go about achieving it. You and your lab group (with help) will decide the specifics of what to do, what to measure (and how to measure it), and how to make sure you are getting good results. This is a form of ***guided inquiry***.

Use this space for summary and/or additional notes.

Framing Your Experiment

Experiments are motivated by something you want to find out, observe, or calculate.

Independent, Dependent, and Control Variables

In an experiment, there is usually something you are doing, and something you are measuring or observing.

independent variable: the conditions you are setting up. These are the parameters that you specify when you set up the experiment. Because you chose these values, they are *independent* of what happens in the experiment. For example, if you are dropping a ball from different heights to find out how long it takes to hit the ground, you are choosing the heights, so height is the *independent* variable.

dependent variable: the things that happen in the experiment. These are the numbers you measure, which are *dependent* on what happens in the experiment. For example, if you are dropping a ball from different heights to find out how long it takes to hit the ground, you are measuring the time, which depends on the height. This means time is the *dependent* variable.

control variable: other things that could vary but are being kept constant. These are usually parameters that could be independent variables in other experiments, but are kept constant so they do not affect the relationship between the independent variable being tested and the dependent variable being measured. For example, if you are dropping a ball from different heights to find out how long it takes to hit the ground, you want to make sure the wind is the same speed and direction for each trial, so wind does not affect the outcome of the experiment. This means wind speed and direction are *control* variables.

If someone asks what your independent, dependent, and control variables are, the question simply means, “What did you vary (independent variable), what did you measure (dependent variable), and what did you keep constant (control variables)?”

Use this space for summary and/or additional notes.

Qualitative Experiments

If the goal of your experiment is to find out **whether or not** something happens at all, you need to set up a situation in which the phenomenon you want to observe can either happen or not, and then observe whether or not it does. The only hard part is making sure the conditions of your experiment don't bias whether the phenomenon happens or not.

If you want to find out **under what conditions** something happens, what you're really testing is whether or not it happens under different sets of conditions that you can test. In this case, you need to test three situations:

1. A situation in which you are sure the thing will happen, to make sure you can observe it. This is your **positive control**.
2. A situation in which you are sure the thing cannot happen, to make sure your experiment can produce a situation in which it doesn't happen and you can observe its absence. This is your **negative control**.
3. A condition or situation that you want to test to see whether or not the thing happens. The condition is your independent variable, and whether or not the thing happens is your dependent variable.

Quantitative Experiments

If the goal of your experiment is to quantify (find a numerical relationship for) the extent to which something happens (the dependent variable), you need to figure out a set of conditions under which you can measure the thing that happens. Once you know that, you need to figure out how much you can change the parameter you want to test (the independent variable) and still be able to measure the result. This gives you the highest and lowest values of your independent variable. Then perform the experiment using a range of values for the independent value that cover the range from the lowest to the highest (or *vice-versa*).

For quantitative experiments, a good rule of thumb is the **8 & 10 rule**: you should have at least 8 data points, and the range from the highest to the lowest values tested should span at least a factor of 10.

Use this space for summary and/or additional notes.

Determining What to Measure

Determining what to measure usually means determining what you need to know and figuring out how to get from there to *quantities that you can measure*.

For a quantitative experiment, if you have a mathematical formula that includes the quantity you want to measure, you simply need to find the values of the other quantities in the equation. For example, suppose you want to calculate the amount of force needed to bring a moving object to a stop. We can calculate force from the equation for Newton's Second Law:

$$F = ma$$

In order to use this equation to calculate force, we need to know:

- **mass**: we can measure this directly, using a balance
- **acceleration**: we could measure this with an accelerometer, but we do not have one in the lab. This means we will need to calculate acceleration from another formula.

Notice that the necessary procedure has expanded. Instead of just measuring force and acceleration, we now need to:

1. Measure the mass.
2. Perform an experiment in which we apply the force and collect enough information to determine the acceleration.
3. Calculate the force on the object from the mass and the acceleration.

In order to determine the acceleration, we need another equation. One candidate is:

$$v = v_o + at$$

This means in order to calculate acceleration, we need to know:

- **final velocity (v)**: the force is being applied until the object is at rest (stopped), so the final velocity $v = 0$.
- **initial velocity (v_o)**: we need to either measure or calculate this.
- **time (t)**: we can measure this directly with a stopwatch.

Use this space for summary and/or additional notes.

Now we need to expand our experiment further, in order to calculate v_o . We can calculate the initial velocity from the equation:

$$\bar{v} = \frac{d}{t} = \frac{v_o + v}{2}$$

We have already figured out how to measure t , and we set up the experiment so that $v = 0$ at the end. This means that to calculate v_o , the only other quantity we need to measure is:

- **displacement (d)**: the change in the object's position. We can measure this with a meter stick or tape measure.

Now every quantity in the experiment is something we can measure or something we can calculate, so we're all set. Our experiment is therefore:

1. Measure the **mass** of the object.
2. Apply the force and determine the **acceleration** of the object:
 - a. Set up an experiment in which the object starts out moving and then it **accelerates** (negatively) until it stops.
 - b. Measure the **time** it takes the object to come to a complete stop, and the **displacement** (change in position) from when it first started slowing down to where it stopped.
 - c. **Calculate the initial velocity**.
 - d. Using the **initial velocity**, **final velocity** and **time**, **calculate the acceleration**.
3. Using the **mass** and **acceleration**, **calculate the force** on the object.
4. Take multiple data points based on the **8 & 10 rule**—take at least **8 data points**, varying the mass over at least a **factor of 10**.

Use this space for summary and/or additional notes.

Generalized Approach

The generalized approach to experimental design is therefore:

1. Find an equation that contains the quantity you want to find.
2. Work your way from that equation through related equations until every quantity in every equation is either something you can calculate or something you can measure.
3. Determine how to measure the quantities that you need (dependent variables). Decide what your starting conditions need to be (independent variables), and figure out what you need to keep constant (control variables).
4. Set up your experiment and take your data.
5. Calculate the results. Whenever possible, apply the **8 & 10 rule** and calculate your answer graphically (explained in the next section).

Use this space for summary and/or additional notes.

Graphical Solutions

Unit: Laboratory & Measurement

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Skills:

- Use a graph to calculate the relationship between two variables.

Language Objectives:

- Understand and use terms relating to graphs.

Notes:

Most experiments in a high-school physics class involve finding a mathematical relationship between two quantities. While it is possible to simply measure each quantity once and calculate, an approach that measures the relationship across a range of values will provide a better result.

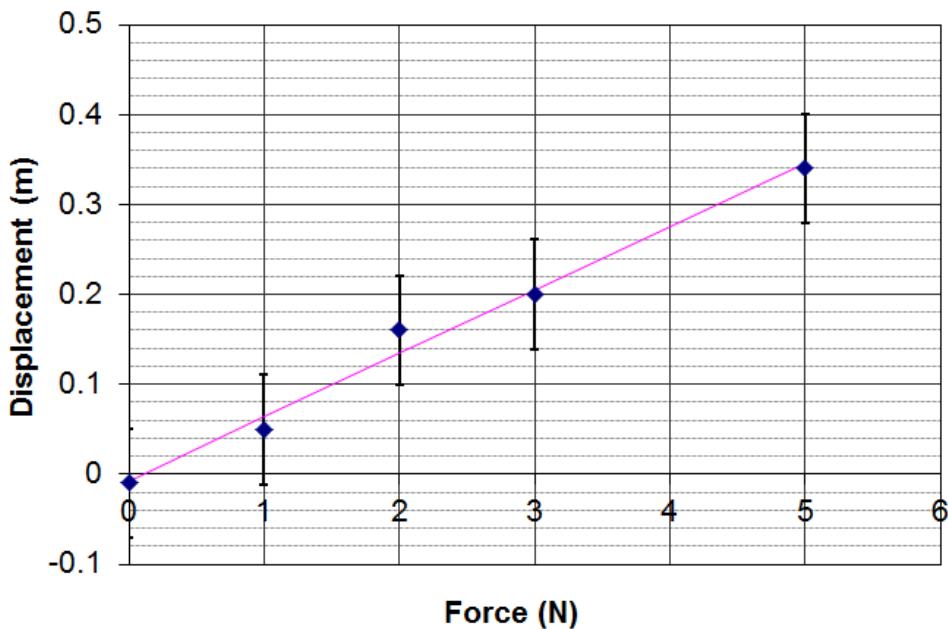
As mentioned above, a good rule of thumb for quantitative experiments is the **8 & 10 rule:** you should have at least 8 data points, and the range from the highest to the lowest values tested should span at least a factor of 10.

Once you have your data points, arrange the equation into $y = mx + b$ form, such that the slope (or $1/\text{slope}$) is the quantity of interest. Then accurately plot your data and draw a best-fit line. The slope of this line will be the quantity of interest (or its reciprocal).

For example, suppose you wanted to calculate the spring constant of a spring by measuring the displacement caused by an applied force. The equation is $F_s = kx$, which means a graph of F_s vs. x will have a slope of k .

Use this space for summary and/or additional notes.

1. Plot your data points, expressing your uncertainties as error bars.
2. Draw a best-fit line that passes through each error bar and minimizes the total accumulated distance away from each data point. (You can use linear regression, provided that the regression line passes through each error bar.) If the line cannot pass through all of the error bars, you need to determine what the problem was with the outlier(s). You may disregard a data point in your calculation *only* if you can explain the problem in the way the data point was taken that caused it to be an outlier.

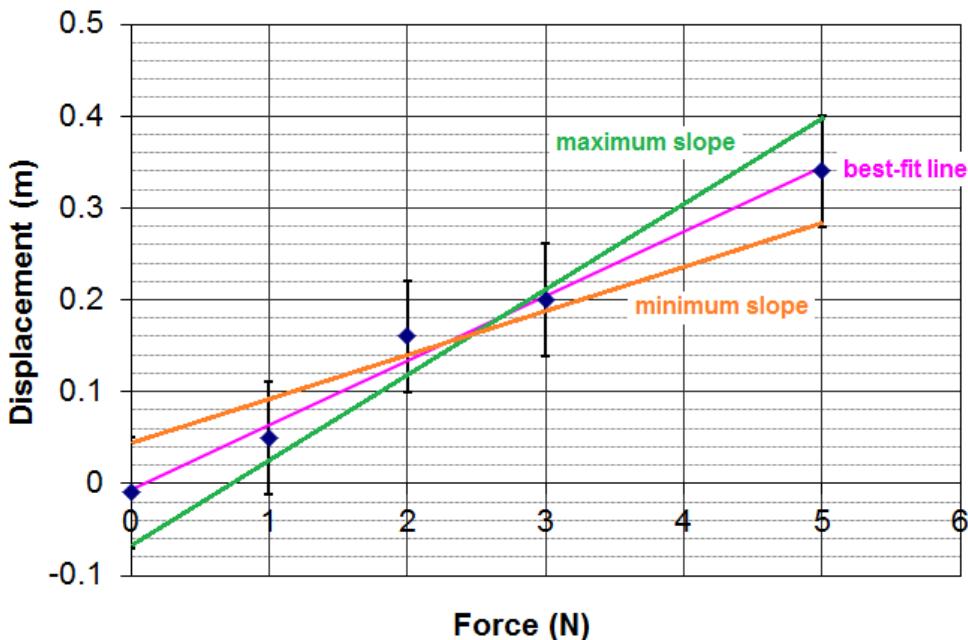


The above graph was plotted with force (the independent variable) on the x-axis and displacement (the dependent variable) on the y-axis. The slope of the best-fit line is 0.072.

Note, however, that the spring constant is defined as $\frac{\text{force}}{\text{displacement}}$ (measured in $\frac{\text{N}}{\text{m}}$), which means the spring constant is the reciprocal of the slope of the above graph. $\frac{1}{0.072} = 14.0 \frac{\text{N}}{\text{m}}$.

Use this space for summary and/or additional notes.

3. Draw the lines of maximum and minimum slope that can pass through all of the error bars. These are the maximum and minimum values of your calculated value. These give you the range, from which you can calculate the uncertainty (\pm , which will be covered later).



4. Your answer is the slope of your best-fit line, and the uncertainty (\pm) is the difference between the slope of the best-fit line and the maximum or minimum slope, whichever gives the larger uncertainty.

The line with the largest slope that goes through the error bars (shown in green) has a slope of $0.093 \frac{N}{m}$, which represents a spring constant of $10.8 \frac{N}{m}$, and the line with the shallowest slope that goes through the error bars (shown in orange) has a slope of $0.046 \frac{N}{m}$, which represents a spring constant of $21.6 \frac{N}{m}$. The larger difference is $7.6 \frac{N}{m}$, so this is our uncertainty. We would therefore express the spring constant as $(14.0 \pm 7.6) \frac{N}{m}$.

Use this space for summary and/or additional notes.

Accuracy & Precision

Unit: Laboratory & Measurement

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

AP Physics 1 Learning Objectives: N/A

Knowledge/Understanding:

- Understand what accuracy and precision mean and the difference between the two.

Language Objectives:

- Understand and be able to differentiate between accuracy and precision.

Notes:

Science relies on making and interpreting measurements, and the accuracy and precision of these measurements affect what you can conclude from them.

Random vs. Systematic Errors

Random errors are natural uncertainties in measurements because of the limits of precision of the equipment used. Random errors are assumed to be distributed around the actual value, without bias in either direction. Systematic errors occur from specific problems in your equipment or your procedure. Systematic errors are often biased in one direction more than another, and can be difficult to identify.

Accuracy vs. Precision

The words “accuracy” and “precision” have specific meanings in science.

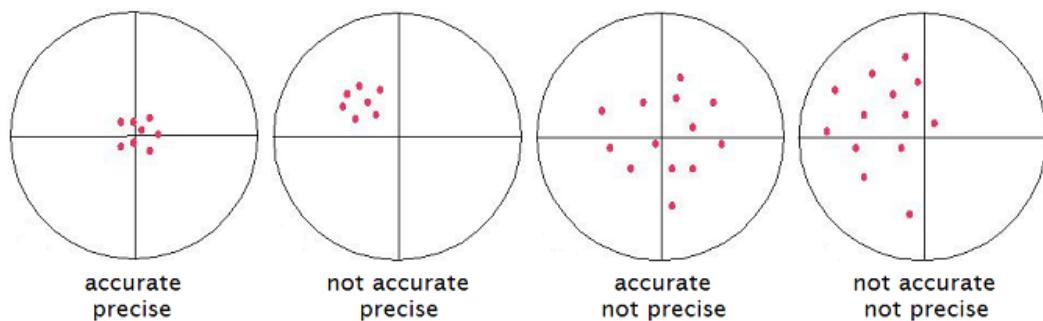
accuracy: for a single measurement, how close the measurement is to the “correct” or accepted value. For a group of measurements, how close the average is to the accepted value. Poor accuracy is often an indication of systematic error.

precision: for a single measurement, how finely the measurement was made. (How many decimal places it was measured to.) For a group of measurements, how close the measurements are to each other. Poor precision tends to cause larger random error.

Use this space for summary and/or adional notes:

Examples:

Suppose the following drawings represent arrows shot at a target.



The first set is both accurate (the average is close to the center) and precise (the data points are all close to each other.)

The second set is precise (close to each other), but not accurate (the average is not close to the correct value). This is an example of *systemic* error—some problem with the experiment caused all of the measurements to be off in the same direction.

The third set is accurate (the average is close to the correct value), but not precise (the data points are not close to each other). This is an example of *random* error—the measurements are not biased in any particular direction, but there is a lot of scatter.

The fourth set is neither accurate nor precise, which means that there are significant random and systematic errors present.

For another example, suppose two classes estimate Mr. Bigler's age. The first class's estimates are 73, 72, 77, and 74 years old. These measurements are fairly precise (close together), but not accurate. (Mr. Bigler is actually about 51 years old.) The second class's estimates are 0, 1, 97 and 98. This set of data is accurate (because the average is 49, which is close to correct), but the set is not precise because the individual values are not close to each other.

Use this space for summary and/or additional notes.

Uncertainty & Error Analysis

Unit: Laboratory & Measurement

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

AP Physics 1 Learning Objectives: N/A

Knowledge/Understanding:

- Understand what uncertainty of measurement means.

Skills:

- Read and interpret uncertainty values.
- Estimate measurement errors.
- Propagate estimate of error/uncertainty through calculations.

Language Objectives:

- Understand and correctly use the terms “uncertainty,” “standard uncertainty,” and “relative error.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Notes:

In science, unlike mathematics, there is no such thing as an exact answer. Ultimately, every quantity is limited by the precision and accuracy of the measurements that it came from. If you can only measure a quantity to within 10%, that means any number that is derived from that measurement can't be any better than $\pm 10\%$.

Error analysis is the practice of determining and communicating the causes and extents of possible errors or uncertainty in your results. Error analysis involves understanding and following the uncertainty in your data from the initial measurement to the final report.

Use this space for summary and/or additional notes.

Uncertainty

The uncertainty of a measurement describes how different the correct value is likely to be from the measured value. For example, if a length was measured to be 22.336 cm, and the uncertainty was 0.030 cm (meaning that the measurement is only known to within ± 0.030 cm), we could represent this measurement in either of two ways:

$$(22.336 \pm 0.030) \text{ cm} \quad 22.336(30) \text{ cm}$$

The first of these states the variation (\pm) explicitly in cm (the actual unit). The second shows the variation in the last digits shown.

Note that scientists use the word “error” to mean uncertainty. This is different from the vernacular usage of “error” to mean a mistake. When you discuss “sources of error” in an experiment, you are explaining the uncertainty in your results, and where it may have come from. **“Sources of error” does not mean “Potential mistakes you might have made.”** This is similar to the difference between scientific use of the word “theory” (“a model that explains every observation that has ever been made”) as opposed to vernacular use (“my own guess about what I think might be going on”).

Absolute Uncertainty

Absolute uncertainty (or absolute error) refers to the uncertainty in the actual measurement, such as (3.64 ± 0.22) cm.

Use this space for summary and/or additional notes.

Relative Error

Relative error (or relative uncertainty) shows the error or uncertainty as a fraction of the measurement. (Percent error, which you used in chemistry last year, is related to relative error.)

$$\text{The formula for relative error is } R.E. = \frac{\text{uncertainty}}{\text{measured value}}$$

For example, a measurement of (3.64 ± 0.22) cm would have a relative error of 0.22 out of 3.64. Mathematically, we express this as:

$$R.E. = \frac{0.22}{3.64} = 0.0604$$

To turn relative error into percent error, multiply by 100. A relative error of 0.0604 is the same as 6.04% error.

Note that relative error does not have any units. This is because the numerator and denominator have the same units, so the units cancel.

To turn relative error back to absolute error, multiply the relative error by the measurement.

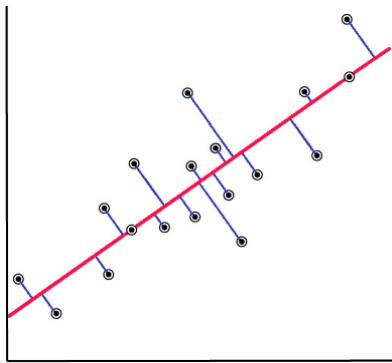
For example, if we multiply the measurement from the problem above (3.64 cm) by the relative error that we just calculated (0.0604), we should get the original absolute error (0.22 cm). Indeed:

$$3.64 \text{ cm} \times 0.0604 = 0.22 \text{ cm}$$

Use this space for summary and/or additional notes.

Standard Deviation

The standard deviation of a set of measurements is the average of how far each one is from the expected value. For example, suppose you had the following data set, represented by a set of data points and a best-fit line:



If you draw a perpendicular line from each data point to the best-fit line, each perpendicular line would represent how far that data point deviates from the best-fit line. The average of all these deviations is the standard deviation, represented by the variable σ .

There are different types of standard deviation. The formula for the sample standard deviation (which you would use if your data are assumed to be a representative sample taken from a larger data set) is:

$$\sigma_s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}$$

Calculators that include statistics functions and spreadsheet programs have the standard deviation function built in, so you will probably never need to calculate a standard deviation by hand.

Uncertainty, in the kinds of experiments you would do in physics, is usually the standard deviation of the mean (*i.e.*, the standard deviation assuming your data are *all* of the data). To calculate the standard deviation of the mean, divide the sample standard deviation by the square root of the number of data points (\sqrt{n}).

$$u = \sigma_m = \frac{\sigma_s}{\sqrt{n}}$$

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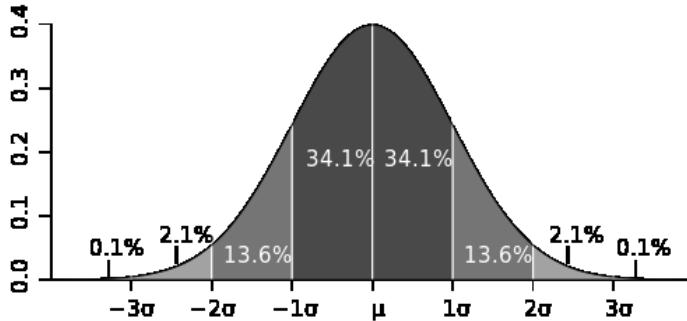
If we had a large number of data points, we could plot a graph of how many data points were how close to the expected value. We would expect that most of the time, the result would be close to the expected value. As you consider results farther and farther from the expected value, you would expect to see these happen less and less often.

This is called a Gaussian or “normalized” distribution. Approximately 68.2% of the measurements would be expected to fall within one standard deviation ($\pm\sigma$) of the expected value. *E.g.*, if we report a measurement of (22.336 ± 0.030) cm, that means:

- The mean value is $\bar{x} = 22.336$ cm
- The standard deviation is $\sigma = 0.030$. This means it is 68% likely that any single measurement we make would fall between 22.306 cm (which equals $22.336 - 0.030$) and 22.366 cm (which equals $22.336 + 0.030$).

Two standard deviations ($\pm 2\sigma$) should include 95.4% of the measurements, and three standard deviations ($\pm 3\sigma$) should include 99.6% of the measurements.

A graph of the Gaussian distribution looks like a bell, and is often called a “bell curve”.



Note that with a very large number of data points, the likelihood that the *average* of the measurements is close to the expected value is much higher than the likelihood that any individual measurement is close.

Use this space for summary and/or additional notes.

Calculating the Uncertainty of a Set of Measurements

When you have measurements of multiple separately-generated data points, the uncertainty is calculated using statistics, so that some specific percentage of the measurements will fall within the average, plus or minus the uncertainty.

Ten or More Independent Measurements

If you have a large enough set of independent measurements (at least 10), then the uncertainty is the standard deviation of the mean. (Independent measurements means you set up the situation that generated the data point on separate occasions. *E.g.*, if you were measuring the length of the dashes that separate lanes on a highway, independent measurements would mean measuring different lines that were likely generated by different line painting apparatus. Measuring the same line ten times would not be considered independent measurements.) That formula is:

$$u = \sigma_m = \frac{\sigma_s}{\sqrt{n}}$$

where:

u = standard uncertainty

σ_m = standard deviation of the mean

$$\sigma_s = \text{sample standard deviation} = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}$$

n = number of measurements

If the variable x represents the measured quantity and \bar{x} represents the arithmetic mean (average) value, you would express your result as:

$$\text{reported value} = \bar{x} \pm u = \bar{x} \pm \frac{\sigma_s}{\sqrt{n}}$$

While this would be the correct formula to use when possible, often we have too few data points (small values of n), which causes the calculated standard deviation to predict a much larger uncertainty than we probably actually have.

Use this space for summary and/or additional notes.

Approximation for Fewer than Ten Independent Measurements

If you have only a few independent measurements (fewer than 10), then you have too few data points to accurately calculate the population standard deviation. In this case, we can estimate of the standard uncertainty by using the formula:

$$u \approx \frac{r}{\sqrt{3}}$$

where:

u = uncertainty

r = range (the difference between the largest and smallest measurement)

If the variable x represents the measured quantity, you would express your result as:

$$\text{reported value} = \bar{x} \pm \frac{r}{\sqrt{3}}$$

Note that we are treating $\sqrt{3}$ as a constant. Whenever you have more than one but fewer than ten data points, find the range and divide it by $\sqrt{3}$ to get the estimated uncertainty.

Example:

Suppose you measured a mass on a balance and the reading drifted between 3.46 g and 3.58 g:

$$\bar{x} = \frac{3.46 + 3.58}{2} = 3.52$$

$$r = 3.58 - 3.46 = 0.12$$

$$u \approx \frac{r}{\sqrt{3}} \approx \frac{0.12}{1.732} \approx 0.07$$

You would record the balance reading as (3.52 ± 0.07) g.

Use this space for summary and/or additional notes.

Uncertainty of a Single Measurement

If you are measuring a quantity that is not changing (such as the mass or length of an object), you can measure it as many times as you like and you should get exactly the same value every time you measure it. This means you have only one data point.

When you have only one data point, you can often assume that the standard uncertainty is the limit of how precisely you can measure it (including any estimated digits). This will be your best educated guess, based on how closely you think you actually measured the quantity. This means you need to take measurements as carefully and precisely as possible, because *every careless measurement needlessly increases the uncertainty of the result*.

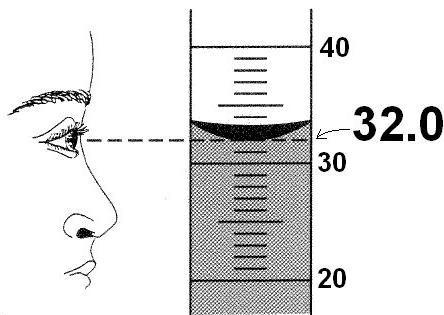
Digital Measurements

For digital equipment, if the reading is stable (not changing), look up the published precision of the instrument in its user's manual. (For example, many balances used in high schools have a readability of 0.01 g but are only precise to within ± 0.02 g.) If there is no published value (or the manual is not available), assume the uncertainty is ± 1 in the last digit.

Use this space for summary and/or additional notes.

Analog Measurements

When making analog measurements, always estimate to one extra digit beyond the finest markings on the equipment. For example, in the diagram below, the graduated cylinder is marked in 1 mL increments. When measuring volume in this graduated cylinder, you would estimate and write down the volume to the nearest 0.1 mL, as shown:



In the above experiment, you should record the volume as 32.0 ± 0.1 mL. It would be inadequate to write the volume as 32 mL. (Note that the zero at the end of the reading of 32.0 mL is not extra. It is necessary because *you measured the volume to the nearest 0.1 mL and not to the nearest 1 mL.*)

When estimating, you can generally assume that the estimated digit has an uncertainty of ± 1 . This means the uncertainty of the measurement is usually $\pm \frac{1}{10}$ of the finest markings on the equipment. Here are some examples:

Equipment	Markings	Estimate To	Uncertainty
triple-beam balance	0.01 g	0.001 g	± 0.001 g
ruler	1 mm	0.1 mm	± 0.1 mm
100 mL graduated cylinder	1 mL	0.1 mL	± 0.1 mL
thermometer	1°C	0.1°C	± 0.1 °C

Use this space for summary and/or additional notes.

Propagating Uncertainty in Calculations

When you perform calculations using numbers that have uncertainty, you need to propagate the uncertainty through the calculation.

Crank Three Times

The simplest way of doing this is the “crank three times” method. The “crank three times” method involves:

1. Perform the calculation using the actual numbers. This gives the result (the part before the \pm symbol).
2. Perform the calculation again, using the end of the range for each value that would result in the smallest result. (Note that with fractions, this means you need to subtract the uncertainty for values in the numerator and add the uncertainty for values in the denominator.) This gives the lower limit of the range.
3. Perform the calculation again using the end of the range for each value that would result in the largest result. This gives the upper limit of the range.
4. If you have fewer than ten data points, use the approximation that the uncertainty = $u \approx \frac{r}{\sqrt{3}}$, where r is the range.

The advantage to “crank three times” is that it’s easy to understand and you are therefore less likely to make a mistake. The disadvantage is that it can become unwieldy when you have multi-step calculations.

Use this space for summary and/or additional notes.

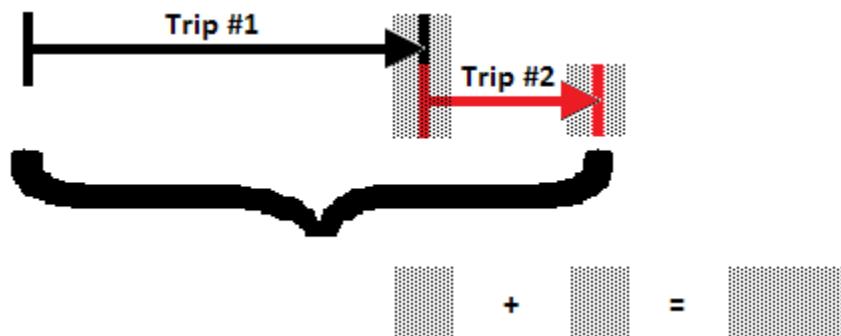
Error Propagation

This method, which we will use throughout the year, requires applying a set of rules based on the formulas you use for the calculations. This is probably the method your college professors will expect you to use in lab experiments.

Addition & Subtraction

If the calculation involves **addition** or **subtraction**, add the absolute errors.

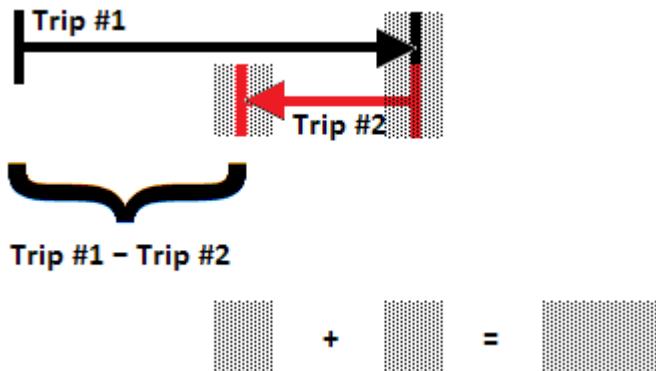
Imagine you walked for a distance and measured it. That measurement has some uncertainty. Then imagine that you started from where you stopped and walked a second distance and measured it. The second measurement also has uncertainty. The total distance is the distance for Trip #1 + Trip #2.



Because there is uncertainty in the distance of Trip #1 and also uncertainty in the distance of Trip #2, it is easy to see that the total uncertainty when the two trips are added together is the sum of the two uncertainties.

Use this space for summary and/or additional notes.

Now imagine that you walked for a distance and measured it, but then you turned around and walked back toward your starting point for a second distance and measured that. Again, both measurements have uncertainty.



Notice that, even though the distances are subtracted to get the answer, the uncertainties still accumulate. As before, the uncertainty in where Trip #1 ended becomes the uncertainty in where Trip #2 started. There is also uncertainty in where Trip #2 ended, so again, the total uncertainty is the *sum* of the two uncertainties.

For a numeric example, consider the problem:

$$(8.45 \pm 0.15) \text{ cm} - (5.43 \pm 0.12) \text{ cm}$$

Rewriting in column format:

$$\begin{array}{r} (8.45 \pm 0.15) \text{ cm} \\ - (5.43 \pm 0.12) \text{ cm} \\ \hline (3.02 \pm 0.27) \text{ cm} \end{array}$$

Notice that even though we had to subtract to find the answer, we had to add the uncertainties.

Use this space for summary and/or additional notes.

Multiplication and Division

If the calculation involves **multiplication or division**, add the relative errors to get the total relative error.

Then, as we saw before, we can multiply the total relative error by the result to get the absolute uncertainty.

For example, if we have the problem $(2.50 \pm 0.15) \text{ kg} \times (0.30 \pm 0.06) \frac{\text{m}}{\text{s}^2}$, we would do the following:

$$\text{The result is } 2.50 \text{ kg} \times 0.30 \frac{\text{m}}{\text{s}^2} = 0.75 \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$

For the error analysis:

$$\text{The relative error of } (2.50 \pm 0.15) \text{ kg is } \frac{0.15}{2.50} = 0.06$$

$$\text{The relative error of } (0.30 \pm 0.06) \frac{\text{m}}{\text{s}^2} \text{ is } \frac{0.06}{0.30} = 0.20$$

$$\text{The total relative error is } 0.06 + 0.20 = 0.26.$$

The total relative error is what we multiply the result by to get its (absolute) uncertainty:

$$0.75 \frac{\text{kg}\cdot\text{m}}{\text{s}^2} \times 0.26 = \pm 0.195 \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$

$$\text{The answer with its uncertainty is } (0.75 \pm 0.195) \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$

We should make sure our answer and uncertainty are reported to the same place value, so we need to round the uncertainty to get our final answer of $(0.75 \pm 0.20) \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$.

Often, you can use the uncertainty to decide where to round your answer. For the kinds of experiments you will do in physics class, it is usually sufficient to show the uncertainty to one or two significant figures, and then round the answer to the same place value.

Use this space for summary and/or additional notes.

Exponents

Calculations that involve **exponents** use the same rule as for multiplication and division. If you think of exponents as multiplying a number by itself the indicated number of times, it means you would need to add the relative error of that number that many times.

In other words, when a value is raised to an exponent, multiply its relative error by the exponent.

For example, consider the problem:

$$(0.5) \times (12 \pm 0.8) \text{ kg} \times ((6.0 \pm 0.3) \frac{\text{m}}{\text{s}})^2$$

The result of the calculation is: $(0.5)(12)(6.0)^2 = 216 \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$

The relative errors of the two measurements are:

$$\frac{0.8}{12} = 0.0667 \quad \text{and} \quad \frac{0.3}{6.0} = 0.05$$

Because the $6.0 \frac{\text{m}}{\text{s}}$ is squared in the calculation, we need to multiply its relative error by two (the exponent). This gives a total relative error of:

$$0.0667 + (0.05 \times 2) = 0.1667$$

Now multiply the total relative error by the result to get the uncertainty:

$$216 \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2} \times 0.1667 = 36 \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$$

Our answer is therefore $(216 \pm 36) \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$.

Rounding

Just as you should never introduce uncertainty by measuring carelessly, you should also never introduce uncertainty by rounding carelessly. You should always keep all digits in the calculator and round *only* at the end.

However, if for some reason you need to off an intermediate value, estimate how many significant figures your answer will have, and carry at least two additional significant figures (guard digits) through the calculation. When in doubt, keep an extra digit or two. It is much better to have a couple of extra digits than to lose significance because of rounding.

Use this space for summary and/or additional notes.

Summary of Uncertainty Calculations

Uncertainty of a Single Quantity

Measured Once

Make your best educated guess of the uncertainty based on how precisely you were able to measure the quantity and the uncertainty of the instrument(s) that you used.

Measured Multiple Times (Independently)

- If you have a lot of data points, the uncertainty is the standard deviation of the mean, which you can get from a calculator that has statistics functions.
- If you have few data points, use the approximation $u \approx \frac{r}{\sqrt{3}}$.

Uncertainty of a Calculated Value

- For addition & subtraction, add the uncertainties of each of the measurements. The sum is the uncertainty of the result.
- For multiplication, division and exponents:
 1. Find the relative error of each measurement.
$$R.E. = \frac{\text{uncertainty}(\pm)}{\text{measured value}}$$
 2. Multiply the relative error by the exponent (if any).
 3. Add each of the relative errors to find the total relative error.
 4. The absolute uncertainty (\pm) is the result times the total R.E.

Use this space for summary and/or additional notes.

Homework Problems

Because the answers are provided, you must show sufficient work in order to receive credit.

1. A pitching machine pitched five baseballs a distance of 18.44 m in the following times: 0.523 s, 0.506 s, 0.541 s, 0.577 s, and 0.518 s.
 - a) What was the average time that it took for each baseball to get from the pitching machine to home plate?

Answer: 0.533 s

- b) Assuming each time interval was measured within ± 0.001 s, use the “fewer than ten data points estimate” to calculate the uncertainty (\pm) in the time it took for the baseballs to travel from the pitching machine to home plate.

Answer: ± 0.041 s

Use this space for summary and/or additional notes.

2. After school, you drove a friend home and then went back to your house. According to your car's odometer, you drove 3.4 miles to your friend's house (going past your house on the way). Then you drove 1.2 miles back to your house. If the uncertainty in your car's odometer reading is 0.1 mile, how far is it from school directly to your house (including the uncertainty)?

Answer: (2.2 ± 0.2) mi.

3. A rock that has a mass of (8.15 ± 0.25) kg is sitting on the top of a cliff that is (27.3 ± 1.1) m high. What is the gravitational potential energy of the rock (including the uncertainty)? The formula for gravitational potential energy is $U_g = mgh$, and g is the acceleration due to gravity on Earth, which is equal to $9.8 \frac{\text{m}}{\text{s}^2}$.

Answer: (2183 ± 155) J

Use this space for summary and/or additional notes.

4. You drive West on the Mass Pike, from Route 128 to the New York state border, a distance of 127 miles. The EZ Pass transponder determines that your car took 1 hour and 54 minutes to complete the trip, and you received a ticket in the mail for driving $66.8 \frac{\text{mi.}}{\text{hr.}}$ in a $65 \frac{\text{mi.}}{\text{hr.}}$ zone. The uncertainty in the distance is ± 1 mile and the uncertainty in the time is ± 5 seconds. Can you fight the ticket and win? (You can win if you prove that because of the uncertainty, your speed *could* have been less than $65 \frac{\text{mi.}}{\text{hr.}}$.)

Answer: No. Your average speed is $(66.8 \pm 0.575) \frac{\text{mi.}}{\text{hr.}}$. Subtracting the uncertainty, the minimum that your speed could have been is $66.8 - 0.575 = 66.2 \frac{\text{mi.}}{\text{hr.}}$.

5. Solve the following expression and round off the answer appropriately, according to the rules for significant figures:

$$23.5 + 0.87 \times 6.02 - 105$$

Answer: $-76.2626 = -76$

Use this space for summary and/or additional notes.

Significant Figures

Unit: Laboratory & Measurement

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

AP Physics 1 Learning Objectives: N/A

Knowledge/Understanding:

- Understand how significant figures work.

Skills:

- Perform calculations and round to the “correct” number of significant figures.

Language Objectives:

- Understand and correctly use the term “significant figure”.
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Notes:

Significant figures are the digits in a number that are not merely placeholders. (*I.e.*, the non-rounded part of the number.)

There are two uses of significant figures, one good and one bad.

The proper use of significant figures is to tell which digits are actually part of the measurement when you record a measurement or data point.

For example, if you record a length as 12 cm, the significant figures indicate that the measurement was recorded to the nearest 1 cm. If you record a length as 12.0 cm, the significant figures indicate that the measurement was recorded to the nearest 0.1 cm.

If you record a length to the nearest 0.1 cm, *you must have a digit holding that place in the number, regardless of whether or not that digit has the value of zero.*

Use this space for summary and/or additional notes.

The improper use of significant figures is as a crude attempt to imply the uncertainty of a number.

This use of significant figures is popular in high school science classes. It is based on the assumption that every measurement has an uncertainty of ± 1 in the last digit. While this assumption is often (though not always) the right order of magnitude, it is rarely a good enough approximation to be useful! For this reason, everyone but high school teachers expresses the uncertainty of a measurement separately from its value, and no one but high school teachers actually use significant figures to express uncertainty.

How Significant Figures Work (as an Approximation)

Note that the rules you learned in chemistry for calculations with significant figures come from the rules for propagating relative error, with the assumption that the uncertainty is ± 1 in the last digit.

For example, consider the following problem:

problem:

$$\begin{array}{r} 123000 \quad \pm 1000 \\ 0.0075 \quad \pm 0.0001 \\ + 1650 \quad \pm 10 \\ \hline 124650.0075 \quad \pm 1010.0001 \end{array}$$

"sig figs" equivalent:

$$\begin{array}{r} 123 \text{ ????.???.?} \\ 0.0075 \\ + 165\text{.?????} \\ \hline 124 \text{ ????.?????} \end{array}$$

↑

↑ (Check this digit for rounding)

If we rounded our answer of 124650 ± 1010 to 125000 ± 1000 , it would agree with the number we got by using the rules for significant figures.

Note, however, that our answer of 124650 ± 1010 suggests that the actual value is between 123 640 and 125 660, whereas the answer based on significant figures suggests that the actual value is between 124 000 and 126 000. This is why scientists frown upon use of "sig figs" instead of genuine error analysis. In the laboratory, if you insist on using sig figs, it is better to report answers to *one more sig fig* than the problem calls for, in order to minimize round-off errors in your results.

Use this space for summary and/or additional notes.

As another example, consider the problem:

$$34.52 \times 1.4$$

Assume this means $(34.52 \pm 0.01) \times (1.4 \pm 0.1)$.

The answer (without its uncertainty) is $34.52 \times 1.4 = 48.328$

Because the calculation involves multiplication, we need to add the relative errors, and then multiply the total relative error by the answer to get the uncertainty.

The relative uncertainties are:

$$\frac{0.01}{34.52} = 0.00029$$

$$\frac{0.1}{1.4} = 0.0714$$

Adding them gives: $0.00029 + 0.0714 = 0.0717$

Multiplying by our answer gives: $0.0717 \times 48.328 = 3.466$

The uncertainty is: ± 3.466 .

Our answer including the uncertainty is therefore 48.328 ± 3.466 , which we can round to 48.3 ± 3.5 .

Doing the calculation with sig figs would give us an answer of 48, which implies 48 ± 1 . Again, this is in the ballpark of the actual uncertainty, but you should notice (and be concerned) that the sig figs method underrepresents the actual uncertainty by a factor of 3.5!

Use this space for summary and/or additional notes.

Rules for Using Significant Figures

The first significant digit is where the “measured” part of the number begins—the first digit that is not zero.

The last significant digit is the last “measured” digit—the last digit whose true value is known or accurately estimated (usually ± 1).

- If the number doesn’t have a decimal point, the last significant digit will be the last digit that is not zero.
- If the number does have a decimal point, the last significant digit will be the last digit.
- If the number is in scientific notation, the above rules tell us (correctly) that all of the digits before the “times” sign are significant.

For any measurement that does not have an explicitly stated uncertainty value, assume the uncertainty is ± 1 in the last significant digit.

In the following numbers, the significant figures have been underlined:

- 13,000
- 0.0275
- 0.0150
- 6804.30500
- 6.0×10^{23}
- 3400. (note the decimal point at the end)

Use this space for summary and/or additional notes.

Math with Significant Figures

Addition & Subtraction:

Line up the numbers in a column. Any column that has an uncertain digit—a zero from rounding—is an uncertain column. (Uncertain digits are shown as question marks in the right column below.) You need to round off your answer to get rid of all of the uncertain columns.

For example:

problem:

$$\begin{array}{r} 123000 \\ 0.0075 \\ + 1650 \\ \hline 124650.0075 \end{array}$$

meaning:

$$\begin{array}{r} 123???.\???? \\ 0.0075 \\ + 165?.\???? \\ \hline 124???.\???? \end{array}$$

Because we can't know which digits go in the hundreds, tens, ones, and decimal places of all of the addends, the exact values of those digits must therefore be unknown in the sum.

This means we need to round off the answer to the nearest 1,000, which gives a final answer of 125,000 (which actually means $125,000 \pm 1,000$).

A silly (but correct) example of addition with significant digits is:

$$100 + 37 = 100$$

Use this space for summary and/or additional notes.

Multiplication, Division, Etc.

For multiplication, division, and just about everything else (except for addition and subtraction, which we have already discussed), round your answer off to the same number of significant digits as the number that has the fewest.

For example, consider the problem $34.52 \times 1.4 = 48.328$

The number 1.4 has the fewest significant digits (2). Remember that 1.4 really means 1.4 ± 0.1 , which means the actual value, if we had more precision, could be anything between 1.3 and 1.5. Using “crank three times,” the actual answer could therefore be anything between $34.52 \times 1.3 = 44.876$ and $34.52 \times 1.5 = 51.780$.

To get from the answer of 48.328 to the largest and smallest answers we would get from “crank three times,” we would have to add or subtract approximately 3.5. (Notice that this agrees with the number we found previously for this same problem by propagating the relative error!) If the uncertainty is in the ones digit (greater than or equal to 1, but less than 10), this means that the ones digit is approximate, and everything beyond it is unknown. Therefore, using the rules of significant figures, we would report the number as 48.

In this problem, notice that the least significant term in the problem (1.4) had 2 significant digits, and the answer (48) also has 2 significant digits. This is where the rule comes from.

A silly (but correct) example of multiplication with significant digits is:

$$147 \times 1 = 100$$

Use this space for summary and/or additional notes.

Mixed Operations

For mixed operations, keep all of the digits until you're finished (so round-off errors don't accumulate), but keep track of the last significant digit in each step by putting a line over it (even if it's not a zero). Once you have your final answer, round it to the correct number of significant digits. Don't forget to use the correct order of operations (PEMDAS)!

For example:

$$\begin{aligned}137.4 \times 52 + 120 \times 1.77 \\(137.4 \times 52) + (120 \times 1.77) \\7,\bar{1}44.8 + 2\bar{1}2.4 = 7,\bar{3}57.2 = 7,400\end{aligned}$$

Note that in the above example, we kept all of the digits until the end. This is to avoid introducing small rounding errors at each step, which can add up to enough to change the final answer. Notice how, if we had rounded off the numbers at each step, we would have gotten the "wrong" answer:

$$\begin{aligned}137.4 \times 52 + 120 \times 1.77 \\(137.4 \times 52) + (120 \times 1.77) \\7,\bar{1}00 + 2\bar{1}0 = 7,\bar{3}10 = 7,300 \quad \leftarrow \text{sad face}\end{aligned}$$

However, if we had done actual error propagation (remembering to add absolute errors for addition/subtraction and relative errors for multiplication/division), we would get the following:

$$\begin{aligned}137.4 \times 52 = 7144.8; \text{ R.E.} = \frac{0.1}{137.4} + \frac{1}{52} = 0.01996 \\ \text{partial answer} = 7144.8 \pm 142.6\end{aligned}$$

$$\begin{aligned}120 \times 1.77 = 212.4; \text{ R.E.} = \frac{1}{120} + \frac{0.01}{1.77} = 0.01398 \\ \text{partial answer} = 212.4 \pm 2.97\end{aligned}$$

The total absolute error is $142.6 + 2.97 = 145.6$

The best answer is therefore 7357.2 ± 145.6 . I.e., the actual value lies between approximately 7200 and 7500.

Use this space for summary and/or additional notes.

Keeping a Laboratory Notebook

Unit: Laboratory & Measurement

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding:

- rules for recording and working with laboratory procedures and data

Skills:

- Ensure that essential information is recorded properly during an experiment.

Language Objectives:

- Understand and be able to describe the sections of a laboratory notebook write-up, and which information goes in each section.

Notes:

A laboratory notebook serves two important purposes:

1. It is a legal record of what you did and when you did it.
2. It is a diary of exactly what you did, so you can look up the details later.

Your Notebook as an Official Record

Laboratory notebooks are kept by scientists in research laboratories and high tech companies. If a company or research institution needs to prove (perhaps in a court case) that you did a particular experiment on a particular date and got a particular set of results, your lab notebook is the primary evidence. While there is no right or wrong way for something to exist as a piece of evidence, the goal is for you to maintain a lab notebook that gives the best chance that it can be used to prove beyond a reasonable doubt exactly what you did, exactly when you did it, and exactly what happened.

Use this space for summary and/or additional notes:

Recording Your Procedure

Recording a procedure in a laboratory notebook is a challenging problem, because on the one hand, you need to have a legal record of what you did that is specific enough to be able to stand as evidence in court. On the other hand, you also need to be able to perform the experiment quickly and efficiently without stopping to write down every detail.

If your experiment is complicated and you need to plan your procedure ahead of time, you can record your intended procedure in your notebook before performing the experiment. Then all you need to do during the experiment is to note any differences between the intended procedure and what you actually did.

If the experiment is quick and simple, or if you suddenly think of something that you want to do immediately, without taking time to plan a procedure beforehand, you can jot down brief notes during the experiment for anything you may not remember, such as instrument settings and other information that is specific to the values of your independent variables. Then, as soon as possible after finishing the experiment, write down *all* of the details of the experiment. Include absolutely *everything*, including the make and model number of any major equipment that you used. Don't worry about presentation or whether the procedure is written in a way that would be easy for someone else to duplicate; concentrate on making sure the specifics are accurate and complete. The other niceties matter in laboratory reports, but not in a notebook.

Recording Your Data

Here are some general rules for working with data. (Most of these are courtesy of Dr. John Denker, at <http://www.av8n.com/physics/uncertainty.htm>):

- Keep *all* of the raw data, whether you will use it or not.
- Never discard a measurement without writing it down, even if you think it is wrong. Record it anyway and put a "?" next to it. You can always choose not to use the data point in your calculations (as long as you give an explanation).
- Never erase or delete a measurement. The only time you should ever cross out recorded data is if you accidentally wrote down the wrong number.
- Record all digits. Never round off original data measurements. If the last digit is a zero, you must record it anyway!

Use this space for summary and/or additional notes.

- For analog readings (e.g., ruler, graduated cylinder, thermometer), always estimate and record one extra digit beyond the smallest marking.
- Always write down the units with every measurement!
- Record every quantity that will be used in a calculation, whether it is changing or not.
- Don't convert units in your head before writing down a measurement. Always record the original data in the units you actually measured it in, and then convert in a separate step.
- Always record uncertainty separately from the measurement. Never rely on "sig figs" to express uncertainty. (In fact, you should never rely on "sig figs" at all!)

Calculations

In general, calculations only need to be included in a laboratory notebook when they lead directly to another data point or another experiment. When this is the case, the calculation should be accompanied by a short statement of the conclusion drawn from it and the action taken. Calculations in support of the after-the-fact analysis of an experiment or set of experiments may be recorded in a laboratory notebook if you wish, or they may appear elsewhere.

Regardless of where calculations appear, you must:

- Use enough digits to avoid unintended loss of significance. (Don't introduce round-off errors in the middle of a calculation.) This usually means use at least two more "guard" digits beyond the number of "significant figures" you expect your answer to have.
- You may round for convenience only to the extent that you do not lose significance.
- Always calculate and express uncertainty separately from the measurement. Never rely on "sig figs" to express uncertainty. (In fact, you should never rely on "sig figs" at all!)
- Leave digits in the calculator between steps. (Don't round until the end.)
- When in doubt, keep plenty of "guard digits" (digits after the place where you think you will end up rounding).

Use this space for summary and/or additional notes.

Integrity of Data

Your data are your data. In classroom settings, people often get the idea that the goal is to report an uncertainty that reflects the difference between the measured value and the “correct” value. That idea certainly doesn’t work in real life—if you knew the “correct” value you wouldn’t need to make measurements!

In all cases—in the classroom and in real life—you need to determine the uncertainty of your own measurement by scrutinizing your own measurement procedures and your own analysis. Then you judge how well they agree. For example, we would say that the quantities 10 ± 2 and 11 ± 2 agree reasonably well, because there is considerable overlap between their probability distributions. However, 10 ± 0.2 does not agree with 11 ± 0.2 , because there is no overlap.

If you get an impossible result or if your results disagree with well-established results, you should look for and comment on possible problems with your procedure and/or measurements that could have caused the differences you observed. You must *never* fudge your data to improve the agreement.

Your Laboratory Notebook is *Not* a Report

Many high school students are taught that a laboratory notebook should be a journal-style book in which they must write perfect after-the-fact reports, but they are not allowed to change anything if they make a mistake. This is not at all what laboratory notebooks were ever meant to be. A laboratory notebook does not need to be anything more than an official signed and dated record of your procedure (what you did) and your data (what happened) at the exact instant that you took it and wrote it down.

Of course, because it is your journal, your laboratory notebook *may* contain anything else that you think is relevant. You may choose to include an explanation of the motivations for one or more experiments, the reasons you chose the procedure that you used, alternative procedures or experiments you may have considered, ideas for future experiments, *etc.* Or you may choose to record these things separately and cross-reference them to specific pages in your lab notebook.

Use this space for summary and/or additional notes.

Informal Laboratory Reports

Unit: Laboratory & Measurement

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

AP Physics 1 Learning Objectives: N/A

Knowledge/Understanding:

- Understand how to do a basic write-up of an experiment

Skills:

- Write up an experiment in an appropriate format that conveys all of the necessary information.

Language Objectives:

- Understand and be able to describe the sections of an informal laboratory report, and which information goes in each section.

Notes:

Every lab you work in, whether in high school, college, research, or industry, will have its own preferred format for laboratory write-ups. It is much more important to understand what *kinds* of information you need to convey than it is to get attached to any one particular format.

Most of the write-ups you will be required to do this year will be informal write-ups as described in this section. The format is meant to follow an outline of the actual experiment.

Title & Date

Each experiment should have the title and date the experiment was performed written at the top. The title should be a descriptive sentence fragment (usually without a verb) that gives some information about the purpose of the experiment.

Objective

This should be a one or two-sentence description of what you are trying to determine or calculate by performing the experiment.

Use this space for summary and/or additional notes.

Experimental Design

Your experimental design needs to explain:

- the process of *determining* what you need to measure (starting from the quantity you are looking for, and using equations and other relationships to relate that quantity to quantities that you can measure)
- the specific quantities that you are going to vary (your independent variables)
- the specific quantities that you are going to keep constant (your control variables)
- the specific quantities that you are going to measure or observe (your dependent variables), and how you are going to measure or observe them
- how you are going to calculate or interpret your results.

Note that your background is different from your experimental procedure. Your background is a much higher-level description of how you set up your experiment and why, whereas your procedure is a specific description of exactly how you took the data points. Your background lays the groundwork for your analysis in the same way that your procedure lays the groundwork for your data and observations.

Procedure

Your procedure is a detailed description of exactly what you did in order to take your measurements. You need to include:

- A step-by-step description of everything you did. The description needs to include the actual values of quantities you used in the experiment (your control and independent variables). For a repeated procedure, write the steps once, then list the differences from one trial to the next. *E.g.,* “Repeat steps 1–4 using distances of 1.5 m, 2.0 m, 2.5 m, and 3.0 m.”
- A *labeled* sketch of your experimental set-up. This is required even if the experiment is simple. The sketch will serve to answer many questions about how you set up the experiment, and will show most of the key equipment you used. All important items must be labeled, and all relevant dimensions must be shown.

Use this space for summary and/or additional notes.

- A list of any equipment that you used other than what you labeled in your sketch.

Data & Observations

This is a section in which you present all of your data. Be sure to record every quantity specified in your procedure, including quantities that are not changing (your control variables), quantities that are changing (your independent variables), and what happens as a result (your dependent variables).

For a high school lab write-up, it is usually sufficient to present one or more data tables that include your measurements for each trial and the quantities that you calculated from them. However, if you have other data or observations that you recorded during the lab, they must be listed here.

You must also include estimates of the uncertainty for each measured quantity, and your calculated uncertainty for the quantity that your experiment is intended to determine.

Analysis

The analysis section is where you interpret your data. (Note that calculated values in the table in the Data & Observations section are actually part of your analysis, even though they appear in the Data & Observations section.) Your analysis should mirror your Background section (possibly in the same order, possibly in reverse), with the goal of guiding the reader from your data to the quantity you ultimately want to calculate or determine.

Your analysis needs to include:

- A narrative description (one or more paragraphs) that guides the reader from your data through your calculations to the quantity you set out to determine.
- One (and only one) sample calculation for each separate equation that you used. For example, if you calculated acceleration for each of five data points, you would write down the formula, and then choose one set of data to plug in and show how you got the answer.
- Any calculated values that did not appear in the data table in your Data & Observations section

Use this space for summary and/or additional notes.

- A carefully-plotted graph showing the data points you took for your dependent vs. independent variables. Often, the quantity you are calculating will be the slope. You need the graph to show the region in which the slope is linear, which tells the range over which your experiment is valid. Note that any graphs you include in your write-up must be drawn accurately to scale, using graph paper, and using a ruler wherever a straight line is needed. (When an accurate graph is required, you will lose points if you include a freehand sketch instead.)
- Quantitative error analysis. You need to:
 1. Measure or estimate the uncertainty of each your measurements.
 2. Calculate the relative error for each measurement.
 3. Combine your relative errors to get the total relative error for your calculated value(s).
 4. Multiply the total relative error by your calculated values to get the absolute uncertainty (\pm) for each one.
- Sources of uncertainty: this is a list of factors *inherent in your procedure* that limit how precise your answer can be. **Never include hypothetical human error!** A statement like “We might have written down the wrong number.” or “We might have done the calculations incorrectly.” is really saying, “We might be stupid and you shouldn’t believe anything we say in this report.” (You will lose points if you include “we might be stupid” statements.)

Note, however, that if a problem *actually occurred*, and if you *used that data point in your calculations anyway*, you need to explain what happened and calculate an estimate of the effects on your results.

Conclusions

Your conclusion should be worded the same way as your objective, but this time including your final calculated result(s) and uncertainty. You do not need to restate sources of uncertainty in your conclusions unless you believe they were significant enough to create some doubt about your results.

Use this space for summary and/or additional notes.

Formal Laboratory Reports

Unit: Laboratory & Measurement

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

AP Physics 1 Learning Objectives: N/A

Knowledge/Understanding:

- Understand the purpose of a formal lab write-up, such as you would submit to a scientific journal

Skills:

- Write up an experiment formally in journal article format.

Language Objectives:

- Understand and correctly use the term “abstract,” as it pertains to a formal laboratory report.
- Understand and be able to describe the sections of a formal laboratory report, and which information goes in each section.

Notes:

A formal laboratory report serves the purpose of communicating the results of your experiment to other scientists outside of your laboratory or institution.

A formal report is a significant undertaking. In a research laboratory, you might submit as many as one or two articles to a scientific journal in a year. Some college professors require students to write their lab reports in journal article format.

The format of a formal journal article-style report is as follows:

Use this space for summary and/or additional notes.

Abstract

This is the most important part of your report. It is a (maximum) 200-word executive summary of *everything* about your experiment—the procedure, results, analysis, and conclusions. In most scientific journals, the abstracts are searchable via the internet, so it needs to contain enough information to enable someone to find your abstract, and after reading it, to know enough about your experiment to determine whether or not to purchase a copy of the full article (which can sometimes cost \$100 or more). It also needs to be short enough that the person doing the search won't just say "TL; DR" ("Too Long; Didn't Read") and move on to the next abstract.

Because the abstract is a complete summary, it is always best to wait to write it until you have already written the rest of your report.

Introduction

Your introduction is a short research paper on its own, with citations. (For a high school lab report, it should be 1–3 pages; for scientific journals, 5–10 pages is not uncommon.) Your introduction needs to describe background information that another scientist might not know, plus all of the background information that specifically led up to your experiment. Assume that your reader has a similar knowledge of physics as you, but does not know anything about this experiment. The introduction is usually the most time-consuming part of the report to write.

Materials and Methods

This section combines both the background and procedure sections of an informal lab write-up. Unlike a lab notebook write-up, the Materials and Methods section of a formal report is written in paragraph form, in the past tense. Again, a labeled drawing of your apparatus is a necessary part of the section, but you need to *also* describe the set-up in the text.

Also unlike the lab notebook write-up, your Materials and Methods section needs to give some explanation of your choices of the values used for your control and independent variables.

Use this space for summary and/or additional notes.

Data and Observations

This section is similar to the same section in the lab notebook write-up, except that:

1. You should present only data you actually recorded/measured in this section. (Calculated values are presented in the Discussion section.)
2. You need to *introduce* the data table. (This means you need to describe the important things someone should notice in the table first, and then say something like “Data are shown in Table 1.”)

Note that all figures and tables in the report need to be numbered consecutively.

Discussion

This section is similar to the Analysis section in the lab notebook write-up, but with some important differences.

Your discussion is essentially a long essay discussing your results and what they mean. You need to introduce and present a table with your calculated values and your uncertainty. After presenting the table, you should discuss the results, uncertainties, and sources of uncertainty in detail. If your results relate to other experiments, you need to discuss the relationship and include citations for those other experiments.

Your discussion needs to include all of the formulas that you used as part of your discussion, but you do not need to show your work for each calculation.

Conclusions

Your conclusions should start by presenting the same information that you included in the Conclusions section of an informal write-up. However, in a formal report, you should explicitly mention significant sources of uncertainty. Your conclusions should also suggest how future experiments might follow up on or expand on your experiment.

Works Cited

As with a research paper, you need to include a complete list of bibliography entries for the references you cited in your introduction and/or discussion sections.

Use this space for summary and/or additional notes.

Introduction: Mathematics

Unit: Mathematics

Topics covered in this chapter:

Standard Assumptions in Physics	69
Assigning & Substituting Variables	72
The Metric System	83
Scientific Notation.....	88
Right-Angle Trigonometry.....	92
Vectors	96
Vectors vs. Scalars in Physics	102
Vector Multiplication	105

The purpose of this chapter is to familiarize you with mathematical concepts and skills that will be needed in physics.

- *Standard Assumptions in Physics* discusses what you can and cannot assume to be true in order to be able to solve the problems you will encounter in this class.
- *Assigning & Substituting Variables* discusses how to determine which quantity and which variable apply to a number given in a problem based on the units, and how to choose which formula applies to a problem.
- *The Metric System* and *Scientific Notation* briefly review skills that you are expected to remember from your middle school math and science classes.
- *Trigonometry, Vectors, Vectors vs. Scalars in Physics, and Vector Multiplication* discuss important mathematical concepts that are widely used in physics, but may be unfamiliar to you.

Depending on your math background, some of the topics, such as trigonometry and vectors, may be unfamiliar. These topics will be taught, but in a cursory manner.

Use this space for summary and/or additional notes.

Textbook:

- *Physics: Principles and Problems* Chapter 2: A Mathematical Toolkit (pp. 12–39)
- *Physics: Principles and Problems* Chapter 6: Vectors (pp. 108–131)

Standards addressed in this chapter:

Next Generation Science Standards (NGSS):

No NGSS standards are addressed in this chapter.

Massachusetts Curriculum Frameworks (2006):

No MA curriculum frameworks are specifically addressed in this chapter. However, this chapter addresses the following mathematical understandings explicitly listed in the MA Curriculum Frameworks as prerequisites for this course:

- Construct and use tables and graphs to interpret data sets.
- Solve simple algebraic expressions.
- Perform basic statistical procedures to analyze the center and spread of data.
- Measure with accuracy and precision (e.g., length, volume, mass, temperature, time)
- Convert within a unit (e.g., centimeters to meters).
- Use common prefixes such as milli-, centi-, and kilo-.
- Use scientific notation, where appropriate.
- Use ratio and proportion to solve problems.

In addition, this chapter addresses the following mathematical understandings. The MA frameworks state that “the following skills are not detailed in the Mathematics Framework, but are necessary for a solid understanding in this course.”

- Determine the correct number of significant figures.
- Determine percent error from experimental and accepted values.

Use this space for summary and/or additional notes.

- Use appropriate metric/standard international (SI) units of measurement for mass (kg); length (m); time (s); force (N); speed (m/s); acceleration (m/s^2); frequency (Hz); work and energy (J); power (W); momentum ($\text{kg}\cdot\text{m/s}$); electric current (A); electric potential difference/voltage (V); and electric resistance (Ω).
- Use the Celsius and Kelvin scales.

Skills learned & applied in this chapter:

- Estimating uncertainty in measurements
- Propagating uncertainty through calculations
- Identifying quantities in word problems and assigning them to variables
- Choosing a formula based on the quantities represented in a problem
- Using trigonometry to calculate the lengths of sides and angles of triangles
- Representing quantities as vectors
- Adding and subtracting vectors
- Multiplying vectors using the dot product and cross product

Use this space for summary and/or additional notes.

Standard Assumptions in Physics

Unit: Mathematics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding Goals:

- what are the usual assumptions in physics problems

Language Objectives:

- understand and correctly use the term “assumption,” as it pertains to setting up and solving physics problems.

Notes:

Many of us have been told not to make assumptions. There is a popular expression that states that “when you assume, you make an ass of you and me”:

ass|u|me

In science, particularly in physics, this adage is crippling. Assumptions are part of everyday life. When you cross the street, you assume that the speed of cars far away is slow enough for you to walk across without getting hit. When you eat your lunch, you assume that the food won’t cause an allergic reaction. When you run down the hall and slide across the floor, you assume that the friction between your shoes and the floor will be enough to stop you before you crash into your friend.

assumption: something that is unstated but considered to be fact for the purpose of making a decision or solving a problem. Because it is impossible to measure and/or calculate everything that is going on in a typical physics or engineering problem, it is almost always necessary to make assumptions.

Use this space for summary and/or additional notes.

In a first-year physics course, in order to make problems and equations easier to understand and solve, we will often assume that certain quantities have a minimal effect on the problem, even in cases where this would not actually be true. The term used for these kinds of assumptions is “ideal”. Some of the ideal physics assumptions we will use include the following. Over the course of the year, you can make each of these assumptions unless you are explicitly told otherwise.

- Constants (such as acceleration due to gravity) have the same value in all parts of the problem.
- Variables change in the manner described by the relevant equation(s).
- Ideal machines and other objects that are not directly considered in the problem have negligible mass, inertia, and friction. (Note that these idealizations may change from problem-to-problem. A pulley may have negligible mass in one problem, but another pulley in another problem may have significant mass that needs to be considered as part of the problem.)
- If the problem does not mention air resistance and air resistance is not a central part of the problem, friction due to air resistance is negligible.
- The mass of an object can often be assumed to exist at a single point in 3-dimensional space.
- Sliding (kinetic) friction between surfaces is negligible unless friction is specifically part of the problem. However, even in friction problems, ice is assumed to be frictionless unless you are explicitly told otherwise.
- Force can be applied in any direction using an ideal rope. (You can even push on it!)
- Collisions between objects are either perfectly elastic or perfectly inelastic, unless the problem states otherwise.
- No energy is lost when energy is converted from one form to another. (This is *always* true, but in an ideal collision, energy lost to heat is usually assumed to be negligible.)
- The amount that solids and liquids expand or contract due to temperature differences is negligible. (This will not be the case in problems involving thermal expansion.)

Use this space for summary and/or additional notes.

- The degree to which solids and liquids can be compressed or expanded due to changes in pressure is negligible.
- Gas molecules do not interact when they collide or are forced together from pressure. (Real gases can form liquids and solids or participate in chemical reactions.)
- Electrical wires have negligible resistance.
- All physics students do all of their homework. ☺ (Of course, real physics students do not always do their homework, which can lead to much more interesting (in a bad way) results on physics tests.)

In some topics, a particular assumption may apply to some problems but not others. In these cases, the problem needs to make it clear whether or not you can make the relevant assumption. (For example, in the “forces” topic, some problems involve friction and others do not. A problem that does not involve friction might state that “a block slides across a frictionless surface.”)

If you are not sure whether you can make a particular assumption, you should ask the teacher. If this is not practical (such as an open response problem on a standardized test), you should decide for yourself whether or not to make the assumption, and explicitly state what you are assuming as part of your answer.

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Assigning & Substituting Variables

Unit: Mathematics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding Goals:

- be able to declare (assign) variables from a word problem
- be able to substitute values for variables in an equation

Language Objectives:

- understand and correctly use the terms “variable” and “subscript.”
- accurately describe and apply the concepts described in this section, using appropriate academic language

Notes:

Math is a language. Like other languages, it has nouns (numbers), pronouns (variables), verbs (operations), and sentences (equations), all of which must follow certain rules of syntax and grammar.

This means that turning a word problem into an equation is translation from English to math.

Mathematical Operations

You have probably been taught translations for most of the common math operations:

word	meaning	word	meaning	word	meaning
and, more than (but not “is more than”)	+	percent (“per” + “cent”)	÷ 100	is at least	≥
less than (but not “is less than”)	-	change in x , difference in x	Δx	is more than	>
of	×	is	=	is at most	≤
per, out of	÷			is less than	<

Use this space for summary and/or additional notes.

Identifying Variables

In science, almost every measurement must have a unit. These units are your key to what kind of quantity the numbers describe. Some common quantities in physics and their units are:

quantity	S.I. unit	variable
mass	kg	m
distance, length	m	d, ℓ
area	m^2	A
acceleration	m/s^2	a
volume	m^3	V
velocity (speed)	m/s	v
pressure	Pa	P^*
momentum	$\text{N}\cdot\text{s}$	p^*
density	kg/m^3	ρ^*
moles	mol	n
time	s	t
temperature	K	T
heat	J	Q
electric charge	C	q

*Note the subtle differences between uppercase "P", lowercase "p", and the Greek letter ρ ("rho").

Any time you see a number in a word problem that has a unit you recognize (such as one listed in this table), notice which quantity the unit is measuring and label the quantity with the appropriate variable.

Be especially careful with uppercase and lowercase letters. In physics, the same uppercase and lowercase letter may be used for completely different quantities.

Use this space for summary and/or additional notes.

Variable Substitution

Variable substitution simply means taking the numbers you have from the problem and substituting those numbers for the corresponding variable in an equation. A simple version of this is a density problem:

If you have the formula:

$$\rho = \frac{m}{V} \quad \text{and you're given: } m = 12.3 \text{ g} \quad \text{and} \quad V = 2.8 \text{ cm}^3$$

simply substitute 12.3 g for m , and 2.8 cm³ for V , giving:

$$\rho = \frac{12.3 \text{ g}}{2.8 \text{ cm}^3} = 4.4 \frac{\text{g}}{\text{cm}^3}$$

Because variables and units both use letters, it is often easier to leave the units out when you substitute numbers for variables and then add them back in at the end:

$$\rho = \frac{12.3}{2.8} = 4.4 \frac{\text{g}}{\text{cm}^3}$$

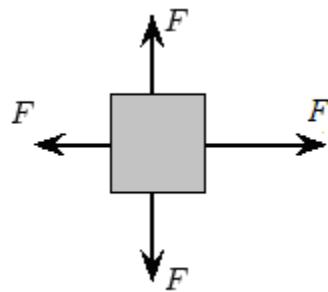
Use this space for summary and/or additional notes.

Subscripts

In physics, one problem can often have several instances of the same quantity. For example, consider a box with four forces on it:

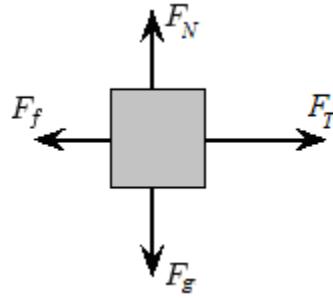
1. The force of gravity, pulling downward.
2. The “normal” force of the table resisting gravity and holding the box up.
3. The tension force in the rope, pulling the box to the right.
4. The force of friction, resisting the motion of the box and pulling to the left.

The variable for force is “F”, so the diagram would look like this:



In order to distinguish between the forces and make the diagram easier to understand, we add subscripts to the variables:

1. F_g is the force of gravity.
2. F_N is the normal force.
3. F_T is the tension in the rope.
4. F_f is friction.



Use this space for summary and/or additional notes.

When writing variables with subscripts, be especially careful that the subscript looks like a subscript—it needs to be smaller than the other letters and lowered slightly. For example, when we write F_g , the variable is F (force) and the subscript g attached to it tells which kind of force it is (gravity). This might occur in the following equation:

$$F_g = mg \quad \leftarrow \quad \text{right } \odot$$

It is important that the subscript g on the left does not get confused with the variable g on the right. Otherwise, the following error might occur:

$$\begin{aligned} Fg &= mg \\ F\cancel{g} &= m\cancel{g} \quad \leftarrow \quad \text{wrong! } \ominus \\ F &= m \end{aligned}$$

Another common use of subscripts is the subscript “0” to mean “initial”. For example, if an object is moving slowly at the beginning of a problem and then it speeds up, we need subscripts to distinguish between the initial velocity and the final velocity. Physicists do this by calling the initial velocity “ v_0 ” where the subscript “0” means “at time zero”, *i.e.*, at the beginning of the problem, when the “time” on the “problem clock” would be zero. The final velocity is simply “ v ” without the zero.

Use this space for summary and/or additional notes.

The Problem-Solving Process

1. Identify the quantities in the problem, based on the units and any other information in the problem.
2. Assign the appropriate variables to those quantities.
3. Find an equation that relates all of the variables.
4. Substitute the values of the variables into the equation.
 - a. If you have only one variable left, it should be the one you're looking for.
 - b. If you have more than one variable left, find another equation that uses one of the variables you have left, plus other quantities that you know.
5. Solve the equation(s), using basic algebra.
6. Apply the appropriate unit(s) to the result.

Use this space for summary and/or additional notes.

Sample Problem

A force of 30 N acts on an object with a mass of 1.5 kg. What is the acceleration of the object?

We have units of N and kg, and we're looking for acceleration. We need to look these up in our reference tables.

From Table D of our Reference Tables ("Quantities, Variables and Units") Table D. Quantities, Variables and Units on page 505, we find:

Symbol	Unit	Quantity	Variable
N	newton	force	\vec{F}
kg	kilogram	mass	m
		acceleration	\vec{a}

Now we know that we need an equation that relates the variables \vec{F} , m , and a . (\vec{F} and \vec{a} are in boldface with an arrow above them because they are vectors. We'll discuss vectors a little later in the course.)

Now that we have the variables, we find a formula that relates them. From the second formula box in Table E ("Mechanics Formulas and Equations") on page 506 of our Reference Tables, we find that:

$$\vec{F} = m \vec{a}$$

So we substitute:

$$30 = 1.5 a$$

$$20 = a$$

Again from Table D, we find that acceleration has units of meters per second squared, so our final answer is $20 \frac{\text{m}}{\text{s}^2}$.

Use this space for summary and/or additional notes.

Homework Problems

To solve these problems, refer to your Physics Reference Tables starting on page 503. To make the equations easier to find, the table and section of the table in your Physics Reference Tables where the equation can be found is given in parentheses.

1. What is the velocity of a car that travels 90. m in 4.5 s?
(mechanics/kinematics)

Answer: $20 \frac{\text{m}}{\text{s}}$

2. If a force of 100. N acts on a mass of 5.0 kg, what is its acceleration?
(mechanics/forces)

Answer: $20 \frac{\text{m}}{\text{s}^2}$

3. If the momentum of a block is $18 \text{ N}\cdot\text{s}$ and its velocity is $3 \frac{\text{m}}{\text{s}}$, what is the mass of the block?
(mechanics/momentum)

Answer: 6 kg

4. What is the potential energy due to gravity of a 95 kg anvil that is about to fall off a 150 m cliff onto Wile E. Coyote's head?
(mechanics/energy, work & power)

Answer: 139 650 J

Use this space for summary and/or additional notes.

5. A $25\ \Omega$ resistor is placed in an electrical circuit with a voltage of 110 V. How much current flows through the resistor? (electricity/circuits)

Answer: 4.4 A

6. What is the frequency of a wave that is traveling at a velocity of $300.\frac{m}{s}$ and has a wavelength of 10. m? (waves/waves)

Answer: 30. Hz

7. What is the energy of a photon that has a frequency of $6\times 10^{15}\text{ Hz}$? Hint: the equation includes a physical constant, which you will need to look up in Table B on page 504 of your Reference Tables. (atomic & particle physics/energy)

Answer: $3.96\times 10^{-18}\text{ J}$

8. A piston with an area of 2.0 m^2 is compressed by a force of 10 000 N. What is the pressure applied by the piston? (fluid mechanics/pressure)

Answer: 5 000 Pa

Use this space for summary and/or additional notes.

9. What is the acceleration of a car whose velocity changes from $60. \frac{\text{m}}{\text{s}}$ to $80. \frac{\text{m}}{\text{s}}$ over a period of 5.0 s?

*Hint: v_0 is the initial velocity and v is the final velocity.
(mechanics/kinematics)*

Answer: $4.0 \frac{\text{m}}{\text{s}^2}$

10. If the normal force on an object is 100. N and the coefficient of kinetic friction between the object and the surface it is sliding on is 0.35, what is the force of friction on the object as it slides along the surface?
(mechanics/forces)

Answer: 35 N

11. A car has a mass of 1 200 kg and kinetic energy of 240 000 J. What is its velocity?
(mechanics/energy)

Answer: $20. \frac{\text{m}}{\text{s}}$

12. A 1200 W hair dryer is plugged into a electrical circuit with a voltage of 110 V. How much electric current flows through the hair dryer?
(electricity/circuits)

Answer: 10.9 A

Use this space for summary and/or additional notes.

13. What is the velocity of a photon (wave of light) through a block of clear plastic that has an index of refraction of 1.40?

*Hint: the equation includes a physical constant, which you will need to look up in Table B on page 504 of your Reference Tables.
(waves/reflection & refraction)*

Answer: $2.14 \times 10^8 \frac{\text{m}}{\text{s}}$

14. If the distance from a mirror to an object is 0.8 m and the distance from the mirror to the image is 0.6 m, what is the distance from the lens to the focus?

(waves/mirrors & lenses)

Answer: 0.343 m

15. What is the momentum of a photon that has a wavelength of 400 nm?

Hint: remember to convert nanometers to meters. Note also that the equation includes a physical constant, which you will need to look up in Table B on page 504 of your Reference Tables.

(modern physics/energy)

Answer: $1.65 \times 10^{-27} \text{ N}\cdot\text{s}$

16. If a pressure of 100 000 Pa is applied to a gas and the volume decreases by 0.05 m^3 , how much work was done on the gas?

(fluid mechanics/work)

Answer: 5 000 J

Use this space for summary and/or additional notes.

The Metric System

Unit: Mathematics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding:

- Understand how units behave and combine algebraically.
- Know the 4 common prefixes and their numeric meanings.

Skills:

- Be able to describe quantities using metric units with & without prefixes.

Language Objectives:

- Understand and correctly use the terms “unit” and “prefix.”
- Accurately describe and apply the concepts described in this section, using appropriate academic language.

Notes:

A unit is a specifically defined measurement. Units describe both the type of measurement, and a base amount.

For example, 1 cm and 1 inch are both lengths. They are used to measure the same dimension, but the specific amounts are different. (In fact, 1 inch is exactly 2.54 cm.)

Every measurement is a number multiplied by its unit. In algebra, the term “ $3x$ ” means “3 times x ”. Similarly, the distance “75 m” means “75 times 1 meter”.

Both the number and its units are necessary to describe any measurement. You **always** need to write the units. Saying that “12 is the same as 12 g” in physics is as ridiculous as saying “12 is the same as $12x$ ” in math.

The metric system is a set of units of measurement that is based on natural quantities (on Earth) and powers of 10.

Use this space for summary and/or additional notes.

The metric system has 7 fundamental “base” units:

- meter (m): length
- kilogram (kg): mass (even though “kilo” is actually a prefix, mass is defined based on the kilogram, not the gram)
- second (s): time
- Kelvin (K): temperature
- mole (mol): amount of substance
- ampere (A): electric current
- candela (cd): intensity of light

Each of these base units is defined in some way that could be duplicated in a laboratory anywhere on Earth (except for the kilogram, which is defined by a physical object that is locked in a vault in the village of Sevres, France). All other metric units are combinations of one or more of these seven.

For example:

Velocity (speed) is a change in distance over a period of time, which would have units of distance/time (m/s).

Force is a mass subjected to an acceleration. Acceleration has units of distance/time² (m/s^2), and force has units of mass \times acceleration. In the metric system this combination of units ($\text{kg}\cdot\text{m/s}^2$) is called a newton (symbol “N”), which means: $1 \text{ N} \equiv 1 \text{ kg}\cdot\text{m/s}^2$

(The symbol “ \equiv ” means “is identical to,” whereas the symbol “=” means “is equivalent to”.)

Use this space for summary and/or additional notes.

The metric system uses prefixes to indicate multiplying a unit by a power of ten. There are prefixes for powers of ten from 10^{-18} to 10^{18} . The most commonly used prefixes are:

- mega (M) = $10^6 = 1\,000\,000$
- kilo (k) = $10^3 = 1\,000$
- centi (c) = $10^{-2} = \frac{1}{100} = 0.01$
- milli (m) = $10^{-3} = \frac{1}{1,000} = 0.001$
- micro (μ) = $10^{-6} = \frac{1}{1\,000\,000} = 0.000\,001$

These prefixes can be used in combination with any metric unit, and they multiply just like units. “35 cm” means “35 times c times m” or “ $(35)(\frac{1}{100})(m)$ ”. If you multiply this out, you get 0.35 m.

Any metric prefix is allowed with any metric unit.

For example, standard atmospheric pressure is 101 325 Pa. This same number could be written as 101.325 kPa or 0.101 325 MPa.

There is a popular geek joke based on the ancient Greek heroine Helen of Troy. She was said to have been the most beautiful woman in the world, and she was an inspiration to the entire Trojan fleet. She was described as having “the face that launched a thousand ships.” Therefore a milliHelen must be the amount of beauty required to launch one ship.

Use this space for summary and/or additional notes.

Conversions

If you need to convert from one prefix to another, the rule of thumb is that if the prefix gets larger, then the number needs to get smaller and vice-versa.

For example, suppose we need to convert 0.25 mg to μg .

The prefix "m" means 10^{-3} and " μ " means 10^{-6} . Therefore, you can replace the prefix "m" with the number 10^{-3} , and the prefix " μ " with the number 10^{-6} .

You can think of the rule in the following way:

$$\begin{array}{c} 0.25 \text{ mg} = x \mu\text{g} \\ \downarrow \qquad \qquad \downarrow \\ (0.25) (0.001) \text{ g} = (x) (0.000\ 001) \text{ g} \\ \\ \frac{(0.25)(0.001)}{0.000\ 001} = 250 \text{ g} = x \end{array}$$

To do this in your head using "common sense," you can realize that when you jump from milli to micro, the prefix is getting *smaller* by 3 decimal places (going from 10^{-3} to 10^{-6}), which means the number that is attached to it needs to get *bigger* by 3 decimal places in order to keep the combined number-and-prefix the same. The answer must therefore be 250 μg .

Use this space for summary and/or additional notes.

The MKS vs. cgs Systems

Because physics heavily involves units that are derived from other units, it is important that you make sure all quantities are expressed in the appropriate units before applying formulas. (This is how we get around having to do factor-label unit-cancelling conversions—like you learned in chemistry—for every single physics problem.)

There are two measurement systems used in physics. In the MKS, or “meter-kilogram-second” system, units are derived from the S.I. units of meters, kilograms, seconds, moles, Kelvins, amperes, and candelas. In the cgs, or “centimeter-gram-second” system, units are derived from the units of centimeters, grams, seconds, moles, Kelvins, amperes, and candelas. The following table shows some examples:

Quantity	MKS Unit	S.I. Equivalent	cgs Unit	S.I. Equivalent
force	newton (N)	$\frac{\text{kg}\cdot\text{m}}{\text{s}^2}$	dyne (dyn)	$\frac{\text{g}\cdot\text{cm}}{\text{s}^2}$
energy	joule (J)	$\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$	erg	$\frac{\text{g}\cdot\text{cm}^2}{\text{s}^2}$
magnetic flux density	tesla (T)	$\frac{\text{N}}{\text{A}}, \frac{\text{kg}\cdot\text{m}}{\text{A}\cdot\text{s}^2}$	gauss (G)	$\frac{0.1 \text{ dyn}}{\text{A}}, \frac{0.1 \text{ g}\cdot\text{cm}}{\text{A}\cdot\text{s}^2}$

In this class, we will use exclusively MKS units. This means you only have to learn one set of derived units. However, you can see the importance, when you solve physics problems, of making sure all of the quantities are in MKS units before you plug them into a formula!

Homework Problems

1. 450 nm = _____ m
2. 18.1 ml = _____ μl
3. $68\,300 \text{ J}$ = _____ kJ
4. $6.56 \times 10^4 \text{ kg}$ = _____ tonne

Use this space for summary and/or additional notes.

Scientific Notation

Unit: Mathematics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Skills:

- Be able to convert numbers to and from scientific notation.
- Be able to enter numbers in scientific notation correctly on your calculator.

Language Objectives:

- Accurately describe and apply the concepts described in this section using appropriate academic language.

Notes:

Scientific notation is a way of writing a very large or very small number in compact form. The value is always written as a number between 1 and 10 multiplied by a power of ten.

For example, the number 1000 would be written as 1×10^3 . The number 0.000075 would be written as 7.5×10^{-5} . The number 602000000000000000000000 would be written as 6.02×10^{23} . The number 0.0000000000000000000000000000663 would be written as 6.6×10^{-34} .

(Note: in science, large numbers are typeset with a space after every three digits, both before and after the decimal point. To avoid confusion, commas are not used because in some countries, the comma is used as a decimal point.)

Scientific notation is really just math with exponents, as shown by the following examples:

$$5.6 \times 10^3 = 5.6 \times 1000 = 5600$$

$$2.17 \times 10^{-2} = 2.17 \times \frac{1}{10^2} = 2.17 \times \frac{1}{100} = \frac{2.17}{100} = 0.0217$$

Use this space for summary and/or additional notes.

Notice that if 10 is raised to a positive exponent means you're multiplying by a power of 10. This makes the number larger, and the decimal point moves to the right. If 10 is raised to a negative exponent, you're actually dividing by a power of 10. This makes the number smaller, and the decimal point moves to the left.

Significant figures are easy to use with scientific notation: all of the digits before the “ \times ” sign are significant. The power of ten after the “ \times ” sign represents the (insignificant) zeroes, which would be the rounded-off portion of the number. In fact, the mathematical term for the part of the number before the “ \times ” sign is the significand.

Math with Scientific Notation

Because scientific notation is just a way of rewriting a number as a mathematical expression, all of the rules about how exponents work apply to scientific notation.

Adding & Subtracting: adjust one or both numbers so that the power of ten is the same, then add or subtract the significands.

$$\begin{aligned} (3.50 \times 10^{-6}) + (2.7 \times 10^{-7}) &= (3.50 \times 10^{-6}) + (0.27 \times 10^{-6}) \\ &= (3.50 + 0.27) \times 10^{-6} = 3.77 \times 10^{-6} \end{aligned}$$

Multiplying & dividing: multiply or divide the significands. If multiplying, add the exponents. If dividing, subtract the exponents.

$$\frac{6.2 \times 10^8}{3.1 \times 10^{10}} = \frac{6.2}{3.1} \times 10^{8-10} = 2.0 \times 10^{-2}$$

Exponents: raise the significand to the exponent. Multiply the exponent of the power of ten by the exponent to which the number is raised.

$$(3.00 \times 10^8)^2 = (3.00)^2 \times (10^8)^2 = 9.00 \times 10^{(8 \times 2)} = 9.00 \times 10^{16}$$

Use this space for summary and/or additional notes.

Using Scientific Notation on Your Calculator

Scientific calculators are designed to work with numbers in scientific notation. It's possible to enter the number as a math problem (always use parentheses if you do this!) but math operations can introduce mistakes that are hard to catch.

Scientific calculators all have some kind of scientific notation button. The purpose of this button is to enter numbers directly into scientific notation and make sure the calculator stores them as a single number instead of a math equation. (This prevents you from making PEMDAS errors when working with numbers in scientific notation on your calculator.) On most Texas Instruments calculators, such as the TI-30 or TI-83, you would do the following:

What you type	What the calculator shows	What you would write
6.6 EE -34	6.6E-34	6.6×10^{-34}
1.52 EE 12	1.52E12	1.52×10^{12}
-4.81 EE -7	-4.81E-7	-4.81×10^{-7}

On some calculators, the scientific notation button is labeled **EXP** or **$\times 10^x$** instead of **EE**.

Important note: many high school students are afraid of the **EE** button because it is unfamiliar. If you are afraid of your **EE** button, you need to get over it and start using it anyway. However, if you insist on clinging to your phobia, you need to at least use parentheses around all numbers in scientific notation, in order to minimize the likelihood of PEMDAS errors in your calculations.

Use this space for summary and/or additional notes.

Homework Problems

Convert the following between scientific notation and algebraic notation.

1. 2.65×10^9 = _____

2. 1.06×10^{-7} = _____

3. 387 000 000 = _____

4. 0.000 000 065 = _____

Solve the following expressions. Be sure to include the correct units.

5. $(\frac{1}{2})(2.5\text{kg})(10.\frac{\text{m}}{\text{s}})^2$ = _____

6. $\frac{3.75 \times 10^8 \text{ m}}{1.25 \times 10^4 \text{ s}}$ = _____

7. $\frac{1.2 \times 10^{-3} \text{ N}}{5.0 \times 10^{-7} \text{ C}}$ = _____

Use this space for summary and/or additional notes.

Right-Angle Trigonometry

Unit: Mathematics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding Goals:

- trigonometry functions that are used heavily in physics

Skills:

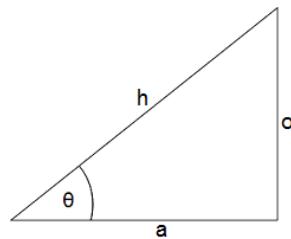
- find the x -component and y -component of a vector

Language Objectives:

- Understand and correctly use the terms “sine,” “cosine,” and “tangent.”
- Accurately describe and apply the concepts described in this section, using appropriate academic language.

Notes:

If we have the following triangle:



- side “ h ” (the longest side, opposite the right angle) is the hypotenuse.
- side “ o ” is the side of the triangle that is opposite (across from) angle θ .
- side “ a ” is the side of the triangle that is adjacent to (connected to) angle θ (and is not the hypotenuse).

Use this space for summary and/or additional notes.

In a right triangle, the ratios of the lengths of the sides will be a function of the angles, and vice-versa.

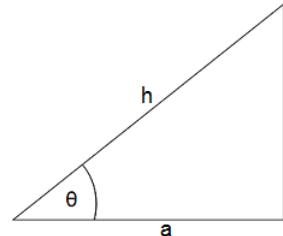
Trigonometry (from “trig” = “triangle” and “ometry” = “measurement”) is the study of these relationships.

The three primary trigonometry functions are defined as follows:

$$\sin\theta = \sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{o}{h}$$

$$\cos\theta = \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{a}{h}$$

$$\tan\theta = \tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{o}{a}$$

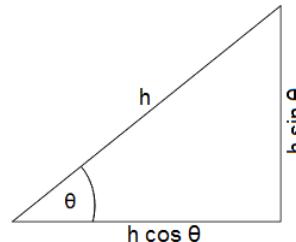


There are a lot of stupid mnemonics for remembering which sides are involved in which functions. My favorite of these is “Oh hell, another hour of algebra!”

The most common use of trigonometry functions in physics is to decompose a vector into its components in the x - and y -directions. In this situation, the vector is the hypotenuse. If we know the angle of the vector, we can use trigonometry and algebra to find the components of the vector in the x - and y -directions:

$$\cos\theta = \frac{a}{h} \text{ which means } a = h\cos\theta$$

$$\sin\theta = \frac{o}{h} \text{ which means } o = h\sin\theta$$

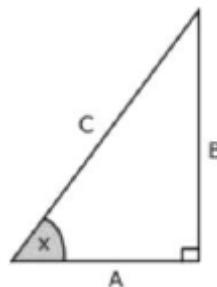


This is often necessary in physics problems involving gravity. Because gravity acts only in the y -direction, the formulas that apply in the y -direction are often different from the ones that apply in the x -direction.

Use this space for summary and/or additional notes.

Homework Problems

Questions 2–5 are based on the following right triangle, with sides A , B , and C , and angle x between A and C .



Note that the drawing is not to scale, and that angle x and the lengths of A , B and C will be different for each problem.

Some problems may also require use of the fact that the angles of a triangle add up to 180° .

1. If $A = 5$ and $C = 13$, what is B ?

2. If $A = 5$ and $C = 13$, what is $\sin x$?

3. If $C = 20$ and $x = 50^\circ$, what are A and B ?

Use this space for summary and/or additional notes.

4. If $A = 100$ and $C = 150$, what is x ?

5. If $B = 100$ and $C = 150$, what is x ?

6. You are a golfer, and your ball is in a sand trap with a hill next to it. You need to hit your ball so that it goes over the hill to the green. If your ball is 10. m away from the side of the hill and the hill is 2.5 m high, what is the minimum angle above the horizontal that you need to hit the ball in order to just get it over the hill?

7. If a force of 80 N is applied at an angle of 40° above the horizontal, how much of that force is applied in the horizontal direction?

Use this space for summary and/or additional notes.

Vectors

Unit: Mathematics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding Goals:

- what a vector is

Skills:

- adding & subtracting vectors

Language Objectives:

- Understand and correctly use the terms “vector,” “scalar,” and “magnitude.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Notes:

vector: a quantity that has both a magnitude (value) and a direction.

scalar: a quantity that has a value but does not have a direction. (A scalar is what you think of as a “regular” number, including its unit.)

magnitude: the scalar part of a vector (*i.e.*, the number and its units, but without the direction). If you have a force of 25 N to the east, the magnitude of the force is 25 N.

The mathematical operation of taking the magnitude of a vector is represented by two double vertical bars (like double absolute value bars) around the vector. For example, if \vec{F} is 25 N to the east, then $\|\vec{F}\|=25\text{ N}$.

Note, however, that the College Board generally uses regular absolute value bars for magnitude, *e.g.*, $|\vec{F}|=25\text{ N}$.

resultant: a vector that results from a mathematical operation (such as the addition of two vectors).

Use this space for summary and/or additional notes.

Variables that represent vectors are traditionally typeset in ***bold Italic***. Vector variables may also optionally have an arrow above the letter:

$$\mathbf{J}, \vec{F}, \mathbf{v}$$

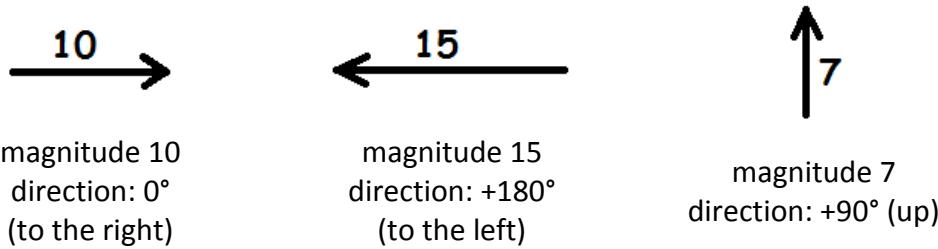
Variables that represent scalars are traditionally typeset in *plain Italics*:

$$V, t, \lambda$$

Note that a variable that represents only the magnitude of a vector quantity is generally typeset as a scalar:

For example, \vec{F} is a vector representing a force of 25 N to the east. (Notice that the vector includes the magnitude or amount **and** the direction.) The magnitude would be 25 N, and would be represented by the variable F .

Vectors are represented graphically using arrows. The length of the arrow represents the magnitude of the vector, and the direction of the arrow represents the direction of the vector:



Use this space for summary and/or additional notes.

Adding & Subtracting Vectors

If the vectors have the same direction or opposite directions, the resultant is easy to envision:

$$\begin{array}{c} \overrightarrow{5} \\ + \end{array} \quad \begin{array}{c} \overrightarrow{5} \\ + \end{array} = \begin{array}{c} \overrightarrow{10} \end{array}$$

$$\begin{array}{c} \overrightarrow{5} \\ + \end{array} \quad \begin{array}{c} \overleftarrow{-5} \\ + \end{array} = \begin{array}{c} \overrightarrow{0} \end{array}$$

$$\begin{array}{c} \overrightarrow{5} \\ + \end{array} \quad \begin{array}{c} \overrightarrow{10} \\ + \end{array} = \begin{array}{c} \overrightarrow{15} \end{array}$$

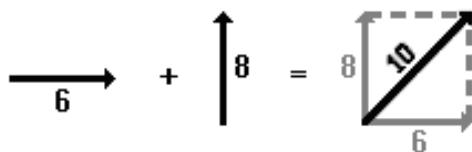
$$\begin{array}{c} \overrightarrow{5} \\ + \end{array} \quad \begin{array}{c} \overleftarrow{-10} \\ + \end{array} = \begin{array}{c} \overleftarrow{-5} \end{array}$$

$$\begin{array}{c} \overrightarrow{5} \\ + \end{array} \quad \begin{array}{c} \overleftarrow{-15} \\ + \end{array} = \begin{array}{c} \overleftarrow{-10} \end{array}$$

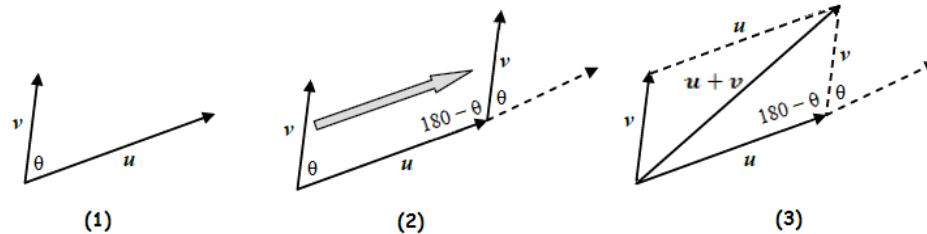
$$\begin{array}{c} \uparrow \\ 10 \\ + \end{array} \quad \begin{array}{c} \downarrow \\ -5 \\ + \end{array} = \begin{array}{c} \uparrow \\ 5 \end{array}$$

Use this space for summary and/or additional notes.

If the vectors are not in the same direction, we move them so they start from the same place and complete the parallelogram. If they are perpendicular, we can add them using the Pythagorean theorem:

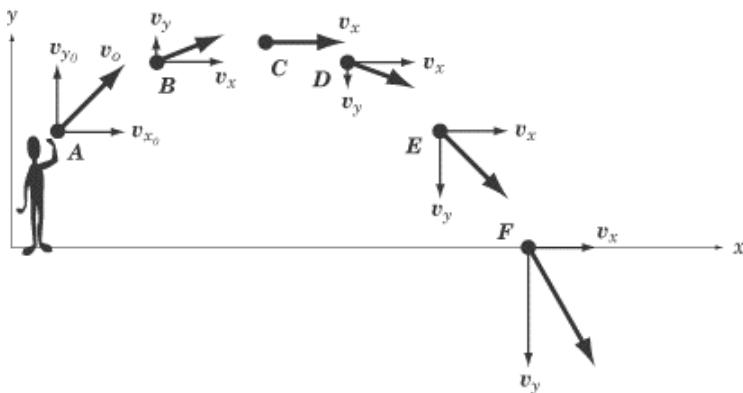


The same process applies to adding vectors that are not perpendicular:



However, the trigonometry needed for the calculations is more involved and is beyond the scope of this course.

One type of physics problem that commonly uses vectors is two-dimensional projectile motion. If the motion of the projectile is represented by a vector, \vec{v} , at angle θ , the vector can be represented as the sum of a horizontal vector \vec{v}_x and a vertical vector \vec{v}_y . This is useful because the horizontal vector gives us the component (portion) of the vector in the x-direction, and the vertical vector gives us the component of the vector in the y-direction.

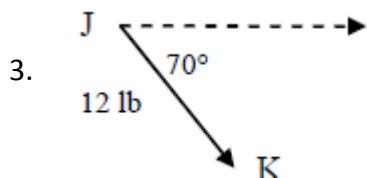
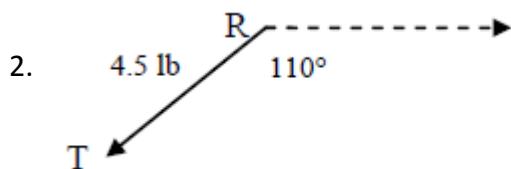
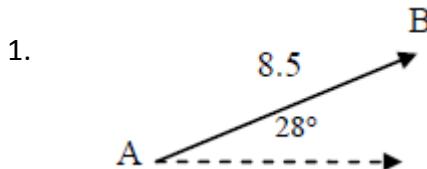


Notice that \vec{v}_x remains constant, but \vec{v}_y changes (because of the effects of gravity).

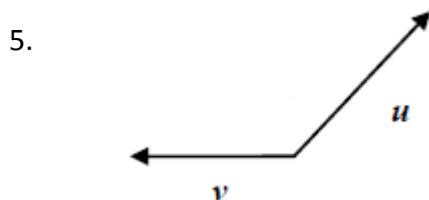
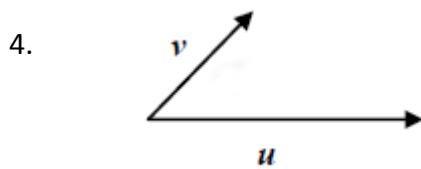
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Homework Problems

State the magnitude and direction of each of the following:

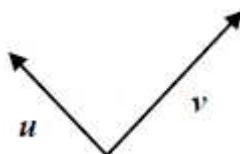


Sketch the resultant of each of the following.

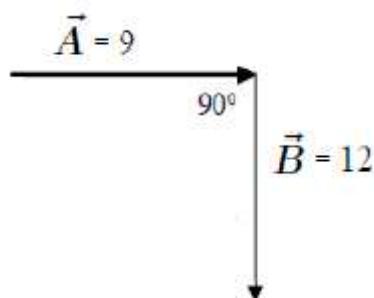


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6.



For the following vectors \vec{A} & \vec{B} :



7. Determine $\vec{A} + \vec{B}$ (both magnitude and direction)

8. Determine $\vec{A} - \vec{B}$ (both magnitude and direction)

Use this space for summary and/or additional notes.

Vectors vs. Scalars in Physics

Unit: Mathematics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding Goals:

- how vectors are used in an algebra-based physics course

Language Objectives:

- Accurately describe and apply the concepts described in this section, including the difference between “vector” and “scalar,” using appropriate academic language.

Notes:

In physics, most numbers represent quantities that can be measured or calculated from measurements. Most of the time, there is no concept of a “deficit” of a measured quantity. For example, quantities like mass, energy, and power can only be nonnegative, because in classical mechanics there is no such thing as “anti-mass,” “anti-energy,” or “anti-power.”

However, vector quantities have a direction as well as a magnitude, and direction can be positive or negative.

A rule of thumb that works *most* of the time in this class is:

Scalar quantities. These are almost always positive. (Note, however, that we will encounter some exceptions during the year. An example is electric charge, which can be positive or negative.)

Vector quantities. Vectors can be positive or negative. In any given problem, you will choose which direction is positive. Vectors in the positive direction will be expressed as positive numbers, and vectors in the opposite (negative) direction will be expressed as negative numbers.

Differences. The difference or change in a variable is indicated by the Greek letter Δ in front of the variable. Any difference can be positive or negative. However, note that a difference can be a vector, indicating a change relative to the positive direction (e.g., $\Delta\mathbf{x}$, which indicates a change in position), or scalar, indicating an increase or decrease (e.g., ΔV , which indicates a change in volume).

Use this space for summary and/or additional notes.

In some cases, you will need to split a vector in two vectors, one vector in the x -direction, and a separate vector in the y -direction. In these cases, you will need to choose which direction is positive and which direction is negative for both the x - and y -axes. Once you have done this, every vector quantity must be assigned a positive or negative value, according to the directions you have chosen.

Example:

Suppose you have a problem that involves throwing a ball straight upwards with a velocity of $15 \frac{\text{m}}{\text{s}}$. Gravity is slowing the ball down with a downward acceleration of $9.8 \frac{\text{m}}{\text{s}^2}$. You want to know how far the ball has traveled in 0.5 s.

Displacement, velocity, and acceleration are all vectors. The motion is happening in the y -direction, so we need to choose whether “up” or “down” is the positive direction. Suppose we choose “up” to be the positive direction. This means:

- When the ball is first thrown, it is moving upwards. This means its velocity is in the positive direction, so we would represent the initial velocity as $\vec{v}_o = +15 \frac{\text{m}}{\text{s}}$.
- Gravity is accelerating the ball downwards, which is the negative direction. We would therefore represent the acceleration as $\vec{a} = -9.8 \frac{\text{m}}{\text{s}^2}$.
- Time is a scalar quantity, so it can only be positive.

If we had to substitute the numbers into the formula:

$$\vec{d} = \vec{v}_o t + \frac{1}{2} \vec{a} t^2$$

we would do so as follows:

$$\vec{d} = (+15)(0.5) + \left(\frac{1}{2}\right)(-9.8)(0.5)^2$$

and we would find out that $\vec{d} = +6.275 \text{ m}$.

The answer is positive. Earlier, we defined positive as “up”, so the answer tells us that the displacement is upwards from the starting point.

Use this space for summary and/or additional notes.

What if, instead, we had chosen “down” to be the positive direction?

- When the ball is first thrown, it is moving upwards. This means its velocity is now in the negative direction, so we would represent the initial velocity as $\vec{v}_o = -15 \frac{\text{m}}{\text{s}}$.
- Gravity is accelerating the ball downwards, which is the positive direction. We would therefore represent the acceleration as $\vec{a} = +9.8 \frac{\text{m}}{\text{s}^2}$.
- Time is a scalar quantity, so it can only be positive.

If we had to substitute the numbers into the formula:

$$\vec{d} = \vec{v}_o t + \frac{1}{2} \vec{a} t^2$$

we would do so as follows:

$$\vec{d} = (-15)(0.5) + \left(\frac{1}{2}\right)(9.8)(0.5)^2$$

and we would find out that $\vec{d} = -6.275 \text{ m}$.

The answer is negative. Remember that “down” was positive, which means “up” is the negative direction. This means the displacement is upwards from the starting point, as before.

Remember: in any problem you solve, the choice of which direction is positive vs. negative is arbitrary. The only requirement is that every vector quantity in the problem needs to be consistent with your choice.

Use this space for summary and/or additional notes.

Vector Multiplication

Unit: Mathematics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Skills:

- dot product & cross product of two vectors

Language Objectives:

- Accurately describe and apply the concepts described in this section using appropriate academic language.

Notes:

With scalar (ordinary) numbers, there is only one way to multiply them, which you learned in elementary school. Vectors, however, can be multiplied in three different ways.

dot product: multiplication of two vectors that results in a scalar.

cross product: multiplication of two vectors that results in a new vector.

tensor product: multiplication of two vectors that results in a tensor. (A tensor is an array of vectors that describes the effect of each vector on each other vector within the array. We will not use tensors in a high school physics course.)

Use this space for summary and/or additional notes.

Multiplying a Vector by a Scalar

Multiplying a vector by a scalar is like multiplying a variable by a number. The magnitude changes, but the direction does not. For example, in physics, displacement equals velocity times time:

$$\vec{d} = \vec{v}t$$

Velocity is a vector; time is a scalar. The magnitude is the velocity times the time, and the direction of the displacement is the same as the direction of the velocity.

The Dot (Scalar) Product of Two Vectors

The scalar product of two vectors is called the “dot product”. Dot product multiplication of vectors is represented with a dot:

$$\vec{A} \bullet \vec{B}$$
^{*}

The dot product of \vec{A} and \vec{B} is:

$$\vec{A} \bullet \vec{B} = AB \cos\theta$$

where A is the magnitude of \vec{A} , B is the magnitude of \vec{B} , and θ is the angle between the two vectors \vec{A} and \vec{B} .

For example, in physics, work (a scalar quantity) is the dot product of the vectors force and displacement (distance):

$$W = \vec{F} \bullet \vec{d} = Fd \cos\theta$$

* pronounced “A dot B”

Use this space for summary and/or additional notes.

The Cross (Vector) Product of Two Vectors

The vector product of two vectors is called the cross product. Cross product multiplication of vectors is represented with a multiplication sign:

$$\vec{A} \times \vec{B}^*$$

The cross product of vectors \vec{A} and \vec{B} that have an angle of θ between them is given by the formula:

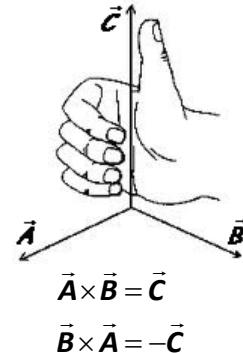
$$\vec{A} \times \vec{B} = AB \sin \theta \hat{n}$$

where the magnitude is $AB \sin \theta$, and the vector \hat{n} is the direction. ($AB \sin \theta$ is a scalar. The unit vector \hat{n} is what gives the vector its direction.)

The direction of the cross product is a little difficult to make sense out of. You can figure it out using the “right hand rule”:

Position your right hand so that your fingers curl from the first vector to the second. Your thumb points in the direction of the resultant vector (\hat{n}).

Note that this means that the resultant vectors for $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$ will point in *opposite* directions, i.e., the cross product of two vectors is not commutative!

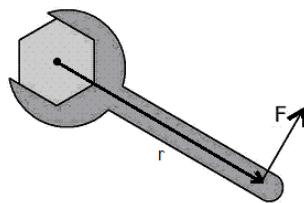


A vector coming out of the page is denoted by a series of symbols, and a vector going into the page is denoted by a series of symbols. The symbols represent an arrow inside a tube. The dot represents the tip of the arrow coming toward you, and the “X” represents the fletches (feathers) on the tail of the arrow going away from you.)

* pronounced “A cross B”

Use this space for summary and/or additional notes.

In physics, torque is a vector quantity that is derived by a cross product.



The torque produced by a force \vec{F} acting at a radius \vec{r} is given by the equation:

$$\vec{\tau} = \vec{r} \times \vec{F} = rF \sin\theta \hat{n}$$

Because the direction of the force is usually perpendicular to the displacement, it is usually true that $\sin \theta = \sin 90^\circ = 1$. This means the magnitude $rF \sin \theta = rF(1) = rF$. Using the right-hand rule, we determine that the *direction* of the resultant torque vector (\hat{n}) is coming out of the page.

Thus, if you are tightening or loosening a nut or bolt that has right-handed (standard) thread, the torque vector will be in the direction that the nut or bolt moves.

Vector Jokes

Now that you understand vectors, here are some bad vector jokes:

Q: What do you get when you cross an elephant with a bunch of grapes?

A: $\sin \theta \hat{n}$

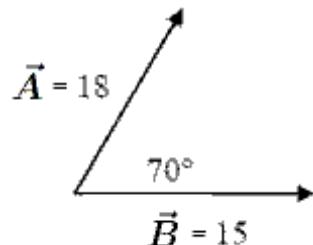
Q: What do you get when you cross an elephant with a mountain climber?

A: You can't do that! A mountain climber is a scalar ("scaler," meaning someone who scales a mountain).

Use this space for summary and/or additional notes.

Homework Problems

For the following vectors \vec{A} & \vec{B} :



1. Determine $\vec{A} \bullet \vec{B}$

2. Determine $\vec{A} \times \vec{B}$ (both magnitude and direction)

Use this space for summary and/or additional notes.

Introduction: Kinematics (Motion)

Unit: Kinematics (Motion)

Topics covered in this chapter:

Linear Motion, Speed & Velocity	112
Linear Acceleration	117
Newton's Equations of Motion.....	128
Centripetal Acceleration	134
Solving Motion Problems.....	136
Projectile Motion	141

In this chapter, you will study how things move and how the relevant quantities are related.

- *Motion, Speed & Velocity* and *Acceleration* deal with understanding and calculating the velocity (change in position) and acceleration (change in velocity) of an object, and with representing and interpreting graphs involving these quantities.
- *Projectile Motion* deals with an object that has two-dimensional motion—moving horizontally and also affected by gravity.

Some of the challenging tasks include identifying quantities from their units, choosing the equation that relates the quantities of interest, and keeping track of positive and negative directions when working with vector quantities.

Textbook:

- *Physics: Principles and Problems* Ch. 3: Describing Motion: Velocity (pp. 40–61)
- *Physics: Principles and Problems* Ch. 4: Acceleration (pp. 62–79)
- *Physics: Principles and Problems* Ch. 7: Motion in Two Dimensions (pp. 132–153)

Use this space for summary and/or additional notes.

Standards addressed in this chapter:

Next Generation Science Standards (NGSS):

No NGSS standards are addressed in this chapter.

Massachusetts Curriculum Frameworks (2006):

1.4 Interpret and apply Newton's three laws of motion.

1.5 Use a free-body force diagram to show forces acting on a system consisting of a pair of interacting objects. For a diagram with only co-linear forces, determine the net force acting on a system and between the objects.

1.6 Distinguish qualitatively between static and kinetic friction, and describe their effects on the motion of objects.

1.7 Describe Newton's law of universal gravitation in terms of the attraction between two objects, their masses, and the distance between them.

Topics from this chapter assessed on the SAT Physics Subject Test:

Kinematics, such as velocity, acceleration, motion in one dimension, and motion of projectiles

1. Displacement
2. Speed, velocity and acceleration
3. Kinematics with graphs
4. One-dimensional motion with uniform acceleration
5. Two-dimensional motion with uniform acceleration

Skills learned & applied in this chapter:

- Choosing from a set of equations based on the quantities present.
- Working with vector quantities.
- Relating the slope of a graph and the area under a graph to equations.
- Using graphs to represent and calculate quantities.
- Keeping track of things happening in two directions at once.

Use this space for summary and/or additional notes.

Linear Motion, Speed & Velocity

Unit: Kinematics (Motion)

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 1.1, 1.2

Knowledge/Understanding Goals:

- understand terms relating to position, speed & velocity
- understand the difference between speed and velocity

Language Objectives:

- Understand and correctly use the terms “position,” “distance,” “displacement,” “speed,” and “velocity.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Labs, Activities & Demonstrations:

- Walk in the positive and negative directions (with positive or negative velocity).
- Walk and change direction to show distance vs. displacement.

Notes:

coördinate system: a framework for describing an object’s position (location), based on its distance (in one or more directions) from a specifically-defined point (the origin). (You should remember these terms from math.)

direction: which way an object is oriented or moving within its coördinate system. Note that direction can be positive or negative.

position (x): the location of an object relative to the origin (zero point) of its coördinate system. We will consider position to be a zero-dimensional vector, which means it can be positive or negative with respect to the chosen coördinate system.

distance (d): [scalar] how far an object has moved.

Use this space for summary and/or additional notes.

displacement (\vec{d} or Δx): [vector] how far an object's current position is from its starting position ("initial position"). Displacement can be positive or negative (or zero), depending on the chosen coordinate system.

rate: the change in a quantity over a specific period of time.

motion: when an object's position is changing over time.

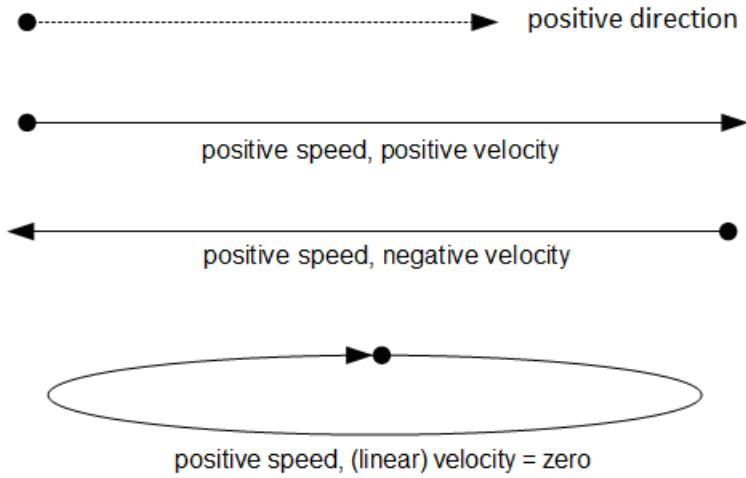
speed: [scalar] the rate at which an object is moving at an instant in time. Speed does not depend on direction, and is always nonnegative.

velocity: (\vec{v}) [vector] an object's displacement over a given period of time.

Because velocity is a vector, it has a direction as well as a magnitude. Velocity can be positive, negative, or zero.

uniform motion: motion at a constant velocity (*i.e.*, with constant speed and direction)

An object that is moving has a positive speed, but its velocity may be positive, negative, or zero, depending on its position.



Use this space for summary and/or additional notes.

Variables Used to Describe Linear Motion

Variable	Quantity	MKS Unit
x	position	m
$d, \Delta x$	distance	m
$\vec{d}, \Delta \vec{x}$	displacement	m
h	height	m
\vec{v}	velocity	$\frac{\text{m}}{\text{s}}$
\bar{v}	average velocity	$\frac{\text{m}}{\text{s}}$

The average velocity of an object is its displacement divided by the time, or its change in position divided by the (change in) time:

$$\bar{v} = \frac{\vec{d}}{t} = \frac{x - x_0}{t} = \frac{\Delta x}{t} = \frac{\Delta x}{\Delta t}$$

(Note that elapsed time is always a difference (Δt), though we usually use t rather than Δt as the variable.)

We can use calculus to turn \bar{v} into v by taking the limit as Δt approaches zero:

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

i.e., velocity is the first derivative of displacement with respect to time.

We can rearrange this formula to show that displacement is average velocity times time:

$$\vec{d} = \bar{v}t$$

Position is the object's starting position plus its displacement:

$$x = x_0 + \vec{d} = x_0 + \bar{v}t$$

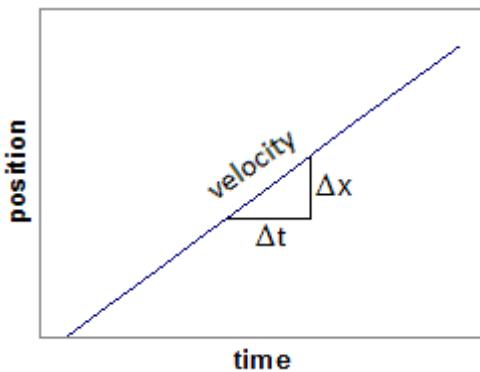
where x_0^* means "position at time = 0". This formula is often expressed as:

$$x - x_0 = \vec{d} = \bar{v}t$$

* x_0 is pronounced "x-zero" or "x-naught".

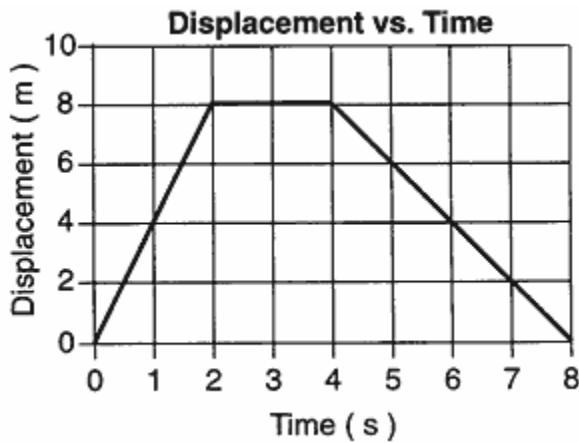
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Note that $\frac{\Delta x}{\Delta t}$ is the slope of a graph of position (x) vs. time (t). Because $\frac{\Delta x}{\Delta t} = v$, this means that the slope of a graph of position vs. time is equal to the velocity.



In fact, *on any graph, the quantity you get when you divide the quantity on the x-axis by the quantity on the y-axis is, by definition, the slope. I.e., the slope is $\frac{\Delta y}{\Delta x}$, which means the quantity defined by $\frac{y - \text{axis}}{x - \text{axis}}$ will always be the slope.*

Recall that velocity is a vector, which means it can be positive, negative, or zero. On the graph below, the velocity is $+4 \frac{\text{m}}{\text{s}}$ from 0 s to 2 s, zero from 2 s to 4 s, and $-2 \frac{\text{m}}{\text{s}}$ from 4 s to 8 s.



Use this space for summary and/or additional notes.

Sample problems:

Q: A car travels 1200 m in 60 seconds. What is its average velocity?

A: $\bar{v} = \frac{d}{t}$

$$\bar{v} = \frac{1200 \text{ m}}{60 \text{ s}} = 20 \frac{\text{m}}{\text{s}}$$

Q: A person walks 320 m at an average velocity of $1.25 \frac{\text{m}}{\text{s}}$. How long did it take?

A: "How long" means what length of time.

$$\bar{v} = \frac{d}{t}$$
$$1.25 = \frac{320}{t}$$
$$t = 256 \text{ s}$$

It took 256 seconds for the person to walk 320 m.

Use this space for summary and/or additional notes.

Linear Acceleration

Unit: Kinematics (Motion)

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 1.1, 1.2

Knowledge/Understanding Goals:

- what linear acceleration means
- what positive vs. negative acceleration means

Skills:

- calculate position, velocity and acceleration for problems that involve movement in one direction

Language Objectives:

- Understand and correctly use the term “acceleration.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Labs, Activities & Demonstrations:

- Walk with different combinations of positive/negative velocity and positive/negative acceleration.
- Drop a dollar bill or meter stick and have someone try to catch it.
- Drop two strings of beads, one spaced at equal distances and the other spaced at equal times.
- Drop a bottle of water with a hole near the bottom or bucket of ping-pong balls.

Notes:

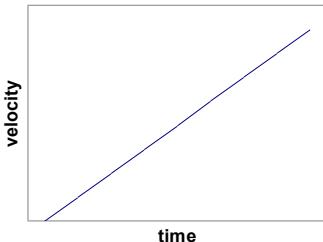
acceleration: a change in velocity over a period of time.

uniform acceleration: when an object’s rate of acceleration (*i.e.*, the rate at which its velocity changes) is constant.

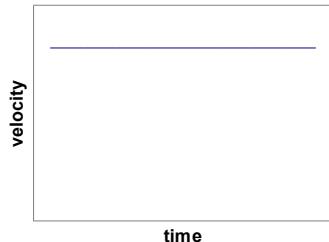
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If an object's velocity is increasing, we say it has positive acceleration.

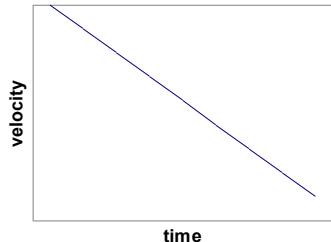
If an object's velocity is decreasing, we say it has negative acceleration.



positive acceleration



acceleration = zero



negative acceleration

Note that if the object's velocity is negative, then increasing velocity (positive acceleration) would mean that the velocity is getting *less negative*, i.e., the object would be slowing down in the negative direction.

Variables Used to Describe Acceleration

Variable	Quantity	MKS Units
\vec{a}	acceleration	$\frac{\text{m}}{\text{s}^2}$
\vec{g}	acceleration due to gravity	$\frac{\text{m}}{\text{s}^2}$

By convention, physicists use the variable \vec{g} to mean acceleration due to gravity, and \vec{a} to mean acceleration caused by something other than gravity.

Use this space for summary and/or additional notes.

Because acceleration is a change in velocity over a period of time, the formula for acceleration is:

$$\bar{a} = \frac{v - v_o}{t} = \frac{\Delta v}{t} = \frac{\Delta v}{\Delta t} \quad \text{and, from calculus: } a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

The units must match the formula, which means the units for acceleration must be velocity (distance/time) divided by time, which equals distance divided by time squared.

Because $v = \frac{dx}{dt}$, this means that acceleration is the second derivative of position

$$\text{with respect to time: } a = \frac{dv}{dt} = \frac{d}{dt}(v) = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

However, in an algebra-based physics course, we will limit ourselves to problems in which acceleration is constant.

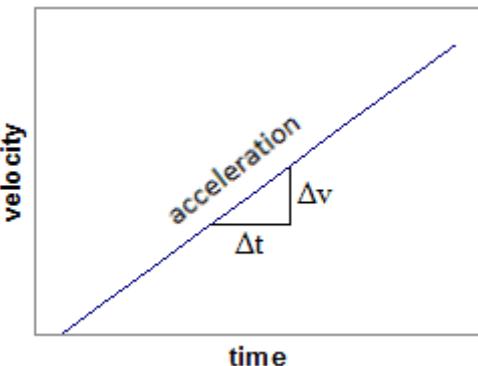
We can rearrange this formula to show that the change in velocity is acceleration times time:

$$\Delta v = v - v_o = at$$

Note that when an object's velocity is changing, the final velocity, v , is not the same as the average velocity, \bar{v} . (This is a common mistake that first-year physics students make.)

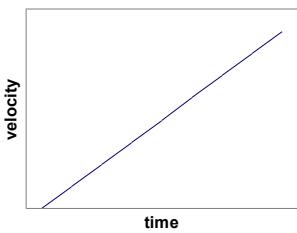
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$\frac{\Delta v}{\Delta t}$ is the slope of a graph of velocity (v) vs. time (t). Because $\frac{\Delta v}{\Delta t} = a$, this means that acceleration is the slope of a graph of velocity vs. time:

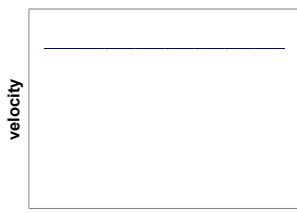


Note the relationship between velocity-time graphs and position-time graphs.

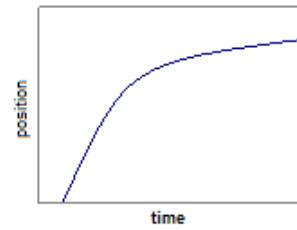
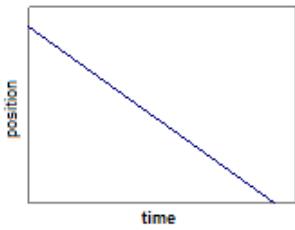
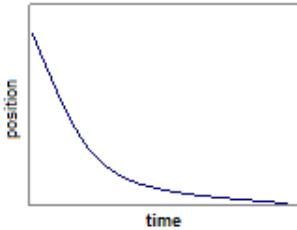
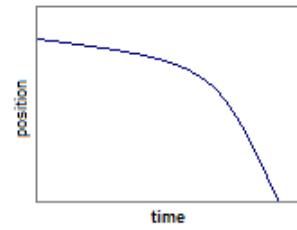
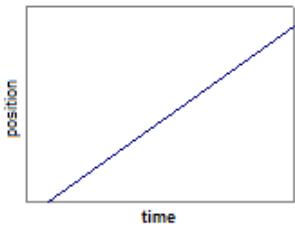
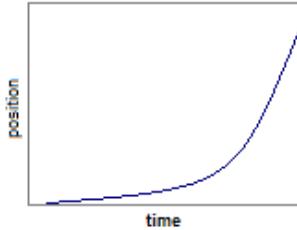
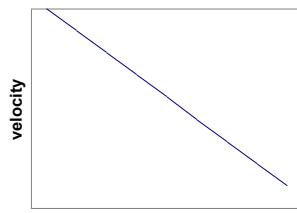
positive acceleration



acceleration = zero



negative acceleration



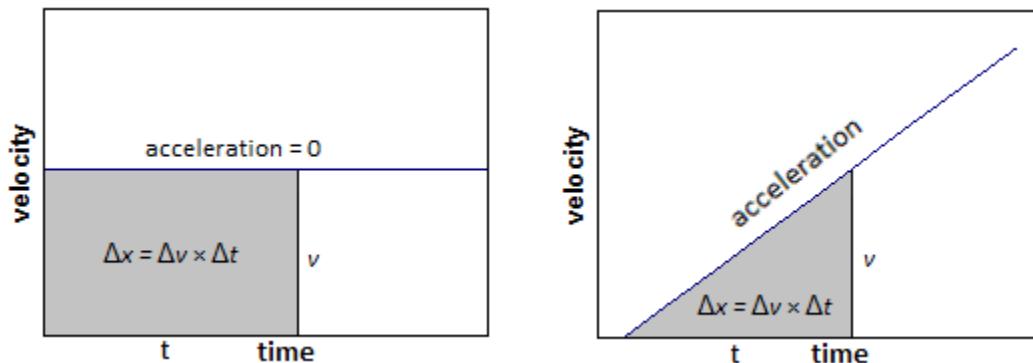
concave up

linear

concave down

Use this space for summary and/or additional notes.

Note also that $\bar{v}t$ is the area under a graph (*i.e.*, the area between the curve and the x-axis) of velocity (v) vs. time (t). Because $\bar{v}t = d$, this means the area under a graph of velocity vs. time is the displacement (Δx). Note that this works both for constant velocity (the graph on the left) and changing velocity (as shown in the graph on the right).



In fact, *on any graph, the quantity you get when you multiply the quantities on the x- and y-axes is, by definition, the area under the graph.*

In calculus, the area under a curve is the integral of the equation for the curve. This means:

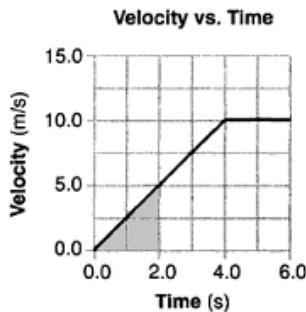
$$d = \int_0^t v dt$$

where v can be any function of t .

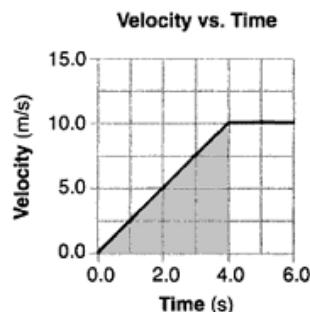
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In the graph below, between 0 s and 4 s the object is accelerating at a rate of $+2.5 \frac{\text{m}}{\text{s}^2}$.

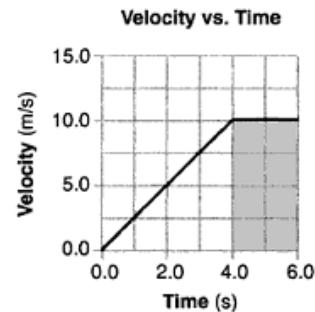
Between 4 s and 6 s the object is moving at a constant velocity (of $+10 \frac{\text{m}}{\text{s}}$), so the acceleration is zero.



$$\begin{aligned}a &= 2.5 \frac{\text{m}}{\text{s}^2} \\d &= \frac{1}{2}(2.5)(2^2) = 5 \text{ m} \\A &= \frac{1}{2}(2)(5) = 5 \text{ m}\end{aligned}$$



$$\begin{aligned}a &= 2.5 \frac{\text{m}}{\text{s}^2} \\d &= \frac{1}{2}(2.5)(4^2) = 20 \text{ m} \\A &= \frac{1}{2}(4)(10) = 20 \text{ m}\end{aligned}$$



$$\begin{aligned}a &= 0 \\d &= \bar{v}t = (10)(2) = 20 \text{ m} \\A &= (2)(10) = 20 \text{ m}\end{aligned}$$

In each case, the area under the velocity-time graph equals the total distance traveled.

Use this space for summary and/or additional notes.

To show the relationship between v and \bar{v} , we can combine the formula for average velocity with the formula for acceleration in order to get a formula for the position of an object that is accelerating.

$$d = \bar{v}t$$

$$v = at$$

However, the problem is that v in the formula $v = at$ is the velocity at the *end*, which is not the same as the *average* velocity \bar{v} .

If the velocity of an object is changing (*i.e.*, the object is accelerating), the average velocity, \bar{v} (the line over the v means “average”), is given by the formula:

$$\bar{v} = \frac{v_o + v}{2}$$

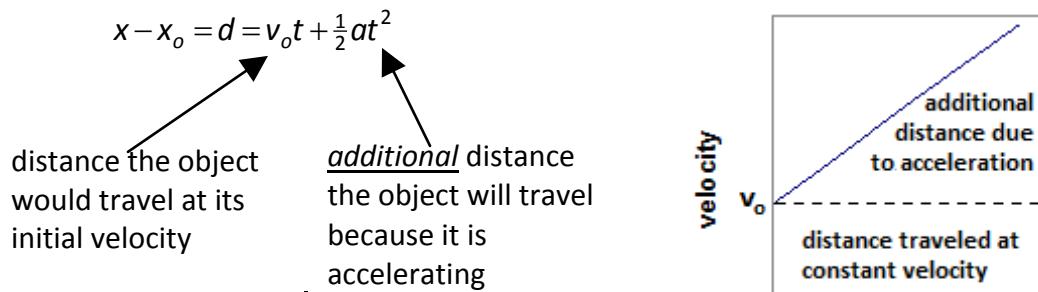
If the object starts at rest (not moving, which means $v_o = 0$) and it accelerates at a constant rate, the average velocity is therefore the average of the initial velocity and the final velocity:

$$\bar{v} = \frac{v_o + v}{2} = \frac{0 + v}{2} = \frac{v}{2} = \frac{1}{2}v$$

Combining all of these gives, for an object starting from rest:

$$d = \bar{v}t = \frac{1}{2}vt = \frac{1}{2}(at)t = \frac{1}{2}at^2$$

If an object was moving before it started to accelerate, it had an initial velocity, or a velocity at time = 0. We will represent this initial velocity as v_o^* . Now, the formula becomes:



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This equation can be combined with the equation for velocity to give the following equation, which relates initial and final velocity and distance:

$$v^2 - v_o^2 = 2ad$$

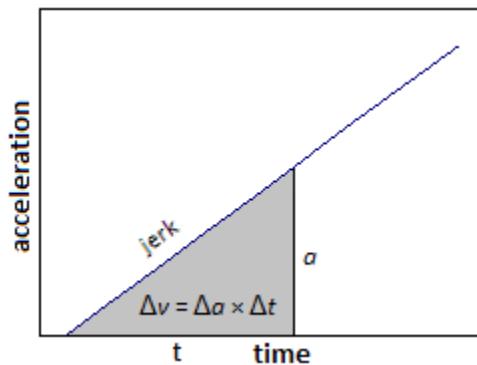
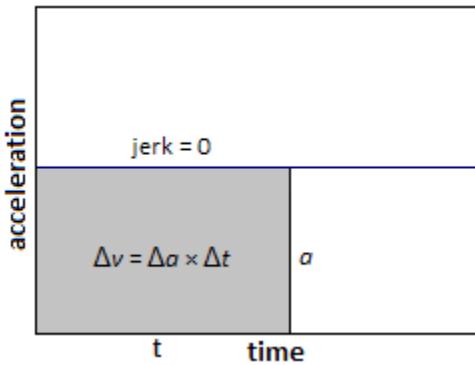
Finally, when an object is accelerating because of gravity, we say that the object is in “free fall”.

On earth, the average acceleration due to gravity is approximately $9.807 \frac{\text{m}}{\text{s}^2}$ at sea level (which we will usually round to $10 \frac{\text{m}}{\text{s}^2}$). Any time gravity is involved (and the problem takes place on Earth), assume that $a = g = 10 \frac{\text{m}}{\text{s}^2}$.

Extensions

Just as a change in velocity is called acceleration, a change in acceleration with respect to time is called “jerk”: $\vec{j} = \frac{\Delta \vec{a}}{\Delta t}$.

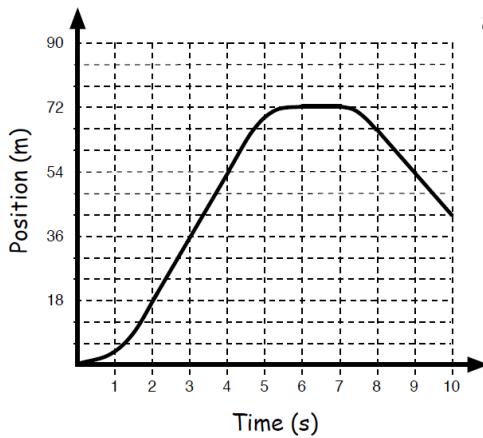
Note that the area under a graph of acceleration vs. time would be the change in velocity (Δv), just as the area under a graph of velocity vs. time is the change in position.



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Homework Problems: Motion Graphs

1. An object's motion is described by the following graph of position vs. time:



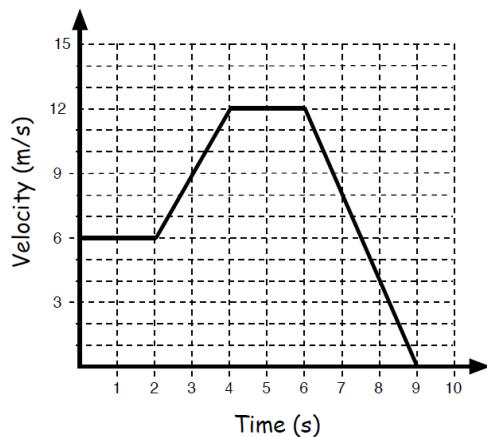
- a. What is the object doing between 2 s and 4 s? What is its velocity during that interval?

- b. What is the object doing between 6 s and 7 s? What is its velocity during that interval?

- c. What is the object doing between 8 s and 10 s? What is its velocity during that interval?

Use this space for summary and/or additional notes.

2. An object's motion is described by the following graph of velocity vs. time:



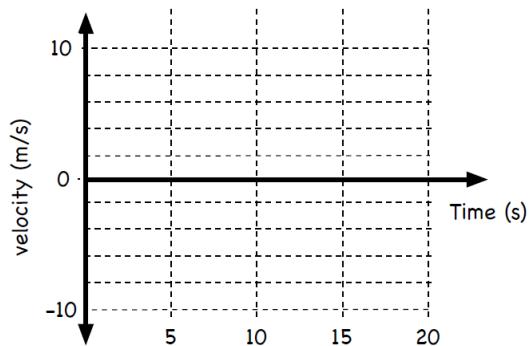
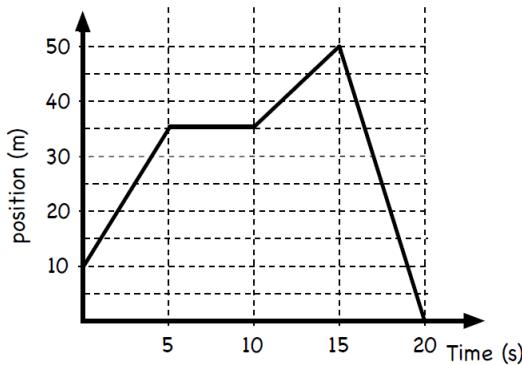
a. What is the object doing between 0 s and 2 s? What are its velocity and acceleration during that interval?

b. What is the object doing between 2 s and 4 s? What is its acceleration during that interval?

c. What is the object doing between 6 s and 9 s? What is its acceleration during that interval?

Use this space for summary and/or additional notes.

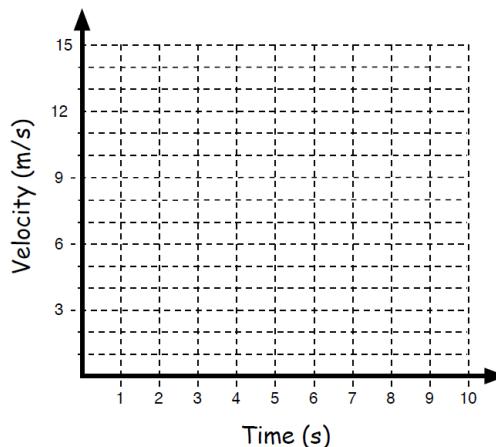
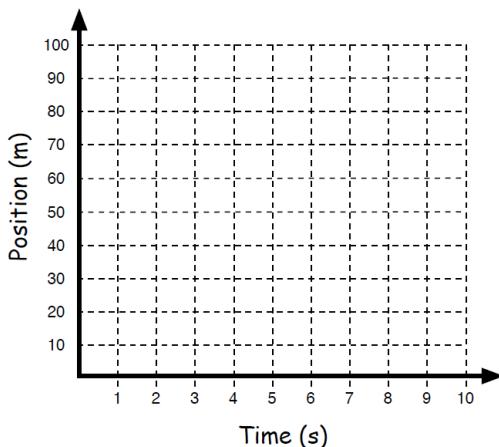
3. The graph on the left below shows the position of an object vs. time. Sketch a graph of velocity vs. time for the same object on a graph similar to the one on the right.



4. In 1991, Carl Lewis became the first sprinter to break the 10-second barrier for the 100 m dash, completing the event in 9.86 s. The chart below shows his time for each 10 m interval.

distance (m)	0	10	20	30	40	50	60	70	80	90	100
time (s)	0	1.88	2.96	3.88	4.77	5.61	6.45	7.29	8.12	8.97	9.86

Plot Lewis's displacement vs. time and velocity vs. time on graphs similar to the ones below.



Use this space for summary and/or additional notes:

Newton's Equations of Motion

Unit: Kinematics (Motion)

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 1.1, 1.2

Skills:

- use Isaac Newton's motion equations to calculate position, velocity and acceleration for problems that involve movement in one direction

Language Objectives:

- Understand, solve and explain problems involving motion and acceleration.

Notes:

Most motion problems can be calculated from Isaac Newton's equations of motion. The following is a summary of the equations presented in the previous sections:

Equation	Description
$\Delta\vec{x} = \vec{d} = \vec{x} - \vec{x}_o$	Definition of displacement.
$\bar{\vec{v}} = \frac{\Delta\vec{x}}{t} = \frac{\vec{d}}{t} = \frac{\vec{v}_o + \vec{v}}{2}$	Average velocity is the distance per unit of time, which also equals the calculated value of average velocity.
$\Delta\vec{v} = \vec{v} - \vec{v}_o = \vec{a}t$	Acceleration is a change in velocity divided by time.
$\vec{x} - \vec{x}_o = \vec{d} = \vec{v}_o t + \frac{1}{2}\vec{a}t^2$	Position at the end equals position at the beginning (x_o) plus the displacement due to velocity ($v_o t$), plus the displacement due to acceleration ($\frac{1}{2}at^2$).
$\vec{v}^2 - \vec{v}_o^2 = 2\vec{a}\vec{d}$	Velocity at the end can be calculated from velocity at the beginning, acceleration, and displacement.

Use this space for summary and/or additional notes.

Selecting the Right Equation

When you are faced with a problem, choose an equation based on the following criteria:

- The equation *must* contain the variable you are looking for.
- All other quantities in the equation must be either given in the problem or assumed from the description of the problem.
 - If an object starts at rest (not moving), that means $\vec{v}_o = 0$.
 - If an object comes to a stop, that means $\vec{v} = 0$. (Remember that \vec{v} is the velocity at the end.)
 - If gravity is involved (*e.g.*, the object is falling), that means $\vec{a} = \vec{g} \approx 10 \frac{\text{m}}{\text{s}^2}$. (Applies to linear acceleration problems only.)

This means you can choose the appropriate equation by making a list of what you are looking for and what you know. The equation in which you know everything except what you are looking for is the one to use.

Representing Vectors with Positive and Negative Numbers

Remember that position (\vec{x}), velocity (\vec{v}), and acceleration (\vec{a}) are all vectors, which means they can be positive or negative, depending on the direction.

- If an object is located is on the positive side of the origin (position zero), then its position, \vec{x} , is positive. If the object is located on the negative side of the origin, its position is negative.
- If an object is moving in the positive direction, then its velocity, \vec{v} , is positive. If the object is moving in the negative direction, then its velocity is negative.
- If an object's velocity is either increasing in the positive direction *or decreasing in the negative direction*, then its acceleration, \vec{a} , is positive. If the object's velocity is either decreasing in the positive direction *or increasing in the negative direction*, then its acceleration is negative.
- An object can have positive velocity and negative acceleration at the same time (*or vice versa*).
- An object can have a velocity of zero for an instant but can still be accelerating.

Use this space for summary and/or additional notes.

Homework Problems: Motion Equations Set #1

1. A racecar, traveling at constant speed, makes one lap around a circular track of radius 100 m. When the car has traveled halfway around the track, what is the magnitude of its displacement from the starting point? (*Hint: it may be helpful to draw a sketch.*)

2. An elevator is moving upward with a speed of $11 \frac{\text{m}}{\text{s}}$. Three seconds later, the elevator is still moving upward, but its speed has been reduced to $5.0 \frac{\text{m}}{\text{s}}$. What is the average acceleration of the elevator during the 3.0 s interval?

Answer: $-2 \frac{\text{m}}{\text{s}^2}$

3. A car, starting from rest, accelerates in a straight-line path at a constant rate of $2.5 \frac{\text{m}}{\text{s}^2}$. How far will the car travel in 12 seconds?

Answer: 180 m

Use this space for summary and/or additional notes.

4. A body initially at rest is accelerated at a constant rate for 5.0 seconds in the positive x direction. If the final speed of the body is $20.0 \frac{\text{m}}{\text{s}}$, what was the body's acceleration?

Answer: $4 \frac{\text{m}}{\text{s}^2}$

5. An object starts from rest and accelerates uniformly in a straight line in the positive x direction. After 10. seconds its speed is $70. \frac{\text{m}}{\text{s}}$.
- Determine the acceleration of the object.

Answer: $7 \frac{\text{m}}{\text{s}^2}$

- How far does the object travel during those first 10 seconds?

Answer: 350 m

6. A racecar has a speed of $80. \frac{\text{m}}{\text{s}}$ when the driver releases a drag chute. If the chute causes a deceleration of $-4 \frac{\text{m}}{\text{s}^2}$, how far will the car travel before it stops?

Answer: 800 m

Use this space for summary and/or additional notes.

7. A brick is dropped from rest from a height of 5.0 m. How long does it take for the brick to reach the ground?

Answer: 1 s

8. A ball is dropped from rest from a tower and strikes the ground 125 m below. Approximately how many seconds does it take for the ball to strike the ground after being dropped? (Neglect air resistance.)

Answer: 5.0 s

9. Water drips from rest from a leaf that is 20 meters above the ground. Neglecting air resistance, what is the speed of each water drop when it hits the ground?

Answer: $20.0 \frac{\text{m}}{\text{s}}$

Use this space for summary and/or additional notes.

10. A ball is shot straight up from the surface of the earth with an initial speed of $30 \frac{m}{s}$. Neglect any effects due to air resistance.

- a. What is the maximum height that the ball will reach?

Answer: 45 m

- b. How much time elapses between the throwing of the ball and its return to the original launch point?

Answer: 6.0 s

11. What is the maximum height that will be reached by a stone thrown straight up with an initial speed of $35 \frac{m}{s}$?

Answer: 61.25 m

Use this space for summary and/or additional notes.

Centripetal Acceleration

Unit: Kinematics (Motion)

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 1.8

Knowledge/Understanding Goals:

- why an object moving in a circle is constantly accelerating

Skills:

- calculate the centripetal force of an object moving in a circle

Language Objectives:

- Understand and correctly use the terms “rotation,” “centripetal force,” and “centrifugal force.”
- Explain the difference between centripetal force and centrifugal force.

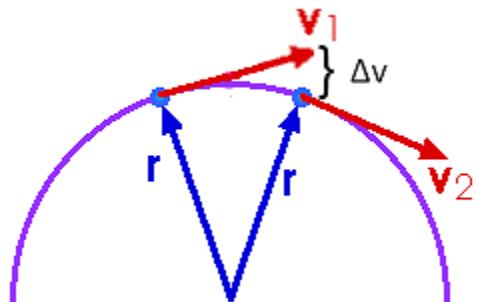
Labs, Activities & Demonstrations:

- Have students swing an object and let it go at the right time to try to hit something. (Have them observe the trajectory.)

Notes:

If an object is moving at a constant speed around a circle, its speed is constant, its direction keeps changing as it goes around. Because velocity is a vector (speed and direction), this means its velocity is constantly changing. (To be precise, the magnitude is staying the same, but the direction is changing.)

Because a change in velocity over time is acceleration, this means the object is constantly accelerating. This continuous change in velocity is toward the center of the circle, which means *there is continuous acceleration toward the center of the circle*.



Use this space for summary and/or additional notes.

centripetal acceleration (a_c): the constant acceleration of an object toward the center of rotation that keeps it rotating around the center at a fixed distance.

The formula for centripetal acceleration (a_c) is:

$$a_c = \frac{v^2}{r}$$

(The derivation of this formula requires calculus, so it will not be presented here.)

Sample Problem:

Q: A weight is swung from the end of a string that is 0.65 m long at a tangential velocity of $6.3 \frac{\text{m}}{\text{s}}$. What is the centripetal acceleration of the weight? How many “g’s” is that? (I.e., how many times the acceleration due to gravity is the centripetal acceleration?)

A: The centripetal acceleration is:

$$a_c = \frac{v^2}{r} = \frac{(6.3)^2}{0.65} = 61 \frac{\text{m}}{\text{s}^2}$$

This is $\frac{61}{10} = 6.1$ times the acceleration due to gravity.

Use this space for summary and/or additional notes.

Solving Motion Problems

Unit: Kinematics (Motion)

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 1.2

Skills:

- solve problems involving motion in one or two dimensions

Language Objectives:

- Set up and solve word problems relating to linear motion.

Notes:

The following is a summary of the variables used for motion problems:

Var.	Unit	Description
x	m	position
$\vec{d}, \Delta x$	m	displacement
\bar{v}	$\frac{m}{s}$	velocity
\vec{a}	$\frac{m}{s^2}$	acceleration
t	s	time

Use this space for summary and/or additional notes.

We have learned the following equations for solving motion problems:

Equation	Comments
$\vec{d} = \Delta \vec{x} = \vec{x} - \vec{x}_o$	Definition of displacement.
$\bar{\vec{v}} = \frac{\Delta \vec{x}}{t} = \frac{\vec{d}}{t} = \frac{\vec{v}_o + \vec{v}}{2}$	Definition of <u>average</u> velocity. Note that you can't use \bar{v} if there is acceleration.
$\vec{a} = \frac{\Delta \vec{v}}{t} = \frac{\vec{v} - \vec{v}_o}{t}$	Definition of acceleration.
$\vec{x} = \vec{x}_o + \vec{v}_o t + \frac{1}{2} \vec{a} t^2$	Position formula.
$\vec{v}^2 = \vec{v}_o^2 + 2 \vec{a} \vec{d}$	Relates velocities, acceleration and distance. Useful if time is not known.
$\vec{a}_c = \frac{\vec{v}^2}{r}$	Centripetal acceleration (acceleration toward the center of a circle.)

Note that vector quantities (shown in bold) can be positive or negative, depending on direction.

Selecting the Right Equation

When you are faced with a problem, choose an equation based on the following criteria:

- The equation must contain the variable you are looking for.
- All other quantities in the equation must be either given in the problem or assumed from the description of the problem.
 - If an object starts at rest (not moving), then $\vec{v}_o = 0$.
 - If an object comes to a stop, then $\vec{v} = 0$.
 - If gravity is involved (e.g., the object is falling), $\vec{a} = \vec{g} = 10 \frac{\text{m}}{\text{s}^2}$.
(Applies to linear acceleration problems only.)

This means you can choose the appropriate equation by making a list of what you are looking for and what you know. The equation in which you know everything except what you are looking for is the one to use.

Use this space for summary and/or additional notes.

Strategies for Motion Problems Involving Gravity

Motion problems in physics often involve gravity. These problems usually fall into one of two categories:

1. If you have an object in free fall, the problem will probably give you either the distance it fell ($x - x_0$), or the time it fell (t). Use the position equation $\vec{x} - \vec{x}_0 = \vec{d} = \vec{v}_o t + \frac{1}{2} \vec{a} t^2$ to calculate whichever one you don't know. (If the object starts from rest, that means $\vec{v}_o = 0$.)
2. If an object is thrown upwards, it will decelerate at a rate of $-10 \frac{\text{m}}{\text{s}^2}$ (assuming "up" is the positive direction) until it stops moving ($\vec{v} = 0$). Then it will fall. This means you need to split the problem into two parts:
 - a. When the object is moving upward, the initial velocity, \vec{v}_o , is usually given and \vec{v} (at the top) = zero. From these, you can use $\vec{v}^2 = \vec{v}_o^2 - 2\vec{a}\vec{d}$ to figure out the maximum height.
 - b. Once you know the maximum height, you know the distance to the ground, $\vec{v}_o = 0$, and you can use the position equation (this time with $\vec{a} = +10 \frac{\text{m}}{\text{s}^2}$) to find the time it spends falling. The total time (up + down) will be twice as much.

Use this space for summary and/or additional notes.

Sample Problems:

Q: If a cat jumps off a 1.8 m tall refrigerator, how long does it take to hit the ground?



A: The problem gives us $d = 1.8 \text{ m}$. The cat is starting from rest ($v_0 = 0$), and gravity is accelerating the cat at a rate of $a = g = 10 \frac{\text{m}}{\text{s}^2}$. We need to find t .

Looking at the equations, the one that has what we need (t) and only quantities we know is:

$$d = v_0 t + \frac{1}{2} a t^2$$

$v_0 = 0$, so this reduces to:

$$\begin{aligned} d &= \frac{1}{2} a t^2 \\ 1.8 &= (\frac{1}{2})(10) t^2 \\ \frac{1.8}{5} &= 0.36 = t^2 \\ t &= \sqrt{0.36} = 0.6 \text{ s} \end{aligned}$$

Use this space for summary and/or additional notes.

Q: An apple falls from a tree branch at a height of 5 m and lands on Isaac Newton's head. (Assume Isaac Newton was 1.8 m tall.)



How fast was the apple traveling at the time of impact?

A: We know $d = x - x_0 = 5 - 1.8 = 3.2\text{ m}$. We also know that the apple is starting from rest ($v_0 = 0$), and gravity is accelerating the apple at a rate of $a = g = 10 \frac{\text{m}}{\text{s}^2}$. We want to find v .

The equation that relates all of our variables is:

$$v^2 - v_0^2 = 2ad$$

Substituting, we get:

$$\begin{aligned}v^2 - 0 &= (2)(10)(3.2) \\v^2 &= 64 \\v &= \sqrt{64} = 8 \frac{\text{m}}{\text{s}}\end{aligned}$$

Use this space for summary and/or additional notes.

Projectile Motion

Unit: Kinematics (Motion)

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 1.2

Skills:

- solve problems involving linear motion in two dimensions

Language Objectives:

- Understand and correctly use the term “projectile.”
- Set up and solve word problems involving projectiles.

Labs, Activities & Demonstrations:

- Play “catch.”
- Drop one ball and punch the other at the same time.
- “Shoot the monkey.”

Notes:

projectile: an object that is propelled (thrown, shot, etc.) horizontally and also falls due to gravity.

Gravity affects projectiles the same way regardless of whether the projectile is moving horizontally. Gravity does not affect the horizontal motion of the projectile. This means the vertical and horizontal motion of the projectile can be considered separately, using a separate set of equations for each.

Use this space for summary and/or additional notes.

Assuming we can neglect friction and air resistance (which is usually the case in first-year physics problems), we make two important assumptions:

1. All projectiles have a constant horizontal velocity, v_h , in the positive horizontal direction. The equation for the horizontal motion (without acceleration) is:

$$d = vt$$

However, we have displacement and velocity in both the vertical and horizontal directions. This means we need add a subscript “ h ” to the horizontal quantities and a subscript “ v ” to the vertical quantities so we can tell them apart. The horizontal equation becomes:

$$d_h = v_h t$$

2. All projectiles have a constant downward acceleration of $g = \pm 10 \frac{\text{m}}{\text{s}^2}$ (in the vertical direction), due to gravity. (You need to choose whether the positive vertical direction is up or down, depending on the situation.) The equation for the vertical motion is:

$$d = v_o t + \frac{1}{2} at^2$$

Adding a subscript “ v ”, (and using “ g ” instead of “ a ”) because gravity is causing the acceleration, this becomes:

$$d_v = v_{o,v} t + \frac{1}{2} gt^2$$

Notice that we have **two** subscripts on the velocity, because it is **both** the initial velocity v_o **and also** the vertical velocity v_v .

3. The time that the projectile spends falling must be the same as the time that the projectile spends moving horizontally. This means time (t) is the same in both equations, which means time is the variable that links the vertical problem to the horizontal problem.

Use this space for summary and/or additional notes.

The consequences of these assumptions are:

- The *time* that the object takes to fall is determined by its movement only in the vertical direction. (I.e., all motion stops when the object hits the ground.)
- The *horizontal distance* that the object travels is determined by the time (calculated above) and its velocity in the horizontal direction.

Therefore, the general strategy for most projectile problems is:

1. Solve the vertical problem first, to get the time.
2. Use the time from the vertical problem to solve the horizontal problem.

Use this space for summary and/or additional notes.

Sample problem:

Q: A ball is thrown horizontally at a velocity of $5 \frac{\text{m}}{\text{s}}$ from a height of 1.5 m. How far does the ball travel (horizontally)?

A: We're looking for the horizontal distance, d_h . We know the vertical distance, $d_v = 1.5 \text{ m}$, and we know that $v_{o,v} = 0$ (there is no initial vertical velocity because the ball is thrown horizontally), and we know that $a = g = 10 \frac{\text{m}}{\text{s}^2}$.

We need to separate the problem into the horizontal and vertical components.

Horizontal:

$$d_h = v_h t$$

$$d_h = 5 t$$

At this point we can't get any farther, so we need to turn to the vertical problem.

Vertical:

$$d_v = v_{o,v} t + \frac{1}{2} g t^2$$

$$d_v = \frac{1}{2} g t^2$$

$$1.5 = (\frac{1}{2})(10) t^2$$

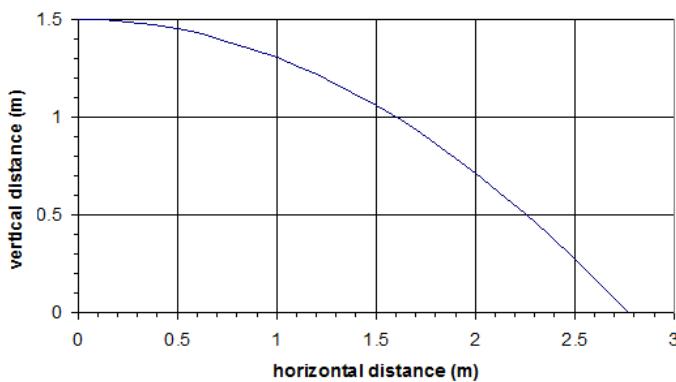
$$\frac{1.5}{5.0} = 0.30 = t^2$$

$$t = \sqrt{0.30} = 0.55 \text{ s}$$

Now that we know the time, we can substitute it back into the horizontal equation, giving:

$$d_h = (5)(0.55) = 2.74 \text{ m}$$

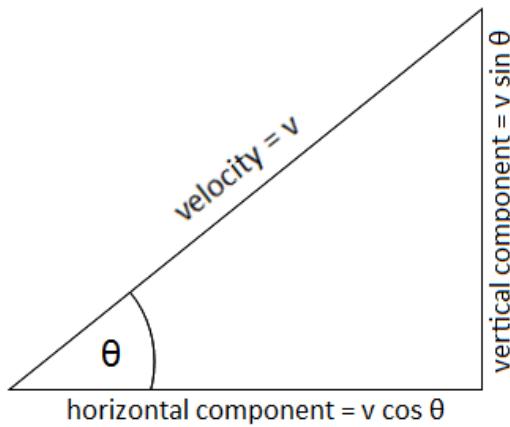
A graph of the vertical vs. horizontal motion of the ball looks like this:



Use this space for summary and/or additional notes.

Projectiles Launched at an Angle

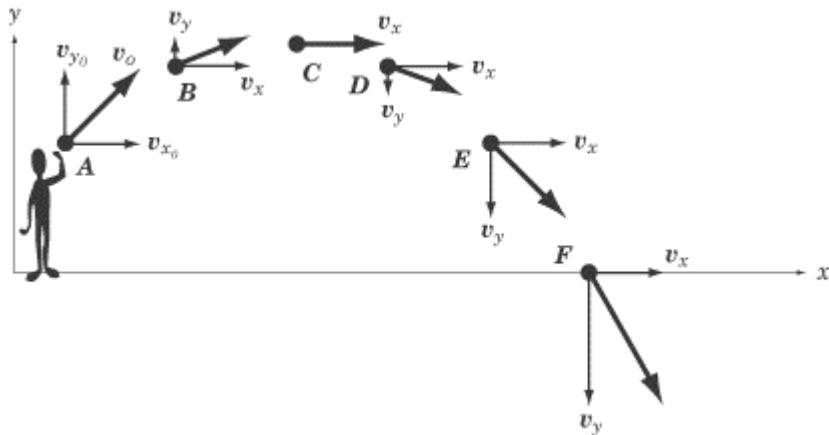
If the object is thrown/launched at an angle, you need to use trigonometry to separate the velocity vector into its horizontal (x) and vertical (y) components:



Thus:

- horizontal velocity = $v_h = v \cos \theta$
- *initial* vertical velocity = $v_{o,v} = v \sin \theta$

Note that the horizontal component of the velocity (note that this diagram uses v_x instead of v_h), is constant, but the vertical component of the velocity (v_y instead of v_v) is constantly changing because of acceleration due to gravity.



You will not be required to solve mathematical problems involving projectiles at an angle in this course, but you should understand qualitatively how they move.

Use this space for summary and/or additional notes.

Homework Problems

Horizontal (level) projectile problems:

1. A diver running $1.6 \frac{\text{m}}{\text{s}}$ dives out horizontally from the edge of a vertical cliff and reaches the water below 3.0 s later.
 - a. How high was the cliff?

Answer: 44 m

- b. How far from the base did the diver hit the water?

Answer: 4.8 m

2. A tiger leaps horizontally from a 7.5 m high rock with a speed of $4.5 \frac{\text{m}}{\text{s}}$.
How far from the base of the rock will he land?

Answer: 5.6 m

Use this space for summary and/or additional notes.

3. A ball is thrown horizontally from the roof of a building 56 m tall and lands 45 m from the base. What was the ball's initial speed?

Answer: $13 \frac{\text{m}}{\text{s}}$

4. The pilot of an airplane traveling $45 \frac{\text{m}}{\text{s}}$ wants to drop supplies to flood victims isolated on a patch of land 160 m below. The supplies should be dropped when the plane is how far from the island?

Answer: 257 m

Use this space for summary and/or additional notes.

Introduction: Dynamics (Forces) & Gravitation

Unit: Dynamics (Forces) & Gravitation

Topics covered in this chapter:

Newton's Laws of Motion	151
Linear Forces	154
Gravitational Fields	160
Free-Body Diagrams.....	162
Newton's Second Law	171
Force Applied at an Angle	180
Friction	191
Aerodynamic/Hydrodynamic Drag	198
Universal Gravitation	200
Kepler's Laws of Planetary Motion	205

In this chapter you will learn about different kinds of forces and how they relate.

- *Newton's Laws and Forces* describe basic scientific principles of how objects affect each other.
- *Free-Body Diagrams* describes a way of drawing a picture that represents forces acting on an object.
- *Forces Applied at an Angle, Ramp Problems, and Pulleys & Tension* describe some common situations involving forces and how to calculate the forces involved.
- *Friction and Aerodynamic Drag* describe situations in which a force is created by the action of another force.
- *Newton's Law of Universal Gravitation* describes how to calculate the force of gravity caused by massive objects such as planets and stars.

Use this space for summary and/or additional notes.

One of the first challenges will be working with variables that have subscripts. Each type of force uses the variable F . Subscripts will be used to keep track of the different kinds of forces. This chapter also makes extensive use of vectors.

Another challenge in this chapter will be to “chain” equations together to solve problems. This involves finding the equation that has the quantity you need, and then using a second equation to find the quantity that you are missing from the first equation.

Textbook:

- *Physics: Principles and Problems* Ch. 5: Forces (pp. 86–107)
- *Physics: Principles and Problems* Ch. 8: Universal Gravitation (pp. 154–173)

Standards addressed in this chapter:**Next Generation Science Standards (NGSS):**

HS-PS2-1. Analyze data to support the claim that Newton’s second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration.

HS-PS2-3. Apply scientific and engineering ideas to design, evaluate, and refine a device that minimizes the force on a macroscopic object during a collision.

HS-PS2-4. Use mathematical representations of Newton’s Law of Gravitation and Coulomb’s Law to describe and predict the gravitational and electrostatic forces between objects.

Massachusetts Curriculum Frameworks (2006):

1.4 Interpret and apply Newton’s three laws of motion.

1.5 Use a free-body force diagram to show forces acting on a system consisting of a pair of interacting objects. For a diagram with only co-linear forces, determine the net force acting on a system and between the objects.

1.6 Distinguish qualitatively between static and kinetic friction, and describe their effects on the motion of objects.

1.7 Describe Newton’s law of universal gravitation in terms of the attraction between two objects, their masses, and the distance between them.

Use this space for summary and/or additional notes.

Topics from this chapter assessed on the SAT Physics Subject Test:

- **Dynamics**, such as force, Newton's laws, statics, and friction.
 - **Gravity**, such as the law of gravitation, orbits, and Kepler's laws.
1. What are Forces?
 2. Types of Forces
 3. Newton's Laws
 4. Problem Solving With Newton's Laws
 5. Pulleys
 6. Inclined Planes
 7. Newton's Law of Universal Gravitation
 8. Weightlessness
 9. Kepler's Laws

Skills learned & applied in this chapter:

- Solving chains of equations.
- Using trigonometry to extract a vector in a desired direction.
- Rotational forces (torque).
- Working with material-specific constants from a table.
- Estimating the effect of changing one variable on another variable in the same equation.

Use this space for summary and/or additional notes.

Newton's Laws of Motion

Unit: Dynamics (Forces) & Gravitation

NGSS Standards: HS-PS2-1

MA Curriculum Frameworks (2006): 1.4

Knowledge/Understanding Goals:

- Newton's three laws of motion

Language Objectives:

- Understand and correctly use the terms "at rest," "in motion," and "force."
- Accurately describe and apply the concepts described in this section, using appropriate academic language.
- Set up and solve word problems relating to forces.

Labs, Activities & Demonstrations:

- Mass with string above & below.
- Tablecloth with dishes (or equivalent).
- "Levitating" globe.
- Fan cart.
- Fire extinguisher & skateboard.
- Forces on two masses hanging (via pulleys) from the same rope.

Notes:

force: a push or pull on an object.

Use this space for summary and/or additional notes.

Newton's First Law: (the law of inertia)

Everything keeps doing what it was doing unless a force acts to change it. "An object at rest remains at rest, unless acted upon by a net force. An object in motion remains in motion, unless acted upon by a net force."

For example, a brick sitting on the floor will stay at rest on the floor forever unless an outside force moves it.

Wile E. Coyote, on the other hand, remains in motion...



Inertia is a property of mass. Everything with mass has inertia. The more mass an object has, the more inertia it has.

Newton's Second Law: Forces cause acceleration (a change in velocity). "A net force, \vec{F} , acting on an object causes the object to accelerate in the direction of the net force." In equation form:

$$\ddot{\mathbf{a}} = \frac{\vec{F}_{\text{net}}}{m} \quad \text{or} \quad \vec{F}_{\text{net}} = m\ddot{\mathbf{a}}$$

Newton's second law is probably the most important concept regarding forces.

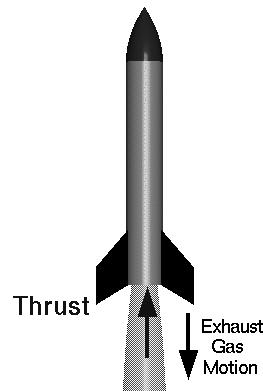
- If there is a net force, the object accelerates (its velocity changes). If the object's velocity is changing (*i.e.*, if it accelerates), then there must be a net force acting on it.
- If there is no net force, the object's velocity will not change (Newton's First Law). If the object's velocity is not changing (*i.e.*, if it is not accelerating), there must be no net force. (If there are no net forces, either there are no forces acting on the object at all, or all of the forces on the object are equal and opposite, and their effects cancel.)

Use this space for summary and/or additional notes.

Newton's Third Law: Every force involves two objects. The first object exerts a force on the second, and the second object exerts the same force back on the first. "For every action, there is an equal and opposite reaction."

Burning fuel in a rocket causes exhaust gases to escape from the back of the rocket. The force from the gases exiting (the action) applies thrust to the rocket (the reaction), which propels the rocket forward.

If you punch a hole in a wall and break your hand in the process, your hand applied the force that broke the wall (the action). This caused a force from the wall, which broke your hand (the reaction). This may seem obvious, though you will find that someone who has just broken his hand by punching a hole in a wall is unlikely to be receptive to a physics lesson!



Systems

system: the collection of objects being considered in a problem.

For example, gravity is the force of attraction between two objects because of their mass. If you jump off the roof of the school, the Earth attracts you, and you attract the Earth. (Because the Earth has a lot more mass than you do, you move much farther toward the Earth than the Earth moves toward you.)

If the system is you, then the Earth exerts a net force on you, causing you to move. However, if the system is you plus the Earth, the force exerted by the Earth on you is equal to the force exerted by you on the Earth. Because the forces are equal in strength but in opposite directions ("equal and opposite"), their effects cancel, which means there is no net force on the system. (Yes, there are forces within the system, but that's not the same thing.)

Use this space for summary and/or additional notes.

Linear Forces

Unit: Dynamics (Forces) & Gravitation

NGSS Standards: HS-PS2-1

MA Curriculum Frameworks (2006): 1.5

Knowledge/Understanding Goals:

- what force is
- net force
- types of forces

Skills:

- identify the forces acting on an object

Language Objectives:

- Understand and correctly use the terms “force,” “normal force,” “contact force,” “opposing force,” and “weight.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Labs, Activities & Demonstrations:

- Tie a rope to a chair or stool and pull it.

Notes:

force: (vector) a push or pull on an object.



weight: the force of gravity pulling an object

downward. In physics, we represent weight as the vector \vec{F}_g . Note that from Newton's second law,

$\vec{F}_g = m\vec{g}$, which means on Earth, $\vec{F}_g = m(10)$.

opposing force: a force in the opposite direction of another force that reduces the effect of the original force.

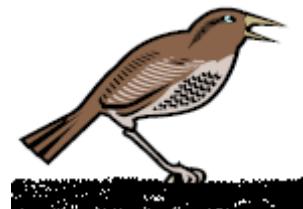
net force: the overall force on an object after opposing forces cancel out.

contact force: a opposing force that exists only while another force is acting on an object. Examples include friction and the normal force.

normal force: a force exerted by a surface (such as the ground or a wall) that resists the force of gravity on an object.

Use this space for summary and/or additional notes.

Because force is a vector, two forces in opposite directions counteract each other; if the magnitudes are equal, the forces completely cancel. For example, in the following picture:



the forces acting on the bird are:

- gravity, which is pulling the bird down, and
- the force from the ground, which is pushing the bird up.

The force from the ground is called the normal force. (The term “normal” is borrowed from math and means “perpendicular”.)

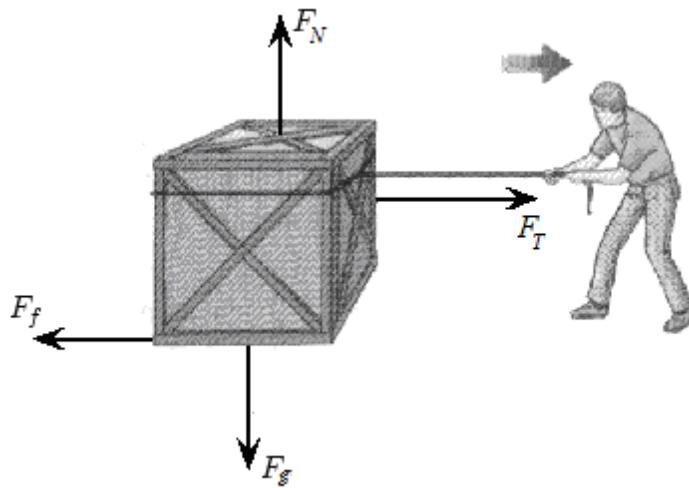
The normal force is the force exerted by a surface (such as the ground, a table, or a wall) that counteracts another force on the object. The normal force is called a contact force, because it is caused by the action of another force, and it exists *only* while the objects are in contact. The normal force is also an opposing force because it acts in the opposite direction from the applied force, and acts to lessen or diminish the applied force.

For example, if you push on a wall with a force of 10 N and the wall doesn't move, that means the force you apply to the wall causes a normal force of 10 N pushing back from the wall. This normal force continues for as long as you continue pushing.

friction: Like the normal force, friction is also both a contact force (caused by the action of another force) and an opposing force. The direction of friction is parallel to the surfaces that are in contact, and opposite to the direction of motion.

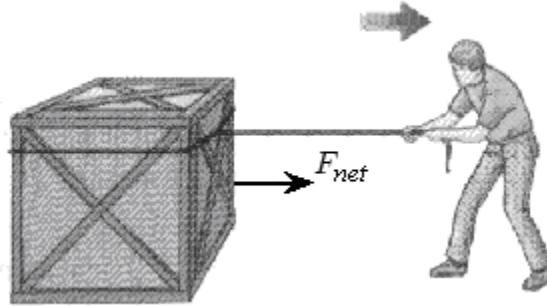
Use this space for summary and/or additional notes.

An object can have several forces acting on it at once:



On the box in the above diagram, the forces are gravity (\vec{F}_g), the normal force (\vec{F}_N), the tension in the rope (\vec{F}_T), and friction (\vec{F}_f). Notice that in this problem, the arrow for tension is longer than the arrow for friction, because the force of tension is stronger than the force of friction.

net force: the remaining force on an object after canceling opposing forces. The net force on the box (after canceling out gravity and the normal force, and subtracting friction from the tension) would be represented as:



Because there is a net force to the right, the box will accelerate to the right as a result of the force.

Use this space for summary and/or additional notes.

You can think of forces as participants in a multi-direction tug-of-war:



In the above situation, the net force is in the direction that the ropes will move.

Forces are what cause acceleration. If a net force acts on an object, the object will speed up, slow down or change direction. Remember that *if the object's velocity is not changing, there is no net force, which means all of the forces on the object cancel.*

In the MKS system, the unit of force is the newton (N). One newton is defined as the amount of force that it would take to cause a 1 kg object to accelerate at a rate of $1 \frac{\text{m}}{\text{s}^2}$.

$$1 \text{ N} \equiv 1 \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$

Because the acceleration due to gravity on Earth is approximately $10 \frac{\text{m}}{\text{s}^2}$, $\vec{F} = m\vec{a}$ indicates that a 1 kg mass on Earth would have a weight of approximately 10 N.

In more familiar terms, one newton is approximately 3.6 ounces, which is the weight of a medium-sized apple. One pound is approximately 4.5 N.

Use this space for summary and/or additional notes.

Common Types of Forces

Force	Symbol	Definition	Direction
gravity (weight)	$\vec{F}_g, \vec{w}, m\vec{g}$	pull of gravity by the Earth (or some other celestial body) on an object with mass	toward the ground (or center of mass of the celestial body)
tension	\vec{F}_T, \vec{T}	pull exerted by a rope/string/cable	away from the object in the direction of the string/rope/cable
normal	\vec{F}_N, \vec{N}	contact force by a surface on an object	perpendicular to and away from surface
friction	\vec{F}_f, \vec{f}	contact force that opposes sliding between surfaces	parallel to surface; opposite to direction of applied force
thrust	\vec{F}_t	push that accelerates objects such as rockets, planes & cars	in the same direction as acceleration
spring	\vec{F}_s	the push or pull exerted by a spring	opposite the displacement of the object
buoyancy	\vec{F}_B	the upward force by a fluid on objects less dense than the fluid	opposite to gravity
drag	\vec{F}_D	friction caused by the molecules of a fluid as an object moves through it	opposite to the direction of motion
lift	\vec{F}_ℓ	the upward push (reaction force) by a fluid on an object (such as an airplane wing) moving through it at an "angle of attack"	opposite to gravity.

Use this space for summary and/or additional notes.

	Force	Symbol	Definition	Direction
	electrostatic	\vec{F}_e	the attraction or repulsion between two objects with an electric charge	like charges repel; opposite charges attract
	magnetic	\vec{F}_B, \vec{F}_M	the magnetic attraction or repulsion between two objects	like magnetic poles repel; opposite poles attract

You may have noticed that both buoyancy and magnetic force can use the same subscript (\vec{F}_B). This should not cause too much confusion, because there are very few situations in which both forces would be applied to the same object. If that were to happen, we would use \vec{F}_B for the buoyant force and \vec{F}_M for the magnetic force.

Extension

The rate of change of force with respect to time is called “yank”: $\bar{Y} = \frac{\Delta \vec{F}}{\Delta t}$. Just as $\vec{F} = m\vec{a}$, yank is the product of mass times jerk: $\bar{Y} = m\vec{j}$.

Use this space for summary and/or additional notes.

Gravitational Fields

Unit: Dynamics (Forces) & Gravitation

NGSS Standards: HS-PS2-1

MA Curriculum Frameworks (2006): 1.5

Knowledge & Understanding:

- Understand the representation of gravity as a force field.

Language Objectives:

- Understand the specific use of the level 2 word “field” in the context of gravity.
- Understand and correctly use the terms “force field” and “gravity field”.

Labs, Activities & Demonstrations:

- Miscellaneous falling objects

Notes:

Gravity is an attractive force between two or more objects that have mass. For reasons that are not yet understood, masses exert attraction to each other in proportion to the masses of the objects and in inverse proportion to the square of the distance between them. (This concept will be discussed more fully in the section on Universal Gravitation on page 200.)

When considering objects near the Earth’s surface (give or take a few hundred meters) that are small in relation to the size and mass of the Earth, gravity may be considered to be a force field.

force field: a region in which a force acts upon all objects that have some particular characteristic or property.

In the case of gravity, that property is the mass of the object, and the force acts in proportion to the strength of the field.

Other types of force fields include electric fields, in which an electric force acts on all objects that have electric charge, and magnetic fields, in which a magnetic force acts on all objects that have magnetic susceptibility (the property that causes them to be attracted to or repelled by a magnet).

Use this space for summary and/or additional notes.

The strength of a gravitational field is a vector quantity described by the variable \vec{g} and is equal to the force that the field applies on an object per unit of the object's mass:

$$\frac{\vec{F}_g}{m} = \frac{m\vec{g}}{m} = \vec{g}$$

Conveniently, force per unit of mass equals acceleration, which is why \vec{g} represents **both** the strength of the gravitational field **and** the acceleration that the gravitational field causes. This is why the value of \vec{g} and how it is used in equations is the same, regardless of how we choose to derive it.

The value of \vec{g} depends on the mass of the object causing the field (usually the Earth) and the distance to its center of mass (usually the radius of the Earth).

The direction of \vec{g} is always toward the center of mass of the large object (*i.e.*, toward the center of the Earth or toward the ground).

Use this space for summary and/or additional notes.

Free-Body Diagrams

Unit: Dynamics (Forces) & Gravitation

NGSS Standards: HS-PS2-1

MA Curriculum Frameworks (2006): 1.5

Skills:

- draw a free-body diagram representing the forces on an object

Language Objectives:

- Understand and correctly use the term “free-body diagram.”
- Identify forces by name and draw a free-body diagram.

Labs, Activities & Demonstrations:

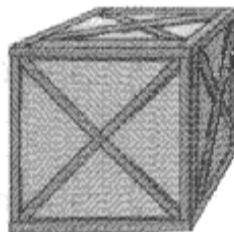
- Human force board activity.

Notes:

free-body diagram (force diagram): a diagram representing the forces acting on an object.

In a free-body diagram, we represent the object as a dot, and each force as an arrow. The direction of the arrow represents the direction of the force, and the relative lengths of the arrows represent the relative magnitudes of the forces.

Consider a box that is at rest on the floor:



The forces on the box are:

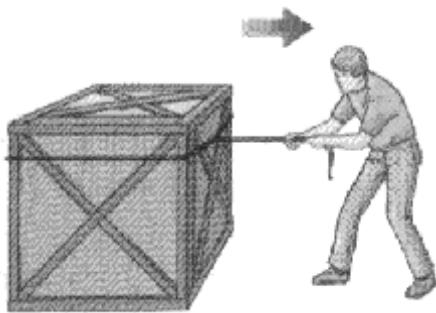
- gravity, which is pulling the box toward the center of the Earth
- the normal force from the floor opposing gravity and holding up the box.

Use this space for summary and/or additional notes.

The free-body diagram for the box looks like this:



Now consider the following situation of a box that accelerates to the right as it is pulled across the floor by a rope:



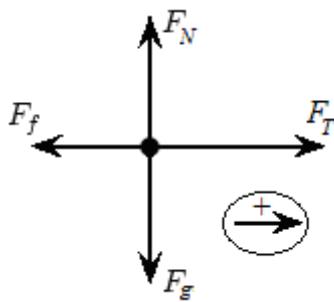
From the picture and description, we can assume that:

- The box has weight, which means gravity is pulling down on it.
- The floor is holding up the box.
- The rope is pulling on the box.
- Friction between the box and the floor is resisting the force from the rope.
- Because the box is accelerating to the right, the force applied by the rope must be stronger than the force from friction.

In the free-body diagram, we again represent the object (the box) as a point, and the forces (vectors) as arrows. Because there is a net force, we need to include a legend that shows which direction is positive.

Use this space for summary and/or additional notes:

The forces are:



\vec{F}_g = the force of gravity pulling down on the box

\vec{F}_N = the normal force (the floor holding the box up)

\vec{F}_T = the force of tension from the rope. (This might also be designated F_a because it is the force applied to the object.)

\vec{F}_f = the force of friction resisting the motion of the box.

In the bottom right corner of the diagram, the arrow with the "+" sign shows that we have chosen to make the positive direction to the right.

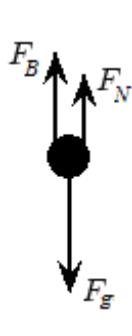
Notice that the arrows for the normal force and gravity are equal in length, because in this problem, these two forces are equal in magnitude.

Notice that the arrow for friction is shorter than the arrow for tension, because in this problem, the tension is stronger than the force from friction. The difference between the lengths of these two vectors represents the net force, which is what causes the box to accelerate to the right.

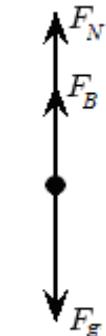
Use this space for summary and/or additional notes:

Note that if you have multiple forces in the same direction, each force vector must originate from the point that represents the object, and must be as close as is practical to the *exact* direction of the force.

For example, consider a stationary object with three forces on it: F_B and F_N both acting upwards, and F_g acting downwards.



correct



incorrect



incorrect



The first representation is correct because all forces originate from the point that represents the object, the directions represent the exact directions of the forces, and the length of each is proportional to its strength. Even though the object is represented by a point (which technically has no dimensions), it is acceptable to draw the representation of the point large enough to show each of the forces acting on it.

The second representation is incorrect because it is unclear whether F_N starts from the object (the dot), or from the tip of the arrow representing F_B .

The third representation is incorrect because it implies that F_B and F_N each have a slight horizontal component, which is not true.

Use this space for summary and/or additional notes.

Steps for Drawing Free-Body Diagrams

In general, the following are the steps for drawing most free-body diagrams.

1. Is gravity involved?
 - Represent gravity as \vec{F}_g or $m\vec{g}$ pointing straight down.
2. Is something holding the object up?
 - If it is a flat surface, it is the normal force (\vec{F}_N or \vec{N}), perpendicular to the surface.
 - If it is a rope, chain, etc., it is the force of tension (\vec{F}_T or \vec{T}) acting along the rope, chain, etc.
3. Is there an external force pulling or pushing on the object?
4. Is there an opposing force?
 - If there are two surfaces in contact, there is almost always friction (\vec{F}_f or \vec{f}), unless the problem specifically states that the surfaces are frictionless.
 - At low velocities, air resistance is very small and can usually be ignored, even if the problem does not explicitly say so.
 - Usually, all sources of friction are shown as one combined force. E.g., if there is sliding friction along the ground and also air resistance, the \vec{F}_f vector includes both.
5. Are forces acting in opposing directions?
 - If the problem requires calculations involving opposing forces, you need to indicate which direction is positive.

Use this space for summary and/or additional notes.

Homework Problems

For each picture, draw a free-body diagram that shows all of the forces acting upon the object in the picture.

1. A bird sits motionless on a perch.



2. A hockey player glides at constant velocity across frictionless ice.
(Ignore air resistance.)



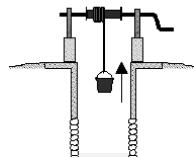
3. A baseball player slides head-first into second base.



4. A chandelier hangs from the ceiling, suspended by a chain.



5. A bucket of water is raised out of a well at constant velocity.



Use this space for summary and/or additional notes.

6. A skydiver has just jumped out of an airplane and is accelerating toward the ground.



7. A skydiver falls through the air at terminal velocity.



8. A hurdler is moving horizontally as she clears a hurdle. (Ignore air resistance.)



9. An airplane moves through the air in level flight at constant velocity.

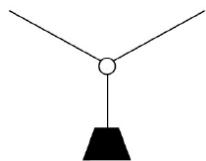


10. A sled is pulled through the snow at constant velocity. (Note that the rope is at an angle.)



Use this space for summary and/or additional notes.

11. A stationary metal ring is held by three ropes, one of which has a mass hanging from it. (Draw the force diagram for the metal ring.)



12. A child swings on a swing. (Ignore all sources of friction, including air resistance.)



13. A squirrel sits motionless on a sloped roof.



14. A skier moves down a slope at constant velocity.



15. A skier accelerates down a slope.



Use this space for summary and/or additional notes.

Newton's Second Law

Unit: Dynamics (Forces) & Gravitation

NGSS Standards: HS-PS2-1

MA Curriculum Frameworks (2006): 1.4

Skills:

- Solve problems relating to Newton's Second Law ($F = ma$)
- Solve problems that combine kinematics (motion) and forces

Language Objectives:

- Identify and correctly use the quantities involved in a Newton's Second Law problem.
- Identify and correctly use the quantities involved in a problem that combines kinematics and forces.

Labs, Activities & Demonstrations:

- Handstands in an elevator.

Notes:

Newton's Second Law: Forces cause acceleration (a change in velocity). "A net force, \vec{F} , acting on an object causes the object to accelerate in the direction of the net force."

If there is a net force, the object accelerates (its velocity changes). If there is no net force, the object's velocity remains the same.

If an object accelerates (its velocity changes), there was a net force on it. If an object's velocity remains the same, there was no net force on it.

Remember that forces are vectors. No net force can either mean that there are no forces at all, or it can mean that there are equal forces in opposite directions and their effects cancel.

Use this space for summary and/or additional notes:

In equation form:

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$$

This form is preferred, because acceleration is what results from a force applied to a mass. (*I.e.*, force and mass are the independent variables and acceleration is the dependent variable. Forces cause acceleration, not the other way around.) However, the equation is more commonly written:

$$\vec{F}_{\text{net}} = m\vec{a}$$

Sample Problems

Most of the physics problems involving forces require the application of

Newton's Second Law, $\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$.

Q: A net force of 50 N in the positive direction is applied to a cart that has a mass of 35 kg. How fast does the cart accelerate?

A: Applying Newton's Second Law:

$$\begin{aligned}\vec{a} &= \frac{\vec{F}_{\text{net}}}{m} \\ \vec{a} &= \frac{50}{35} \\ \vec{a} &= 1.43 \frac{\text{m}}{\text{s}^2}\end{aligned}$$

Q: What is the weight of (*i.e.*, the force of gravity acting on) a 10 kg block?

A: $\vec{a} = \vec{g}$

$$\vec{F}_g = m\vec{a}_g = m\vec{g}$$

$$\vec{F}_g = (10.)(10) = 100 \text{ N}$$

(Remember that we use the variable \vec{g} instead of \vec{a} when the acceleration is caused by a gravity field.)

Use this space for summary and/or additional notes.

Free Body Diagrams and Newton's Second Law

Free-body diagrams are often used in combination with Newton's second law ($\vec{a} = \frac{\vec{F}_{net}}{m}$); the free-body diagram enables you to calculate the net force, from which you can calculate mass or acceleration.

Sample Problem:

Q: Two children are fighting over a toy.



Jamie pulls to the left with a force of 40 N, and Edward pulls to the right with a force of 60 N. If the toy has a mass of 0.6 kg, what is the resulting acceleration of the toy?

A: Let us decide that the positive direction is to the right. (This is convenient because the force to the right is larger, which means the net force will come out to a positive number.)

The force diagram looks like this:



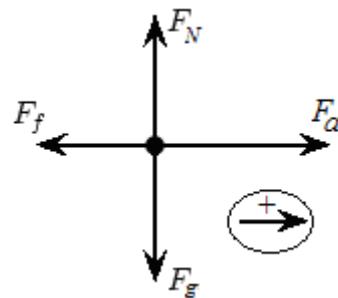
$$\vec{a} = \frac{\vec{F}_{net}}{m} = \frac{+20}{0.6} = +33.3 \frac{\text{m}}{\text{s}^2} (\text{to the right})$$

Use this space for summary and/or additional notes.

Q: A 5.0 kg block is resting on a horizontal, flat surface. How much force is needed to overcome a force of 2.0 N of friction and accelerate the block from rest to a velocity of $6.0 \frac{\text{m}}{\text{s}}$ over a 1.5-second interval?

A: This is a combination of a Newton's second law problem, and a motion problem. We will need a free-body diagram to be able to visualize what's going on.

The free-body diagram for the block looks like this:



3. The net force is given by:

$$\begin{aligned}F_{net} &= F_a - F_f = F_a - 2 \\F_a &= F_{net} + 2\end{aligned}$$

This means we need to find the net force, and then add 2 N to get the applied force.

4. To find the net force, we need the equation:

$$\vec{F}_{net} = m\vec{a}$$

5. We know that $m = 5.0 \text{ kg}$, but we don't know a . We need to find a in order to calculate F_{net} . For this, we will turn to the motion problem.
6. The problem tells us that $v_o = 0$, $v = 6.0 \frac{\text{m}}{\text{s}}$, and $t = 1.5 \text{ s}$. Looking at the motion equations, we see that we have all of the variables except for a in the equation:

$$v - v_o = at$$

Use this space for summary and/or additional notes.

3. Our strategy is to solve this equation for a , then substitute into $F_{net} = ma$ to find F_{net} , then use the relationship we found from the free-body diagram to find F_a .

The acceleration is:

$$\begin{aligned}v - v_0 &= at \\6.0 - 0 &= a(1.5) \\6.0 &= 1.5 a \\a &= 4.0 \frac{\text{m}}{\text{s}^2}\end{aligned}$$

Substituting back into $F_{net} = ma$ gives:

$$\begin{aligned}F_{net} &= ma \\F_{net} &= (5.0)(4.0) \\F_{net} &= 20. \text{ N}\end{aligned}$$

Finally:

$$\begin{aligned}F_a &= F_{net} + 2 \\F_a &= 20 + 2 \\F_a &= 22 \text{ N}\end{aligned}$$

Problems like this are straightforward to solve, but they are challenging because you need to keep chasing the quantities that you don't know until you have enough information to calculate them. However, you need to keep track of each step, because once you have found the last equation you need, you have to follow the steps in reverse order to get back to the answer.

Use this space for summary and/or additional notes.

Homework Problems

1. Two horizontal forces, 225 N and 165 N are exerted on a canoe. If these forces are both applied eastward, what is the net force on the canoe?

2. Two horizontal forces are exerted on a canoe, 225 N westward and 165 N eastward. What is the net force on the canoe?

Questions 3 & 4 refer to the following situation:

Three confused sled dogs are trying to pull a sled across the snow in Alaska. Alutia pulls to the east with a force of 135N. Seward pulls to the east with a force of 142N. Kodiak pulls to the west with a force of 153N.

3. What is the net force on the sled?

Answer: 124 N east

4. If the sled has a mass of 150. kg and the driver has a mass of 100. kg, what is the acceleration of the sled? (Assume there is no friction between the runners of the sled and the snow.)

Answer: $0.496 \frac{\text{m}}{\text{s}^2}$

Use this space for summary and/or additional notes.

5. When a net force of 10.N acts on a hockey puck, the puck accelerates at a rate of $50 \frac{m}{s^2}$. Determine the mass of the puck.

Answer: 0.20 kg

6. A 15N net force is applied for 6.0 s to a 12 kg box initially at rest. What is the speed of the box at the end of the 6.0 s interval?

Answer: $7.5 \frac{m}{s}$

7. An 810 kg car accelerates from rest to $27 \frac{m}{s}$ in a distance of 120m. What is the magnitude of the average net force acting on the car?

Answer: 2460 N

8. A 44 kg child places one foot on each of two scales side-by-side. What is the child's weight? If the child distributes equal amounts of weight between the two scales, what is the reading on each scale? (Assume the scales display weights in newtons.)

Use this space for summary and/or additional notes.

9. A 70.0 kg astronaut pushes on a spacecraft with a force \bar{F} in space. The spacecraft has a total mass of 1.0×10^4 kg. During the push, the astronaut accelerates to the right with an acceleration of $0.36 \frac{\text{m}}{\text{s}^2}$. Determine the magnitude of the acceleration of the spacecraft.

Answer: $0.0025 \frac{\text{m}}{\text{s}^2}$

10. How much force will it take to accelerate a 60 kg student, wearing special frictionless roller skates, across the ground from rest to $16 \frac{\text{m}}{\text{s}}$ in 4 s?

Answer: 240 N

11. How much force would it take to accelerate a 60 kg student upwards at $2 \frac{\text{m}}{\text{s}^2}$?

Answer: 720 N

Use this space for summary and/or additional notes.

12. How much will a 400 N air conditioner weigh on the planet Mercury, where the value of \vec{g} is only $3.6 \frac{\text{m}}{\text{s}^2}$?

Answer: 146.9N

13. A person pushes a 500 kg crate with a force of 1200 N and the crate accelerates at $0.5 \frac{\text{m}}{\text{s}^2}$. What is the force of friction acting on the crate?

Answer: 950N

Use this space for summary and/or additional notes.

Force Applied at an Angle

Unit: Dynamics (Forces) & Gravitation

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 1.5

Skills:

- calculate forces applied at different angles using trigonometry

Language Objectives:

- Set up and solve word problems involving forces applied at an angle.

Labs, Activities & Demonstrations:

- Mass hanging from one or two scales. Change angle and observe changes in force.
- Fan cart with fan at an angle.
- For rope attached to heavy object, pull vs. anchor rope at both ends & push middle.

Notes:

An important property of vectors is that a vector has no effect on another vector that is perpendicular to it. As we saw with projectiles, this means that the velocity of an object in the horizontal direction has no effect on the velocity of the same object in the vertical direction. This allowed us to solve for the horizontal and vertical velocities as separate problems.

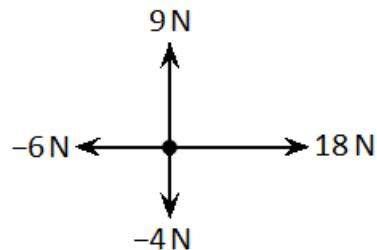
The same is true for forces. If forces are perpendicular to each other, they act independently, and the two can be separated into separate, independent mathematical problems:

$$\vec{F}_{\text{net,h}} = m\vec{a}_h$$

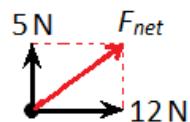
$$\vec{F}_{\text{net,v}} = m\vec{a}_v$$

Use this space for summary and/or additional notes.

For example, if we have the following forces acting on an object:



The net horizontal force (F_h) would be $18 \text{ N} + (-6 \text{ N}) = +12 \text{ N}$, and the net vertical force (F_v) would be $9 \text{ N} + (-4 \text{ N}) = +5 \text{ N}$. The total net force would be the resultant of the net horizontal and net vertical forces:



Using the Pythagorean Theorem:

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 5^2 + 12^2 &= F_{net}^2 \\ 169 &= F_{net}^2 \\ \sqrt{169} &= F_{net} = 13 \text{ N} \end{aligned}$$

We can get the angle from trigonometry:

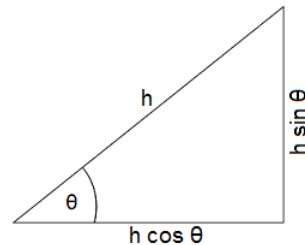
$$\begin{aligned} \tan \theta &= \frac{\text{opposite}}{\text{adjacent}} = \frac{5}{12} = 0.417 \\ \theta &= \tan^{-1}(\tan \theta) = \tan^{-1}(0.417) = 22.6^\circ \end{aligned}$$

(Of course, because you have just figured out the length of the hypotenuse, you could get the same answer by using \sin^{-1} or \cos^{-1} .)

Use this space for summary and/or additional notes.

If we have one or more forces that is neither vertical nor horizontal, we can use trigonometry to split the force into a vertical component and a horizontal component. Once all of the forces are either vertical or horizontal, we can add them as above.

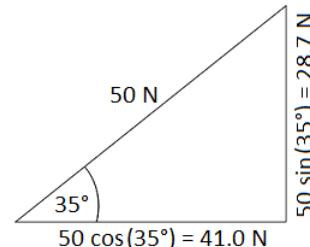
Recall the following relationships from trigonometry:



Suppose we have a force of 50 N at a direction of 35° above the horizontal. In the above diagram, this would mean that $h = 50 \text{ N}$ and $\theta = 35^\circ$:

$$\text{The horizontal force is } h \cos(\theta) = 50 \cos(35^\circ) = 41.0 \text{ N}$$

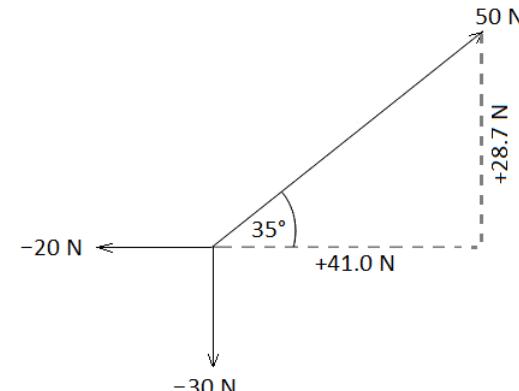
$$\text{The vertical force is } h \sin(\theta) = 50 \sin(35^\circ) = 28.7 \text{ N}$$



Now, suppose an object was subjected to the same 50 N force at an angle of 35° above the horizontal, but also a 20 N force to the left and a 30 N force downward.

The net horizontal force would therefore be $41 + (-20) = 21 \text{ N}$ to the right.

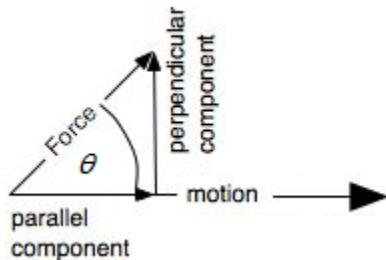
The net vertical force would therefore be $28.7 + (-30) = -1.3 \text{ N}$ upwards, which is the same as 1.3 N downwards.



Once you have calculated the net vertical and horizontal forces, you can resolve them into a single net force, as in the previous example.

Use this space for summary and/or additional notes.

In some physics problems, a force is applied at an angle but the object can move in only one direction. A common problem is a force applied at an angle to an object resting on a flat surface, which causes the object to move horizontally:



In this situation, only the horizontal (parallel) component of the applied force actually causes the object to move. If magnitude of the total force is F , then the horizontal component of the force is given by:

$$F_h = F \cos\theta$$

If the object accelerates horizontally, that means only the horizontal component is causing the acceleration, which means the net force must be $F \cos\theta$ and we can ignore the vertical component.

For example, suppose the worker in the diagram at the right pushes on the hand truck with a force of 200 N, at an angle of 60°.

The force in the direction of motion (horizontally) would be:

$$F \cos\theta = 200 \cos(60^\circ) = (200)(0.5) = 100 \text{ N}$$

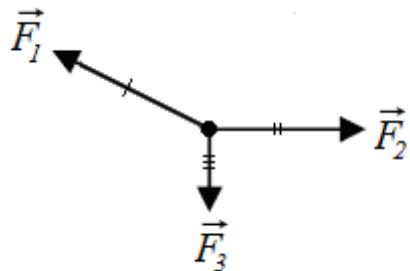
In other words, if the worker applies 200 N of force at an angle of 60°, the resulting horizontal force will be 100 N.



Use this space for summary and/or additional notes.

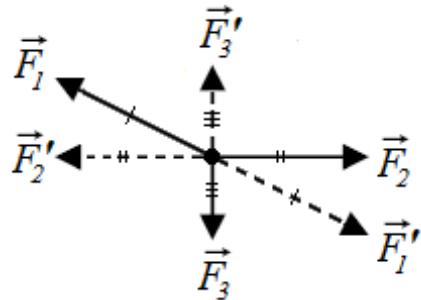
Static Problems Involving Forces at an Angle

Many problems involving forces at an angle are based on an object with no net force (either a stationary object or an object moving at constant velocity) that has three or more forces acting at different angles. In the following diagram, the forces are \vec{F}_1 , \vec{F}_2 and \vec{F}_3 .



\vec{F}_1 needs to cancel the combination of \vec{F}_2 and \vec{F}_3 . This means:

1. If we split \vec{F}_1 into its horizontal and vertical components, those components would be exactly opposite \vec{F}_2 and \vec{F}_3 , so we will call them \vec{F}'_2 and \vec{F}'_3 .
2. If we calculate the resultant of vectors \vec{F}_2 and \vec{F}_3 , this would be exactly opposite \vec{F}_1 , so we will call this vector \vec{F}'_1 .



As you can see, each vector is canceled by the resultant of the other two vectors, which shows why there is no net force.

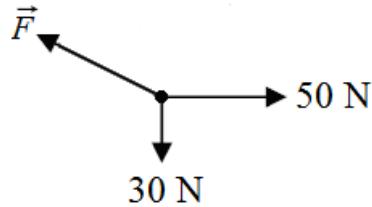
Use this space for summary and/or additional notes.

Strategy

1. Resolve all known forces into their horizontal and vertical components.
2. Add the horizontal and vertical components separately.
3. Use the Pythagorean Theorem to find the magnitude of forces that are neither horizontal nor vertical.
4. Because you know the vertical and horizontal components of the resultant force, use arcsine (\sin^{-1}), arccosine (\cos^{-1}) or arctangent (\tan^{-1}) to find the angle.

Sample Problems:

Q: A stationary object has three forces acting on it, as shown in the diagram below (which is not to scale):



What are the magnitude and direction of \vec{F} ?

A: \vec{F}' is equal and opposite to the resultant of the other two vectors. The magnitude of the resultant is:

$$\|\vec{F}'\| = \sqrt{30^2 + 50^2} = \sqrt{3400} = 58.3 \text{ N}$$

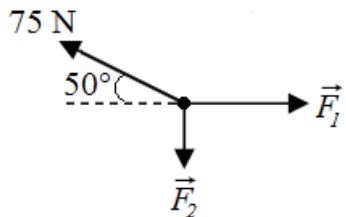
The direction is:

$$\tan \theta = \frac{30}{50} = 0.6$$

$$\theta = \tan^{-1}(0.6) = 31.0^\circ \text{ up from the left (horizontal)}$$

Use this space for summary and/or additional notes.

Q: A stationary object has three forces acting on it, as shown in the diagram below (which is not to scale):



What are the magnitudes of \vec{F}_1 and \vec{F}_2 ?

A: \vec{F}_1 and \vec{F}_2 are equal and opposite to the vertical and horizontal components of the 75 N force, which we can find using trigonometry:

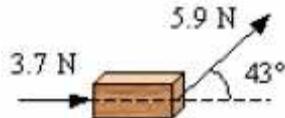
$$\|\vec{F}_1\| = \text{horizontal} = 75 \cos(50^\circ) = (75)(0.643) = 48.2 \text{ N}$$

$$\|\vec{F}_2\| = \text{vertical} = 75 \sin(50^\circ) = (75)(0.766) = 57.5 \text{ N}$$

Use this space for summary and/or additional notes.

Homework Problems

1. An object has three forces acting on it, a 15 N force pushing to the right, a 10. N force pushing to the right, and a 20. N force pushing to the left.
 - a. Draw a free-body diagram for the object (including a legend showing which direction is positive).
 - b. Calculate the magnitude of the net force on the object.
 - c. Draw a separate diagram showing only the net force (magnitude and direction) on the object.
2. A force of 3.7 N horizontally and a force of 5.9 N at an angle of 43° act on a 4.5-kg block that is resting on a frictionless surface, as shown in the following diagram:

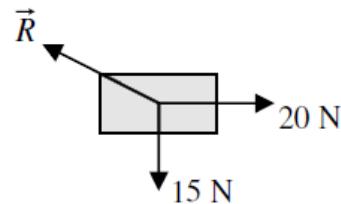


What is the magnitude of the horizontal acceleration of the block?

Answer: $1.8 \frac{\text{m}}{\text{s}^2}$

Use this space for summary and/or additional notes.

3. A stationary block has three forces acting on it: a 20 N force to the right, a 15 N force downwards, and a third force, \vec{R} of unknown magnitude and direction, as shown in the diagram to the right:



a. What are the horizontal and vertical components of \vec{R} ?

b. What is the magnitude of \vec{R} ?

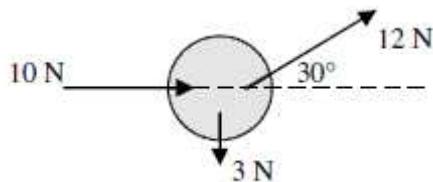
Answer: 25 N

c. What is the direction (angle up from the horizontal) of \vec{R} ?

Answer: 36.9°

Use this space for summary and/or additional notes.

4. Three forces act on an object. One force is 10 N to the right, one force is 3.0N downwards, and one force is 12 N at an angle of 30° above the horizontal, as shown in the diagram below.



- a. What are the net vertical and horizontal forces on the object?

Answer: positive directions are up and to the right.

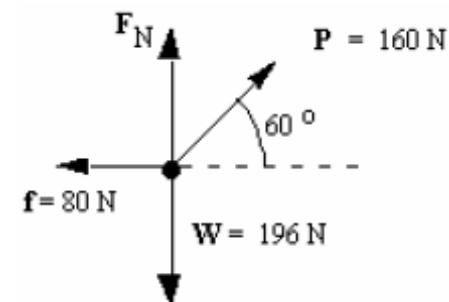
vertical: +3.0 N; horizontal: +20.4 N

- b. What is the net force (magnitude and direction) on the object?

Answer: 20.6 N at an angle of $+8.4^\circ$

Use this space for summary and/or additional notes.

5. A force of 160 N (\vec{P}) pulls at an angle of 60° on a crate that is sitting on a rough surface. The weight of the crate (\vec{W}) is 196 N. The force of friction on the crate (\vec{f}) is 80 N. These forces are shown in the diagram to the right:



- a. What is the magnitude of the normal force (\vec{F}_N) on the crate?

Answer: 57 N

- b. What is the acceleration of the crate?

Answer: zero

Use this space for summary and/or additional notes.

Friction

Unit: Dynamics (Forces) & Gravitation

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 1.6

Knowledge/Understanding Goals:

- difference between static & kinetic friction
- direction of the vector representing friction

Skills:

- calculate the frictional force on an object
- calculate net force in problems involving friction

Language Objectives:

- Understand and correctly use the terms “friction,” “static friction,” “kinetic friction,” and “coefficient of friction.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving friction.

Labs, Activities & Demonstrations:

- Drag a heavy object attached to a spring scale.
- Friction board (independent of surface area of contact).

Notes:

friction: a contact force caused by the roughness of the materials in contact, deformations in the materials, and molecular attraction between materials. Frictional forces are always parallel to the plane of contact between two surfaces, and opposite to the direction of motion or applied force.

static friction: friction acting on an object at rest that resists its ability to start moving.

kinetic friction: friction resisting the motion of an object.

In almost all situations, static friction is a stronger force than kinetic friction.

Use this space for summary and/or additional notes.

coefficient of friction: a constant that relates the frictional force on an object to the normal force. The coefficient of friction is represented by the Greek letter μ (mu). It is a dimensionless number, which means that it has no units. (This is because μ is a ratio of two forces, so the units cancel.)

coefficient of static friction: μ_s represents the coefficient of friction for an object that is stationary (not moving)

coefficient of kinetic friction: μ_k represents the coefficient of friction for an object that is moving.

The coefficient of friction takes into account the surface areas and surface characteristics of the objects in contact.

The force of friction on an object is given by the equations:

$$F_f \leq \mu_s F_N \quad \text{for an object that is stationary, and}$$

$$F_f = \mu_k F_N \quad \text{for an object that is moving,}$$

Where F_f is the magnitude of the force of friction, μ_s and μ_k are the coefficients of static and kinetic friction, respectively, and F_N is the magnitude of the normal force.

Note that the force of static friction is an inequality. For a stationary object, the force that resists sliding is, of course, equal to the force applied. However, once the applied force exceeds $\mu_s F_N$, the object starts moving and the equation for kinetic friction applies.

Use this space for summary and/or additional notes.

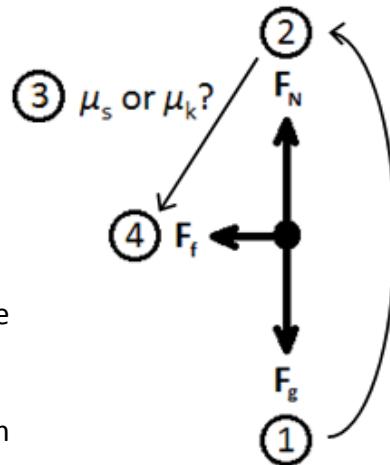
Solving Simple Friction Problems

Because friction is a contact force, all friction problems involve friction in addition to some other (usually externally applied) force.

To calculate the force from friction, you need to:

1. Calculate the force of gravity. On Earth, $F_g = m(9.8)$
2. Calculate the normal force. If the object is resting on a horizontal surface (which is usually the case), the normal force is usually equal in magnitude to the force of gravity. This means that for an object sliding across a horizontal surface:

$$F_N = F_g$$



3. Figure out whether the friction is static (there is an applied force, but the object is not moving), or kinetic (the object is moving). Look up the appropriate coefficient of friction (μ_s for static friction, or μ_k for kinetic friction).
4. Calculate the force of friction from the equation:

$$F_f \leq \mu_s F_N \quad \text{or} \quad F_f = \mu_k F_N$$

Make the force of friction positive or negative, as appropriate.

5. If the problem is asking for net force, remember to go back and calculate it now that you have calculated the force of friction.

If friction is causing the object to slow down and eventually stop and there is no separate applied force, then:

$$F_{net} = F_f$$

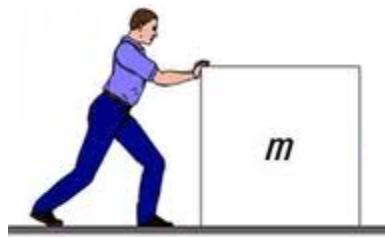
However, if there is an applied force and friction is opposing it, then the net force would be:

$$F_{net} = F_{applied} - F_f$$

Use this space for summary and/or additional notes.

Sample Problem:

Q: A person pushes a box at a constant velocity across a floor:



The box has a mass of 40 kg, and the coefficient of kinetic friction between the box and the floor is 0.35. What is the magnitude of the force that the person exerts on the box?

A: The box is moving at a constant velocity, which means there is no acceleration, and therefore no net force on the box. This means the force exerted by the person is exactly equal to the force of friction.

The force of friction between the box and the floor is given by the equation:

$$F_f = \mu_k F_N$$

The normal force is equal in magnitude to the weight of the box (F_g), which is given by the equation:

$$F_N = F_g = ma = (40)(10) = 400 \text{ N}$$

Therefore, the force of friction is:

$$F_f = \mu_k F_N$$

$$F_f = (0.35)(400) = 140 \text{ N}$$

Use this space for summary and/or additional notes.

Homework Problems

For these problems, you may need to look up coefficients of friction from Table C of your Physics Reference Tables on page 504.

1. A student wants to slide a steel 15 kg mass across a steel table.
 - a. How much force must the student apply in order to start the box moving?

Answer: 111 N

- b. How much force must the student apply to keep the mass moving at a constant velocity?

Answer: 85.5 N

2. A wooden desk has a mass of 74 kg.
 - a. How much force must be applied to the desk to start it moving across a wooden floor?

Answer: 310.8 N

- b. Once the desk is in motion, how much force must be used to keep it moving at a constant velocity?

Answer: 222 N

Use this space for summary and/or additional notes.

3. A large sport utility vehicle has a mass of 1850 kg and is traveling at $15 \frac{\text{m}}{\text{s}}$ (a little over 30 MPH). The driver slams on the brakes, causing the vehicle to skid.
- a. How far would the SUV travel before it stops on dry asphalt? (*Hint: this is a combination of a motion problem and a dynamics problem.*)

Answer: 16.8 m

- b. How far would the SUV travel if it were skidding to a stop on ice?

Answer: 75 m

Use this space for summary and/or additional notes.

4. A curling stone with a mass of 18 kg slides 38 m across a sheet of ice in 8.0 s before it stops because of friction. What is the coefficient of kinetic friction between the ice and the stone?

Answer: 0.12

Use this space for summary and/or additional notes.

Aerodynamic/Hydrodynamic Drag

Unit: Dynamics (Forces) & Gravitation

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding Goals:

- an intuitive sense of how aerodynamic drag works
- how aerodynamic drag is calculated

Language Objectives:

- Understand and correctly use the term “drag” when it refers to an object that is slowed down by a fluid.
- Accurately describe and apply the concepts described in this section, using appropriate academic language.
- Set up and solve word problems relating to aerodynamic drag.

Labs, Activities & Demonstrations:

- Crumpled piece of paper or tissue vs. golf ball (drag force doesn't depend on mass).
- Projectiles with same mass but different shapes.

Notes:

Most of the physics problems that involve aerodynamic drag fall into two categories:

1. The drag force is small enough that we ignore it.
2. The drag force is equal to some other force that we can measure or calculate.
3. The question asks only for a qualitative comparison of forces with and without aerodynamic drag.

However, it is possible to estimate the drag force on an object based on its shape, its velocity, cross-sectional area, and the density of the fluid (*e.g.*, air or water) it is moving through.

Use this space for summary and/or additional notes.

For simple situations involving aerodynamic drag, the drag force is given by the following equation:

$$\vec{F}_D = -\frac{1}{2} \rho \vec{v}^2 C_D A$$

where:

\vec{F}_D = drag force

ρ = density of the fluid that the object is moving through

\vec{v} = velocity of the object (relative to the fluid)

C_D = drag coefficient of the object (based on its shape)

A = cross-sectional area of the object in the direction of motion

This equation applies when the object has a blunt form factor, and the object's velocity relative to the properties of the fluid (such as viscosity) causes turbulence in the object's wake (*i.e.*, behind the object).

The drag coefficient, C_D , is a dimensionless number (meaning that it has no units) that encompasses all of the types of friction associated with aerodynamic drag. It serves the same purpose in drag problems that the coefficient of friction, μ , serves in problems involving friction between solid surfaces.

Approximate drag coefficients for simple shapes are given in the table to the right, assuming that the fluid motion relative to the object is in the direction of the arrow.

Measured Drag Coefficients

Shape	Drag Coefficient
Sphere	0.47
Half-sphere	0.42
Cone	0.50
Cube	1.05
Angled Cube	0.80
Long Cylinder	0.82
Short Cylinder	1.15
Streamlined Body	0.04
Streamlined Half-body	0.09

Use this space for summary and/or additional notes.

Universal Gravitation

Unit: Dynamics (Forces) & Gravitation

NGSS Standards: HS-PS2-4

MA Curriculum Frameworks (2006): 1.7

Skills:

- solve problems involving Newton's Law of Universal Gravitation

Language Objectives:

- Understand and correctly use the terms "gravity" and "gravitation."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to Newton's Law of Universal Gravitation.

Notes:

Gravity is a force of attraction between two objects because of their mass. The cause of this attraction is not currently known. More mass causes a stronger force, but the force gets weaker as the object gets farther away.

This means that the gravitational pull of a single object is directly proportional to its mass, and inversely proportional to its distance, *i.e.*,

$$F_g \propto \frac{m}{d}$$

(The symbol \propto means "is proportional to".)

If we have two objects, "1" and "2":

$$F_{g,1} \propto \frac{m_1}{d_1} \quad \text{and} \quad F_{g,2} \propto \frac{m_2}{d_2}$$

Because each object is pulling on the other one, the total force is therefore:

$$F_g \propto \frac{m_1}{d_1} \cdot \frac{m_2}{d_2} = \frac{m_1 m_2}{d_1 d_2} = \frac{m_1 m_2}{d^2}$$

Use this space for summary and/or additional notes.

Finally, if we are using MKS units, the masses are in kilograms, the distance is in meters. If we want the force in newtons to be correct, we have to multiply by the appropriate conversion factor, which turns out to be $6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}$. (The units are chosen because they cancel the m^2 and kg^2 from the formula and give newtons, which is the desired unit. Thus the formula becomes:

$$F_g = (6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}) \frac{m_1 m_2}{d^2}$$

The number $6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}$ is called the universal gravitation constant, and is represented by the symbol "G". Thus we have Isaac Newton's Law of Universal Gravitation in equation form:

$$F_g = \frac{G m_1 m_2}{d^2}$$

If we divide this expression by the mass of the object being attracted (m_2), we get the equation:

$$\frac{\vec{F}_g}{m_2} = \frac{G m_1 \cancel{m_2}}{d^2 \cancel{m_2}} = \frac{G m_1}{d^2}$$

Recall from the section on Gravitational Fields on page 160 that $\vec{g} = \frac{\vec{F}_g}{m}$. This means:

$$\vec{g} = \frac{G m_1}{d^2}$$

In the above expression, G is $6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}$, m_1 is the mass of the Earth ($5.97 \times 10^{24} \text{ kg}$) and d is the radius of the Earth ($6.37 \times 10^6 \text{ m}$). If you plug these numbers into the formula, you get $g = 9.81 \frac{\text{m}}{\text{s}^2} \approx 10 \frac{\text{m}}{\text{s}^2}$ as expected.

Use this space for summary and/or additional notes.

Sample Problems:

Q: Find the force of gravitational attraction between the Earth and a person with a mass of 75 kg. The mass of the Earth is 5.97×10^{24} kg, and its radius is 6.37×10^6 m.

A: $F_g = \frac{Gm_1m_2}{d^2}$

$$F_g = \frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})(75)}{(6.37 \times 10^6)^2}$$

$$F_g = 736 \text{ N}$$

This is the same number that we would get using $F_g = mg$, with $g = 9.81 \frac{\text{m}}{\text{s}^2}$.

Our approximation of $g = 10 \frac{\text{m}}{\text{s}^2}$ gives $F_g = 750 \text{ N}$, which is within 2%.

Q: Find the acceleration due to gravity on the moon.

A: $g_{\text{moon}} = \frac{Gm_{\text{moon}}}{d_{\text{moon}}^2}$

$$g_{\text{moon}} = \frac{(6.67 \times 10^{-11})(7.35 \times 10^{22})}{(1.74 \times 10^6)^2} = 1.62 \frac{\text{m}}{\text{s}^2}$$

Use this space for summary and/or additional notes.

Homework Problems

You will need planetary data from Table S and Table T on page 512 of your Reference Tables. The value of the universal gravitation constant G may be found in Table B on page 504 of your Reference Tables.

1. Find the force of gravity between the earth and the sun.

Answer: $3.52 \times 10^{22} \text{ N}$

2. Find the acceleration due to gravity (the value of g) on the planet Mars.

Answer: $3.73 \frac{\text{m}}{\text{s}^2}$

3. A mystery planet in another part of the galaxy has an acceleration due to gravity of $5.0 \frac{\text{m}}{\text{s}^2}$. If the radius of this planet is $2.0 \times 10^6 \text{ m}$, what is its mass?

Answer: $3.0 \times 10^{23} \text{ kg}$

Use this space for summary and/or additional notes.

4. A person has a mass of 80. kg.
- What is the weight of this person on the surface of the Earth?

Answer: 785 N

- What is the weight of the same person when orbiting the Earth at a height of 4.0×10^6 m above its surface? (*Hint: Remember that gravity acts from the center of the Earth. It may be helpful to draw a sketch.*)

Answer: 296N

Use this space for summary and/or additional notes.

Kepler's Laws of Planetary Motion

Unit: Dynamics (Forces) & Gravitation

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 1.1, 1.2

Knowledge/Understanding Goals:

- understand terms relating to angular position, speed & velocity

Skills Goals:

- solve problems using Kepler's Laws

Language Objectives:

- Accurately describe and apply the concepts described in this section using appropriate academic language.

Notes:

The German mathematician and astronomer Johannes Kepler derived the laws and equations that govern planetary motion, which were published in three volumes between 1617 and 1621.

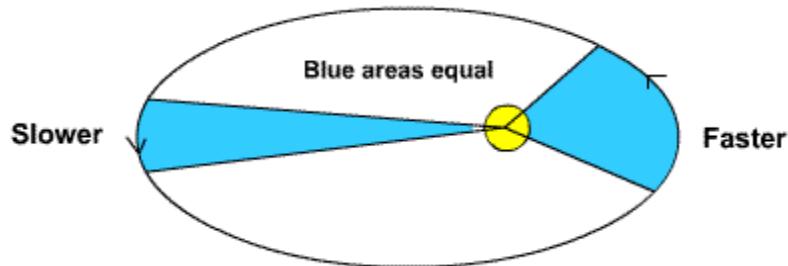
Kepler's First Law

The orbit of a planet is an ellipse, with the sun at one focus.

Use this space for summary and/or additional notes.

Kepler's Second Law

A line that joins a planet with the sun will sweep out equal areas in equal amounts of time.



i.e., the planet moves faster as it moves closer to the sun and slows down as it gets farther away. If the planet takes exactly 30 days to sweep out one of the blue areas above, then it will take exactly 30 days to sweep out the other blue area, and any other such area in its orbit.

While we now know that the planet's change in speed is caused by the force of gravity, Kepler's Laws were published fifty years before Isaac Newton published his theory of gravity.

Kepler's Third Law

If T is the period of time that a planet takes to revolve around a sun and \bar{r} is the average radius of the planet from the sun (the length of the semi-major axis of its elliptical orbit) then:

$$\frac{T^2}{\bar{r}^3} = \text{constant for every planet in that solar system}$$

As it turns out, $\frac{T^2}{\bar{r}^3} = \frac{4\pi^2}{GM}$, where G is the universal gravitational constant and M is the mass of the star in question, which means this ratio is different for every planetary system. For our solar system, the value of $\frac{T^2}{\bar{r}^3}$ is approximately $9.5 \times 10^{-27} \frac{s^2}{m^3}$ or $3 \times 10^{-34} \frac{y^2}{m^3}$.

Use this space for summary and/or additional notes.

Introduction: Rotational Dynamics

Unit: Rotational Dynamics

Topics covered in this chapter:

Centripetal Force	209
Center of Mass	214
Rotational Inertia	217
Torque	219

This chapter deals with the ability of a moving object (or potential for an object to move) to affect other objects.

- *Linear Momentum* describes a way to represent the movement of an object and what happens when objects collide, and the equations that relate to it. *Impulse* describes changes in momentum.
- *Work* and *Energy* describe the ability to cause something to move and the related equations. *Power* describes the rate at which energy is applied.
- *Escape Velocity* and *Newton's Cradle* describe interesting applications of energy and momentum.

New challenges in this chapter involve keeping track of the same quantity applied to the same object, but at different times.

Textbook:

- *Physics: Principles and Problems* Ch. 7: Motion in Two Dimensions, §7.2 (pp. 142–146)

Standards addressed in this chapter:

Next Generation Science Standards (NGSS):

No NGSS standards are addressed in this chapter.

Massachusetts Curriculum Frameworks (2006):

- 1.8 Describe conceptually the forces involved in circular motion.

Use this space for summary and/or additional notes:

Topics from this chapter assessed on the SAT Physics Subject Test:

- **Energy and Momentum**, such as potential and kinetic energy, work, power, impulse, and conservation laws.

1. Uniform Circular Motion
2. Rotational Dynamics

Skills learned & applied in this chapter:

- Working with more than one instance of the same quantity in a problem.
- Conservation laws (before/after problems).

Use this space for summary and/or additional notes.

Centripetal Force

Unit: Rotational Dynamics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 1.8

Knowledge/Understanding Goals:

- the difference between centripetal and centrifugal force

Skills:

- calculate the centripetal force of an object moving in a circle

Language Objectives:

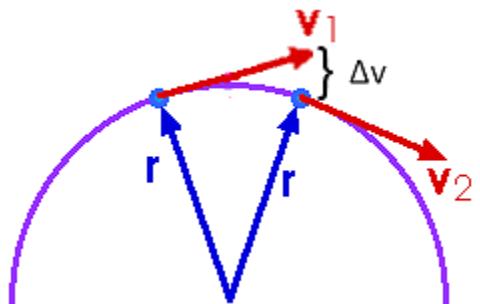
- Understand and correctly use the terms “rotation,” “centripetal force,” and “centrifugal force.”
- Explain the difference between centripetal force and centrifugal force.

Labs, Activities & Demonstrations:

- Swing a bucket of water in a circle.
- Golf ball loop-the-loop.
- Spin a weight on a string and have the weight pull up on a mass or spring scale.

Notes:

As we saw previously, when an object is moving at a constant speed around a circle, its direction keeps changing toward the center of the circle as it goes around, which means *there is continuous acceleration toward the center of the circle*.

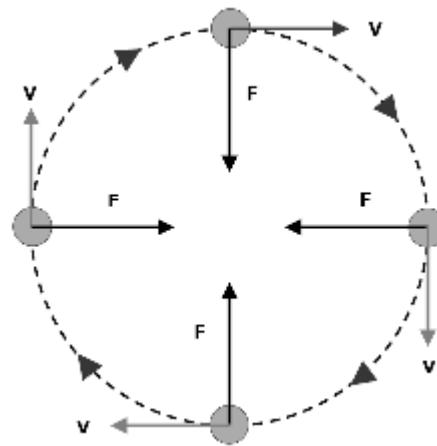


Use this space for summary and/or additional notes.

Centripetal Force

Because acceleration is caused by a net force (Newton's second law of motion), if there is continuous acceleration toward the center of the circle, then there must be a continuous force toward the center of the circle.

This force is called "centripetal force".



centripetal force: the inward force that keeps an object moving in a circle. If the centripetal force were removed, the object would fly away from the circle in a straight line that starts from a point tangent to the circle.

Recall that the formula for centripetal acceleration (a_c) is:

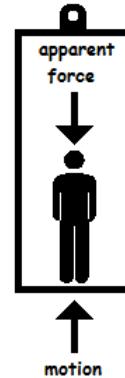
$$a_c = \frac{v^2}{r} = r\omega^2$$

Given that $F = ma$, the equation for centripetal force is therefore:

$$F_c = ma_c = \frac{mv^2}{r} = mr\omega^2$$

centrifugal "force": the apparent outward force felt by an object that is moving in a circle.

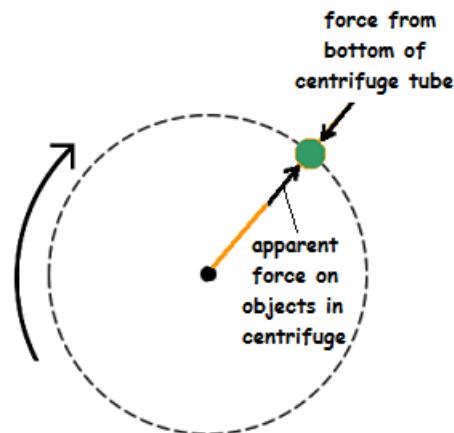
Centrifugal "force" is technically not a force as we would define it in physics. Centrifugal "force" is actually the inertia of objects resisting motion as they are continuously pulled toward the center of a circle by centripetal acceleration.



Use this space for summary and/or additional notes.

As an analogy, imagine that you are standing in an elevator. While the elevator is accelerating upward, the force between you and the floor of the elevator increases. An increase in the normal force from the floor because of the upward acceleration of the elevator feels the same as an increase in the downward force of gravity.

Similarly, a sample being spun in a centrifuge is subjected to the force *from the bottom of the centrifuge tube* as the tube is accelerated toward the center. The faster the rotation, the stronger the force. Again, an increase in the normal force from the bottom of the centrifuge tube would "feel" the same as a downward force toward the bottom of the centrifuge tube.



Sample Problems:

Q: A 300 kg roller coaster car reaches the bottom of a hill traveling at a speed of $20 \frac{\text{m}}{\text{s}}$. If the track curves upwards with a radius of 50 m, what is the total force exerted by the track on the car?

A: The total force on the car is the normal force needed to resist the force of gravity on the car (equal to the weight of the car) plus the centripetal force exerted on the car as it moves in a circular path.

$$F_g = mg = (300)(10) = 3000 \text{ N}$$

$$F_c = \frac{mv^2}{r} = \frac{(300)(20)^2}{50} = 2400 \text{ N}$$

$$F_N = F_g + F_c = 3000 + 2400 = 5400 \text{ N}$$

Use this space for summary and/or additional notes.

Q: A 20 g ball attached to a 60 cm long string is swung in a horizontal circle 80 times per minute. Neglecting gravity, what is the tension in the string?

A: Converting to MKS units, the mass of the ball is 0.02 kg and the string is 0.6 m long.

$$\omega = \frac{80 \text{ revolutions}}{1 \text{ min}} \times \frac{2\pi \text{ rad}}{\text{revolution}} \times \frac{1 \text{ min}}{60 \text{ s}} = \frac{8\pi}{3} = 8.38 \frac{\text{rad}}{\text{s}}$$

$$F_T = F_c = mr\omega^2$$

$$F_T = F_c = (0.02)(0.6)(8.38)^2 = 0.842 \text{ N}$$

Homework Problems

- Find the force needed to keep a 0.5 kg ball spinning in a 0.70 m radius circle with an angular velocity of 15 revolutions every 10 s.

Answer: 31.1 N

- Find the force of friction needed to keep a 3 000 kg car traveling with a speed of $22 \frac{\text{m}}{\text{s}}$ around a highway exit ramp curve that has a radius of 100 m.

Answer: 14 520 N

Use this space for summary and/or additional notes.

3. A passenger on an amusement park ride is cresting a hill in the ride at $15 \frac{\text{m}}{\text{s}}$. If the top of the hill has a radius of 30 m, what force will a 50 kg passenger feel from the seat? What fraction of the passenger's weight is this?

Answer: 125 N; $\frac{1}{4}$

4. A roller coaster has a vertical loop with a 40 m radius. What speed at the top of the loop will make a 60 kg rider feel "weightless?"

Answer: $20 \frac{\text{m}}{\text{s}}$

5. A ride called "The Rotor" at Six Flags is a cylinder that spins at 56 RPM, which is enough to "stick" people to the walls. What force would a 90 kg rider feel from the walls of the ride, if the ride has a diameter of 6 m?

Answer: 9285 N

Use this space for summary and/or additional notes.

Center of Mass

Unit: Rotational Dynamics

NGSS Standards: HS-PS2-1

MA Curriculum Frameworks (2006): 1.5

Knowledge/Understanding Goals:

- center of mass

Skills:

- find the center of mass of an object

Language Objectives:

- Understand and correctly use the term “center of mass.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Labs, Activities & Demonstrations:

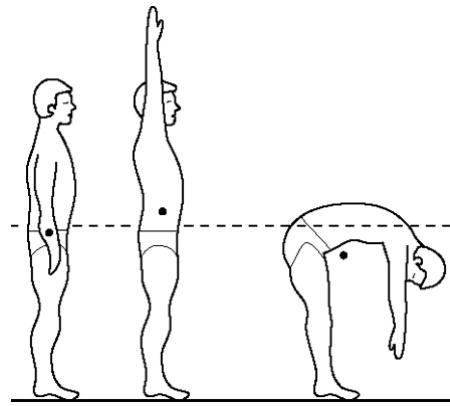
- Spin a hammer or drill team rifle.

Notes:

center of mass: the point where all of an object’s mass could be placed without changing the overall forces on the object or its rotational inertia.

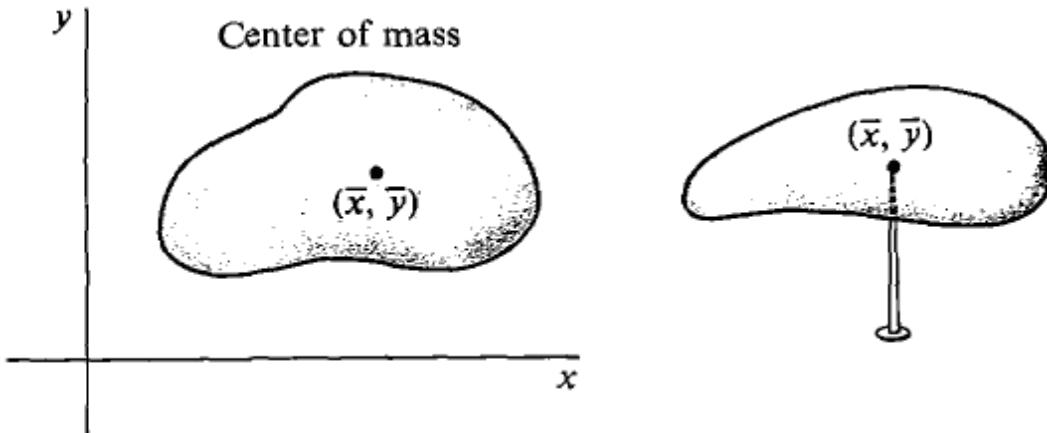
Use this space for summary and/or additional notes.

Objects have nonzero volumes. For any object, some of the mass of the object will always be closer to the center of rotation, and some of the mass will always be farther away. In most of the problems that you will see in this course, we can simplify the problem by pretending that all of the mass of the object is at a single point.



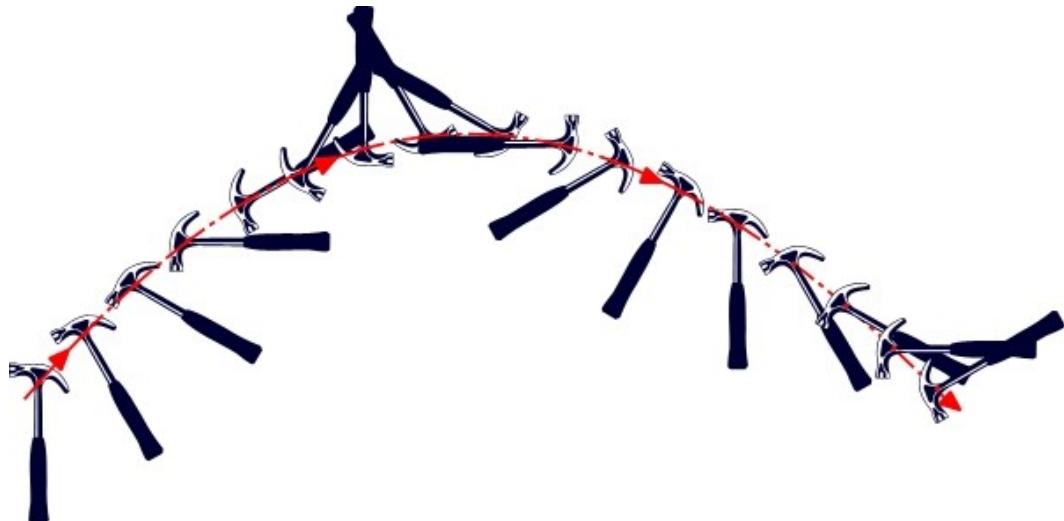
Because an object at rest remains at rest, this means that an object's center of mass is also the point at which the object will balance on a sharp point.

(Technically, because gravity is involved, the reason the object balances there is because the torques cancel.)



Use this space for summary and/or additional notes.

Finally, note that an object that is rotating freely in space will always rotate about its center of mass:



Use this space for summary and/or additional notes.

Rotational Inertia

Unit: Rotational Dynamics

NGSS Standards: HS-PS2-1

MA Curriculum Frameworks (2006): 1.5

Knowledge/Understanding Goals:

- rotational inertia

Skills:

- calculate the rotational inertia of a system that includes one or more masses at different radii from the center of rotation

Language Objectives:

- Understand and correctly use the terms “rotational inertia” and “torque.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Labs, Activities & Demonstrations:

- Bicycle wheel.
- Gyroscope.

Notes:

inertia: the tendency for an object to continue to do what it is doing (remain at rest or remain in motion).

rotational inertia (or angular inertia): the tendency for a rotating object to continue rotating.

Inertia in linear systems is a fairly easy concept to understand. The more mass an object has, the more it tends to remain at rest or in motion, and the more force is required to change its motion. *I.e.*, in a linear system, inertia depends only on mass.

Use this space for summary and/or additional notes.

Rotational Inertia

Rotational inertia is somewhat more complicated than inertia in a non-rotating system. Suppose we have a mass that is being rotated at the end of a string. (Let's imagine that we're doing this in space, so we can neglect the effects of gravity.) The mass's inertia keeps it moving around in a circle at the same speed. If you suddenly shorten the string, the mass continues moving at the *same speed through the air*, but because the radius is shorter, the mass makes more revolutions around the circle in a given amount of time.

In other words, the object has the same linear speed (*not* the same *velocity* because its *direction* is constantly changing), but its angular velocity (degrees per second) has increased.

This must mean that an object's moment of inertia (its tendency to continue moving at a constant angular velocity) must depend on its distance from the center of rotation as well as its mass.

Rotational inertia is covered quantitatively in AP Physics. For this class, it is sufficient to have a conceptual understanding of rotation and rotational inertia.

Use this space for summary and/or additional notes.

Torque

Unit: Rotational Dynamics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding Goals:

- center of mass and torque

Skills:

- calculate torque

Language Objectives:

- Understand and correctly use the terms “torque,” “axis of rotation,” “fulcrum,” “lever arm,” and “center of mass.”
- Accurately describe and apply the concepts described in this section, using appropriate academic language.
- Set up and solve word problems involving torque.

Labs, Activities & Demonstrations:

- Balance an object on two fingers and slide both toward the center.
- Clever wine bottle stand.

Notes:

torque ($\vec{\tau}$): a vector quantity that measures the effectiveness of a force in causing rotation. Take care to distinguish the Greek letter “ τ ” from the Roman letter “t”. Torque is measured in units of newton-meters:

$$1 \text{ N}\cdot\text{m} = 1 \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$$

(Note that work and energy (which we will study later) are also measured in Newton-meters. However, work and energy are scalar quantities. The derived S.I. unit Joule (J) is a scalar unit that measures scalar Newton-meters. Because torque is a *vector* quantity, one Newton-meter of torque can not be described as one Joule. This is true even if we are only concerning ourselves with the magnitude.)

axis of rotation: the point around which an object rotates.

Use this space for summary and/or additional notes.

fulcrum: the point around which a lever pivots. Also called the pivot.

lever arm: the distance from the axis of rotation that a force is applied, causing a torque.

Torque is the cross product of distance from the center of rotation (“lever arm”) \times force:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

For our purposes, this is equal to the lever arm times the component of the force that is perpendicular to the lever arm:

$$\tau = r F_{\perp}$$

(We use the variable r for the lever arm, which is a distance, because torque causes rotation, and r is the distance from the center of the circle (radius) at which the force is applied.

Sample Problems

Q: If a perpendicular force of 20 N is applied to a wrench with a 25 cm handle, what is the torque applied to the bolt?

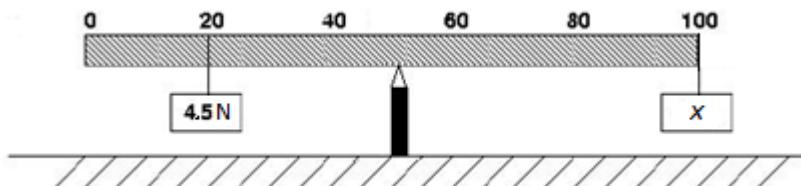
A: $\tau = r F_{\perp}$

$$\tau = (0.25\text{m})(20\text{N})$$

$$\tau = 4 \text{ N}\cdot\text{m}$$

Use this space for summary and/or additional notes.

Q: A 100 cm meter stick is balanced at its center (the 50-cm mark) with two objects hanging from it, as shown below:



One of the objects weighs 4.5 N, and is hung from the 20-cm mark (30 cm = 0.3 m from the fulcrum). A second object is hung at the opposite end (50 cm = 0.5 m from the fulcrum). What is the weight of the second object?

A: In order for the ruler to balance, the torque on the left side (which is trying to rotate the ruler counter-clockwise) must be equal to the torque on the right side (which is trying to rotate the ruler clockwise). The torques from the two halves of the ruler are the same (because the ruler is balanced in the middle), so this means the torques applied by the objects also must be equal.

The torque applied by the object on the left is:

$$\tau = rF = (0.30)(4.5) = 1.35 \text{ N}\cdot\text{m}$$

The torque applied by the object on the right must also be 1.35 N·m, so we can calculate the force:

$$\tau = rF$$

$$1.35 = 0.50 F$$

$$F = \frac{1.35}{0.50} = 2.7 \text{ N}$$

Extension

Just as yank is the rate of change of force with respect to time, the rate of change of torque with respect to time is called rotatum: $\vec{P} = \frac{\Delta \vec{\tau}}{\Delta t} = \vec{r} \times \vec{Y}$.

Rotatum is also sometimes called the “moment of a yank,” because it is the rotational analogue to yank.

Use this space for summary and/or additional notes.

Introduction: Work, Energy & Momentum

Unit: Work, Energy & Momentum

Topics covered in this chapter:

Work.....	225
Energy	233
Conservation of Energy.....	237
Escape Velocity	248
Power	251
Linear Momentum	256
Momentum and Kinetic Energy	269
Impulse.....	273
Angular Momentum.....	279

This chapter deals with the ability of a moving object (or potential for an object to move) to affect other objects.

- *Linear Momentum* describes a way to represent the movement of an object and what happens when objects collide, and the equations that relate to it. *Impulse* describes changes in momentum.
- *Work* and *Energy* describe the ability to cause something to move and the related equations. *Power* describes the rate at which energy is applied.
- *Escape Velocity* and *Newton's Cradle* describe interesting applications of energy and momentum.

New challenges in this chapter involve keeping track of the same quantity applied to the same object, but at different times.

Use this space for summary and/or additional notes.

Textbook:

- *Physics: Principles and Problems* Ch. 9: Momentum and Its Conservation (pp. 174–195)
- *Physics: Principles and Problems* Ch. 10: Work, Energy, and Simple Machines (pp. 196–215)
- *Physics: Principles and Problems* Ch. 11: Energy (pp. 216–239)

Standards addressed in this chapter:

Next Generation Science Standards (NGSS):

HS-PS2-2. Use mathematical representations to support the claim that the total momentum of a system of objects is conserved when there is no net force on the system.

HS-PS3-1. Create a computational model to calculate the change in the energy of one component in a system when the change in energy of the other component(s) and energy flows in and out of the system are known.

HS-PS3-2. Develop and use models to illustrate that energy at the macroscopic scale can be accounted for as either motions of particles or energy stored in fields.

HS-PS3-3. Design, build, and refine a device that works within given constraints to convert one form of energy into another form of energy.

Massachusetts Curriculum Frameworks (2006):

2.1 Interpret and provide examples that illustrate the law of conservation of energy.

2.2 Interpret and provide examples of how energy can be converted from gravitational potential energy to kinetic energy and vice versa.

2.3 Describe both qualitatively and quantitatively how work can be expressed as a change in mechanical energy.

2.4 Describe both qualitatively and quantitatively the concept of power as work done per unit time.

2.5 Provide and interpret examples showing that linear momentum is the product of mass and velocity, and is always conserved (law of conservation of momentum). Calculate the momentum of an object.

Use this space for summary and/or additional notes.

Topics from this chapter assessed on the SAT Physics Subject Test:

- **Energy and Momentum**, such as potential and kinetic energy, work, power, impulse, and conservation laws.
- 1. What is Linear Momentum?
- 2. Impulse
- 3. Conservation of Momentum
- 4. Collisions
- 5. Center of Mass
- 6. Work
- 7. Energy
- 8. Forms of Energy
- 9. Power

Skills learned & applied in this chapter:

- Working with more than one instance of the same quantity in a problem.
- Conservation laws (before/after problems).

Use this space for summary and/or additional notes.

Work

Unit: Work, Energy & Momentum

NGSS Standards: HS-PS3-1

MA Curriculum Frameworks (2006): 2.3

Knowledge/Understanding Goals:

- understand the definition of work

Skills:

- calculate work done by a force applied to an object

Language Objectives:

- Understand and correctly use the term “work.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving work.

Notes:

work: the effort applied over a distance against a force.

For example, if you lift a heavy object off the ground, you are doing work against the force of gravity.

Mathematically, work is the dot product of the force vector and the displacement vector:

$$W = \vec{F} \bullet \vec{d}$$

The dot product is one of three ways of multiplying vectors. The dot product is a scalar (a number without a direction), and is equal to the product of the magnitudes of the force and distance, and the cosine of the angle between them. This means:

$$W = F d \cos \theta = F_{||} d$$

Where F is the magnitude of the force vector \vec{F} , d is the magnitude of the displacement vector \vec{d} , and θ is the angle between the two vectors.

Sometimes $F \cos \theta$ is written as $F_{||}$, which means “the component of the force that is parallel to the direction of motion.”

Use this space for summary and/or additional notes.

Note that if the force and displacement are in the same direction, the angle $\theta = 0^\circ$ which means $\cos\theta = \cos(0^\circ) = 1$. In this case, $F_{||} = F\cos\theta = (F)(1) = F$ and the equation reduces to $W = Fd$.

Work is measured in newton-meters (N·m).

$$1 \text{ N}\cdot\text{m} \equiv 1 \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$$

As we will see later, work is a change in the energy of a system, which means work and energy must have the same units. This means that a newton-meter is equivalent to a Joule.

Note that:

- If the displacement is zero, no work is done by the force. *E.g.*, if you hold a heavy box without moving it, you are exerting a force (counteracting the force of gravity) but you are not doing work.
- If the net force is zero, no work is done by the displacement (change in location) of the object. *E.g.*, if a cart is sliding across a frictionless air track at a constant velocity, the net force on the cart is zero, which means no work is being done.
- If the displacement is perpendicular to the direction of the applied force ($\theta = 90^\circ$, which means $\cos\theta = 0$), no work is done by the force. *E.g.*, you can slide a very heavy object along a roller conveyor (pictured below), because the force of gravity is acting vertically and the object's displacement is horizontal, which means gravity is doing no work, and therefore you do not have to do any work against gravity.

Use this space for summary and/or additional notes.

In industry, gravity roller conveyors are usually set up at a slight downward angle:

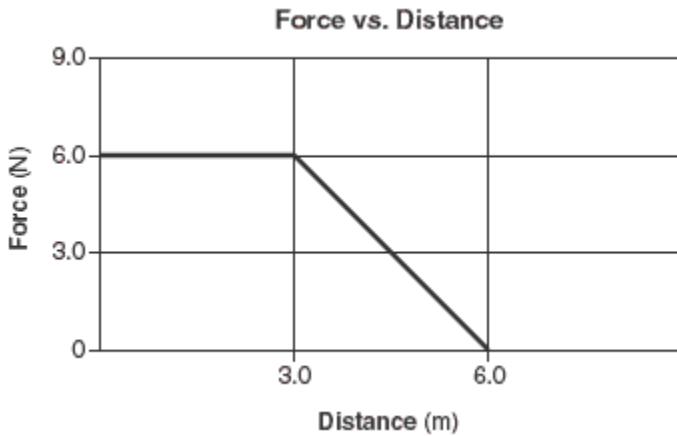


In this situation, gravity is doing work to pull the boxes downward. (Vertical force; vertical displacement). There is friction in the bearings of the rollers, which keeps the boxes from accelerating. However, friction cannot *cause* displacement, which means friction cannot do work. The angle is designed so that the work done by gravity is exactly the work done against friction, which means the workers do not have to do any work (in the physics sense ☺) to keep the boxes moving.

Use this space for summary and/or additional notes.

Force vs. Distance Graphs

If the amount of force is changing as an object moves, work is the area under a graph of force vs. distance:



In the above example, $18 \text{ N}\cdot\text{m}$ of work was done as the object moved from 0 m to 3 m, and $9 \text{ N}\cdot\text{m}$ of work was done as the object moved from 3 m to 6 m, for a total of $27 \text{ N}\cdot\text{m}$ of work.

Once again, note the general rule that we have a formula for a quantity (work) that equals the quantity on the y-axis (force) times the change in the quantity on the x-axis (distance). Therefore, the value of the quantity (work) equals the area under the graph (force vs. distance)

Use this space for summary and/or additional notes.

Sample Problems

Q: How much work does it take to lift a 60. kg box 1.5 m off the ground at a constant velocity over a period of 3.0 s?

A: The box is being lifted, which means the work is done against the force of gravity.

$$W = F_{\parallel} \cdot d = F_g d$$

$$W = F_g d = [mg]d$$

$$W = [(60)(10)](1.5)$$

$$W = [600](1.5) = 900 \text{ N}\cdot\text{m}$$

Note that the amount of time it took to lift the box has nothing to do with the amount of work done.

It may be tempting to try to use the time to calculate velocity and acceleration in order to calculate the force. However, because the box is lifted at a constant velocity, the only force needed to lift the box is enough to overcome the weight of the box (F_g).

In general, if work is done to move an object vertically, the work is done against gravity, and you need to use $a = g = 10 \frac{\text{m}}{\text{s}^2}$ for the acceleration when you calculate $F = ma$.

Similarly, if work is done to move an object horizontally, the work is *not* against gravity and either you need to know the force applied or you need to find it from the acceleration of the object using $F = ma$.

Use this space for summary and/or additional notes.

Q: In the picture below, the adult is pulling on the handle of the wagon with a force of 150. N at an angle of 60.0°.



If the adult pulls the wagon for a distance of 500. m, how much work does he do?

A: $W = F_{||}d$

$$W = [F \cos \theta]d$$

$$W = [(150.) \cos 60.0^\circ](500.)$$

$$W = [(150.)(0.500)](500.\text{m}) = 37500 \text{ N}\cdot\text{m}$$

Homework Problems

1. How much work is done against gravity by a weightlifter lifting a 30. kg barbell 1.5 m upwards at a constant speed?

Answer: 450 N·m

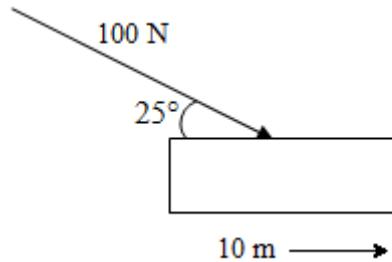
2. A 3000. kg car is moving across level ground at $5.0 \frac{\text{m}}{\text{s}}$ when it begins an acceleration that ends with the car moving at $15.0 \frac{\text{m}}{\text{s}}$. Is work done in this situation? How do you know?

Use this space for summary and/or additional notes.

3. A 60. kg man climbs a 3.0 m tall flight of stairs. How much work was done by the man against the force of gravity?

Answer: 1800 N·m

4. Find the work done when a 100. N force at an angle of 25° pushes a cart 10. m to the right, as shown in the diagram below.



Answer: 906 N·m

Use this space for summary and/or additional notes.

5. An Alaskan huskie pulls a sled using a 500. N force across a 10. m wide street. The force of friction on the 90. kg sled is 200. N. How much work is done by the huskie? How much work is done on the sled? How much work is done by friction? How much work is done by gravity?

Answers: by dog: $5000 \text{ N} \cdot \text{m}$
on sled: $3000 \text{ N} \cdot \text{m}$ (Find this by calculating F_{net} on the sled.)
by friction: $-2000 \text{ N} \cdot \text{m}$ (Work done *by* friction is the negative of work done *against* friction. We use a negative number to show the conservation of energy.)
by gravity: zero (because gravity is not causing any displacement)

Use this space for summary and/or additional notes.

Energy

Unit: Work, Energy & Momentum

NGSS Standards: HS-PS3-1

MA Curriculum Frameworks (2006): 2.1, 2.2, 2.3

Knowledge/Understanding Goals:

- define energy
- conservation of energy
- work-energy theorem

Skills:

- calculate gravitational potential energy
- calculate kinetic energy

Language Objectives:

- Understand and correctly use the terms “energy,” “kinetic energy,” and “potential energy.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving energy and the conservation of energy.

Labs, Activities & Demonstrations:

- “Happy” and “sad” balls.
- Popper.

Notes:

energy: the ability to do work

Energy is a scalar quantity that exists in several forms, including:

kinetic energy (K): the energy an object has because of its motion.

potential energy (U) the unrealized energy that an object has because of its position, temperature, chemical reactions that could occur, etc.

Use this space for summary and/or additional notes.

In mechanics, kinetic energy usually refers to the energy of an object because of its mass and velocity. Potential energy usually refers to an object's position, and the ability of the force of gravity to cause it to move.

heat (Q): the energy an object has because of the kinetic energy of its molecules.

work (W): the amount of change in the energy of one or more objects. $W = \Delta E$

electrical work (W): the work done by applying an electric current over a period of time.

Energy is measured in Joules (J):

$$1 \text{ J} \equiv 1 \text{ N} \cdot \text{m} \equiv 1 \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}$$

Systems and Potential Energy

A system is a collection of objects for the purpose of describing the interaction of objects within vs. outside of that collection. The surroundings is all of the objects outside of the system ("everything else").

Potential energy is a property of the energy relationship between two objects within a system, because of a field that can change the relationship between two or more objects within the system. A single object cannot have potential energy.

Gravitational Potential Energy

As discussed earlier, a gravity field is a region (near a massive object like the Earth) in which the force of gravity acts on all objects that have mass.

Gravitational potential energy is the work that the gravity field has the potential to do on the object because of its mass.

Use this space for summary and/or additional notes.

The gravitational potential energy of an object is determined by the strength of the gravitational field, the mass of the object, and the object's distance above the ground (height, which is the distance over which the force of gravity is able to do work on the object).

$$U_g = F_g h = mgh$$

(Remember that g is the strength of the gravity field near the surface of the Earth, which is equal to $10 \frac{\text{m}}{\text{s}^2}$, the acceleration due to gravity on Earth.)

Although most physicists use the above equation for potential energy, the AP formula sheet uses the variable "y" for height and " Δy " for "difference in height," which gives the following representation of the same formula:

$$U_g = mg\Delta y$$

Remember that gravitational potential energy exists only when there are two or more objects in a system, and at least one of the objects has a significant gravity field. For example, if an anvil is sitting on top of a cliff, the anvil has gravitational potential energy relative to the Earth, provided that the anvil and the Earth are both part of the system. (We would call this "the anvil-Earth system".)

Kinetic Energy

The translational (linear) kinetic energy of an object is related to its mass and velocity:

$$K = \frac{1}{2}mv^2$$

Note that a single object can have kinetic energy. An entire system can also have kinetic energy if the center of mass of the system is moving (has nonzero mass and velocity).

Kinetic energy exists both in linear systems and rotating systems. The above equation is for translational kinetic energy; rotational kinetic energy will be discussed in a separate topic.

Mechanical Energy

Mechanical energy is gravitational potential energy plus kinetic energy. Because potential energy and kinetic energy are easily interconverted, it is convenient to have a term that represents the combination of the two.

Use this space for summary and/or additional notes.

Heat

Kinetic energy is both a macroscopic property of a large object (*i.e.*, something that is at least large enough to see), and a microscopic property of the individual particles (atoms or molecules) that make up an object. Heat is the macroscopic energy that an object has due to the combined kinetic energies of its individual particles.

As we will see when we study thermal physics, temperature is the average microscopic kinetic energy of the individual particles that an object is made of. (Macroscopic) kinetic energy can be converted into heat if the kinetic energy of a macroscopic object is turned into the individual kinetic energies of the molecules of that object and/or some other object. This can occur via friction or via a collision.

Chemical Potential Energy

In chemistry, chemical potential energy comes from the electromagnetic forces attracting the atoms in a chemical bond. The energy absorbed or given off in a chemical reaction is the difference between the energies of those bonds before vs. after the reaction. If energy is given off, it is absorbed by the particles, increasing their kinetic energy, which means the temperature increases. If energy is absorbed, that energy must come from the kinetic energy of the particles, which means the temperature decreases.

Electric Potential

Electric potential is the energy that moves electric charges, enabling them to do work. The energy for this must ultimately come from some other source, such as chemical potential (*i.e.*, a battery), mechanical energy (*i.e.*, a generator), etc.

Use this space for summary and/or additional notes.

Conservation of Energy

Unit: Work, Energy & Momentum

NGSS Standards: HS-PS3-1

MA Curriculum Frameworks (2006): 2.1, 2.2, 2.3

Knowledge/Understanding Goals:

- conservation of energy
- work-energy theorem

Skills:

- solve problems involving conversion of energy between one form and another

Language Objectives:

- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving the conservation of energy.

Labs, Activities & Demonstrations:

- Golf ball loop-the-loop.
- Marble raceways.
- Bowling ball pendulum.

Notes:

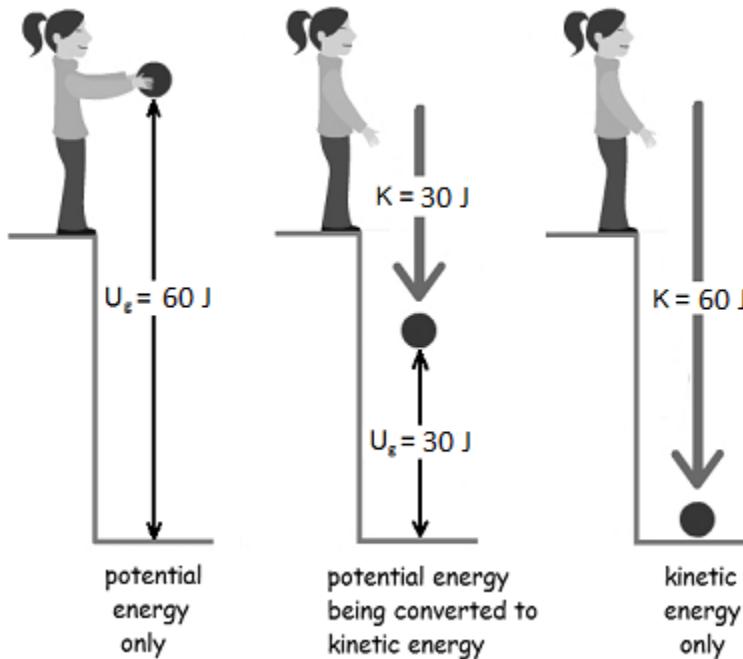
In a *closed system* (meaning a system in which there is no exchange of matter or energy between the system and the surroundings), the total energy is constant. Energy can be converted from one form to another. When this happens, the increase in any one form of energy is the result of a corresponding decrease in another form of energy.

Use this space for summary and/or additional notes.

In a system that has potential energy, kinetic energy and heat, the total energy is given by:

$$E_{total} = U + K + Q$$

In the following diagram, suppose that the girl drops a ball with a mass of 2 kg from a height of 3 m.



Before the girl lets go of the ball, it has 60 J of potential energy. As the ball falls to the ground, potential energy is gradually converted to kinetic energy. The potential energy continuously decreases and the kinetic energy continuously increases, but the total energy is always 60 J. After the ball hits the ground, 60 N·m of work was done by gravity, and the 60 J of kinetic energy is converted to other forms, such as thermal energy (the temperatures of the ball and the ground increase infinitesimally), sound, etc.

Use this space for summary and/or additional notes.

Work-Energy Theorem

We have already seen that work is the action of a force applied over a distance. A broader and more useful definition is that work is the change in the energy of an object or system. If we think of a system as having imaginary boundaries, then work is the flow of energy across those boundaries, either into or out of the system.

For a system that has only mechanical energy, work changes the amount of potential and/or kinetic energy in the system.

$$W = \Delta K + \Delta U$$

Although work is a scalar quantity, we *use a positive number for work coming into the system* (“work is done on the system”), and a *negative number for work going out of the system* (“the system does work on its surroundings”).

Note that the units for work and energy—newton-meters and joules—are equivalent.

$$1 \text{ J} \equiv 1 \text{ N} \cdot \text{m} \equiv 1 \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}$$

Work-energy theorem problems will give you information related to the gravitational potential and/or kinetic energy of an object (such as its mass and a change in velocity) and ask you how much work was done.

A simple rule of thumb (meaning that it is *not always strictly true*) is:

- Potential energy is energy in the *future* (energy that is available for use).
- Kinetic energy is energy in the *present* (the energy of an object that is currently in motion).
- Work is the result of energy in the *past* (the result of potential kinetic energy having acted on an object).

Use this space for summary and/or additional notes.

Solving Conservation of Energy Problems

Conservation of energy problems involve recognizing that energy is changing from one form to another. Once you have figured out what is being converted, calculate the amount energy that is converted, and use the equation for the new form to calculate the desired quantity.

In mechanics, conservation of energy problems usually involve work, gravitational potential energy, and kinetic energy:

$$W = \vec{F} \bullet \vec{d} = Fd \cos\theta \quad (= \text{Force and displacement are in the same direction})$$

$$U_g = mgh$$

$$K = \frac{1}{2}mv^2$$

There are two common types of conservation of energy problems:

One type gives you the energy at the start, and you have to calculate the desired quantity (such as velocity or height) from the energy after the conversion.

$$\text{Energy of one type} = \text{Energy of another type}$$

$$\text{Equation for first type} = \text{Equation for second type}$$

The second type requires you to calculate the same type of energy at two different times or locations. You have to use the difference to calculate the desired quantity (such as velocity or height).

$$\text{Energy after} - \text{Energy before} = \text{Energy Change}$$

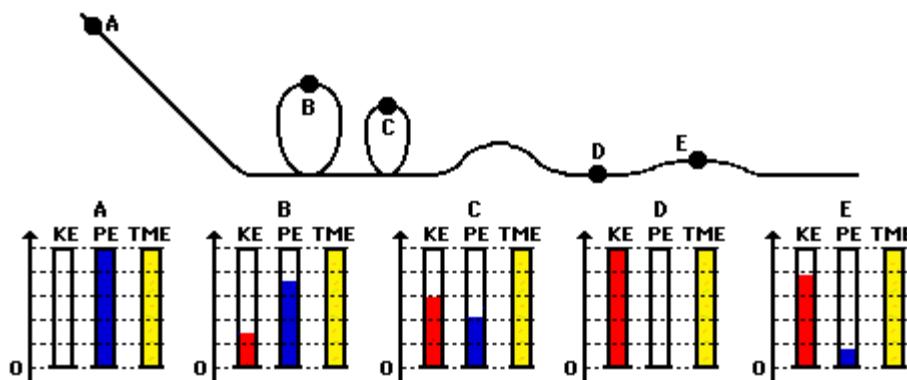
$$\text{Energy change of one type} = \text{Energy change of another type or work done}$$

Use this space for summary and/or additional notes.

Energy Bar Charts

A useful way to represent conservation of energy is through bar graphs that represent kinetic energy (KE), gravitational potential energy (PE), and total mechanical energy (TME). (We use the term “chart” rather than “graph” because the scale is arbitrary and the chart is not meant to be used quantitatively.)

The following is an energy bar chart for a roller coaster, starting from point A and traveling through points B, C, D, and E.



Notice, in this example, that:

1. The total mechanical energy always remains the same. (This is often the case in conservation of energy problems.)
2. KE is zero at point A because the roller coaster is not moving. All of the energy is PE, so PE = TME.
3. PE is zero at point D because the roller coaster is at its lowest point. All of the energy is KE, so KE = TME.
4. At all points (including points A and D), $KE + PE = TME$

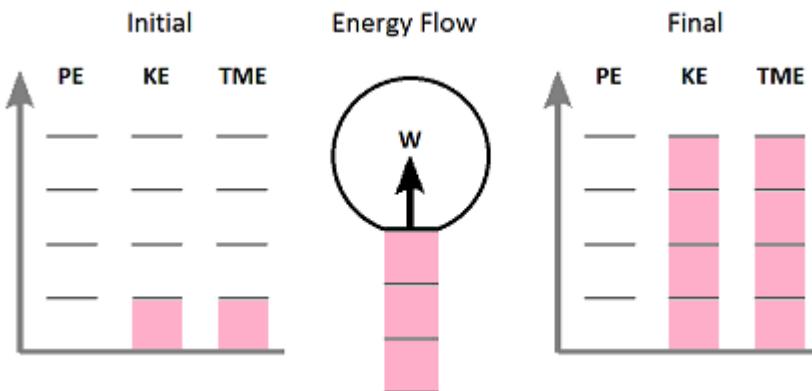
It can be helpful to sketch energy bar charts representing the different points in complicated conservation of energy problems. If energy is being added to or removed from the system, add an Energy Flow diagram to show energy that is being added to or removed from the system.

Use this space for summary and/or additional notes.

Sample Problems

Q: An 875 kg car accelerates from $22 \frac{m}{s}$ to $44 \frac{m}{s}$.

- a. Draw an energy bar chart representing the initial and final energies and the flow of energy into or out of the system.



Notice that:

- *KE = TME, both before and after. There is no gravitational potential energy described in the problem.*
- *TME is greater at the end than at the beginning, which means energy must have been added to the system. This is represented by the flow of three “units” of energy into the system. (The flow of energy into or out of a system is “work”).*

- b. What were the actual initial and final kinetic energies of the car?

$$K_i = \frac{1}{2}mv_1^2 = \frac{1}{2}(875\text{kg})(22\frac{m}{s})^2 = 211750\text{J}$$

$$K_f = \frac{1}{2}mv_2^2 = \frac{1}{2}(875\text{kg})(44\frac{m}{s})^2 = 847000\text{J}$$

(Because kinetic energy is proportional to v^2 , notice that doubling the velocity resulted in four times as much kinetic energy.)

- c. How much work did the engine do to accelerate it?

$$W = \Delta K = 847000\text{J} - 211750\text{J} = 635250\text{J} = 635250\text{N}\cdot\text{m}$$

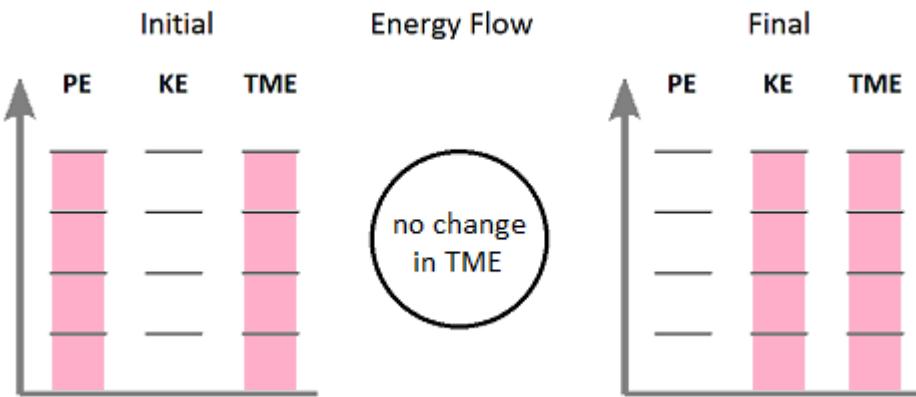
The engine did 635250 N·m of work.

Use this space for summary and/or additional notes.

Q: An 80 kg physics student falls off the roof of a 15 m high school building. How much kinetic energy does he have just before he hits the ground? What is his final velocity?

A: There are two approaches to answer this question.

1. Recognize that the student's potential energy at the top of the building is entirely converted to kinetic energy just before he hits the ground.



Calculate his potential energy at the top of the building, and that will be his kinetic energy when he hits the ground.

$$U = mgh = (80 \text{ kg})(10 \frac{\text{m}}{\text{s}^2})(15 \text{ m}) = \boxed{12000 \text{ J}}$$

We can now plug 12 000 J into the formula for kinetic energy and solve for velocity:

$$K = \frac{1}{2} mv^2$$

$$12000 = \left(\frac{1}{2}\right)(80)v^2$$

$$\frac{12000}{40} = 300 = v^2$$

$$v = \sqrt{300} = \boxed{17.3 \frac{\text{m}}{\text{s}}}$$

Use this space for summary and/or additional notes.

2. Use motion equations to find the student's velocity when he hits the ground, based on the height of the building and acceleration due to gravity. Then use the formula $K = \frac{1}{2}mv^2$.

$$d = \frac{1}{2}at^2$$

$$15\text{m} = \frac{1}{2}(10\frac{\text{m}}{\text{s}^2})t^2$$

$$t^2 = 3$$

$$t = \sqrt{3} = 1.73\text{ s}$$

$$v = at$$

$$v = (10\frac{\text{m}}{\text{s}^2})(1.73\text{s}) = \boxed{17.3\frac{\text{m}}{\text{s}}}$$

$$K = \frac{1}{2}mv^2$$

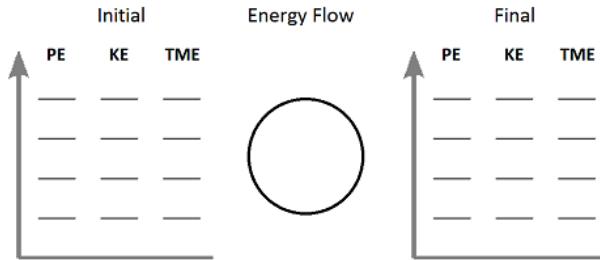
$$K = \frac{1}{2}(80\text{kg})(17.32\frac{\text{m}}{\text{s}})^2$$

$$K = \boxed{12000\text{ J}}$$

Use this space for summary and/or additional notes.

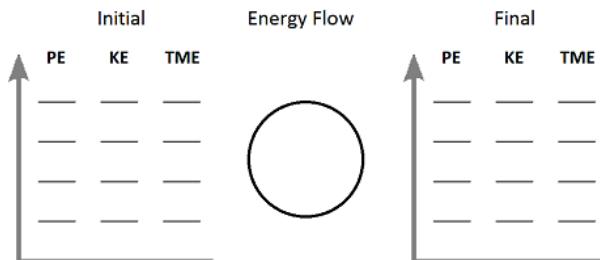
Homework Problems

1. A 0.200 kg model rocket is observed to rise 100. m above the ground after launch. If no additional thrust was applied to the rocket after launch, what was its launch speed at the ground?



Answer: $44.3 \frac{\text{m}}{\text{s}}$

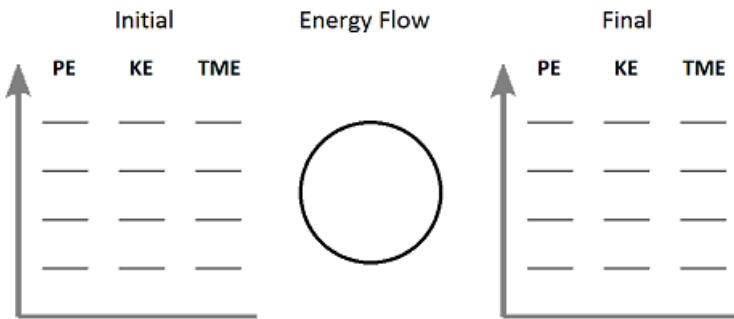
2. A 70. kg pole vaulter converts the kinetic energy of running at ground level into the potential energy needed to clear the crossbar at a height of 4.0 m above the ground. What is the minimum velocity that the pole vaulter must have when taking off from the ground in order to clear the bar?



Answer: $8.9 \frac{\text{m}}{\text{s}}$

Use this space for summary and/or additional notes.

3. A 500. kg roller coaster car is launched, from ground level, at $20. \frac{m}{s}$. Neglecting friction, how fast will it be moving when it reaches the top of a loop, which is 15 m above the ground?



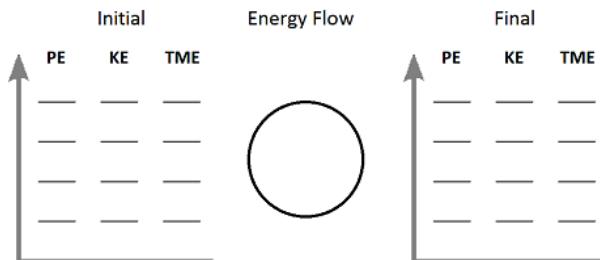
Answer: $10 \frac{m}{s}$

4. A roller coaster car with mass m is launched, from ground level with a velocity of v_o . Neglecting friction, how fast will it be moving when it reaches the top of a loop, which is a distance of h above the ground? (You may use your work from problem #3 above to guide your algebra.)

Answer: $\sqrt{v_o^2 - 2gh}$

Use this space for summary and/or additional notes.

5. A 10.0 kg monkey swings on a vine from a point which is 40.0 m above the jungle floor to a point which is 15.0 m above the floor. If the monkey was moving at $2.0 \frac{m}{s}$ initially, what will be its velocity at the 15.0 m point?



Answer: $22.4 \frac{m}{s}$

Use this space for summary and/or additional notes.

Escape Velocity

Unit: Work, Energy & Momentum

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 2.1, 2.2, 2.3

Knowledge/Understanding Goals:

- how fast a rocket or space ship needs to travel to escape Earth's (or any other planet's) gravity
- how escape velocity relates to gravitational potential energy, kinetic energy, and Newton's Law of Universal Gravitation

Language Objectives:

- Understand and correctly use the term "escape velocity."
- Set up and solve word problems involving escape velocity.

Notes:

Escape Velocity

If you want to send a rocket or space ship to explore the rest of the solar system or beyond, the rocket needs enough kinetic energy to escape from the force of Earth's gravity.

To explain the calculation, it is necessary to understand that our formula for gravitational potential energy is actually a simplification. Because the force of gravity attracts objects to the center of the Earth, the equation should be:

$$U_g = mg\Delta h = mg(h - h_o)$$

where h_o is the distance to the center of the Earth (radius of the Earth), and h is the total distance between the object and the center of the Earth. Normally, we calculate gravitational potential energy based on a difference in height between some height above the ground (h) and the ground ($h = 0$).

However, when we consider the problem of escaping from the gravity of the entire planet, we need to consider the potential energy relative to the center of the Earth, where the total force of gravity would be zero.

Use this space for summary and/or additional notes.

To do this, the rocket's kinetic energy ($\frac{1}{2}mv^2$) must be greater than or equal to the Earth's gravitational potential energy (mgh). This means $\frac{1}{2}mv^2 = F_g h$. We can write this in terms of Newton's Law of Universal Gravitation:

$$\frac{1}{2}mv^2 = F_g h = \left(\frac{Gm_1m_2}{d^2} \right) h$$

However, because we need to escape the gravitational pull of the entire planet, we need to measure h from the center of the Earth, not the surface. This means that at the surface of the earth, h in the above equation is the same as r in Newton's Law of Universal Gravitation. Similarly, the mass of the spaceship is one of the masses (let's choose m_2) in Newton's Law of Universal Gravitation. This gives:

$$\begin{aligned} F_g h &= F_g r = \frac{Gm_1m_2}{r^2} r = \frac{Gm_1m_2}{r} \\ \frac{1}{2}m_2v_e^2 &= \frac{Gm_1m_2}{r} \\ \frac{1}{2}v_e^2 &= \frac{Gm_1}{r} \\ v_e &= \sqrt{\frac{2Gm_{planet}}{r}} \end{aligned}$$

At the surface of the Earth, where $m_{planet} = 5.97 \times 10^{24}$ kg and $r = 6.37 \times 10^6$ m, $v_e = 1.12 \times 10^4 \frac{\text{m}}{\text{s}} = 11200 \frac{\text{m}}{\text{s}}$. (This equals approximately 25 100 miles per hour.)

Use this space for summary and/or additional notes.

Sample Problem:

Q: When Apollo 11 went to the moon, the space ship needed to achieve the Earth's escape velocity of $11200 \frac{\text{m}}{\text{s}}$ to escape Earth's gravity. What velocity did the space ship need to achieve in order to escape the moon's gravity and return to Earth? (*I.e.*, what is the escape velocity on the surface of the moon?)

$$\text{A: } v_e = \sqrt{\frac{2Gm_{\text{moon}}}{d_{\text{moon}}}}$$

$$v_e = \sqrt{\frac{(2)(6.67 \times 10^{-11})(7.35 \times 10^{22})}{1.74 \times 10^6}}$$

$$v_e = 2370 \frac{\text{m}}{\text{s}}$$

Use this space for summary and/or additional notes.

Power

Unit: Work, Energy & Momentum

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 2.4

Skills:

- calculate power

Language Objectives:

- Understand and correctly use the term “power.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving power.

Notes:

power: a measure of the rate at which energy is applied or work is done. Power is calculated by dividing work (or energy) by time.

$$P = \frac{W}{t} = \frac{\Delta K}{t} = \frac{\Delta U}{t}$$

Power is a scalar quantity and is measured in Watts (W).

$$1 \text{ W} = 1 \frac{\text{J}}{\text{s}} = 1 \frac{\text{N}\cdot\text{m}}{\text{s}} = 1 \frac{\text{kg}\cdot\text{m}^2}{\text{s}^3}$$

Note that utility companies measure energy in kilowatt-hours. This is because

$$P = \frac{W}{t}, \text{ which means energy} = W = Pt.$$

Because $1 \text{ kW} = 1000 \text{ W}$ and $1 \text{ h} = 3600 \text{ s}$, this means

$$1 \text{ kWh} = (1000 \text{ W})(3600 \text{ s}) = 3600000 \text{ J}$$

$$\text{Because } W = Fd, \text{ this means } P = \frac{Fd}{t} = F \left(\frac{d}{t} \right) = Fv$$

Use this space for summary and/or additional notes.

Solving Power Problems

Many power problems require you to calculate the amount of work done or the change in energy, which you should recall is:

$$W = F_{||}d \quad \text{if the force is caused by linear displacement}$$

$$\Delta K = \frac{1}{2} m(v^2 - v_o^2) \quad \text{if the change in energy was caused by a change in velocity}$$

$$\Delta U_g = mg \Delta h \quad \text{if the change in energy was caused by a change in height}$$

Once you have the work or energy, you can plug it in for either W , ΔE_k or ΔU , use the appropriate parts of the formula:

$$P = \frac{W}{t} = \frac{\Delta K}{t} = \frac{\Delta U}{t} = Fv$$

and solve for the missing variable.

Sample Problems

Q: What is the power output of an engine that pulls with a force of 500. N over a distance of 100. m in 25 s?

A: $W = Fd = (500)(100) = 50\,000 \text{ J}$

$$P = \frac{W}{t} = \frac{50\,000}{25} = 2\,000 \text{ W}$$

Use this space for summary and/or additional notes.

Q: A 60. W incandescent light bulb is powered by a generator that is powered by a falling 1.0 kg mass on a rope. Assuming the generator is 100% efficient (*i.e.*, no energy is lost when the generator converts its motion to electricity), how far must the mass fall in order to power the bulb at full brightness for 1.0 minute?

A:

$$P = \frac{\Delta U_g}{t} = \frac{mg \Delta h}{t}$$
$$60 = \frac{(1)(10) \Delta h}{60}$$
$$3600 = 10 \Delta h$$
$$\Delta h = \frac{3600}{10} = 360 \text{ m}$$

Note that 360 m is approximately the height of the Empire State Building. This is why changing from incandescent light bulbs to more efficient compact fluorescent or LED bulbs can make a significant difference in energy consumption!

Homework Problems

1. A small snowmobile has a 9 000 W (12 hp) engine. It takes a force of 300. N to move a sled load of wood along a pond. How much time will it take to tow the wood across the pond if the distance is measured to be 850 m?

Answer: 28.3 s

Use this space for summary and/or additional notes.

2. A winch, which is rated at 720 W, is used to pull an all-terrain vehicle (ATV) out of a mud bog for a distance of 2.3 m. If the average force applied by the winch is 1500 N, how long will the job take?

Answer: 4.8 s

3. What is your power output if you have a mass of 65 kg and you climb a 5.2 m vertical ladder in 10.4 s?

Answer: 325 W

4. The maximum power output of a particular crane is 12 kW. What is the fastest time in which this crane could lift a 3500 kg crate to a height of 6.0 m?

Answer: 17.5 s

Use this space for summary and/or additional notes.

5. Jack and Jill went up the hill. (The hill was 23m high.) Jack was carrying a 21 kg pail of water. If Jack has a mass of 75 kg and he made the trip in 45 s, how much power did he apply?

Answer: 490.7 W

6. Jill, who has a mass of 55 kg, made the same trip as Jack did in problem #3, but she took 10 seconds less. How much power did she apply?

Answer: 499.4 W

Use this space for summary and/or additional notes.

Linear Momentum

Unit: Work, Energy & Momentum

NGSS Standards: HS-PS2-2

MA Curriculum Frameworks (2006): 2.5

Knowledge/Understanding Goals:

- definition of momentum

Skills:

- calculate the momentum of an object
- solve problems involving the conservation of momentum

Language Objectives:

- Understand and correctly use the terms “collision,” “elastic collision,” “inelastic collision,” “inertia” and “momentum.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving collisions and momentum.

Labs, Activities & Demonstrations:

- Collisions on air track.
- “Happy” and “sad” balls knocking over a board.
- Students riding momentum cart.

Notes:

collision: when two or more objects come together and hit each other.

elastic collision: a collision in which the objects bounce off each other after they collide, without any loss of kinetic energy.

inelastic collision: a collision in which the objects remain together after colliding.

In an inelastic collision, total energy is still conserved, but some of the energy is changed into other forms, so the amount of kinetic energy is different before vs. after the collision.

Use this space for summary and/or additional notes.

Any macroscopic collision (meaning a collision between objects that are larger than individual atoms or molecules) will convert some of the kinetic energy into internal energy and other forms of energy. This means that no large scale impacts are ever perfectly elastic.

In the 17th century, the German mathematician Gottfried Leibnitz recognized the fact that in some cases, the mass and velocity of objects before and after a collision were related by kinetic energy ($\frac{1}{2}mv^2$, which he called the “quantity of motion”); in other cases, however, the “quantity of motion” was not preserved but another quantity (mv , which he called the “motive force”) was the same before and after. Debate about “quantity of motion” and “motive force” continued through the 17th and 18th centuries.

We now realize that the two quantities are different, but related. “Quantity of motion” is what we now call “kinetic energy”; “motive force” is what we now call “momentum”.

While *total energy* is always conserved in a collision, kinetic energy is not; during the collision, kinetic energy may be converted to other forms of energy, such as heat. Momentum, however, is always conserved in a collision, regardless of what happens to the energy.

momentum (\vec{p}): the amount of force that a moving object could transfer in a given time in a collision. (Formerly called “motive force”.)

Momentum is a vector quantity given by the formula:

$$\vec{p} = m\vec{v}$$

and is measured in units of N·s, or $\frac{\text{kg}\cdot\text{m}}{\text{s}}$.

Note that an object at rest ($\vec{v} = 0$) has a momentum of zero.

Momentum is the quantity that is transferred in all collisions. Any problem that involves one or more collisions is a momentum problem.

The net force on an object is its change in momentum with respect to time:

$$\vec{F}_{net} = \frac{\Delta \vec{p}}{\Delta t} = \frac{d\vec{p}}{dt}$$

Use this space for summary and/or additional notes.

inertia: an object's ability to resist the action of a force.

Inertia and momentum are related, but are not the same thing; an object has inertia even at rest. An object's momentum changes if either its mass or its velocity changes, but an the inertia of an object can change only if its mass changes.

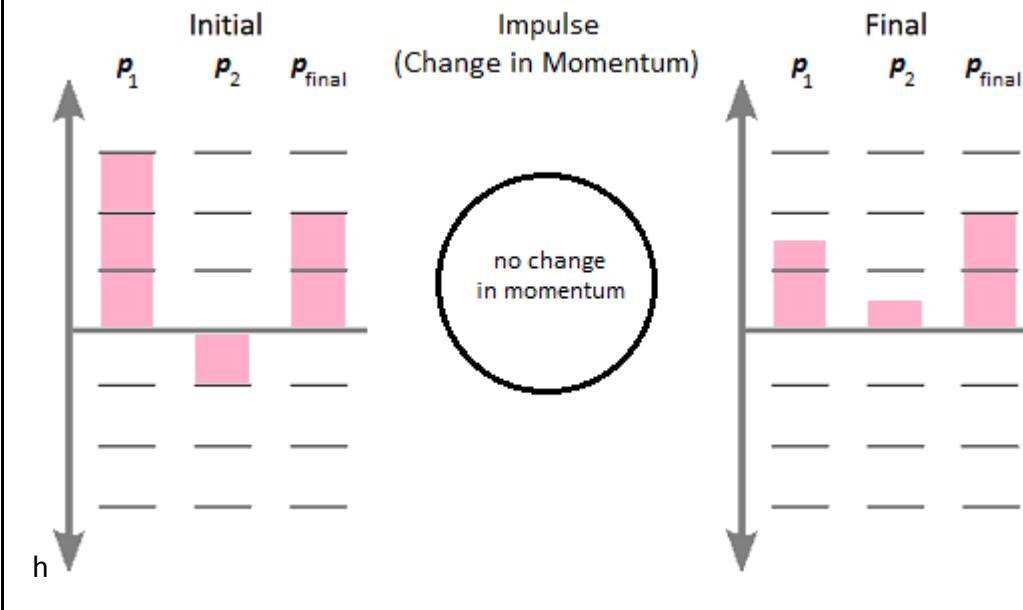
Conservation of Momentum

In a closed system, momentum is **conserved**. This means that unless there is an outside force, *the combined momentum of objects after they collide is equal to the combined momentum of the objects before the collision*.

elastic collision: a collision in which the objects are separate both before and after the collision.

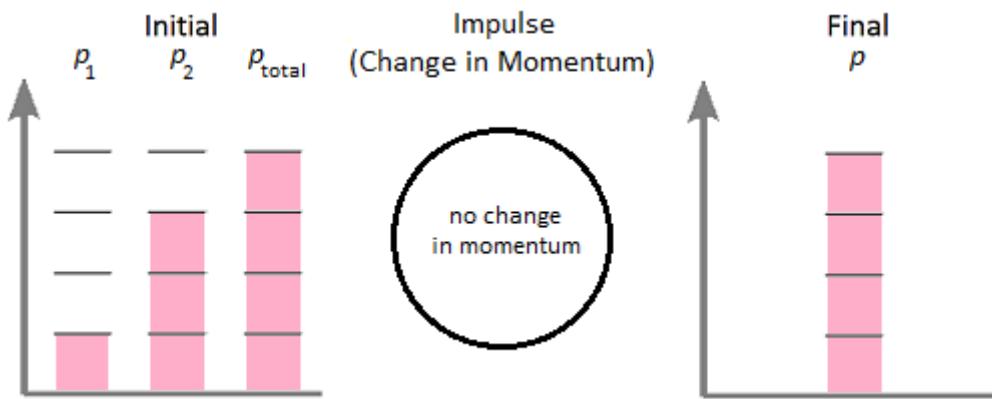
In the following momentum bar chart, imagine that two objects are moving in opposite directions and then collide. Before the collision, the first object has a momentum of +3 arbitrary units, and the second has a momentum of -1. The total momentum is therefore $+3 + (-1) = +2$.

After the collision, the first object has a momentum of +1.5 and the second has a momentum of +0.5. Because there are no forces changing the momentum of the system, the final momentum **must** also be +2.



Use this space for summary and/or additional notes.

inelastic collision: a collision in which the objects are joined (or are a single object) either before or after the collision.



Examples of inelastic collisions include a person catching a ball and a bullet being fired from a gun. (The bullet and gun are together initially and separate after the “collision”.)

Solving Momentum Problems

Almost all momentum problems involve the conservation of momentum law:

before = after

$$\sum \vec{p}_i = \sum \vec{p}_f$$

The symbol \sum is the Greek capital letter “sigma”. In mathematics, the symbol \sum means “summation”. $\sum \vec{p}$ means the sum of the momentums. The subscript “ i ” means initial (before the collision), and the subscript “ f ” means final (after the collision). In plain English, $\sum \vec{p}$ means find each individual value of \vec{p} (positive or negative, depending on the direction) and then add them all up to find the total. The conservation of momentum law means that the total before a collision must be equal to the total after.

Use this space for summary and/or additional notes.

For example, if you had a momentum problem with two objects, the law of conservation of momentum becomes:

before = after

$$\vec{p}_{1,i} + \vec{p}_{2,i} = \vec{p}_{1,f} + \vec{p}_{2,f}$$

Notice that we have two subscripts after each “*p*”, because we have two separate things to keep track of. The “*i*” and “*f*” mean “initial” and “final,” and the “1” and “2” mean object #1 and object #2.

Because $\vec{p} = m\vec{v}$, we can replace each \vec{p} with $m\vec{v}$.

For our momentum problem with two objects, this becomes:

before = after

$$m_1\vec{v}_{1,i} + m_2\vec{v}_{2,i} = m_1\vec{v}_{1,f} + m_2\vec{v}_{2,f}$$

Note that there are six separate quantities in this problem: m_1 , m_2 , $\vec{v}_{1,i}$, $\vec{v}_{2,i}$, $\vec{v}_{1,f}$, and $\vec{v}_{2,f}$. A typical momentum problem will give you (or enable you to calculate) five of these, and will ask you for the sixth.

Note also that most momentum problems do not mention the word “momentum.” The problems usually give information about masses and velocities before and after some sort of collision, and it is up to you to realize that the problem involves conservation of momentum.

Most momentum problems involve collisions. Usually, there are two objects initially, and the objects either bounce off each other (elastic collision) or stick together (inelastic collision).

For an elastic collision between two objects, the problem is exactly as described above: there are six quantities to consider: the two masses, the two initial velocities, and the two final velocities. The equation relating them is:

before = after

$$m_1\vec{v}_{1,i} + m_2\vec{v}_{2,i} = m_1\vec{v}_{1,f} + m_2\vec{v}_{2,f}$$

To solve the problem, you need to obtain the quantities given in the word problem and solve for the missing one.

Use this space for summary and/or additional notes.

For an inelastic collision between two objects, the objects stick together after the collision, which means there is only one “object” afterwards. The total mass of the object is $m_T = m_1 + m_2$, and there is only one “object” with a final velocity.

there are five quantities: the two masses, the two initial velocities, and the final velocity of the combined object. The equation relating them is:

before = **after**

$$m_1 \vec{v}_{1,i} + m_2 \vec{v}_{2,i} = m_T \vec{v}_f$$

To solve this problem, you need to obtain the quantities given in the word problem, add the two masses to find m_T , and solve for the missing quantity.

Use this space for summary and/or additional notes.

Sample Problems

Q: What is the momentum of a 15 kg object moving at a velocity of + 3.0 $\frac{\text{m}}{\text{s}}$?

A: $\vec{p} = m\vec{v}$

$$\vec{p} = (15 \text{ kg})(+3.0 \frac{\text{m}}{\text{s}}) = 45 \frac{\text{kg}\cdot\text{m}}{\text{s}} = +45 \text{ N}\cdot\text{s}$$

The answer is given as +45 N·s because momentum is a vector, and we indicate the direction as positive or negative.

Q: An object with a mass of 8.0 kg moving with a velocity of + 5.0 $\frac{\text{m}}{\text{s}}$ collides with a stationary object with a mass of 12 kg. If the two objects stick together after the collision, what is their velocity?



A: The momentum of the moving object before the collision is:

$$\vec{p} = m\vec{v} = (8.0)(+5.0) = +40 \text{ N}\cdot\text{s}$$

The stationary object has a momentum of zero, so the total momentum of the two objects combined is +40 N·s.

After the collision, the total mass is 8.0 kg + 12 kg = 20 kg. The momentum after the collision must still be +40 N·s, which means the velocity is:

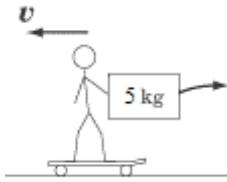
$$\vec{p} = m\vec{v} \quad 40 = 20\vec{v} \quad \vec{v} = +2 \frac{\text{m}}{\text{s}}$$

Using the equation, we would solve this as follows:

$$\begin{aligned} \text{before} &= \text{after} \\ \vec{p}_{1,i} + \vec{p}_{2,i} &= \vec{p}_f \\ m_1 \vec{v}_{1,i} + m_2 \vec{v}_{2,i} &= m_f \vec{v}_f \\ (8)(5) + (12)(0) &= (8 + 12) \vec{v}_f \\ 40 &= 20 \vec{v}_f \\ \vec{v}_f &= \frac{40}{20} = +2 \frac{\text{m}}{\text{s}} \end{aligned}$$

Use this space for summary and/or additional notes.

Q: Mr. Stretchy has a mass of 60. kg and is holding a 5.0 kg box as he rides on a skateboard toward the west at a speed of $3.0 \frac{\text{m}}{\text{s}}$. (Assume the 60. kg is the mass of Mr. Stretchy and the skateboard combined.) He throws the box behind him, giving it a velocity of $2.0 \frac{\text{m}}{\text{s}}$ to the east.



What is Mr. Stretchy's velocity after throwing the box?

A: This problem is like an inelastic collision in reverse; Mr. Stretchy and the box are together before the “collision” and apart afterwards. The equation would therefore look like this:

$$m_T \vec{v}_i = m_s \vec{v}_{s,f} + m_b \vec{v}_{b,f}$$

Where the subscript “*s*” is for Mr. Stretchy, and the subscript “*b*” is for the box. Note that after Mr. Stretchy throws the box, he is moving one direction and the box is moving the other, which means we need to be careful about our signs. Let’s choose the direction Mr. Stretchy is moving (west) to be positive. Because the box is thrown to the east, this means the final velocity of the box will be $\vec{v}_{b,f} = -2.0 \frac{\text{m}}{\text{s}}$

Plugging values from the problem into the equation for the law of conservation of momentum, we get:

before = **after**

$$\begin{aligned}\vec{p}_i &= \vec{p}_{s,f} + \vec{p}_{b,f} \\ m_T \vec{v}_i &= m_s \vec{v}_{s,f} + m_b \vec{v}_{b,f} \\ (60 + 5)(+3) &= 60 \vec{v}_{s,f} + (5)(-2) \\ + 195 &= 60 \vec{v}_{s,f} + (-10) \\ + 205 &= 60 \vec{v}_{s,f} \\ \vec{v}_{s,f} &= \frac{+205}{60} = +3.4 \frac{\text{m}}{\text{s}}\end{aligned}$$

Use this space for summary and/or additional notes.

Q: A soccer ball that has a mass of 0.43 kg is rolling east with a velocity of $5.0 \frac{\text{m}}{\text{s}}$. It collides with a volleyball that has a mass of 0.27 that is rolling west with a velocity of $6.5 \frac{\text{m}}{\text{s}}$. After the collision, the soccer ball is rolling to the west with a velocity of $3.87 \frac{\text{m}}{\text{s}}$. Assuming the collision is perfectly elastic and friction between both balls and the ground is negligible, what is the velocity (magnitude and direction) of the volleyball after the collision?

A: This is an elastic collision, so the soccer ball and the volleyball are separate both before and after the collision. The equation is:

$$m_s \vec{v}_{s,i} + m_v \vec{v}_{v,i} = m_s \vec{v}_{s,f} + m_v \vec{v}_{v,f}$$

Where the subscript “s” is for the soccer ball and the subscript “v” is for the volleyball. In all elastic collisions, assume we need to keep track of the directions, which means we need to be careful about our signs. We don’t know which direction the volleyball will be moving after the collision (though a good guess would be that it will probably bounce off the soccer ball and move to the east). So let us arbitrarily choose east to be positive and west to be negative. This means:

quantity	direction	value
initial velocity of soccer ball	east	$+5.0 \frac{\text{m}}{\text{s}}$
initial velocity of volleyball	west	$-6.5 \frac{\text{m}}{\text{s}}$
final velocity of soccer ball	west	$-3.87 \frac{\text{m}}{\text{s}}$

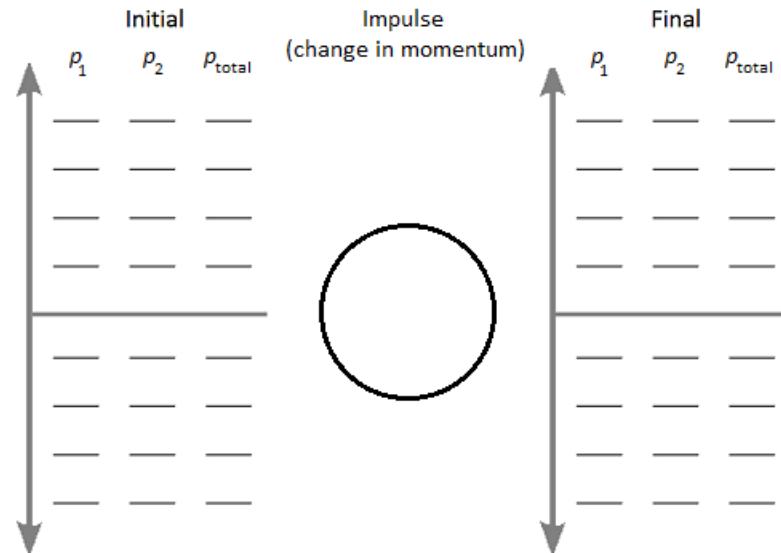
Plugging values from the problem into the equation for the law of conservation of momentum, we get:

$$\begin{aligned}
 \text{before} &= \text{after} \\
 \vec{p}_{s,i} + \vec{p}_{v,i} &= \vec{p}_{s,f} + \vec{p}_{v,f} \\
 m_s \vec{v}_{s,i} + m_v \vec{v}_{v,i} &= m_s \vec{v}_{s,f} + m_v \vec{v}_{v,f} \\
 (0.43)(5.0) + (0.27)(-6.5) &= (0.43)(-3.87) + (0.27) \vec{v}_{v,f} \\
 2.15 + (-1.755) &= -1.664 + 0.27 \vec{v}_{v,f} \\
 0.395 &= -1.664 + 0.27 \vec{v}_{v,f} \\
 2.059 &= 0.27 \vec{v}_{v,f} \\
 \vec{v}_{s,f} &= \frac{+2.059}{0.27} = +7.63 \frac{\text{m}}{\text{s}} \text{ or } 7.63 \frac{\text{m}}{\text{s}} \text{ to the east}
 \end{aligned}$$

Use this space for summary and/or additional notes.

Homework Problems

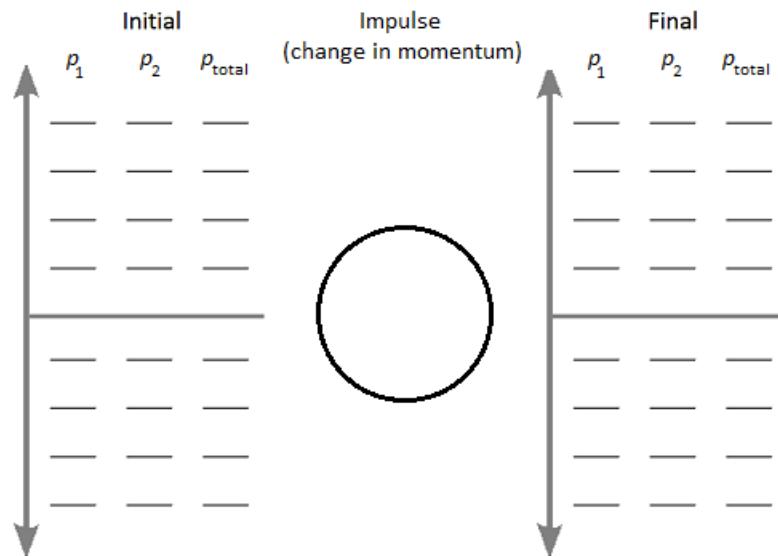
1. A large 10. kg sports ball is caught by a 70. kg student on the track team. If the ball was moving at $4.0 \frac{\text{m}}{\text{s}}$ and the student was stationary before catching the ball, how fast will the student be moving after catching the ball?



Answer: $0.5 \frac{\text{m}}{\text{s}}$

Use this space for summary and/or additional notes.

2. An 0.010 kg bullet is fired down the barrel of a gun, which is pointed to the left. The bullet accelerates from rest to a velocity of $400 \frac{\text{m}}{\text{s}}$. What velocity does the 1.5 kg gun acquire as a result of this impulse? (Note that neither the bullet nor the gun was moving before the “collision.”)



Answer: $2.7 \frac{\text{m}}{\text{s}}$ to the right

Use this space for summary and/or additional notes.

Although Blank momentum bar charts are not provided for the remaining problems, feel free to draw them if you would find them helpful.

3. A 730 kg Mini runs into a stationary 2500 kg sport utility vehicle. If the Mini was moving at $10 \frac{m}{s}$ initially, how fast will it be moving after making a completely inelastic collision with the SUV?

Answer: $2.3 \frac{m}{s}$

4. A 6.0 kg bowling ball moving at $3.5 \frac{m}{s}$ to the right makes a collision, head-on, with a stationary 0.70 kg bowling pin. If the ball is moving $2.77 \frac{m}{s}$ to the right after the collision, what will be the velocity (magnitude and direction) of the pin?

Answer: $6.25 \frac{m}{s}$ to the right

5. A pair of 0.20 kg billiard balls make an elastic collision. Before the collision, the 4-ball was moving $0.50 \frac{m}{s}$ to the right, and the 8-ball was moving $1.0 \frac{m}{s}$ to the left. After the collision, the 4-ball is now moving at $1.0 \frac{m}{s}$ to the left. What is the velocity (magnitude and direction) of the 8-ball after the collision?

Answer: $0.50 \frac{m}{s}$ to the right

Use this space for summary and/or additional notes.

6. A pair of billiard balls, each with mass m , make an elastic collision. Before the collision, the 4-ball was moving with a velocity of $\vec{v}_{4,i}$, and the 8-ball was moving with a velocity of $\vec{v}_{8,i}$. After the collision, the 4-ball is now moving with a velocity of $\vec{v}_{4,f}$. What is the velocity of the 8-ball after the collision? (You may use your work from problem #5 above to guide your algebra.)

Answer: $\vec{v}_{8,f} = \vec{v}_{4,i} + \vec{v}_{8,i} - \vec{v}_{4,f}$

7. A 75 kg astronaut on a space walk pushes to the right on a 1000. kg satellite. If the velocity of the satellite after the push is $0.75 \frac{\text{m}}{\text{s}}$, what is the velocity of the astronaut?

Answer: $10 \frac{\text{m}}{\text{s}}$ to the left

Use this space for summary and/or additional notes.

Momentum and Kinetic Energy

Unit: Work, Energy & Momentum

NGSS Standards: HS-PS3-1

MA Curriculum Frameworks (2006): 2.5

Knowledge/Understanding Goals:

- how the conservation of momentum and conservation of energy are related

Language Objectives:

- Accurately describe how Newton's cradle illustrates the principles of conservation of energy and momentum, using appropriate academic language.

Labs, Activities & Demonstrations:

- Newton's Cradle.
- Ballistic pendulum.

Notes:

For those of you taking calculus, you may recognize that momentum is the derivative of kinetic energy with respect to velocity.

$$\frac{d}{dv} \left(\frac{1}{2} mv^2 \right) = mv$$
$$\frac{d}{dv} (K) = p$$

In an algebra-based physics class, this becomes:

$$p = \frac{\Delta K}{\Delta v}$$

which means momentum is the slope of a graph of kinetic energy as a function of velocity.

Use this space for summary and/or additional notes.

We can also use $p=mv$ to eliminate v from the kinetic energy equation, giving the equation:

$$K = \frac{p^2}{2m}$$

The relationship between momentum and kinetic energy explains why the velocities of objects after a collision are determined by the collision.

Because kinetic energy and momentum must *both* be conserved in an elastic collision, the two final velocities are actually determined by the masses and the initial velocities. The masses and initial velocities are determined before the collision. The only variables are the two velocities after the collision. This means there are two equations (conservation of momentum and conservation of kinetic energy) and two unknowns ($v_{1,f}$ and $v_{2,f}$).

For a perfectly elastic collision, conservation of momentum states:

$$m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f}$$

and conservation of kinetic energy states:

$$\frac{1}{2} m_1 v_{1,i}^2 + \frac{1}{2} m_2 v_{2,i}^2 = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 v_{2,f}^2$$

If we use these two equations to solve for $v_{1,f}$ and $v_{2,f}$ in terms of the other variables, the result is the following:

$$v_{1,f} = \frac{v_{1,i}(m_1 - m_2) + 2m_2 v_{2,i}}{m_1 + m_2}$$

$$v_{2,f} = \frac{v_{2,i}(m_2 - m_1) + 2m_1 v_{1,i}}{m_1 + m_2}$$

For an inelastic collision, there is no solution that satisfies both the conservation of momentum and the conservation of kinetic energy; the total kinetic energy after the collision is always less than the total kinetic energy before. This matches what we observe, which is that momentum is conserved, but some of the kinetic energy is converted to heat energy during the collision.

Use this space for summary and/or additional notes.

Newton's Cradle

Newton's Cradle is the name given to a set of identical balls that are able to swing suspended from wires:



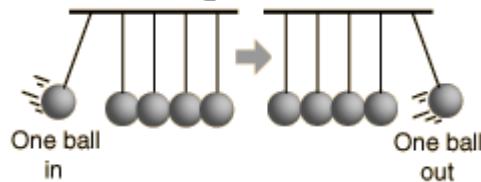
When one ball is swung and allowed to collide with the rest of the balls, the momentum transfers through the balls and one ball is knocked out from the opposite end. When two balls are swung, two balls are knocked out from the opposite end. And so on.

This apparatus demonstrates the relationship between the conservation of momentum and conservation of kinetic energy. When the balls collide, the collision is mostly elastic collision, meaning that all of the momentum and most of the kinetic energy are conserved.

Before the collision, the moving ball(s) have momentum (mv) and kinetic energy ($\frac{1}{2}mv^2$). There are no external forces, which means momentum must be conserved. The collision is mostly elastic, which means kinetic energy is mostly also conserved. The only way for the same momentum and kinetic energy to be present after the collision is for the same number of balls to swing away from the opposite end with the same velocity.

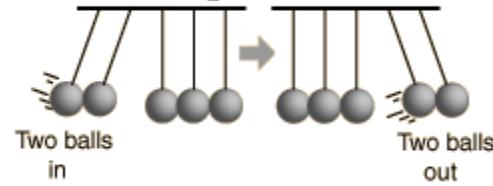
Momentum in: $mv = \text{momentum out}$

Kinetic energy in: $\frac{1}{2}mv^2 = \text{kinetic energy out}$



Momentum in: $2mv = \text{momentum out}$

Kinetic energy in: $\frac{1}{2}2mv^2 = \text{kinetic energy out}$

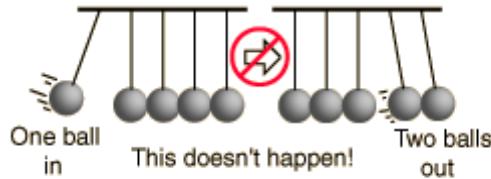


Use this space for summary and/or additional notes.

If only momentum had to be conserved, it would be possible to pull back one ball but for two balls to come out the other side at $\frac{1}{2}$ of the original velocity. However, this can't actually happen.

$$\text{Momentum in: } mv = \text{momentum out}$$

$$\text{Kinetic energy in: } \frac{1}{2}mv^2 \cancel{\neq} \text{ kinetic energy out!}$$



Conserving momentum in this case requires that the two balls come out with half the speed.

$$\text{Momentum out} = 2m \frac{v}{2}$$

But this gives

$$\text{Kinetic energy out} = \frac{1}{2}2m \frac{v^2}{4}$$

Which amounts to a loss of half of the kinetic energy!

Note also that if there were no friction, the balls would continue to swing forever. However, because of friction (between the balls and air molecules, within the strings as they stretch, etc.) and conversion of some of the kinetic energy to other forms (such as heat), the balls in a real Newton's Cradle will, of course, slow down and eventually stop.

Use this space for summary and/or additional notes.

Impulse

Unit: Work, Energy & Momentum

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 2.5

Knowledge/Understanding Goals:

- what impulse is

Skills:

- calculate the impulse given to an object
- calculate the change in momentum as the result of an impulse

Language Objectives:

- Understand and correctly use the term “impulse.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving impulse.

Notes:

impulse (\vec{J}): a force applied to an object over an interval of time. Impulse is a vector quantity.

An impulse causes a change in momentum, and the amount of the impulse is equal to the change in momentum. The impulse also equals force times time:

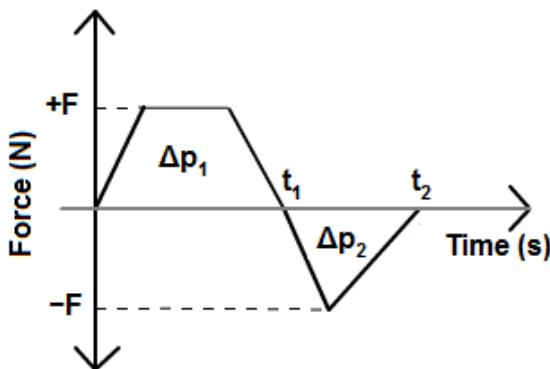
$$\vec{J} = \vec{F}\Delta t = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

Impulse is measured in the same unit as momentum (newton-seconds):

$$1 \text{ N}\cdot\text{s} = 1 \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

Use this space for summary and/or additional notes.

If the force is changing with time, the area under a force-time graph is the impulse or change in momentum:



In the above graph, the impulse from time zero to t_1 would be Δp_1 . The impulse from t_1 to t_2 would be Δp_2 , and the total impulse would be $\Delta p_1 + \Delta p_2$ (keeping in mind that Δp_2 is negative).

Sample Problem

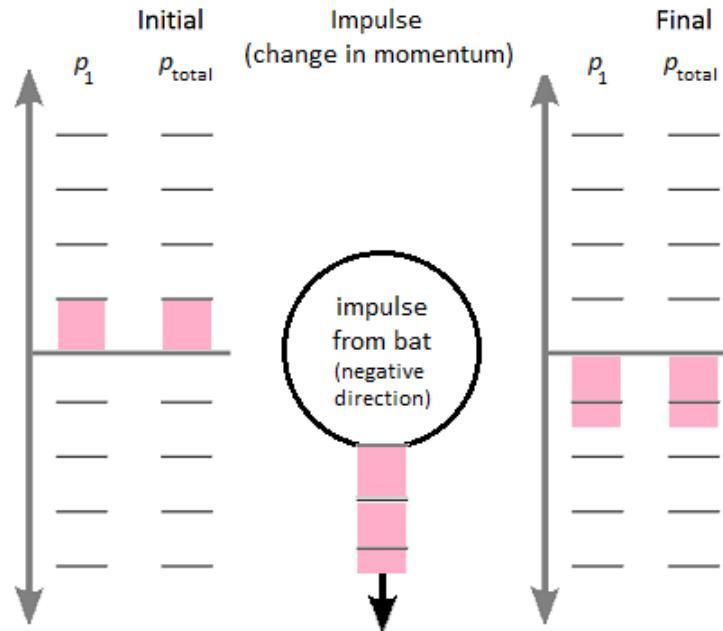
Q: A baseball has a mass of 0.145 kg and is pitched with a velocity of $38 \frac{m}{s}$ toward home plate. After the ball is hit, its velocity is $52 \frac{m}{s}$ toward the outfield fence. If the impact between the ball and bat takes place over an interval of 3.0 ms, find the impulse given to the ball by the bat, and the force applied to the ball by the bat.

A: The ball starts out moving toward home plate. The bat applies an impulse in the **opposite** direction. As with any vector quantity, opposite directions means we will have opposite signs. If we choose the initial direction of the ball (toward home plate) as the positive direction, then the initial velocity is $+38 \frac{m}{s}$, and the final velocity is $-52 \frac{m}{s}$. Because mass is scalar and always positive, this means the initial momentum is positive and the final momentum is negative.

Furthermore, because the final velocity is about $1\frac{1}{2}$ times as much as the initial velocity (in the opposite direction) and the mass doesn't change, this means the impulse needs to be enough to negate the ball's initial momentum plus enough in addition to give the ball about $1\frac{1}{2}$ times as much momentum in the opposite direction.

Use this space for summary and/or additional notes.

In the momentum bar chart, notice that we are adding an impulse in the negative direction, and the final momentum ends up in the negative direction. By convention, we define the positive direction as “into the system” and the negative direction as “out of the system”. This means that in this problem, we draw the “momentum flow” diagram to show momentum “flowing out of the system” (*i.e.*, in the negative direction).



Now, solving the problem numerically:

$$\begin{aligned}\vec{J} &= \Delta \vec{p} = \vec{p}_f - \vec{p}_i \\ \vec{J} &= m \vec{v}_f - m \vec{v}_i = m(\vec{v}_f - \vec{v}_i) \\ \vec{J} &= (0.145)(52 - (-38)) = +13.05 \text{ N}\cdot\text{s}\end{aligned}$$

The collision takes place over a time interval of 3.0 ms = 0.0030 s.

$$\begin{aligned}\vec{J} &= \vec{F} \Delta t \\ + 13.05 &= \vec{F}(0.0030) \\ \vec{F} &= \frac{13.05}{0.0030} = +4350 \text{ N}\end{aligned}$$

Use this space for summary and/or additional notes.

Homework Problems

1. A student is rushing to class and collides with a vice principal, who was standing still just before the collision. The vice principal has a mass of 100. kg, and the student has a mass of 50. kg, and had a velocity of $2.0 \frac{\text{m}}{\text{s}}$ before the collision.
 - a. What is the velocity of the entangled vice principal and student after they collide?

Answer: $0.67 \frac{\text{m}}{\text{s}}$

- b. If the collision lasted for 0.020 s, calculate the impulse and the force applied to the vice principal.

Answer: impulse: $67 \text{ N} \cdot \text{s}$

force: 3333 N

Use this space for summary and/or additional notes.

2. An 800 kg car travelling at $10 \frac{\text{m}}{\text{s}}$ comes to a stop in 0.50 s in an accident.

- a. What was the impulse applied to the car?

Answer: $-8000 \text{ N}\cdot\text{s}$

- b. What was the average net force on the car as it came to a stop?

Answer: -16000 N

3. A 0.80 kg ball was dropped from a height of 2.0 m above the ground. It rebounded to a height of 1.6 m. The contact between the ball and the ground lasted for 0.045 s.

- a. What was the impulse applied to the ball?

Answer: $9.59 \text{ N}\cdot\text{s}$

- b. What was the average net force on the ball?

Answer: 213 N upwards

Use this space for summary and/or additional notes.

4. A ball with mass m was dropped from a height h_o above the ground. It rebounded to a height of h . The contact between the ball and the ground lasted for time Δt . (You may use your work from problem #3 above to guide your algebra.)

a. What was the impulse applied to the ball?

$$\text{Answer: } \vec{J} = \Delta \vec{p} = m\sqrt{2g} \left(\sqrt{h_o} - \sqrt{h} \right)$$

b. What was the average net force on the ball?

$$\text{Answer: } F = \frac{m\sqrt{2g} \left(\sqrt{h_o} - \sqrt{h} \right)}{\Delta t}$$

Use this space for summary and/or additional notes.

Angular Momentum

Unit: Work, Energy & Momentum

NGSS Standards: HS-PS3-1

MA Curriculum Frameworks (2006): 2.3

Knowledge/Understanding Goals:

- understand angular momentum

Skills:

- calculate the angular momentum of an object
- apply the law of conservation of angular momentum

Language Objectives:

- Understand and correctly use the term “angular momentum”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving angular momentum.

Labs, Activities & Demonstrations:

- Try to change the direction of rotation of a bicycle wheel.
- Spin on a turntable with weights at arm’s length.
- Sit on a turntable with a spinning bicycle wheel and invert the wheel.

Notes:

angular momentum (\vec{L}): the momentum of a rotating object in the direction of rotation. Angular momentum is the property of an object that resists changes in the speed or direction of rotation.

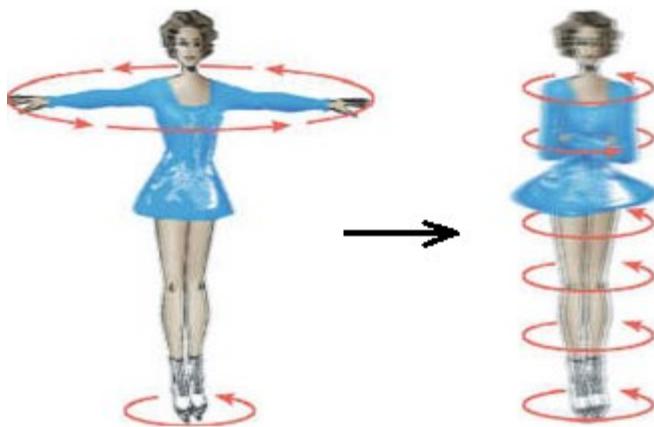
As was the case with torque, this course will involve only a conceptual look at angular momentum.

Use this space for summary and/or additional notes.

Conservation of Angular Momentum

Just as linear momentum is conserved unless an external force is applied, angular momentum is conserved unless an external torque is applied. This means that the total angular momentum before some change (that occurs entirely within the system) must equal the total angular momentum after the change.

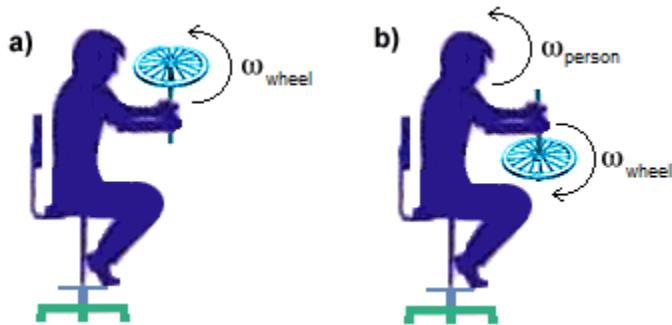
An example of this occurs when a person spinning (e.g., an ice skater) begins the spin with arms extended, then pulls the arms closer to the body. This causes the person to spin faster. (In physics terms, it increases the angular velocity, which means it causes angular acceleration.)



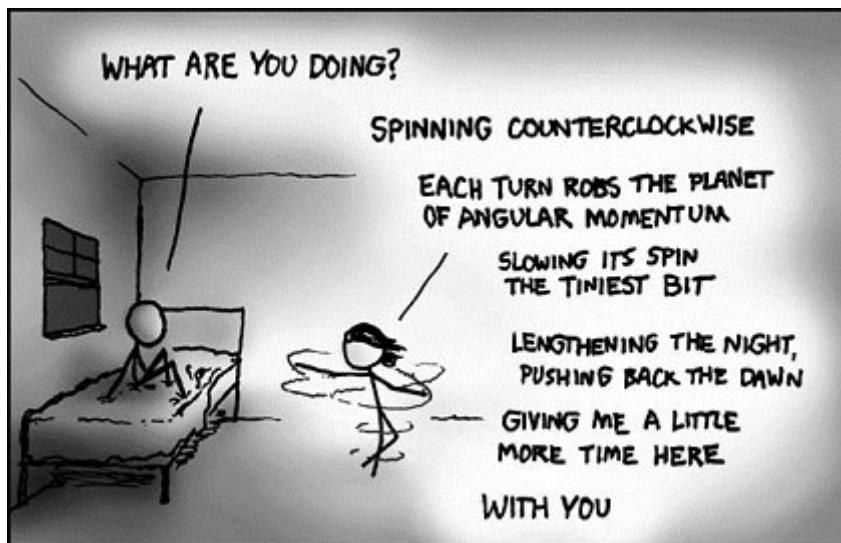
When the skater's arms are extended, the moment of inertia of the skater is greater (because there is more mass farther out) than when the arms are close to the body. When she brings her arms in, the average of her mass is closer to her body. Because her angular momentum stays the same, this causes her to spin faster.

Use this space for summary and/or additional notes.

Another popular example, which shows the vector nature of angular momentum, is the demonstration of a person holding a spinning bicycle wheel on a rotating chair. The person then turns over the bicycle wheel, causing it to rotate in the opposite direction:



Because the wheel was the only thing rotating, the angular momentum was in the direction that the wheel was turning. When the person flipped the wheel, this caused the direction of rotation to be the opposite. However, because angular momentum must be conserved, the only way for this to happen is for the person to start rotating in the direction that the wheel was originally rotating!



Use this space for summary and/or additional notes.

Introduction: Simple Harmonic Motion

Unit: Simple Harmonic Motion

Topics covered in this chapter:

Simple Harmonic Motion	284
Springs.....	287
Pendulums	293

This chapter discusses the physics of simple harmonic (repetitive) motion.

- *Simple Harmonic Motion* (SHM) describes the concept of repetitive back-and-forth motion and situations that apply to it.
- *Springs* and *Pendulums* describe specific examples of SHM and the specific equations relating to each.

Textbook:

- *Physics: Principles and Problems* Ch. 7: Motion in Two Dimensions, §7.2 (pp. 141–150)

Standards addressed in this chapter:

Next Generation Science Standards (NGSS):

No NGSS standards are addressed in this chapter.

Massachusetts Curriculum Frameworks (2006):

- ~~4.1 Describe the measurable properties of waves (velocity, frequency, wavelength, amplitude, period) and explain the relationships among them.~~ Recognize examples of simple harmonic motion.

Use this space for summary and/or additional notes.

Topics from this chapter assessed on the SAT Physics Subject Test:

- **Simple Harmonic Motion**, such as mass on a spring and the pendulum
 1. Periodic Motion (Simple Harmonic Motion)
 2. Frequency and Period
 3. Springs
 4. Pendulums

Skills learned & applied in this chapter:

- Understanding and representing repetitive motion.

Use this space for summary and/or additional notes.

Simple Harmonic Motion

Unit: Simple Harmonic Motion

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 4.1

Knowledge/Understanding Goals:

- what simple harmonic motion is
- examples of simple harmonic motion

Language Objectives:

- Understand and correctly use the term “simple harmonic motion” and be able to give examples.

Labs, Activities & Demonstrations:

- Show & tell with springs & pendulums.

Notes:

simple harmonic motion; motion consisting of regular, periodic back-and-forth oscillation.

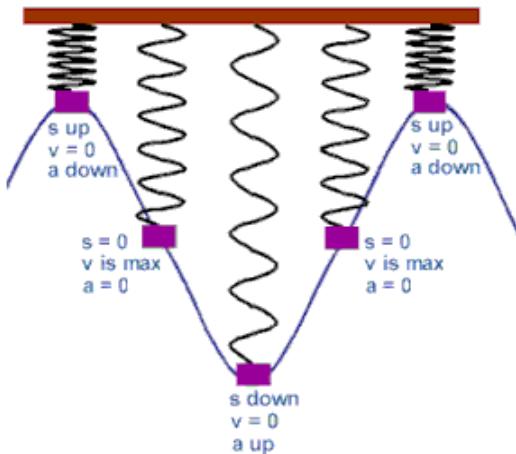
Requirements:

- The acceleration is always in the opposite direction from the displacement. This means the acceleration always slows down the motion and reverses the direction.
- In an ideal system (no friction), once simple harmonic motion is started, it would continue forever.
- A graph of displacement vs. time will result in the trigonometric function sine or cosine.

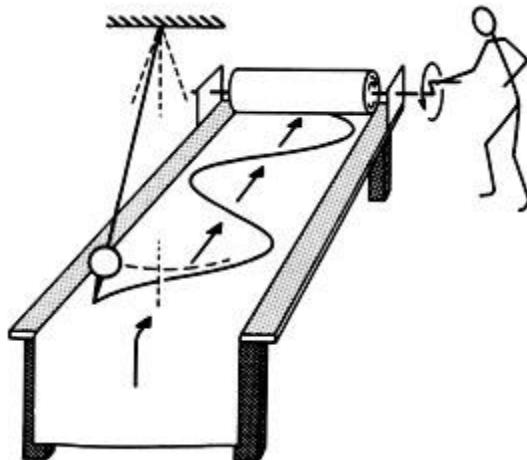
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Examples of Simple Harmonic Motion

- **Springs:** as the spring compresses or stretches, the spring force accelerates it back toward its equilibrium position.

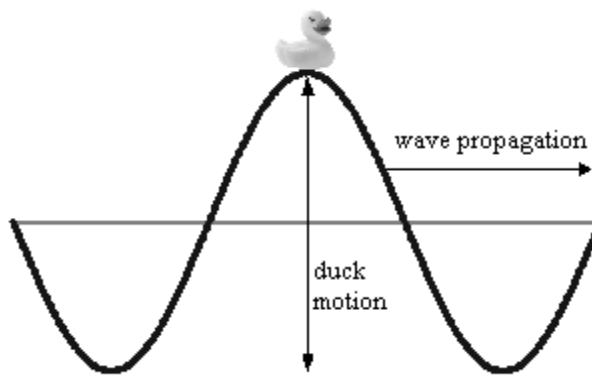


- **Pendulums:** as the pendulum swings, gravity accelerates it back toward its equilibrium position.

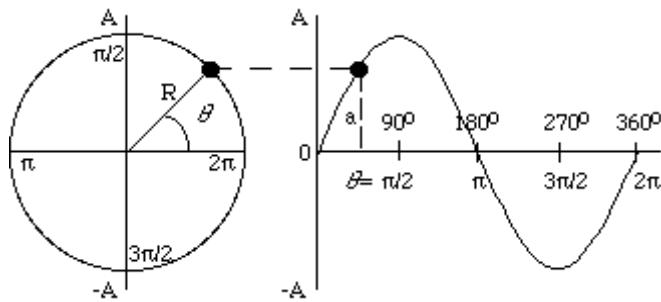


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- **Waves:** waves passing through some medium (such as water or air) cause the medium to oscillate up and down, like a duck sitting on the water as waves pass by.



- **Uniform circular motion:** as an object moves around a circle, its vertical position (y -position) is continuously oscillating between $+r$ and $-r$.



Use this space for summary and/or additional notes.

Springs

Unit: Simple Harmonic Motion

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding Goals:

- springs and spring constants
- spring force as a vector quantity

Skills:

- calculate the force and potential energy of a spring

Language Objectives:

- Understand and correctly use the term “spring.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Labs, Activities & Demonstrations:

- Spring mounted to lab stands with paper taped somewhere in the middle as an indicator.

Notes:

spring: a coiled object that resists motion parallel with the direction of propagation of the coil.

Spring Force

The equation for the force (vector) from a spring is given by Hooke’s Law, named for the British physicist Robert Hooke:

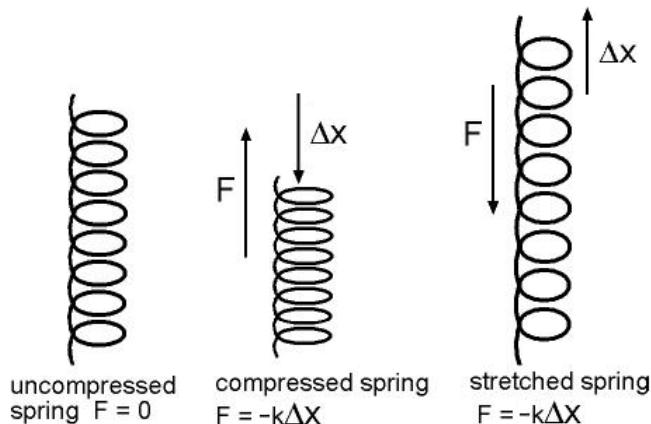
$$\vec{F}_s = -k\vec{x}$$

Where \vec{F}_s is the spring force (vector quantity representing the force exerted by the spring), \vec{x} is the displacement of the end of the spring (also a vector quantity), and k is the spring constant, an intrinsic property of the spring based on its mass, thickness, and the elasticity of the material that it is made of.

The negative sign in the equation is because the force is always in the opposite (negative) direction from the displacement.

Use this space for summary and/or additional notes.

A Slinky has a spring constant of $0.5 \frac{\text{N}}{\text{m}}$, while a heavy garage door spring might have a spring constant of $500 \frac{\text{N}}{\text{m}}$.



Potential Energy

The potential energy stored in a spring is given by the equation:

$$U = \frac{1}{2}kx^2$$

Where U is the potential energy (measured in joules), k is the spring constant, and x is the displacement. Note that the potential energy is always positive (or zero); this is because energy is a scalar quantity. A stretched spring and a compressed spring both have potential energy.

Use this space for summary and/or additional notes.

The Period of a Spring

period or period of oscillation: the time it takes a spring to move from its maximum displacement in one direction to its maximum displacement in the opposite direction and back again. The variable for the period is T , and the unit is usually seconds.

The period of a spring depends on the mass of the spring and its spring constant, and is given by the equation:

$$T_s = 2\pi \sqrt{\frac{m}{k}}$$

Frequency

frequency: the number of times something occurs in a given amount of time.

Frequency is usually given by the variable f , and is measured in units of hertz (Hz). One hertz is the inverse of one second:

$$1 \text{ Hz} \equiv \frac{1}{1 \text{ s}} \equiv 1 \text{ s}^{-1}$$

Note that period and frequency are reciprocals of each other:

$$T = \frac{1}{f} \quad \text{and} \quad f = \frac{1}{T}$$

Use this space for summary and/or additional notes.

Sample Problem:

Q: A spring with a mass of 0.1 kg and a spring constant of $2.7 \frac{\text{N}}{\text{m}}$ is compressed 0.3 m. Find the force needed to compress the spring, the potential energy stored in the spring when it is compressed, and the period of oscillation.

A: The force is given by Hooke's Law.

Substituting these values gives:

$$\vec{F} = -k\vec{x}$$

$$\vec{F} = -(2.7 \frac{\text{N}}{\text{m}})(+0.3 \text{ m}) = -0.81 \text{ N}$$

The potential energy is:

$$U = \frac{1}{2} kx^2$$

$$U = (0.5)(2.7 \frac{\text{N}}{\text{m}})(0.3 \text{ m})^2 = 0.12 \text{ J}$$

The period is:

$$T_s = 2\pi \sqrt{\frac{m}{k}}$$

$$T_s = (2)(3.14) \sqrt{\frac{0.1}{2.7}}$$

$$T_s = 6.28 \sqrt{0.037} = (6.28)(0.19) = 1.2 \text{ s}$$

Use this space for summary and/or additional notes.

Homework Problems

1. A 100.0 g mass is suspended from a spring whose constant is $50.0 \frac{N}{m}$. The mass is then pulled down 1.0 cm and then released.
 - a. How much force was applied in order to pull the spring down the 1.0 cm?

Answer: 0.5 N

- b. What is the frequency of the resulting oscillation?

Answer: 3.56 Hz

2. A 1000. kg car bounces up and down on its springs once every 2.0 s. What is the spring constant of its springs?

Answer: $9870 \frac{N}{m}$

Use this space for summary and/or additional notes.

3. A 4.0 kg block is released from a height of 5.0 m on a frictionless ramp. When the block reaches the bottom of the ramp, it slides along a frictionless surface and hits a spring with a spring constant of $3.92 \times 10^4 \frac{\text{N}}{\text{m}}$ as shown in the diagram below:



What is the maximum distance that the spring is compressed after the impact?

Answer: 0.101 m

4. A 1.6 kg block is attached to a spring that has a spring constant of $1.0 \times 10^3 \frac{\text{N}}{\text{m}}$. The spring is compressed a distance of 2.0 cm and the block is released from rest onto a frictionless surface. What is the speed of the block as it passes through the equilibrium position?

Answer: $0.5 \frac{\text{m}}{\text{s}}$

Use this space for summary and/or additional notes.

Pendulums

Unit: Simple Harmonic Motion

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding Goals:

- factors that affect the period and motion of a pendulum

Skills:

- calculate the period of a pendulum

Language Objectives:

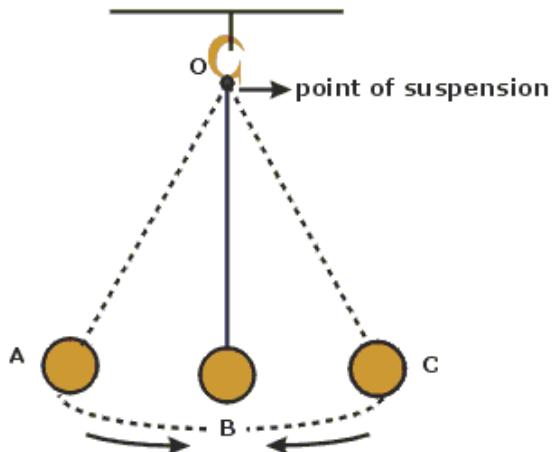
- Understand and correctly use the terms “pendulum” and “period.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Labs, Activities & Demonstrations:

- Pendulum made from a mass hanging from a lab stand.

Notes:

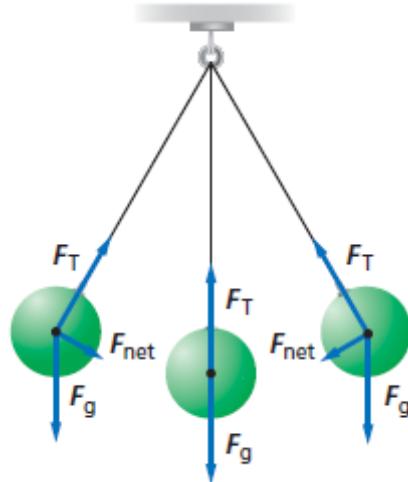
pendulum: a lever that is suspended from a point such that it can swing back and forth.



Use this space for summary and/or additional notes.

The Forces on a Pendulum

As the pendulum swings, its mass remains constant, which means the force of gravity pulling it down remains constant. The tension on the pendulum (which we can think of as a rope or string, though the pendulum can also be solid) also remains constant as it swings.



However, as the pendulum swings, the angle of the tension force changes. When the pendulum is not in the center (bottom), the vertical component of the tension is $F_T \cos \theta$, and the horizontal component is $F_T \sin \theta$. Because the angle is between 0° and 90° , $\cos \theta < 1$, which means F_g is greater than the upward component of F_T . This causes the pendulum to eventually stop. Also because the angle is between 0° and 90° , $\sin \theta > 0$, This causes the pendulum to start swinging in the opposite direction.

Use this space for summary and/or additional notes.

The Period of a Pendulum

period or period of oscillation: the time it takes a pendulum to travel from its maximum displacement in one direction to its maximum displacement in the opposite direction and back again. The variable for the period is T , and the unit is usually seconds.

Note that the time between pendulum “beats” (such as the tick-tock of a pendulum clock) are $\frac{1}{2}$ of the period of the pendulum. Thus a “grandfather” clock with a pendulum that beats seconds has a period $T = 2$ s.

The period of a pendulum depends on the force of gravity, the length of the pendulum, and the maximum angle of displacement. For small angles ($\theta < 15^\circ$), the period is given by the equation:

$$T = 2\pi \sqrt{\frac{\ell}{g}}$$

where T is the period of oscillation, ℓ is the length of the pendulum in meters, and g is the acceleration due to gravity ($9.8 \frac{\text{m}}{\text{s}^2}$ on Earth).

Note that the potential energy of a pendulum is simply the gravitational potential energy of the pendulum’s center of mass at its maximum displacement.

The velocity of the pendulum at its lowest point (where the potential energy is zero and all of the energy is kinetic) can be calculated using the conservation of energy.

Use this space for summary and/or additional notes.

Sample Problem:

Q: An antique clock has a pendulum that is 0.20 m long. What is its period?

A: The period is given by the equation:

$$T = 2\pi \sqrt{\frac{\ell}{g}}$$

$$T = 2(3.14) \sqrt{\frac{0.20}{10}}$$

$$T = 6.28 \sqrt{0.02}$$

$$T = (6.28)(0.141)$$

$$T = 0.889 \text{ s}$$

Homework Problems

1. A 20.0 kg chandelier is suspended from a high ceiling with a cable 6.0 m long. What is its period of oscillation as it swings?

Answer: 4.87 s

2. What is the length of a pendulum that oscillates 24.0 times per minute?

Answer: 1.58 m

Use this space for summary and/or additional notes.

3. The ceiling in our physics classroom is approximately 3.6 m high. How long did it take the bowling ball pendulum to swing across the room and back?

Answer: 3.77 s

Use this space for summary and/or additional notes.

Introduction: Electricity & Magnetism

Unit: Electricity & Magnetism

Topics covered in this chapter:

Electric Charge	303
Coulomb's Law	310
Electric Fields	314
Electric Current & Ohm's Law.....	317
Electrical Components	326
Circuits	329
Kirchhoff's Rules	337
Series Circuits.....	340
Parallel Circuits.....	346
Mixed Series & Parallel Circuits	355
Measuring Voltage, Current & Resistance.....	356
Magnetism	360
Magnetic Fields	364
Electromagnetism	369
Devices that Use Electromagnetism	371

This chapter discusses electricity and magnetism, how they behave, and how they relate to each other.

- *Electric Change, Coulomb's Law, and Electric Fields* describe the behavior of individual charged particles and how to calculate the effects of these particles on each other.
- *Electric Current & Ohm's Law* describes equations and calculations involving the flow of charged particles (electric current).

Use this space for summary and/or additional notes:

- *Electrical Components, EMF & Internal Resistance of a Battery, Circuits, Series Circuits, Parallel Circuits, Mixed Series & Parallel Circuits, and Measuring Voltage, Current & Resistance* describe the behavior of electrical components in a circuit and how to calculate quantities relating to the individual components and the entire circuit, based on the way the components are arranged.
- *Magnetism* describes properties of magnets and what causes objects to be magnetic. *Electricity & Magnetism* describes how electricity and magnetism affect each other.

One of the new challenges encountered in this chapter is interpreting and simplifying circuit diagrams, in which different equations may apply to different parts of the circuit.

Textbook:

- *Physics: Principles and Problems* Ch. 20: Static Electricity (pp. 406–423)
- *Physics: Principles and Problems* Ch. 21: Electric Fields (pp. 424–445)
- *Physics: Principles and Problems* Ch. 22: Current Electricity (pp. 446–467)
- *Physics: Principles and Problems* Ch. 23: Series and Parallel Circuits (pp. 468–489)
- *Physics: Principles and Problems* Ch. 24: Magnetic Fields (pp. 490–513)
- *Physics: Principles and Problems* Ch. 25: Electromagnetic Induction (pp. 514–533)

Standards addressed in this chapter:

Next Generation Science Standards (NGSS):

HS-PS2-4. Use mathematical representations of ~~Newton's Law of Gravitation~~ and Coulomb's Law to describe and predict the gravitational and electrostatic forces between objects.

HS-PS2-6. Communicate scientific and technical information about why the molecular-level structure is important in the functioning of designed materials.

HS-PS3-3. Design, build, and refine a device that works within given constraints to convert one form of energy into another form of energy.

Use this space for summary and/or additional notes.

HS-PS3-5. Develop and use a model of two objects interacting through electric or magnetic fields to illustrate the forces between objects and the changes in energy of the objects due to the interaction.

Massachusetts Curriculum Frameworks (2006):

- 5.1** Recognize that an electric charge tends to be static on insulators and can move on and in conductors. Explain that energy can produce a separation of charges.
- 5.2** Develop qualitative and quantitative understandings of current, voltage, resistance, and the connections among them (Ohm's law).
- 5.3** Analyze simple arrangements of electrical components in both series and parallel circuits. Recognize symbols and understand the functions of common circuit elements (battery, connecting wire, switch, fuse, resistance) in a schematic diagram.
- 5.4** Describe conceptually the attractive or repulsive forces between objects relative to their charges and the distance between them (Coulomb's law).
- 5.5** Explain how electric current is a flow of charge caused by a potential difference (voltage), and how power is equal to current multiplied by voltage.

Use this space for summary and/or additional notes.

Topics from this chapter assessed on the SAT Physics Subject Test:

- **Electric Fields, Forces, and Potentials**, such as Coulomb's law, induced charge, field and potential of groups of point charges, ~~and charged particles in electric fields~~
- **Circuit Elements and DC Circuits**, such as resistors, light bulbs, series and parallel networks, Ohm's law, and Joule's law
 1. Electric Charge
 2. Electric Force
 3. Electric Potential
 4. Conductors and Insulators
 5. Voltage
 6. Current
 7. Resistance
 8. Energy, Power, ~~and Heat~~
 9. Circuits

Skills learned & applied in this chapter:

- Working with material-specific constants from a table.
- Identifying electric circuit components.
- Simplifying circuit diagrams.

Use this space for summary and/or additional notes.

Electric Charge

Unit: Electricity & Magnetism

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 5.1, 5.4

Knowledge/Understanding Goals:

- electric charge
- properties of electric charges
- conductors vs. insulators

Language Objectives:

- Understand and correctly use the terms “electricity,” “charge,” “current,” “conductor,” “insulator,” and “induction.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Labs, Activities & Demonstrations:

- Charged balloon making hairs repel, attracting water molecules.
- Water sprayed on balloon neutralizes the charge.
- Wimshurst machine.
- Van de Graaff generator.

Notes:

electric charge: a physical property of matter which causes it to experience a force when near other electrically charged matter. Electric charge is measured in coulombs (C).

positive charge: the type of charge carried by protons. Originally defined as the charge left on a piece of glass when rubbed with silk. The glass becomes positively charged because the silk pulls electrons off the glass.

negative charge: the type of charge carried by electrons. Originally defined as the charge left on a piece of amber (or rubber) when rubbed with fur (or wool). The amber becomes negatively charged because the amber pulls the electrons off the fur.

Use this space for summary and/or additional notes.

elementary charge: the magnitude (amount) of charge on one proton or one electron. One elementary charge equals 1.60×10^{-19} C. Because ordinary matter is made of protons and electrons, the amount of charge carried by any object must be a multiple of the elementary charge.

Note however, that the quarks that protons and neutrons are made of carry fractional charges; up-type quarks carry a charge of $+\frac{2}{3}$ of an elementary charge, and down-type quarks carry a charge of $-\frac{1}{3}$ of an elementary charge. A proton is made of two up quarks and one down quark and carries a charge of +1 elementary charge. A neutron is made of one up quark and two down quarks and carries no charge.

static electricity: stationary electric charge, such as the charge left on silk or amber in the above definitions.

electric current (sometimes called electricity): the movement of electrons through a medium (substance) from one location to another. Note, however, that electric current is defined as the direction a positively charged particle would move. Thus electric current “flows” in the opposite direction from the actual electrons.

Use this space for summary and/or additional notes.



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Devices that Produce, Use or Store Charge

capacitor: a device that stores electric charge.

battery: a device that uses chemical reactions to produce an electric current.

generator: a device that converts mechanical energy (motion) into an electric current.

motor: a device that converts an electric current into mechanical energy.

Use this space for summary and/or additional notes.

Conductors vs. Insulators

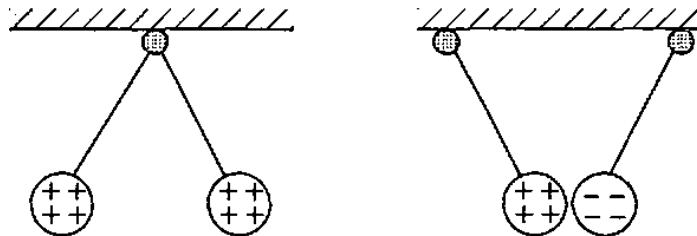
conductor: a material that allows charges to move freely through it. Examples of conductors include metals and liquids with positive and negative ions dissolved in them (such as salt water). When charges are transferred to a conductor, the charges distribute themselves evenly throughout the substance.

insulator: a material that does not allow charges to move freely through it. Examples of insulators include nonmetals and most pure chemical compounds (such as glass or plastic). When charges are transferred to an insulator, they cannot move, and remain where they are placed.

Behavior of Charged Particles

Like charges repel. A pair of the same type of charge (two positive charges or two negative charges) exert a force that pushes the charges away from each other.

Opposite charges attract. A pair of opposite types of charge (a positive charge and a negative charge) exert a force that pulls the charges toward each other.



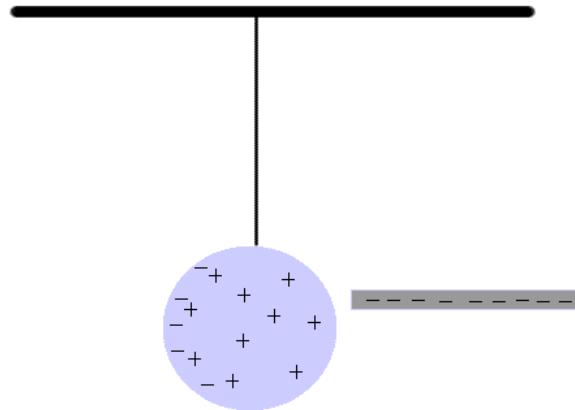
Charge is conserved. Electric charges cannot be created or destroyed, but can be transferred from one location or medium to another. (This is analogous to the laws of conservation of mass and energy.)

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Charging by Induction

induction: when an electrical charge on one object causes a charge in a second object.

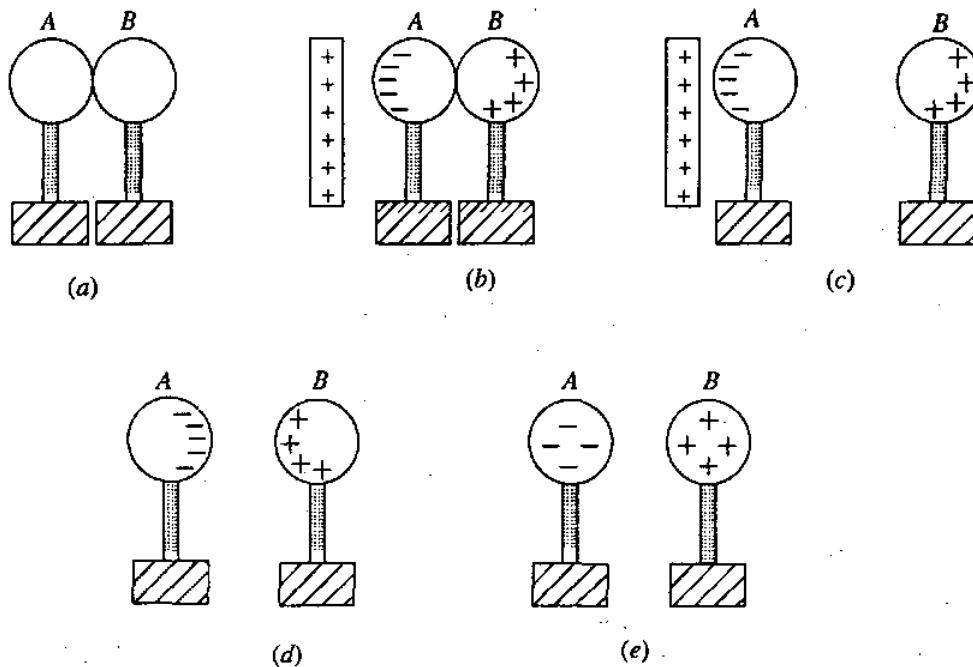
When a charged rod is brought near a neutral object, the charge on the rod attracts opposite charges and repels like charges that are near it. The diagram below shows a negatively-charged rod repelling negative charges.



If the negatively-charged rod above were touched to the sphere, some of the charges from the rod would be transferred to the sphere at the point of contact, and the sphere would acquire an overall negative charge.

Use this space for summary and/or additional notes.

A process for inducing charges in a pair of metal spheres is shown below:



- Metal spheres *A* and *B* are brought into contact.
- A positively charged object is placed near (but not in contact with) sphere *A*. This induces a negative charge in sphere *A*, which in turn induces a positive charge in sphere *B*.
- Sphere *B* (which is now positively charged) is moved away.
- The positively charged object is removed.
- The charges distribute themselves throughout the metal spheres.

AP questions dealing with charging by induction are common, so you should be sure to understand how and why every step of this procedure works.

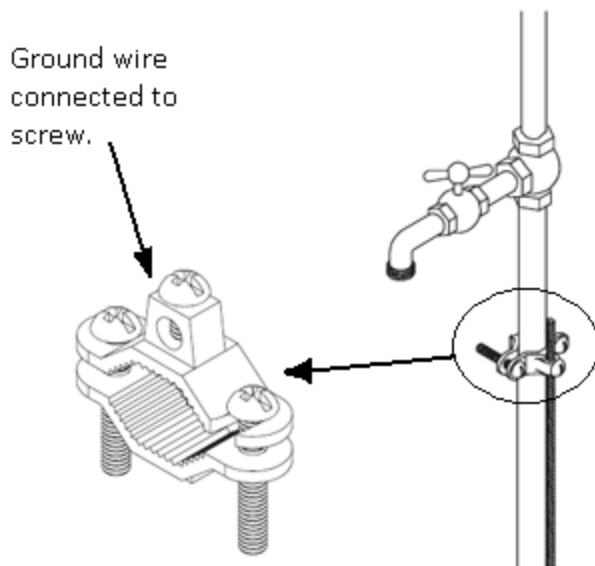
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Grounding

For the purposes of our use of electric charges, the ground (Earth) is effectively an endless supply of both positive and negative charges. Under normal circumstances, if a charged object is touched to the ground, electrons will move to neutralize the charge, either by flowing from the object to the ground or from the ground to the object.

Grounding a charged object or circuit means neutralizing the electrical charge on an object or portion of the circuit. *The charge of any object that is connected to ground is zero, by definition.*

In buildings, the metal pipes that bring water into the building are often used to ground the electrical circuits. The metal pipe is a good conductor of electricity, and carries the unwanted charge out of the building and into the ground outside.



Use this space for summary and/or additional notes.

Coulomb's Law

Unit: Electricity & Magnetism

NGSS Standards: HS-PS2-4, HS-PS3-5

MA Curriculum Frameworks (2006): 5.4

Skills:

- understand & solve problems using Coulomb's Law

Language Objectives:

- Accurately describe Coulomb's Law using appropriate academic language.
- Set up and solve word problems relating to Coulomb's Law.

Labs, Activities & Demonstrations:

- Van de Graaff generator with inertia balance pan.
- Charged balloon or Styrofoam sticking to wall.
- Charged balloon pushing meter stick.

Notes:

Electric charge is measured in Coulombs (abbreviation "C"). One Coulomb is the amount of electric charge transferred by a current of 1 ampere for a duration of 1 second.

1 C is the charge of 6.2415×10^{18} protons.

-1 C is the charge of 6.2415×10^{18} electrons.

One proton or electron (elementary charge) therefore has a charge of 1.6022×10^{-19} C.

Use this space for summary and/or additional notes.

Because charged particles exert a force on each other, that force can be measured and quantified. The force is directly proportional to the strengths of the charges and inversely proportional to the square of the distance. The formula is therefore:

$$F_e = \frac{kq_1q_2}{r^2}$$

where:

F_e = electrostatic force of repulsion between electric charges. A positive value of \vec{F}_e denotes that the charges are repelling (pushing away from) each other; a negative value of \vec{F}_e denotes that the charges are attracting (pulling towards) each other.

k = electrostatic constant = $8.9876 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2} \approx 9.0 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$

q_1 and q_2 = charges 1 and 2 respectively

r = distance (radius, because it goes outward in every direction) between the centers of the two charges

This formula is Coulomb's Law, named for its discoverer, the French physicist Charles-Augustin de Coulomb.

Use this space for summary and/or additional notes.

Sample problem:

Q: Find the force of electrostatic attraction between the proton and electron in a hydrogen atom if the radius of the atom is 37.1 pm

A: The charge of a single proton is 1.60×10^{-19} C, and the charge of a single electron is -1.60×10^{-19} C.

$$37.1 \text{ pm} = 3.71 \times 10^{-11} \text{ m}$$

$$F_e = \frac{kq_1q_2}{d^2}$$

$$F_e = \frac{(9.0 \times 10^9)(1.60 \times 10^{-19})(-1.60 \times 10^{-19})}{(3.71 \times 10^{-11})^2}$$

$$F_e = -1.67 \times 10^{-7} \text{ N}$$

The value of the force is negative, which signifies that the force is attractive.

Homework Problems

- What is the magnitude of the electric force between two objects, each with a charge of $+2.00 \times 10^{-6}$ C, which are separated by a distance of 1.50 m? Is the force attractive or repulsive?

Answer: 0.016 N, repulsive

Use this space for summary and/or additional notes.

2. An object with a charge of $+1.50 \times 10^{-2}$ C is separated from a second object with an unknown charge by a distance of 0.500 m. If the objects attract each other with a force of 1.35×10^6 N, what is the charge on the second object?

Answer: -2.50×10^{-3} C

3. The distance between an alpha particle and an electron is 2.00×10^{-25} m. What is the force of electrostatic attraction between the charges?

Answer: -1.15×10^{22} N

Use this space for summary and/or additional notes.

Electric Fields

Unit: Electricity & Magnetism

NGSS Standards: HS-PS3-2

MA Curriculum Frameworks (2006): N/A

Skills:

- drawing electric field lines

Language Objectives:

- Understand and correctly use the term “electric field.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Labs, Activities & Demonstrations:

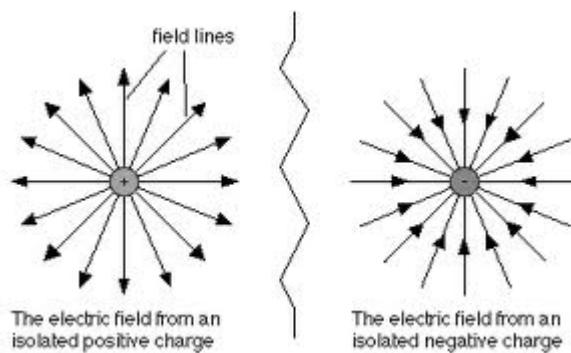
- students holding copper pipe in one hand and zinc-coated steel pipe in other—measure with voltmeter. (Can chain students together.)

Notes:

force field: a region in which an object experiences a force because of some intrinsic property of the object that enables the force to act on it. Force fields are vectors, which means they have both a magnitude and a direction.

electric field: an electrically charged region (force field) that exerts a force on any charged particle within the region.

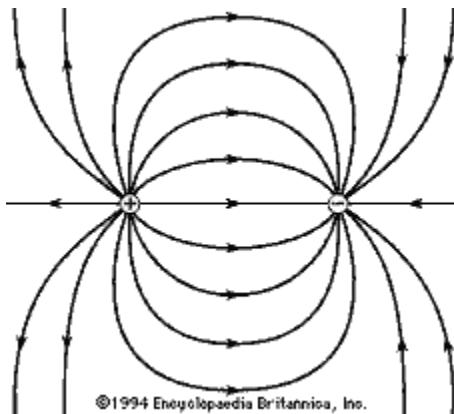
The simplest electric field is the region around a single charged particle:



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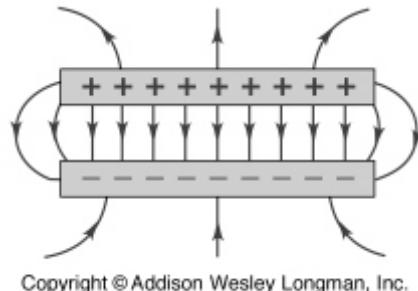
Field lines are vectors that show the directions of force on an object. In an electric field, the object is a positively-charged particle. This means that *the direction of the electric field is from positive to negative, i.e., field lines go outward in all directions from a positively-charged particle, and inward from all directions toward a negatively-charged particle.*

If a positive and a negative charge are near each other, the field lines go from the positive charge toward the negative charge:



(Note that even though this is a two-dimensional drawing, the field itself is three-dimensional. Some field lines come out of the paper from the positive charge and go into the paper toward the negative charge, and some go behind the paper from the positive charge and come back into the paper from behind toward the negative charge.)

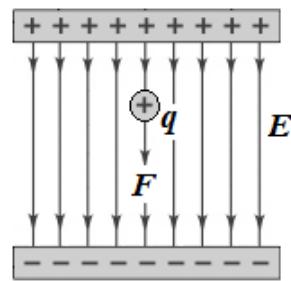
In the case of two charged plates (flat surfaces), the field lines would look like the following:



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We can measure the strength of an electric field by placing a particle with a positive charge (q) in the field, and measuring the force (\vec{F}) on the particle.

Use this space for summary and/or additional notes.



Coulomb's Law tells us that the force on the charge is due to the charges from the electric field:

$$F_e = \frac{kq_1q_2}{d^2}$$

If the positive and negative charges on the two surfaces that make the electric field are equal, *the force is the same everywhere in between the two surfaces.* (This is because as the particle gets farther from one surface, it gets closer to the other.) This means that the force on the particle is related only to the charges that make up the electric field and the charge of the particle.

We can therefore describe the electric field (\vec{E}) as the force between the electric field and our particle, divided by the charge of our particle:

$$\vec{E} = \frac{\vec{F}}{q} \quad \text{or} \quad \vec{F} = q\vec{E}$$

Use this space for summary and/or additional notes.

Electric Current & Ohm's Law

Unit: Electricity & Magnetism

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 5.2, 5.5

Knowledge/Understanding Goals:

- electric potential (voltage)
- electric current
- resistance

Skills:

- perform calculations involving voltage, current, resistance and power

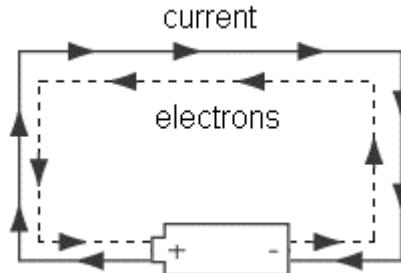
Language Objectives:

- Understand and correctly use the terms “current,” “direct current,” “alternating current,” “potential difference,” “voltage,” “resistance,” “power,” and “work.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to Ohm’s Law.

Notes:

electric current: the flow of charged particles from one place to another, caused by a difference in electric potential. The direction of the electric current is defined as the direction that a positively-charged particle would move. Note, however, that the particles that are actually moving are electrons, which are negatively charged.

This means that electric current “travels” in the opposite direction from the electrons.



Use this space for summary and/or additional notes.

Electric current (\bar{I}) is a vector quantity and is measured in amperes (A), often abbreviated as "amps". One ampere is one coulomb per second.

$$I = \frac{\Delta q}{t}$$

voltage (potential difference): the difference in electric potential energy between two locations, per unit of charge.

Potential difference is the work (W) done on a charge per unit of charge (q). Potential difference (V) is a scalar quantity (in DC circuits) and is measured in volts (V), which are equal to joules per coulomb.

$$V = \frac{W}{q}$$

The total voltage in a circuit is usually determined by the power supply that is used for the circuit (usually a battery in DC circuits).

resistance: the amount of electromotive force (electric potential) needed to force a given amount of current through an object.

Resistance (R) is a scalar quantity and is measured in ohms (Ω). One ohm is one volt per ampere.

$$R = \frac{V}{I}$$

This relationship is Ohm's Law, named for the German physicist Georg Ohm. Ohm's Law is more commonly written:

$$I = \frac{V}{R} \quad \text{or} \quad V = IR$$

Simply put, Ohm's Law states that an object has an ability to resist electric current flowing through it. The more resistance an object has, the more voltage you need to force electric current through it. Or, for a given voltage, the more resistance an object has, the less current will flow through it.

Resistance is an intrinsic property of a substance. In this course, we will limit problems that involve calculations to ohmic resistors, which means their resistance does not change with temperature.

Choosing the voltage and the arrangement of objects in the circuit (which determines the resistance) is what determines how much current will flow.

Electrical engineers use arrangements of resistors in circuits in order to adjust the amount of current that flows through the components.

Use this space for summary and/or additional notes.

resistivity: the innate ability of a substance to offer electrical resistance. The resistance of an object is therefore a function of the resistivity of the substance (ρ), and of the length (L) and cross-sectional area (A) of the object. In MKS units, resistivity is measured in ohm-meters ($\Omega \cdot \text{m}$).

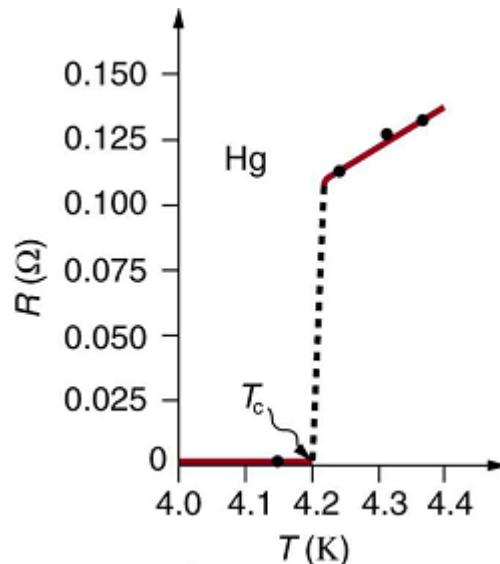
$$R = \frac{\rho L}{A}$$

Resistivity changes with temperature. For small temperature differences (less than 100°C), resistivity is given by:

$$\rho = \rho_0(1 + \alpha \Delta T)$$

where ρ_0 is the resistivity at some reference temperature and α is the temperature coefficient of resistivity for that substance. For conductors, α is positive (which means their resistivity increases with temperature). For metals at room temperature, resistivity typically varies from $+0.003$ to $+0.006 \text{ K}^{-1}$.

Some materials become superconductors (essentially zero resistance) at very low temperatures. The temperature below which a material becomes a superconductor is called the critical temperature (T_c). For example, the critical temperature for mercury is 4.2 K, as shown in the graph to the right.



conductivity: the innate ability of a substance to conduct electricity. Conductivity (σ) is the inverse of resistivity, and is measured in siemens (S). Siemens used to be called mhos (symbol ℧). ("Mho" is "ohm" spelled backwards.)

$$\sigma = \frac{1}{\rho}$$

Use this space for summary and/or additional notes.

ohmic resistor: a resistor whose resistance is the same regardless of voltage and current. The filament of an incandescent light bulb is an example of a non-ohmic resistor, because the current heats up the filament, which increases its resistance. (This is necessary in order for the filament to also produce light.)

capacitance: the ability of an object to hold an electric charge. Capacitance (C) is a scalar quantity and is measured in farads (F). One farad equals one coulomb per volt.

$$C = \frac{q}{V}$$

power: as discussed in the mechanics section of this course, power (P) is the work done per unit of time and is measured in watts (W).

In electric circuits:

$$P = \frac{W}{t} = VI = I^2R = \frac{V^2}{R}$$

work: recall from mechanics that work (W) equals power times time, and is measured in either newton-meters ($N\cdot m$) or joules (J):

$$W = Pt = VI t = I^2 R t = \frac{V^2 t}{R} = Vq$$

Electrical work or energy is often measured in kilowatt-hours ($kW\cdot h$).

$$1 \text{ kW}\cdot \text{h} \equiv 3.6 \times 10^6 \text{ J} \equiv 3.6 \text{ MJ}$$

Summary of Terms and Variables

Term	Variable	Unit	Term	Variable	Unit
charge	q or Q	coulomb (C)	resistance	R	ohm (Ω)
current	I	ampere (A)	power	P	watt (W)
voltage	V	volt (V)	work	W	joule (J)

Use this space for summary and/or additional notes.

Alternating Current vs. Direct Current

Electric current can move in two ways.

direct current: electric current flows through the circuit, starting at the positive terminal of the battery or power supply, and ending at the negative terminal. Batteries supply direct current. A typical AAA, AA, C, or D battery supplies 1.5 volts DC.

alternating current: electric current flows back and forth in one direction and then the other, like a sine wave. The current alternates at a particular frequency. In the U.S., household current is 110 volts AC with a frequency of 60 Hz.

Alternating current requires higher voltages in order to operate devices, but has the advantage that the voltage drop is much less over a length of wire than with direct current.

Sample Problems:

Q: A simple electrical device uses 1.5 A of current when plugged into a 110 V household electrical outlet. How much current would the same device draw if it were plugged into a 12 V outlet in a car?

A: Resistance is a property of a specific object. Because we are not told otherwise, we assume the device is ohmic and the resistance is the same regardless of the current.

To solve this problem, we need to find the resistance from the numbers when the device is plugged into the household outlet, and use that resistance along with the car's voltage to find the current it draws when it is plugged into the car's 12 V plug.

In the household outlet:

$$R = \frac{V}{I} = \frac{110}{1.5} = 73.3 \Omega$$

In the car:

$$I = \frac{V}{R} = \frac{12}{73.3} = 0.163 \text{ A}$$

Use this space for summary and/or additional notes.

Q: A laptop computer uses 10 W of power. The laptop's power supply adjusts the current so that the power is the same regardless of the voltage supplied. How much current would the computer draw from a 110 V household outlet? How much current would the same laptop computer need to draw from a 12 V car outlet?

A: Household outlet:

$$P = VI$$

$$I = \frac{P}{V} = \frac{10}{110} = 0.091 \text{ A}$$

Car outlet:

$$I = \frac{P}{V} = \frac{10}{12} = 0.83 \text{ A}$$

Q: A 100 Ω resistor is 0.70 mm in diameter and 6.0 mm long. Suppose you wanted to make a 470 Ω resistor out of the same material. If you wanted to use a piece of the same diameter material, what would the length need to be? If, instead, you wanted to make a resistor the same length, what would the new diameter need to be?

A: In both cases, $R = \frac{\rho \ell}{A}$.

For a resistor of the same diameter (same cross-sectional area), ρ and A are the same, which gives:

$$\frac{R'}{R} = \frac{\ell'}{\ell}$$

$$\ell' = \frac{R' \ell}{R} = \frac{(470)(6.0)}{100} = 28.2 \text{ mm}$$

For a resistor of the same length:

$$\frac{R'}{R} = \frac{A}{A'} = \frac{\pi r^2}{\pi (r')^2} = \frac{\pi (\frac{d}{2})^2}{\pi (\frac{d'}{2})^2} = \frac{d^2}{(d')^2}$$

$$d' = \sqrt{\frac{R d^2}{R'}} = d \sqrt{\frac{R}{R'}} = 0.70 \sqrt{\frac{100}{470}} = 0.70 \sqrt{0.213} = 0.323 \text{ mm}$$

Use this space for summary and/or additional notes.

Homework Problems

1. An MP3 player uses a standard 1.5 V battery. How much resistance is in the circuit if it uses a current of 0.010 A?

Answer: 150 Ω

2. How much current flows through a hair dryer plugged into a 110 V circuit if it has a resistance of 25 Ω ?

Answer: 4.4 A

3. A battery pushes 1.2 A of charge through the headlights in a car, which has a resistance of 10 Ω . What is the potential difference across the headlights?

Answer: 12 V

4. A 0.7 mm diameter by 60 mm long pencil "lead" is made of graphite, which has a resistivity of approximately $1.0 \times 10^{-4} \Omega \cdot \text{m}$. What is its resistance?

Answer: 15.6 Ω

Use this space for summary and/or additional notes.

5. A circuit used for electroplating copper applies a current of 3.0 A for 16 hours. How much charge is transferred?

Answer: 172 800 C

6. What is the power when a voltage of 120 V drives a 2.0 A current through a device?

Answer: 240W

7. What is the resistance of a 40. W light bulb connected to a 120 V circuit?

Answer: 360 Ω

8. If a component in an electric circuit dissipates 6.0 W of power when it draws a current of 3.0 A, what is the resistance of the component?

Answer: 0.67 Ω

Use this space for summary and/or additional notes.

9. Some children are afraid of the dark and ask their parents to leave the hall light on all night. Suppose the hall light in a child's house has two 60. W incandescent light bulbs (120 W total), the voltage is 120 V, and the light is left on for 8.0 hours.

a. How much current flows through the light fixture?

Answer: 1.0 A

b. How many kilowatt-hours of energy would be used in one night?

Answer: 0.96 kW·h

c. If the power company charges 22 ¢ per kilowatt-hour, how much does it cost to leave the light on overnight?

Answer: 21.1 ¢

d. If the two 60 W incandescent bulbs are replaced by LED bulbs that use 8.5 W each, how much money would the family save each night?

Answer: 18.1 ¢

Use this space for summary and/or additional notes.

Electrical Components

Unit: Electricity & Magnetism

NGSS Standards: HS-PS2-6

MA Curriculum Frameworks (2006): 5.3

Knowledge/Understanding Goals:

- recognize common components of electrical circuits

Language Objectives:

- Recognize and be able to name and draw symbols for each of the electrical components described in this section.

Labs, Activities & Demonstrations:

- Show & tell with actual components.

Notes:

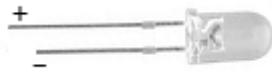
electrical component: an object that performs a specific task in an electric circuit.

A circuit is a collection of components connected together so that the tasks performed by the individual components combine in some useful way.

circuit diagram: a picture that represents a circuit, with different symbols representing the different components.

Use this space for summary and/or additional notes.

The following table describes some of the common components of electrical circuits, what they do, and the symbols that are used to represent them in circuit diagrams.

Component	Symbol	Picture	Description
wire	—		Carries current in a circuit.
junction	+		Connection between two or more wires.
unconnected wires	+		Wires cross but are not connected.
battery			Supplies current at a fixed voltage.
resistor			Resists flow of current.
potentiometer (rheostat, dimmer)			Provides variable (adjustable) resistance.
capacitor	+		Stores charge.
diode			Allows current to flow in only one direction (from + to -).
light-emitting diode (LED)			Diode that gives off light.

Use this space for summary and/or additional notes.

Electrical Components

Component	Symbol	Picture	Description
switch			Opens / closes circuit.
incandescent lamp (light)			Provides light (and resistance).
transformer			Increases or decreases voltage.
voltmeter			Measures voltage (volts).
ammeter			Measures current (amperes).
ohmmeter			Measures resistance (ohms).
fuse			Opens circuit if too much current flows through it.
ground			Neutralizes charge.

Use this space for summary and/or additional notes.

Circuits

Unit: Electricity & Magnetism

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 5.3

Knowledge/Understanding Goals:

- how resistance limits current in a circuit
- the difference between series and parallel circuits

Language Objectives:

- Explain why resistors are necessary in electric circuits.
- Understand and correctly use the terms “series” and “parallel” as applied to electric circuits.

Labs, Activities & Demonstrations:

- Example circuit with light bulbs & switches.
- Fuse demo using a single strand from a multi-strand wire.

Notes:

circuit: an arrangement of electrical components that allows electric current to pass through them so that the tasks performed by the individual components combine in some useful way.

closed circuit: a circuit that has a complete path for current to flow from the positive terminal of the battery or power supply through the components and back to the negative terminal.

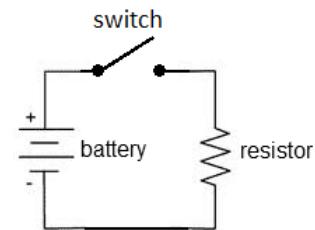
open circuit: a circuit that has a gap such that current cannot flow from the positive terminal to the negative terminal.

short circuit: a circuit in which the positive terminal is connected directly to the negative terminal with no load (resistance) in between.

Use this space for summary and/or additional notes.

A diagram of a simple electric circuit might look like the diagram to the right.

When the switch is closed, the electric current flows from the positive terminal of the battery through the switch, through the resistor, and back to the negative terminal of the battery.



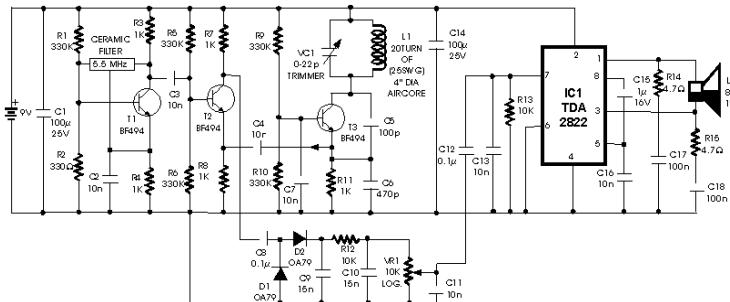
An electric circuit needs a power supply (often a battery) that provides current at a specific difference in electric potential (voltage), and one or more components that use the energy provided by the battery.

The battery continues to supply current, provided that:

1. There is a path for the current to flow from the positive terminal to the negative terminal, and
2. The total resistance of the circuit is small enough to allow the current to flow.

If the circuit is broken, current cannot flow and the chemical reactions inside the battery stop.

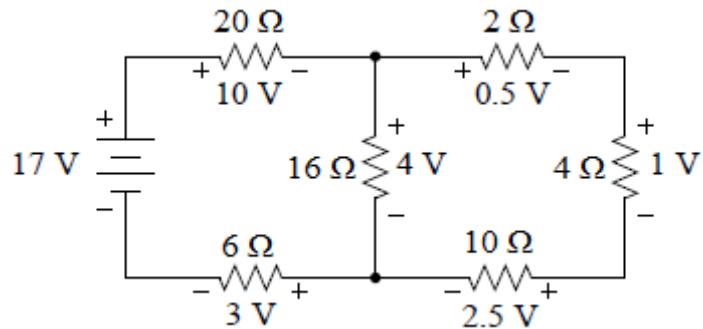
Of course, as circuits become more complex, the diagrams reflect this increasing complexity. The following is a circuit diagram for a metal detector:



Analyzing an electrical circuit means figuring out the potential difference (voltage), current, and/or resistance contributed by each component of a circuit.

Use this space for summary and/or additional notes.

The following is an example of a circuit with one 17 V battery and six resistors. Notice that the voltages and resistances are all labeled.

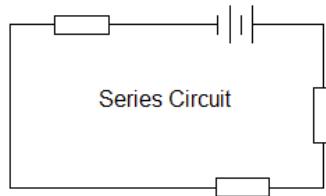


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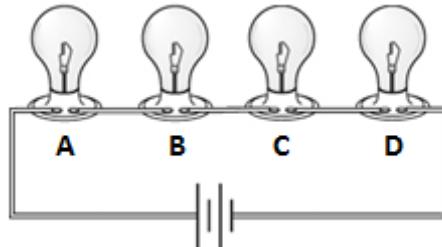
Series vs. Parallel Circuits

If a circuit has multiple components, they can be arranged in series or parallel.

series: Components in **series** lie along the same path, one after the other.



In a series circuit, all of the current flows through every component, one after another. If the current is interrupted anywhere in the circuit, no current will flow. For example, in the following series circuit, if any of light bulbs A, B, C, or D is removed, no current can flow and none of the light bulbs will be illuminated.

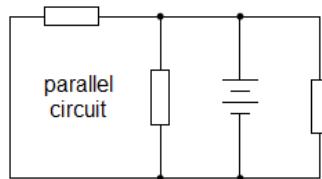


Because some voltage is “used up” by each bulb in the circuit, each additional bulb means the voltage is divided among more bulbs and is therefore less for each bulb. This is why light bulbs get dimmer as you add more bulbs in series.

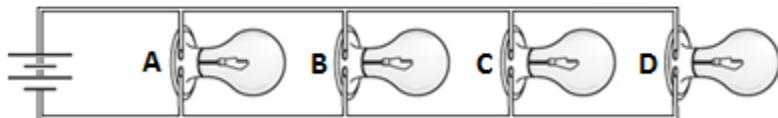
Christmas tree lights used to be wired in series. This caused a lot of frustration, because if one bulb burned out, the entire string went out, and it could take several tries to find which bulb was burned out.

Use this space for summary and/or additional notes.

parallel: Components in **parallel** lie in separate paths.

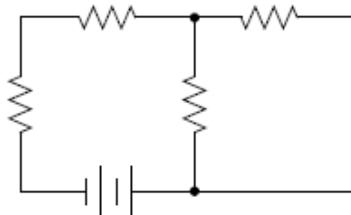


In a parallel circuit, the current divides at each junction, with some of the current flowing through each path. If the current is interrupted in one path, current can still flow through the other paths. For example, in the following parallel circuit, if any of light bulbs A, B, C, or D is removed, current still flows through the remaining bulbs.



Because the voltage across each branch is equal to the total voltage, all of the bulbs will light up with full brightness, regardless of how many bulbs are in the circuit. (However, each separate light bulb draws the same amount of current as if it were the only thing in the circuit, so the total current in the circuit increases with each new branch. This is why you trip a circuit breaker or blow a fuse if you have too many high-power components plugged into the same circuit.)

Note that complex circuits may have some components that are in series with each other and other components that are in parallel.

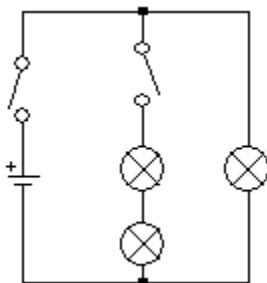


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Sample Problem:

Q: A circuit consists of a battery, two switches, and three light bulbs. Two of the bulbs are in series with each other, and the third bulb is in parallel with the others. One of the switches turns off the two light bulbs that are in series with each other, and the other switch turns off the entire circuit. Draw a schematic diagram of the circuit, using the correct symbol for each component.

A:

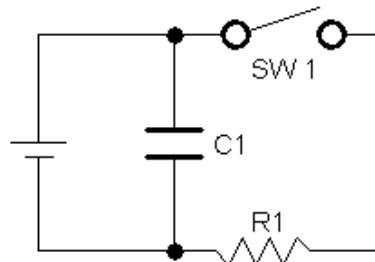


Note that no sensible person would intentionally wire a circuit this way. It would make much more sense to have the second switch on the branch with the one light bulb, so you could turn off either branch separately or both branches by opening both switches. This is an example of a strange circuit that a physics teacher would use to make sure you really can follow exactly what the question is asking!

Use this space for summary and/or additional notes.

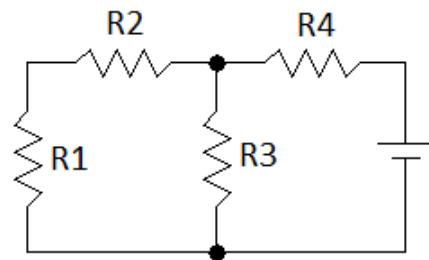
Homework Problems

1. The following circuit contains a battery, switch (SW1), capacitor (C1), and resistor (R1).



Which of components C1 and SW1 are in series with R1? Which are in parallel with R1?

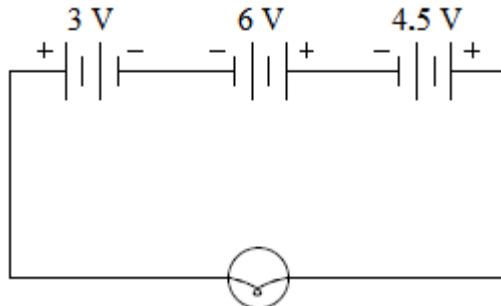
2. The following circuit contains a battery and four resistors (R1, R2, R3, and R4).



Which resistors are in series with R1? Which are in parallel with R1?

Use this space for summary and/or additional notes.

3. The following bizarre circuit contains three batteries and a light bulb.
What is the potential difference across the light bulb?
(Hint: remember to check the +/- orientation of the batteries.)



4. A circuit is powered by a 9 V battery. The circuit has a 100Ω resistor in series with the battery, and then the circuit splits. One branch contains only a switch. The second branch contains a $100 \mu\text{F}$ capacitor in series with another switch. Draw a diagram for this circuit.

Use this space for summary and/or additional notes.

Kirchhoff's Rules

Unit: Electricity & Magnetism

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 5.3

Knowledge & Understanding:

- Understand Kirchhoff's junction rule and Kirchhoff's loop rule.

Skills:

- Use Kirchhoff's rules to determine voltage, current and resistance in complex circuits.

Language Objectives:

- Accurately describe how to measure voltage, current and resistance in an electric circuit, using appropriate academic language.

Notes:

In 1845, the German physicist Gustav Kirchhoff came up with two simple rules that describe the behavior of current in complex circuits. Those rules are:

Kirchhoff's junction rule: the total current coming into any junction must equal the total current coming out of the junction.

The junction rule is based on the concept that electric charge cannot be created or destroyed. Current is simply the flow of electric charge, so any charges that come into a junction must also come out of it.

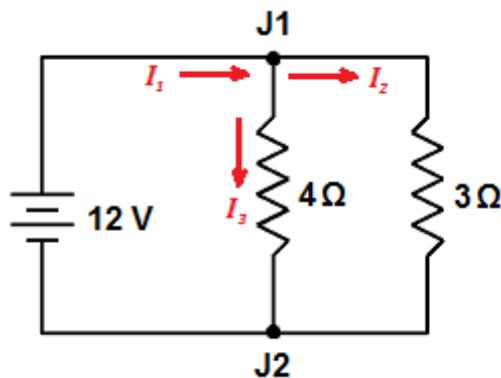
Kirchhoff's loop rule: the sum of the voltages around any closed loop must add up to zero.

The loop rule is based on the concept that voltage is the difference in electric potential between one location in the circuit and another. If you come back to the same point in the circuit, the difference in electric potential between where you started and where you ended (the same place) must be zero. Therefore, any increases and decreases in voltage around the loop must cancel.

Use this space for summary and/or additional notes.

Junction Rule Example:

As an example of the junction rule, consider the following circuit:



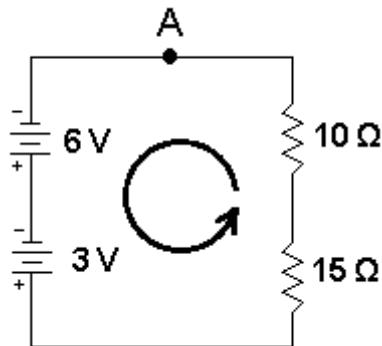
The junction rule tells us that the current flowing into junction J1 must equal the current flowing out. If we assume current I_1 flows into the junction, and currents I_2 and I_3 flow out of it, then $I_1 = I_2 + I_3$.

We know that the voltage across both resistors is 12 V. From Ohm's Law we can determine that the current through the 3 Ω resistor is $I_2 = 4 \text{ A}$, and the current through the 4 Ω resistor is $I_3 = 3 \text{ A}$. The junction rule tells us that the total current must therefore be $I_1 = I_2 + I_3 = 4 \text{ A} + 3 \text{ A} = 7 \text{ A}$.

Use this space for summary and/or additional notes.

Loop Rule Example:

For the loop rule, consider the following circuit:



If we start at point A and move counterclockwise around the loop (in the direction of the arrow), the voltage should be zero when we get back to point A.

For this example, we are moving around the circuit in the same direction that the current flows, because that makes the most intuitive sense. However, it wouldn't matter if we moved clockwise instead—just as with vector quantities, we choose a positive direction and assign each quantity to a positive or negative number accordingly, and the math tells us what is actually happening.

Starting from point A, we first move through the 6 V battery. We are moving from the negative pole to the positive pole of the battery, so the voltage increases by +6 V. When we move through the second battery, the voltage increases by +3 V.

Next, we move through the 15Ω resistor. When we move through a resistor in the positive direction (of current flow), the voltage drops, so we assign the resistor a voltage of $-15I$ (based on $V = IR$, where I is the current through the resistor). Similarly, the voltage across the 10Ω resistor is $-10I$. Applying the loop rule gives:

$$\begin{aligned} 6 + 3 + (-15I) + (-10I) &= 0 \\ 9 - 25I &= 0 \\ 9 &= 25I \\ I &= \frac{9}{25} = 0.36 \text{ A} \end{aligned}$$

Use this space for summary and/or additional notes.

Series Circuits

Unit: Electricity & Magnetism

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 5.3

Knowledge/Understanding Goals:

- the difference between series and parallel circuits

Skills:

- calculate voltage, current, resistance, and power in series circuits.

Language Objectives:

- Understand and correctly use the term “series circuit.”
- Set up and solve word problems relating to electrical circuits with components in series.

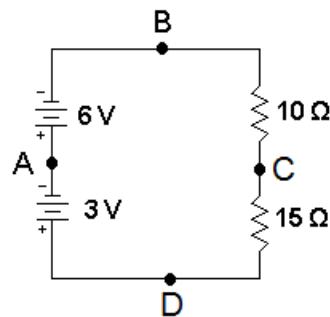
Labs, Activities & Demonstrations:

- Circuit with light bulbs wired in series.

Notes:

Analyzing Series Circuits

The following circuit shows two batteries and two resistors in series:



Use this space for summary and/or additional notes.

Current

Because there is only one path, all of the current flows through every component. This means the current is the same through every component in the circuit:

$$I_{total} = I_1 = I_2 = I_3 = \dots$$

Voltage

In a series circuit, if there are multiple voltage sources (e.g., batteries), the voltages add:

$$V_{total} = V_1 + V_2 + V_3 + \dots$$

In the above circuit, there are two batteries, one that supplies 6V and one that supplies 3V. The voltage from A to B is +6V, the voltage from A to D is -3V (note that A to D means measuring from negative to positive), and the voltage from D to B is (+3V) + (+6V) = +9V.

Resistance

If there are multiple resistors, each one contributes to the total resistance and the resistances add:

$$R_{total} = R_1 + R_2 + R_3 + \dots$$

In the above circuit, the resistance between points B and D is $10\Omega + 15\Omega = 25\Omega$.

Power

In all circuits (series and parallel), any component that has resistance dissipates power whenever current passes through it. The total power consumed by the circuit is the sum of the power dissipated by each component:

$$P_{total} = P_1 + P_2 + P_3 + \dots$$

Use this space for summary and/or additional notes.

Calculations

You can calculate the voltage, current, resistance, and power of each component separately and also the entire circuit using the equations:

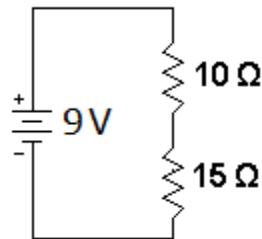
$$V = IR \quad P = VI = I^2R = \frac{V^2}{R}$$

“Solving” the circuit for these quantities is much like solving a Sudoku puzzle. You systematically decide which variables (for each component and/or the entire circuit) you have enough information to solve for. Each result enables you to determine more and more of the, until you have found all of the quantities you need.

Sample Problem:

Suppose we are given the following circuit:

and we are asked to fill in the table:



	R ₁	R ₂	Total
Voltage (V)			9 V
Current (I)			
Resistance (R)	10 Ω	15 Ω	
Power (P)			

First, we recognize that resistances in series add, which gives us:

	R ₁	R ₂	Total
Voltage (V)			9 V
Current (I)			
Resistance (R)	10 Ω	15 Ω	25 Ω
Power (P)			

Use this space for summary and/or additional notes.

Now, we know two variables in the “Total” column, so we use $V = IR$ to find the current. Because this is a series circuit, the total current is also the current through R_1 and R_2 .

$$V = IR$$

$$9 = (I)(25)$$

$$I = \frac{9}{25} = 0.36 \text{ A}$$

	R_1	R_2	Total
Voltage (V)			9 V
Current (I)	0.36 A	0.36 A	0.36 A
Resistance (R)	10 Ω	15 Ω	25 Ω
Power (P)			

As soon as we know the current, we can find the voltage across R_1 and R_2 , again using $V = IR$.

	R_1	R_2	Total
Voltage (V)	3.6 V	5.4 V	9 V
Current (I)	0.36 A	0.36 A	0.36 A
Resistance (R)	10 Ω	15 Ω	25 Ω
Power (P)			

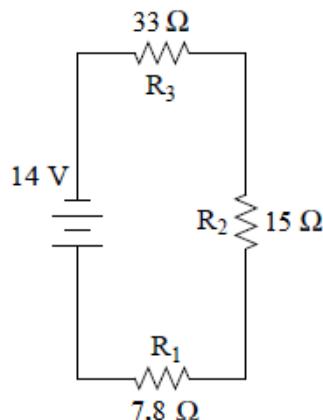
Finally, we can fill in the power, using $P = VI$, $P = I^2R$, and/or $P = \frac{V^2}{R}$:

	R_1	R_2	Total
Voltage (V)	3.6 V	5.4 V	9 V
Current (I)	0.36 A	0.36 A	0.36 A
Resistance (R)	10 Ω	15 Ω	25 Ω
Power (P)	1.30 W	1.94 W	3.24 W

Use this space for summary and/or additional notes.

Homework Problems

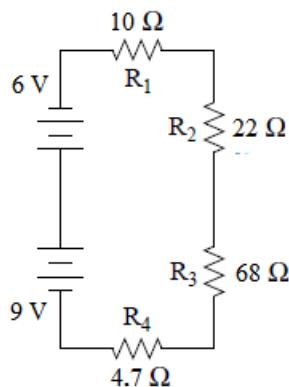
1. Fill in the table for the following circuit:



	R ₁	R ₂	R ₃	Total
Voltage (V)				14 V
Current (I)				
Resist. (R)	7.8 Ω	15 Ω	33 Ω	
Power (P)				

Use this space for summary and/or additional notes.

2. Fill in the table for the following circuit:



	R ₁	R ₂	R ₃	R ₄	Total
Voltage (V)					
Current (I)					
Resist. (R)	10Ω	22Ω	68Ω	4.7Ω	
Power (P)					

Use this space for summary and/or additional notes:

Parallel Circuits

Unit: Electricity & Magnetism

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 5.3

Skills:

- calculate voltage, current, resistance, and power in parallel circuits.

Language Objectives:

- Understand and correctly use the term “parallel circuit.”
- Set up and solve word problems relating to electrical circuits with components in parallel.

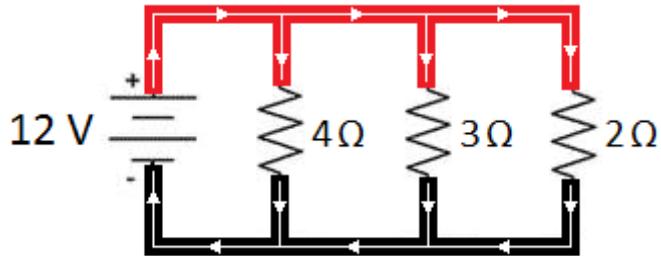
Labs, Activities & Demonstrations:

- Circuit with light bulbs wired in parallel.

Notes:

Parallel Circuits

The following circuit shows a battery and three resistors in parallel:



Current

The current divides at each junction (as indicated by the arrows). This means the current through each path must add up to the total current:

$$I_{total} = I_1 + I_2 + I_3 + \dots$$

Use this space for summary and/or additional notes.

Voltage

In a parallel circuit, the potential difference (voltage) across the battery is always the same (12 V in the above example). Therefore, the potential difference between *any point* on the top wire and *any point* on the bottom wire must be the same. This means the voltage is the same across each path:

$$V_{total} = V_1 = V_2 = V_3 = \dots$$

Power

Just as with series circuits, in a parallel circuit, any component that has resistance dissipates power whenever current passes through it. The total power consumed by the circuit is the sum of the power dissipated by each component:

$$P_{total} = P_1 + P_2 + P_3 + \dots$$

Resistance

If there are multiple resistors, the effective resistance of each path becomes less as there are more paths for the current to flow through. The total resistance is given by the formula:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

Some students find it confusing that the combined resistance of a group of resistors in series is always less than any single resistor by itself.

Use this space for summary and/or additional notes.

Electric current is analogous to water in a pipe:

- The current corresponds to the flow rate.
- The voltage corresponds to the pressure between one side and the other.
- The resistance would correspond to how small the pipe is (i.e., how hard it is to push water through the pipes). A smaller pipe has more resistance; a larger pipe will let water flow through more easily than a smaller pipe.



The voltage (pressure) drop is the same between one side and the other because less water flows through the smaller pipes and more water flows through the larger ones until the pressure is completely balanced. The same is true for electrons in a parallel circuit.

The water will flow through the set of pipes more easily than it would through any one pipe by itself. The same is true for resistors. As you add more resistors, you add more pathways for the current, which means less total resistance.

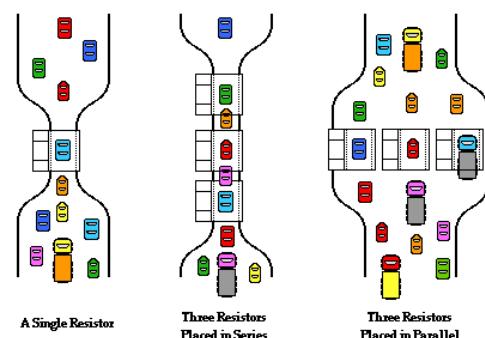
Another common analogy is to compare resistors with toll booths on a highway.

One toll booth slows cars down while the drivers pay the toll.

Multiple toll booths in series would slow traffic down more.

Multiple toll booths in parallel make traffic flow faster because there are more paths for the cars to follow. Each additional toll booth further reduces the resistance to the flow of traffic.

Influencing the Flow Rate on a Tollway



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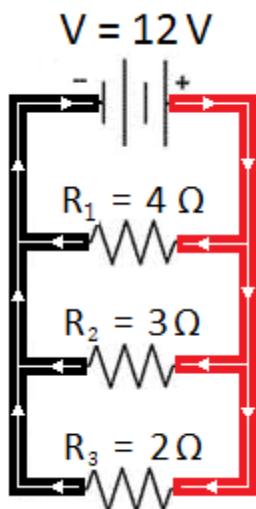
Calculations

Just as with series circuits, you can calculate the voltage, current, resistance, and power of each component and the entire circuit using the equations:

$$V = IR \quad P = VI = I^2R = \frac{V^2}{R}$$

Sample Problem

Suppose we are given the following circuit:



and we are asked to fill in the table:

	R_1	R_2	R_3	Total
Voltage (V)				12 V
Current (I)				
Resistance (R)	4 Ω	3 Ω	2 Ω	
Power (P)				

Because this is a parallel circuit, the total voltage equals the voltage across all three branches, so we can fill in 12 V for each resistor.

The next thing we can do is use $V = IR$ to find the current through each resistor:

	R_1	R_2	R_3	Total
Voltage (V)	12 V	12 V	12 V	12 V
Current (I)	3 A	4 A	6 A	13 A
Resistance (R)	4 Ω	3 Ω	2 Ω	
Power (P)				

In a parallel circuit, the current adds, so the total current is $3 + 4 + 6 = 13\text{ A}$.

Use this space for summary and/or additional notes.

Now, we have two ways of finding the total resistance.

We can use $V = IR$ for the voltage and total current:

$$V = IR$$

$$12 = 13 R$$

$$R = \frac{12}{13} = 0.923 \Omega$$

Or we can use the formula for resistances in parallel:

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_{total}} = \frac{1}{4} + \frac{1}{3} + \frac{1}{2}$$

$$\frac{1}{R_{total}} = \frac{3}{12} + \frac{4}{12} + \frac{6}{12} = \frac{13}{12}$$

$$R_{total} = \frac{12}{13} = 0.923 \Omega$$

Now we have:

	R ₁	R ₂	R ₃	Total
Voltage (V)	12 V	12 V	12 V	12 V
Current (I)	3 A	4 A	6 A	13 A
Resistance (R)	4 Ω	3 Ω	2 Ω	0.923 Ω
Power (P)				

Use this space for summary and/or additional notes.

As we did with series circuits, we can calculate the power, using $P = VI$, $P = I^2R$, and/or $P = \frac{V^2}{R}$:

	R ₁	R ₂	R ₃	Total
Voltage (V)	12 V	12 V	12 V	12 V
Current (I)	3 A	4 A	6 A	13 A
Resistance (R)	4 Ω	3 Ω	2 Ω	0.923 Ω
Power (P)	36 W	48 W	72 W	156 W

Batteries in Parallel

One question that has not been answered yet is what happens when batteries are connected in parallel.

If the batteries have the same voltage, the potential difference (voltage) remains the same, but the total current is the combined current from the two batteries.

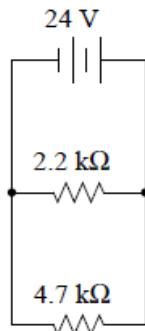
However, if the batteries have different voltages there is a problem, because each battery attempts to maintain a constant potential difference (voltage) between its terminals. This results in the higher voltage battery overcharging the lower voltage battery.

Remember that physically, batteries are electrochemical cells—small solid-state chemical reactors with redox reactions taking place in each cell. If one battery overcharges the other, material is deposited on the cathode (positive terminal) until the cathode becomes physically too large for its compartment, at which point the battery bursts and the chemicals leak out.

Use this space for summary and/or additional notes.

Homework Problems

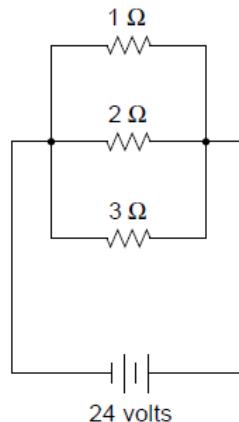
- Fill in the table for the following circuit:



	R ₁	R ₂	Total
Voltage (V)			24 V
Current (I)			
Resist. (R)	2.2 kΩ	4.7 kΩ	
Power (P)			

Use this space for summary and/or additional notes.

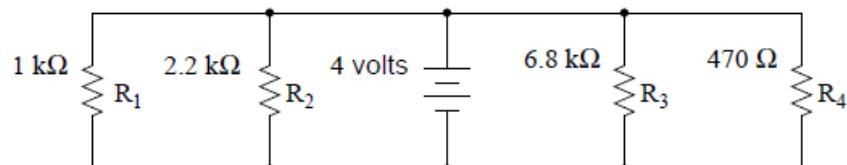
2. Fill in the table for the following circuit:



	R ₁	R ₂	R ₃	Total
Voltage (V)				24 V
Current (I)				
Resist. (R)	1 Ω	2 Ω	3 Ω	
Power (P)				

Use this space for summary and/or additional notes:

3. Fill in the table for the following circuit:



	R ₁	R ₂	R ₃	R ₄	Total
Voltage (V)					4 V
Current (I)					
Resistance (R)	1 kΩ	2.2 kΩ	6.8 kΩ	470 Ω	
Power (P)					

Use this space for summary and/or additional notes:

Mixed Series & Parallel Circuits

Unit: Electricity & Magnetism

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 5.3

Skills:

- analyze circuits by replacing networks of resistors with a single resistor of equivalent resistance.

Language Objectives:

- Set up and solve word problems involving electrical circuits with some components in series and others in parallel with each other.

Notes:

Of course, circuits can have mixed series and parallel sections. If this is the case, you can determine the various voltages, currents and resistances by applying Kirkhoff's Rules and/or by "simplifying the circuit." Simplifying the circuit, in this case, means replacing resistors in series or parallel with a single resistor of equivalent resistance.

However, mixed series & parallel circuits are much more challenging to analyze, and will not be addressed in this course.

Use this space for summary and/or additional notes.

Measuring Voltage, Current & Resistance

Unit: Electricity & Magnetism

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 5.3

Knowledge & Understanding:

- how voltage, current and resistance are measured.
- how positive and negative numbers for voltage and current correspond with the direction of current flow.

Skills:

- measure voltage, current and resistance.

Language Objectives:

- Accurately describe how to measure voltage, current and resistance in an electric circuit, using appropriate academic language.

Labs, Activities & Demonstrations:

- Show & tell with digital multi-meter.

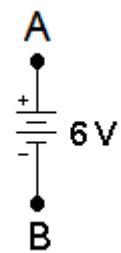
Notes:

Analyzing an electrical circuit means figuring out the potential difference (voltage), current, and/or resistance in each component of a circuit. In order to analyze actual circuits, it is necessary to be able to measure these quantities.

Measuring Voltage

Suppose we want to measure the electric potential (voltage) across the terminals of a 6 V battery. The diagram would look like this:

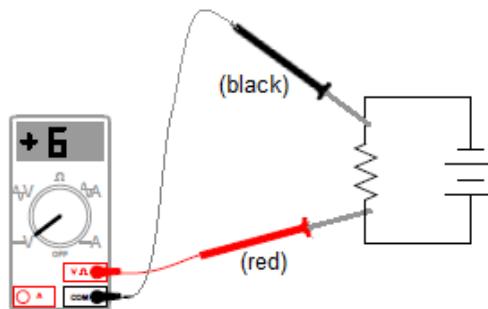
The voltage between points A and B is either +6 V or -6 V, depending on the direction. The voltage from A to B (positive to negative) is +6 V, and the voltage from B to A (negative to positive) is -6 V.



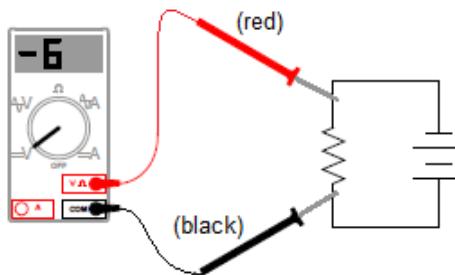
Use this space for summary and/or additional notes.

When measuring voltage, the circuit needs to be powered up with current flowing through it. Make sure that the voltmeter is set for volts (DC or AC, as appropriate) and that the red lead is plugged into the VΩ socket (for measuring volts or ohms). Then touch the two leads *in parallel* with the two points you want to measure the voltage across. (*Remember that voltage is the same across all branches of a parallel circuit. You want the voltmeter in parallel so the voltmeter reads the same voltage as the voltage across the component that you are measuring.*)

On a voltmeter (a meter that measures volts or voltage), the voltage is measured assuming the current is going from the red (+) lead to the black (-) lead. In the following circuit, if you put the red (+) lead on the more positive end of a resistor and the black (-) lead on the more negative end, the voltage reading would be positive. In this circuit, the voltmeter reads a potential difference of +6V:



If you switch the leads, so the black (-) lead is on the more positive end and the red (+) lead is on the more positive end, the voltage reading would be negative. In this circuit, the voltmeter reads -6V:



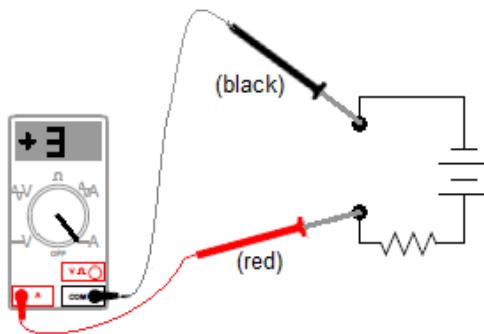
The reading of -6V indicates that the potential difference is 6V, but the current is actually flowing in the opposite direction from the way the voltmeter is measuring—from the black (-) lead to the red (+) lead.

Use this space for summary and/or additional notes.

Measuring Current

When measuring current, the circuit needs to be open between two points. Make sure the ammeter is set for amperes (A), milliamperes (mA) or microamperes (μ A) AC or DC, depending on what you expect the current in the circuit to be. Make sure the red lead is plugged into appropriate socket (A if the current is expected to be 0.5 A or greater, or mA/ μ A if the current is expected to be less than 0.5 A). Then touch one lead to each of the two contact points, so that the ammeter is *in series* with the rest of the circuit. (*Remember that current is the same through all components in a series circuit. You want the ammeter in series so that all of the current flows through it.*)

On an ammeter (a meter that measures current), the current is measured assuming that it is flowing from the red (+) lead to the black (-) lead. In the following circuit, if you put the red (+) lead on the side that is connected to the positive terminal and the black (-) lead on the end that is connected to the negative terminal, the current reading would be positive. In this circuit, the current is +3 A:



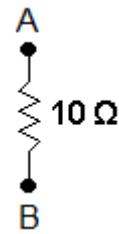
As with the voltage example above, if you switched the leads, the reading would be -3 A instead of +3 A.

Use this space for summary and/or additional notes.

Measuring Resistance

Resistance does not have a direction. If you placed an ohmmeter (a meter that measures resistance) across points A and B, it would read $10\ \Omega$ regardless of which lead is on which point.

However, because an ohmmeter needs to supply a small amount of current across the component and measure the resistance, the reading is more susceptible to measurement problems, such as the resistance of the wire itself, how well the probes are making contact with the circuit, etc. It is often more reliable to measure the voltage and current and calculate resistance using Ohm's Law ($V = IR$).



Use this space for summary and/or additional notes.

Magnetism

Unit: Electricity & Magnetism

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding Goals:

- what magnetism is and how it occurs
- properties of magnets

Language Objectives:

- Understand and correctly use the terms “magnet” and “magnetic field.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Labs, Activities & Demonstrations:

- neodymium magnets
- ring magnets repelling each other on a dowel
- magnets attracting each other across a gap

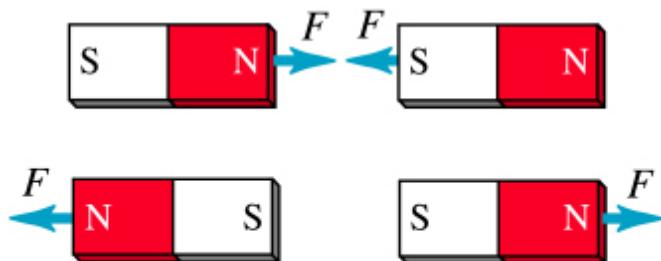
Notes:

magnet: a material with electrons that can align in a manner that attracts or repels other magnets.

A magnet has two ends or “poles”, called “north” and “south”. If a magnet is allowed to spin freely, the end that points toward the north on Earth is called the north end of the magnet. The end that points toward the south on Earth is called the south end of the magnet. (The Earth’s magnetic poles are near, but not in exactly the same place as its geographic poles.)

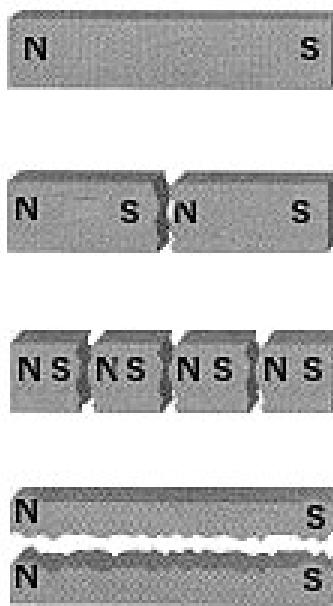
Use this space for summary and/or additional notes.

All magnets have a north and south pole. As with charges, opposite poles attract, and like poles repel.



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If you were to cut a magnet in half, each piece would be a magnet with its own north and south pole:



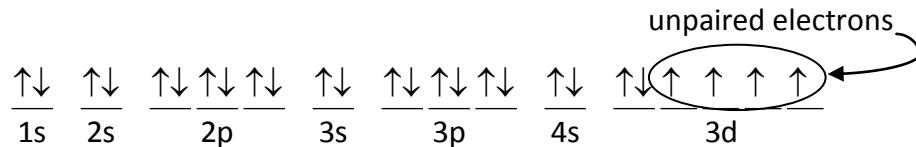
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Electrons and Magnetism

Magnetism is caused by unpaired electrons in atoms. Electrons within atoms reside in energy regions called “orbitals”. Each orbital can hold up to two electrons.

If two electrons share an orbital, they have opposite spins. (Note that the electrons are not actually spinning. “Spin” is the term for an intrinsic property of certain subatomic particles.) This means that if one electron aligns itself with a magnetic field, the other electron in the same orbital becomes aligned to oppose the magnetic field, and there is no net force.

However, if an orbital has only one electron, that electron is free to align with the magnetic field, which causes an attractive force between the magnet and the magnetic material. For example, as you may remember from chemistry, the electron configuration for iron is:



The inner electrons are paired up, but four of the electrons in the $3d$ sublevel are unpaired, and are free to align with an external magnetic field.

Use this space for summary and/or additional notes.

Magnetic measurements and calculations involve fields that act over 3-dimensional space and change continuously with position. This means that most calculations relating to magnetic fields need to be represented using multivariable calculus, which is beyond the scope of this course.

magnetic permeability (magnetic permittivity): the ability of a material to support the formation of a magnetic field. Magnetic permeability is represented by the variable μ . The magnetic permeability of empty space is $\mu_0 = 4\pi \times 10^{-7} \frac{\text{N}}{\text{A}^2}$.

diamagnetic: a material whose electrons cannot align with a magnetic field. Diamagnetic materials have very low magnetic permeabilities.

paramagnetic: a material that has electrons that can align with a magnetic field. Paramagnetic materials have relatively high magnetic permeabilities.

ferromagnetic: a material that can form crystals with permanently-aligned electrons, resulting in a permanent magnet. Ferromagnetic materials can have very high magnetic permeabilities. Some naturally-occurring materials that exhibit ferromagnetism include iron, cobalt, nickel, gadolinium, dysprosium, and magnetite (Fe_3O_4).

Use this space for summary and/or additional notes.

Magnetic Fields

Unit: Electricity & Magnetism

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding Goals:

- what a magnetic field is

Language Objectives:

- Understand and correctly use the term “magnetic field.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Labs, Activities & Demonstrations:

- magnetic field demonstrator plate
- placing various objects into the gap between two magnets
- ferrofluid

Notes:

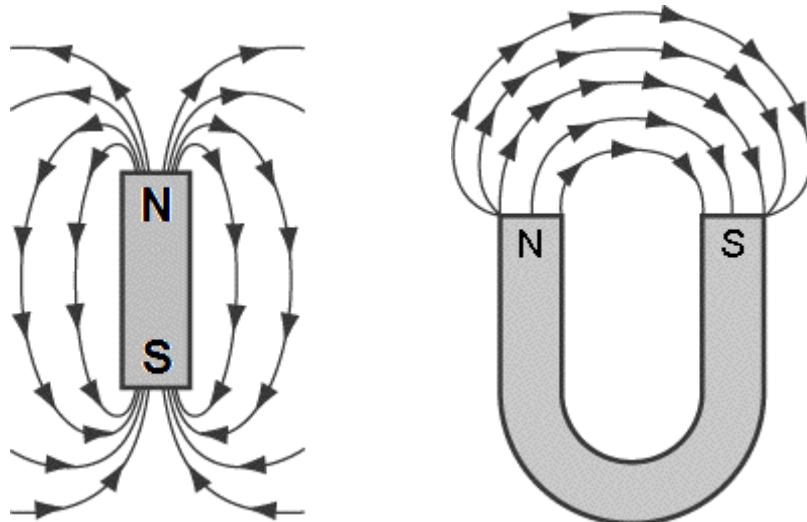
magnetic field: a region in which magnetic attraction and repulsion are occurring.

Similar to gravitational fields and electric fields, a magnetic field is a region in which there is a force on objects that have unpaired electrons that can respond to a magnetic field.

magnetic susceptibility: a measure of the degree of magnetization of a material when it is placed into a magnetic field.

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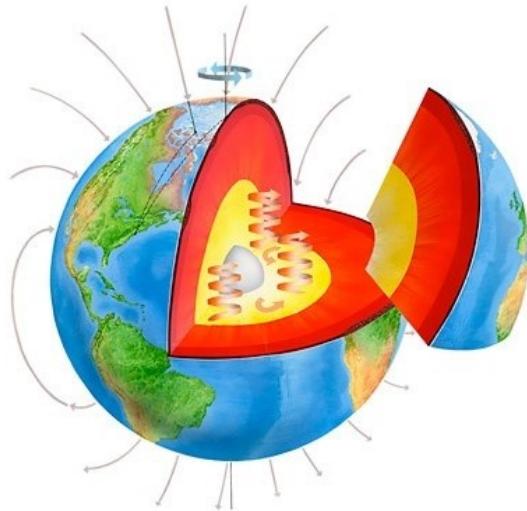
Similar to an electric field, we represent a magnetic field by drawing field lines. Magnetic field lines point from the north pole of a magnet toward the south pole, and they show the direction that the north end of a compass or magnet would be deflected if it was placed in the field:



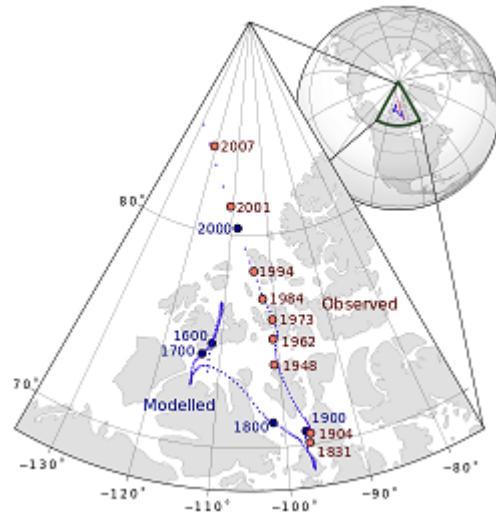
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The Earth's Magnetic Field

The molten iron outer core and very hot inner core of the Earth causes a magnetic field over the entire planet:

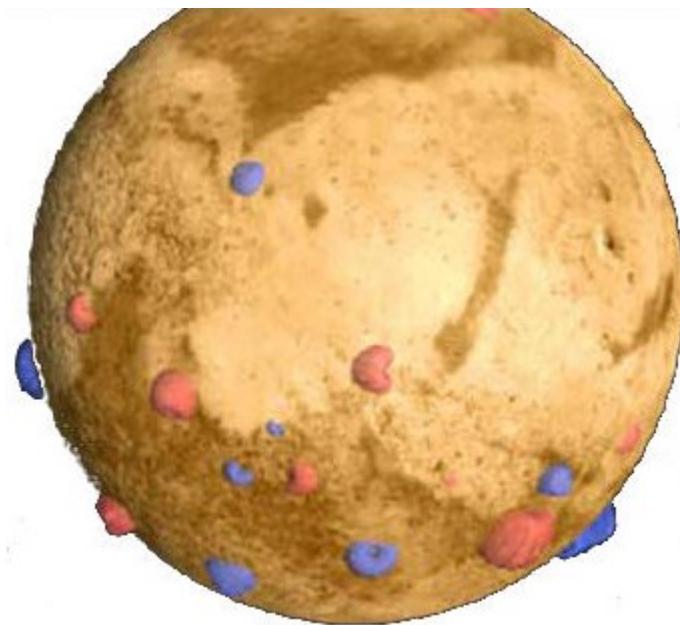


Because the core of the Earth is in constant motion, the Earth's magnetic field is constantly changing. The exact location of the Earth's magnetic north and south poles varies by about 80 km over the course of a day because of the rotation of the Earth. Its average location (shown on the map of Northern Canada below) drifts by about 50 km each year:



Use this space for summary and/or additional notes.

Not all planets have a planetary magnetic field. Mars, for example, is believed to have once had a planetary magnetic field, but the planet cooled off enough to disrupt the processes that caused it. Instead, Mars has some very strong localized magnetic fields that were formed when minerals cooled down in the presence of the planetary magnetic field:



In this picture, the blue and red areas represent regions with strong localized magnetic fields. On Mars, a compass could not be used in the ways that we use a compass on Earth; if you took a compass to Mars, the needle would point either toward or away from each of these regions.

Jupiter, on the other hand, has a planetary magnetic field twenty times as strong as that of Earth. This field may be caused by water with dissolved electrolytes or by liquid hydrogen.

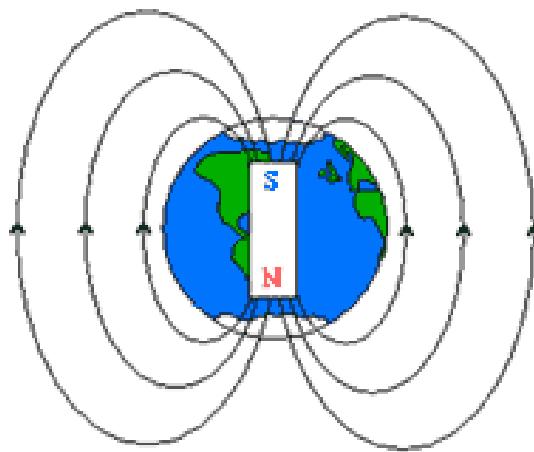
Use this space for summary and/or additional notes.

Recall that the north pole of a magnet is the end that points toward the north end of the Earth. This must mean that if the Earth is a giant magnet, one of its magnetic poles must be near the geographic north pole, and the other magnetic pole must be near the geographic south pole.

For obvious reasons, the Earth's magnetic pole near the north pole is called the Earth's "north magnetic pole" or "magnetic north pole". Similarly, the Earth's magnetic pole near the south pole is called the Earth's "south magnetic pole" or "magnetic south pole".

However, because the north pole of a magnet points toward the north, the Earth's north magnetic pole (meaning its location) must therefore be the south pole of the giant magnet that is the Earth.

Similarly, because the south pole of a magnet points toward the south, the Earth's south magnetic pole (meaning its location) must therefore be the north pole of the giant Earth-magnet.



Unfortunately, the term "magnetic north pole," "north magnetic pole" or any other similar term almost always means the magnetic pole that is in the north part of the Earth. There is no universally-accepted way to name the poles of the Earth-magnet.

Use this space for summary and/or additional notes.

Electromagnetism

Unit: Electricity & Magnetism

NGSS Standards: HS-PS2-5

MA Curriculum Frameworks (2006): 5.6

Knowledge/Understanding Goals:

- how a charge moving through a magnetic field produces a force
- how this property can be used to make electric motors, generators, and transformers

Skills:

- calculate the force on a wire produced by a current moving through a magnetic field
- calculate the voltage and current changes in a step-up or step-down transformer

Language Objectives:

- Understand and correctly use the terms “electromagnet,” “motor,” “generator,” and “transformer.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to the behavior of electric current in a magnetic field.

Labs, Activities & Demonstrations:

- current-carrying wire in a magnetic field
- electromagnet
- electric motor

Use this space for summary and/or additional notes.

Magnetic Fields and Moving Charges

Like gravitational and electric fields, a magnetic field is a force field. (Recall that force fields are vector quantities, meaning that they have both magnitude and direction.) The strength of a magnetic field is measured in Teslas (T), named after the Serbian-American physicist Nikola Tesla.

$$1 \text{ T} = 1 \frac{\text{N}}{\text{A}\cdot\text{m}}$$

In the 1830s, physicists Michael Faraday and Joseph Henry each independently discovered that an electric current could be produced by moving a magnet through a coil of wire, or by moving a wire through a magnetic field. This process is called electromagnetic induction.

The key concepts are:

1. Moving charges produce a magnetic field.
2. A changing magnetic field creates an electric force, which causes electric charges to move. (Recall that current is defined as the movement of electric charges.)

Use this space for summary and/or additional notes.

Devices that Use Electromagnetism

Unit: Electricity & Magnetism

NGSS Standards: HS-PS2-5

MA Curriculum Frameworks (2006): 5.6

Knowledge/Understanding Goals:

- understand the basic design of solenoids, motors, generators, transformers, and mass spectrometers

Skills:

- calculate the voltage and current changes in a step-up or step-down transformer

Language Objectives:

- Understand and correctly use the terms “solenoid,” “electromagnet,” “motor,” “generator,” “transformer,” and “mass spectrometer.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to the behavior of electric current in a magnetic field.

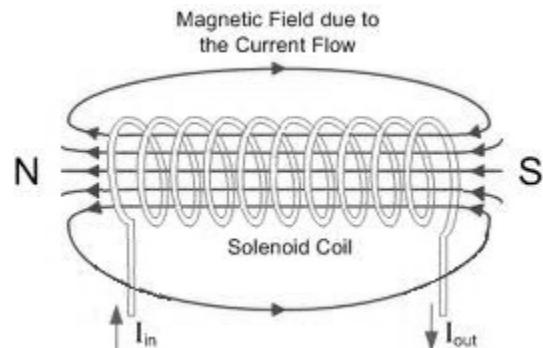
Notes:

Solenoid

A solenoid is a coil made of fine wire. When a current is passed through the wire, it produces a magnetic field through the center of the coil.

When a current is applied, a permanent magnet placed in the center of the solenoid will be attracted or repelled and will move.

One of the most common uses of a solenoid is for electric door locks.

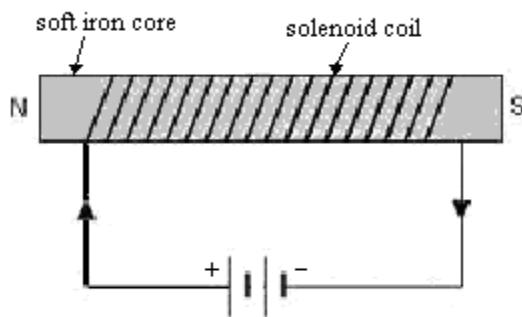


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Electromagnet

An electromagnet is a device that acts as magnet only when electric current is flowing through it.

An electromagnet is made by placing a soft iron core in the center of a solenoid. The high magnetic permeability of iron causes the resulting magnetic field to become thousands of times stronger:



Because the iron core is not a permanent magnet, the electromagnet only works when current is flowing through the circuit. When the current is switched off, the electromagnet stops acting like a magnet and releases whatever ferromagnetic objects might have been attracted to it.

Of course, the above description is a simplification. Real ferromagnetic materials such as iron usually experience magnetic remanence, meaning that some of the electrons in the material remain aligned, and the material is weakly magnetized.

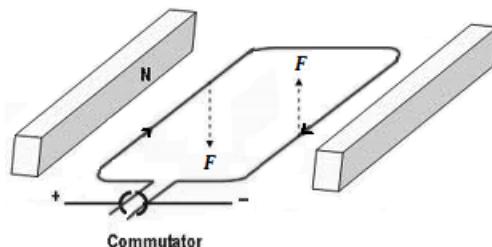
While magnetic remanence is undesirable in an electromagnet, it is the basis for magnetic computer storage media, such as audio and computer tapes and floppy and hard computer disks. To write information onto a disk, a disk head (an electromagnetic that can be moved radially) is pulsed in specific patterns as the disk spins. The patterns are encoded on the disk as locally magnetized regions.

When encoded information is read from the disk, the moving magnetic regions produce a changing electric field that causes an electric current in the disk head.

Use this space for summary and/or additional notes.

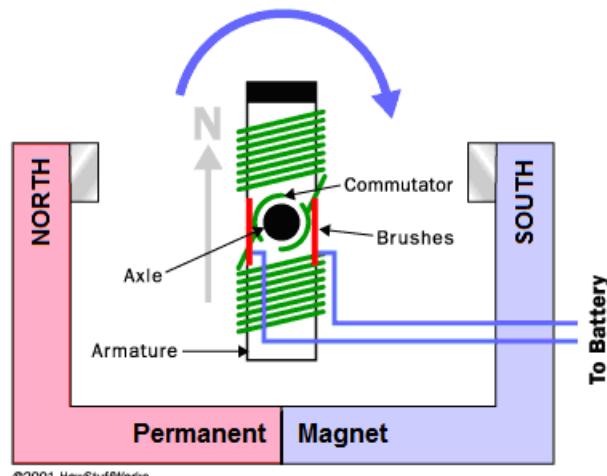
Electric Motor

The force produced by a moving current in a magnetic field can be used to cause a loop of wire to spin:



A commutator is used to reverse the direction of the current as the loop turns, so that the combination of attraction and repulsion always applies force in the same direction.

If we replace the loop of wire with an electromagnet (a coil of wire wrapped around a material such as iron that has both a high electrical conductivity and a high magnetic permeability), the electromagnet will spin with a strong force.

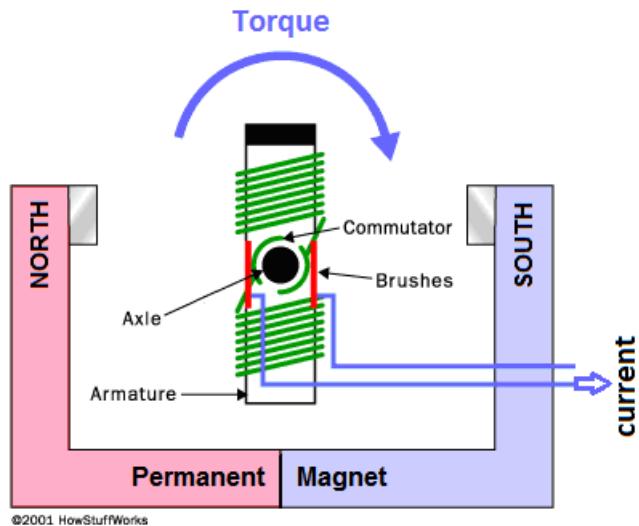


An electromagnet that spins because of its continuously switching attraction and repulsion to the magnetic field produced by a separate set of permanent magnets is called an electric motor. An electric motor turns electric current into rotational motion, which can be used to do work.

Use this space for summary and/or additional notes.

Generator

A *generator* uses the same components and operates under the same principle, except that a mechanical force is used to spin the coil. When the coil moves through the magnetic field, it produces an electric current. Thus a generator is a device that turns rotational motion (work) into an electric current.

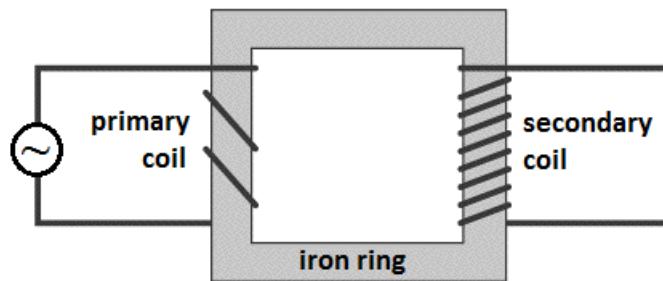


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Inductor (Transformer)

Because electric current produces a magnetic field, a ring made of a ferromagnetic material can be used to move an electric current. An inductor (transformer) is a device that takes advantage of this phenomenon in order to increase or decrease the voltage in an AC circuit.

The diagram below shows an inductor or transformer.



The current on the input side (primary) generates a magnetic field in the iron ring. The magnetic field in the ring generates a current on the output side (secondary).

In this particular transformer, the coil wraps around the output side more times than the input. This means that each time the current goes through the coil, the magnetic field adds to the electromotive force (voltage). This means the voltage will increase in proportion to the increased number of coils on the output side. However, the magnetic field on the output side will produce less current with each turn, which means the current will decrease in the same proportion:

$$\frac{\# \text{turns}_{in}}{\# \text{turns}_{out}} = \frac{V_{in}}{V_{out}} = \frac{I_{out}}{I_{in}}$$

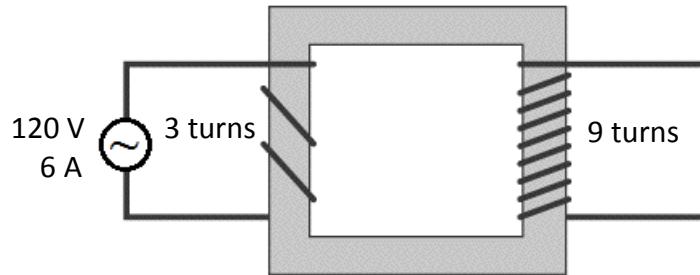
$$P_{in} = P_{out}$$

A transformer like this one, which produces an increase in voltage, is called a step-up transformer; a transformer that produces a decrease in voltage is called a step-down transformer.

Use this space for summary and/or additional notes.

Sample Problem:

If the input voltage to the following transformer is 120 V, and the input current is 6 A, what are the output voltage and current?



The voltage on either side of a transformer is proportional to the number of turns in the coil on that side. In the above transformer, the primary has 3 turns, and the secondary coil has 9 turns. This means the voltage on the right side will be $\frac{9}{3} = 3$ times as much as the voltage on the left, or 360 V. The current will be $\frac{3}{9} = \frac{1}{3}$ as much, or 2 A.

We can also use:

$$\frac{\# \text{turns}_{in}}{\# \text{turns}_{out}} = \frac{V_{in}}{V_{out}}$$

$$\frac{3}{9} = \frac{120 \text{ V}}{V_{out}}$$

$$V_{out} = 360 \text{ V}$$

$$\frac{\# \text{turns}_{in}}{\# \text{turns}_{out}} = \frac{I_{out}}{I_{in}}$$

$$\frac{3}{9} = \frac{I_{out}}{6 \text{ A}}$$

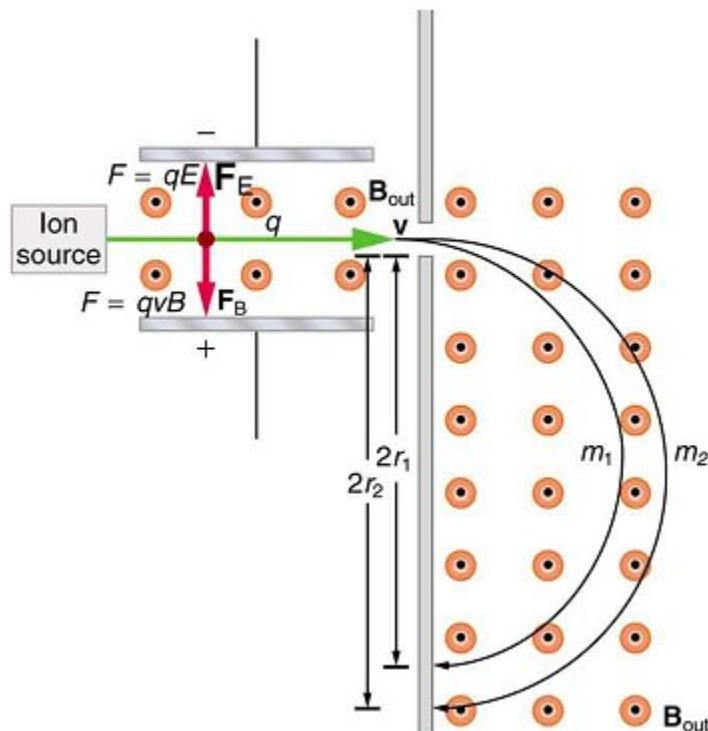
$$I_{out} = 2 \text{ A}$$

Use this space for summary and/or additional notes.

Mass Spectrometer

A mass spectrometer is a device uses the path of a charged particle in a magnetic field to determine its mass.

The particle is first selected for the desired velocity by a combination of externally-applied magnetic and electric fields. Then the particle enters a chamber with only a magnetic field. (In the example below, the magnetic field is directed out of the page.)



The magnetic field applies a force on the particle perpendicular to its path (downward in this example). As the particle's direction changes, the direction of the applied force changes with it, causing the particle to move in a circular path until it hits the detector.

Use this space for summary and/or additional notes.

Introduction: Mechanical Waves & Sound

Unit: Mechanical Waves & Sound

Topics covered in this chapter:

Waves.....	381
Reflection and Superposition.....	392
Sound & Music	399
Sound Levels	413
The Doppler Effect	416
Exceeding the Speed of Sound.....	420

This chapter discusses properties of waves that travel through a medium (mechanical waves).

- *Waves* gives general information about waves, including vocabulary and equations. *Reflection and Superposition* describes what happens when two waves share space within a medium.
- *Sound & Music* describes the properties and equations of waves that relate to music and musical instruments.
- *The Doppler Effect* describes the effects of motion of the source or receiver (listener) on the perception of sound.

Textbook:

- *Physics: Principles and Problems* Ch. 14: Waves and Energy Transfer (pp. 286–305)
- *Physics: Principles and Problems* Ch. 15: Sound (pp. 306–327)

Standards addressed in this chapter:

Next Generation Science Standards (NGSS):

- HS-PS4-1.** Use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling in various media.

Use this space for summary and/or additional notes:

Massachusetts Curriculum Frameworks (2006):

- 4.1** Describe the measurable properties of waves (velocity, frequency, wavelength, amplitude, period) and explain the relationships among them. Recognize examples of simple harmonic motion.
- 4.2** Distinguish between mechanical and electromagnetic waves.
- 4.3** Distinguish between the two types of mechanical waves, transverse and longitudinal.
- 4.4** Describe qualitatively the basic principles of reflection and refraction of waves.
- 4.5** Recognize that mechanical waves generally move faster through a solid than through a liquid and faster through a liquid than through a gas.
- 4.6** Describe the apparent change in frequency of waves due to the motion of a source or a receiver (the Doppler effect).

Topics from this chapter assessed on the SAT Physics Subject Test:

- **General Wave Properties**, such as wave speed, frequency, wavelength, superposition, standing wave diffraction, and the Doppler effect.
 - 1. Wave Motion
 - 2. Transverse Waves and Longitudinal Waves
 - 3. Superposition
 - 4. Standing Waves and Resonance
 - 5. The Doppler Effect

Skills learned & applied in this chapter:

- Visualizing wave motion.

Use this space for summary and/or additional notes.

Waves

Unit: Mechanical Waves & Sound

NGSS Standards: HS-PS4-1

MA Curriculum Frameworks (2006): 4.1, 4.3

Knowledge/Understanding:

- what waves are & how they move/propagate
- transverse vs. longitudinal waves
- mechanical vs. electromagnetic waves

Skills:

- calculate wavelength, frequency, period, and velocity of a wave

Language Objectives:

- Understand and correctly use the terms “wave,” “medium,” “propagation,” “mechanical wave,” “electromagnetic wave,” “transverse,” “longitudinal,” “torsional,” “crest,” “trough,” “amplitude,” “frequency,” “wavelength,” and “velocity.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving the wavelength, frequency and velocity of a wave.

Labs, Activities & Demonstrations:

- Show & tell: transverse waves in a string tied at one end, longitudinal waves in a spring, torsional waves.
- Buzzer in a vacuum.
- Tacoma Narrows Bridge collapse movie.
- Japan tsunami TV footage.

Use this space for summary and/or additional notes.

Notes:

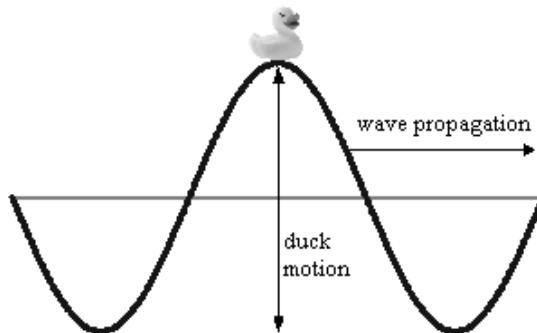
wave: a disturbance that travels from one place to another.

medium: a substance that a wave travels through.

propagation: the process of a wave traveling through space.

mechanical wave: a wave that propagates through a medium via contact between particles of the medium. Some examples of mechanical waves include ocean waves and sound waves.

1. The energy of the wave is transmitted via the particles of the medium as the wave passes through it.
2. The wave travels through the medium. The particles of the medium are moved by the wave passing through, and then return to their original position. (The duck sitting on top of the wave below is an example.)



3. The denser the medium, the more frequently the particles come in contact, and therefore the faster the wave propagates. For example,

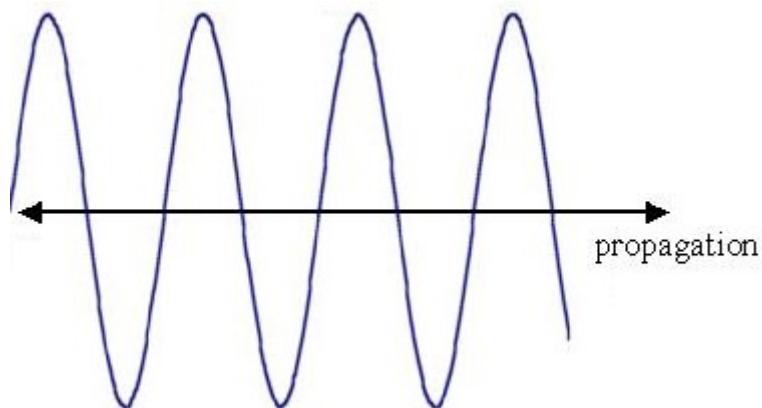
medium	density (kg/m^3)	velocity of sound waves
air (20°C and 1 atm)	1.2	$343 \frac{\text{m}}{\text{s}}$ ($768 \frac{\text{mi}}{\text{hr}}$)
water (20°C)	998	$1\,481 \frac{\text{m}}{\text{s}}$ ($3\,317 \frac{\text{mi}}{\text{hr}}$)
steel (longitudinal wave)	7 800	$6\,000 \frac{\text{m}}{\text{s}}$ ($13\,000 \frac{\text{mi}}{\text{hr}}$)

electromagnetic wave: a wave of electricity and magnetism interacting with each other. Electromagnetic waves can propagate through empty space.

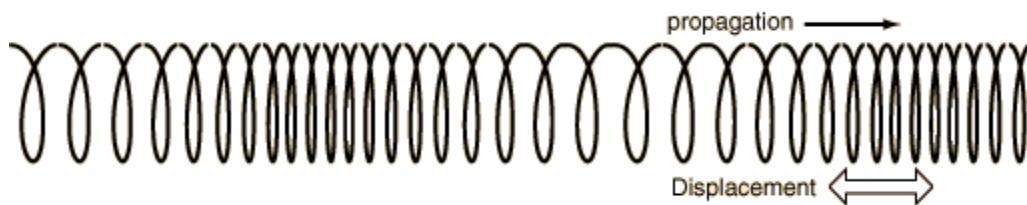
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Types of Waves

transverse wave: moves its medium up & down (or back & forth) as it travels through. Examples: light, ocean waves



longitudinal wave (or compressional wave): compresses and decompresses the medium as it travels through. Example: sound.



Use this space for summary and/or additional notes.

torsional wave: a type of transverse wave that propagates by twisting about its direction of propagation.



The most famous example of the destructive power of a torsional wave was the Tacoma Narrows Bridge, which collapsed on November 7, 1940. On that day, strong winds caused the bridge to vibrate torsionally. At first, the edges of the bridge swayed about eighteen inches. (This behavior had been observed previously, resulting in the bridge acquiring the nickname "Galloping Gertie".) However, after a support cable snapped, the vibration increased significantly, with the edges of the bridge being displaced up to 28 feet! Eventually, the bridge started twisting in two halves, one half twisting clockwise and the other half twisting counterclockwise, and then back again. This opposing torsional motion eventually caused the bridge to twist apart and collapse.

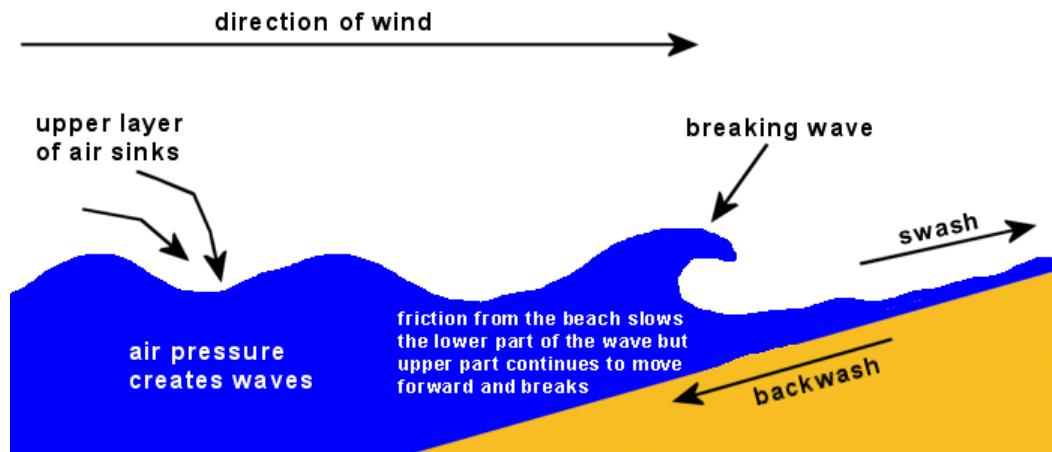


The bridge's collapse was captured on film. Video clips of the bridge twisting and collapsing are available on YouTube. There is a detailed analysis of the bridge's collapse at <http://www.vibrationdata.com/Tacoma.htm>

Use this space for summary and/or additional notes.

surface wave: a transverse wave that travels at the interface between two media.

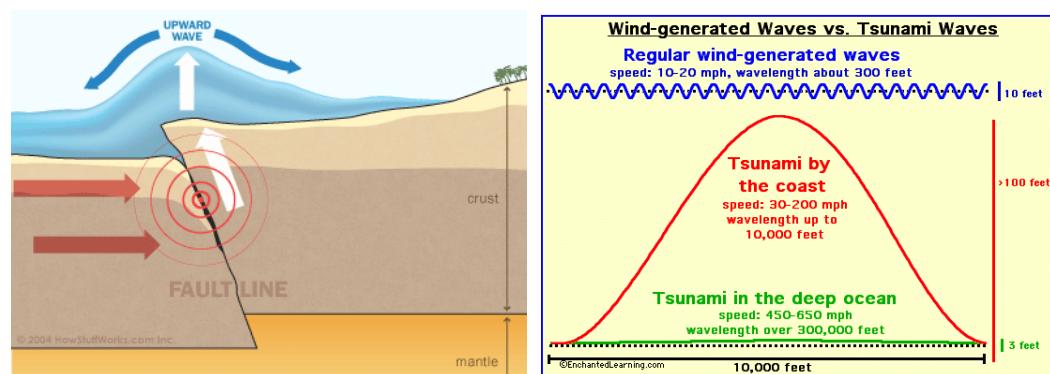
Ocean waves are an example of surface waves, because they travel at the interface between the air and the water. Surface waves on the ocean are caused by wind disturbing the surface of the water. Until the wave gets to the shore, surface waves have no effect on water molecules far below the surface.



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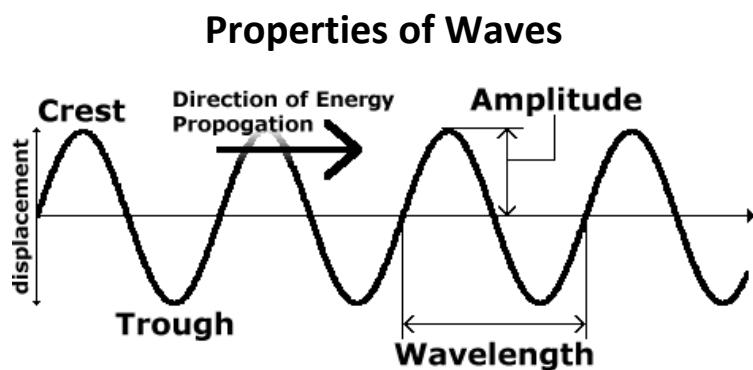
Tsunamis

The reason tsunamis are much more dangerous than regular ocean waves is because tsunamis are created by earthquakes on the ocean floor. The tsunami wave propagates through the entire depth of the water, which means tsunamis carry many times more energy than surface waves.



This is why a 6–12 foot high surface wave breaks harmlessly on the beach; however, a tsunami that extends 6–12 feet above the surface of the water includes a significant amount of energy below the surface and can destroy an entire city.

Use this space for summary and/or additional notes.



crest: the point of maximum positive displacement of a transverse wave. (The highest point.)

trough: the point of maximum negative displacement of a transverse wave. (The lowest point.)

amplitude: the distance of maximum displacement of a point in the medium as the wave passes through it.

wavelength: the length of the wave, measured from a specific point in the wave to the same point in the next wave. Symbol = λ (lambda); unit = distance (m, cm, nm, etc.)

frequency: the number of waves that travel past a point in a given time.
Symbol = f ; unit = $1/\text{time}$ ($\text{Hz} = 1/\text{s}$)

Note that while high school physics courses generally use the variable f for frequency, college courses usually use ν (the Greek letter "nu", which is different from the Roman letter "v").

period or time period: the amount of time between two adjacent waves.
Symbol = T ; unit = time (usually seconds)

$$T = 1/f$$

Use this space for summary and/or additional notes.

velocity: the velocity of a wave depends on its frequency (f) and its wavelength (λ):

$$v = \lambda f$$

The velocity of electromagnetic waves (such as light, radio waves, microwaves, X-rays, etc.) is called the speed of light, which is $3.00 \times 10^8 \frac{\text{m}}{\text{s}}$ in a vacuum. The speed of light is slower in a medium that has an index of refraction greater than 1. (We will discuss index of refraction in more detail in the light and optics topic.)

The velocity of a wave traveling through a string under tension (such as a piece of string, a rubber band, a violin/guitar string, etc.) depends on the tension and the ratio of the mass of the string to its length:

$$v_{\text{string}} = \sqrt{\frac{F_T L}{m}}$$

where F_T is the tension on the string, L is the length, and m is the mass.

Sample Problem:

Q: The radio station WZLX broadcasts waves with a frequency of 100.7 MHz. If the waves travel at the speed of light, what is the wavelength?

A: $f = 100.7 \text{ MHz} = 100,700,000 \text{ Hz} = 1.007 \times 10^8 \text{ Hz}$

$$v = c = 3.00 \times 10^8 \frac{\text{m}}{\text{s}}$$

$$v = \lambda f$$

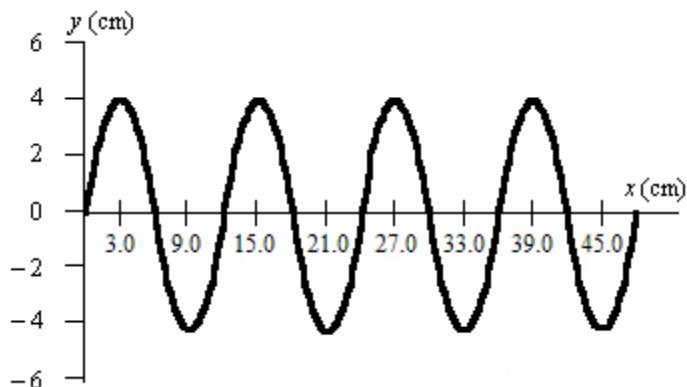
$$3.00 \times 10^8 = \lambda (1.007 \times 10^8)$$

$$\lambda = \frac{3.00 \times 10^8}{1.007 \times 10^8} = 2.98 \text{ m}$$

Use this space for summary and/or additional notes.

Homework Problems

1. Consider the following wave:



- a. What is the amplitude of this wave?
- b. What is its wavelength?
- c. If the velocity of this wave is $30 \frac{\text{m}}{\text{s}}$, what is its period?
2. What is the speed of wave with a wavelength of 0.25 m and a frequency of 5.5 Hz?

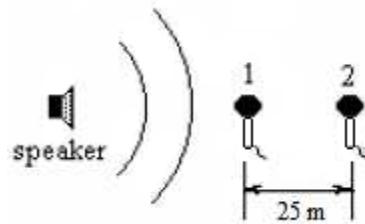
Answer: $1.375 \frac{\text{m}}{\text{s}}$

Use this space for summary and/or additional notes.

3. A sound wave traveling in water at 10°C has a wavelength of 0.65 m. What is the frequency of the wave.
(Note: you will need to look up the speed of sound in water at 10°C in Table P of your Physics Reference Tables, on page 511.)

Answer: 2226Hz

4. Two microphones are placed in front of a speaker as shown in the following diagram:

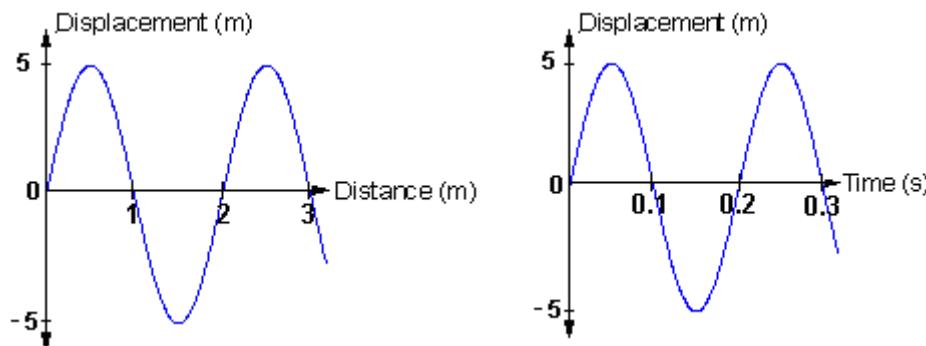


If the air temperature is 30°C , what is the time delay between the two microphones?

Answer: 0.0716 s

Use this space for summary and/or additional notes.

5. The following are two graphs of the same wave. The first graph shows the displacement vs. distance, and the second shows displacement vs. time.



- a. What is the wavelength of this wave?
- b. What is its amplitude?
- c. What is its frequency?
- d. What is its velocity?

Use this space for summary and/or additional notes.

Reflection and Superposition

Unit: Mechanical Waves & Sound

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 4.1, 4.3

Knowledge/Understanding Goals:

- what happens when a wave reflects (“bounces”) off an object or surface
- what happens when two or more waves occupy the same space

Language Objectives:

- Understand and correctly use the terms “reflection,” “superposition,” “constructive interference,” and “destructive interference.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

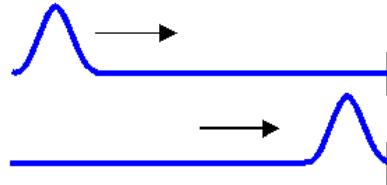
Labs, Activities & Demonstrations:

- waves on a string or spring anchored at one end
- large Slinky with longitudinal and transverse waves passing each other

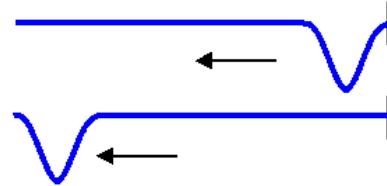
Notes:

Reflection of Waves

reflection: when a wave hits a fixed (stationary) point and “bounces” back.

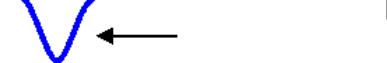


Notice that when the end of the rope is fixed, the reflected wave is inverted. (If the end of the rope were free, the wave would not invert.)



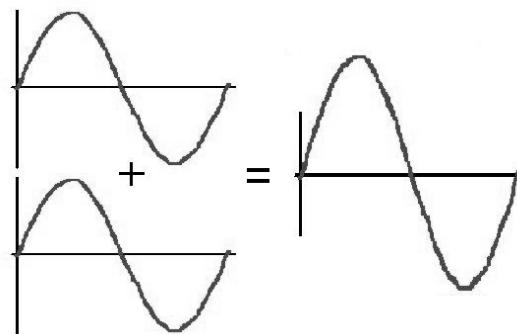
Superposition of Waves

When waves are superimposed (occupy the same space), their amplitudes add:



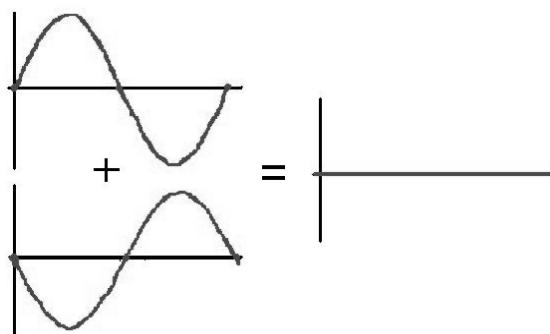
constructive interference: when waves add in a way that the amplitude of the resulting wave is larger than the amplitudes of the component waves.

Use this space for summary and/or additional notes.



Because the wavelengths are the same and the maximum, minimum, and zero points all coincide (line up), the two component waves are said to be “in phase” with each other.

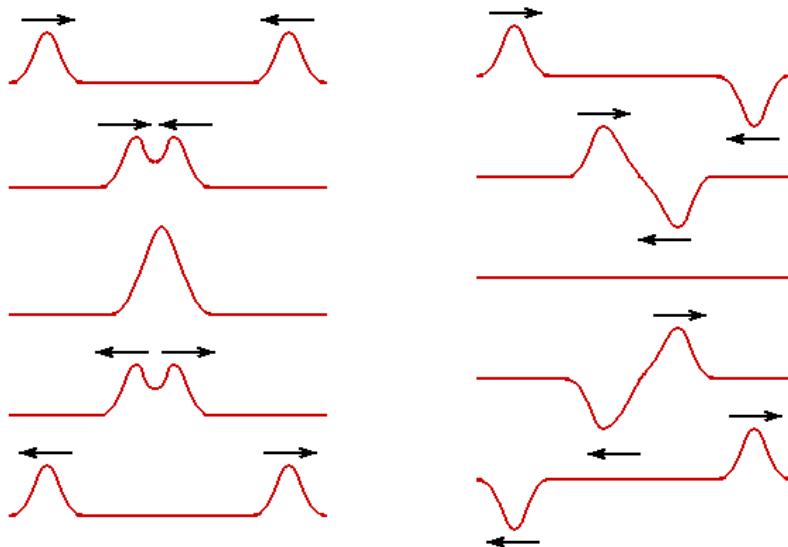
destructive interference: when waves add in a way that the amplitude of the resulting wave is smaller than the amplitudes of the component waves.
(Sometimes we say that the waves “cancel” each other.)



Because the wavelengths are the same but the maximum, minimum, and zero points do not coincide, the waves are said to be “out of phase” with each other.

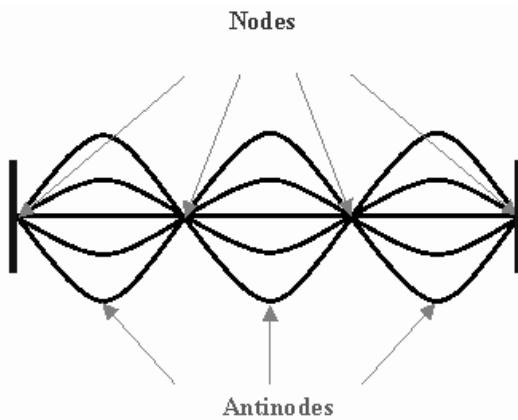
Use this space for summary and/or additional notes.

Note that waves can travel in two opposing directions at the same time. When this happens, the waves pass through each other, exhibiting constructive and/or destructive interference as they pass:



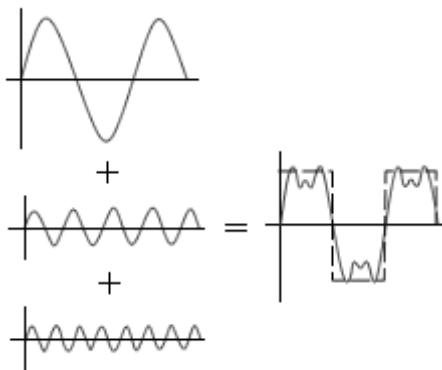
Standing Waves

standing wave: when the wavelength is an exact fraction of the length of a medium that is vibrating, the wave reflects back and the reflected wave interferes constructively with itself. This causes the wave to appear stationary. Points along the wave that are not moving are called "nodes". Points of maximum displacement are called "antinodes".



Use this space for summary and/or additional notes.

When we add waves with different wavelengths and amplitudes, the result can be complex:

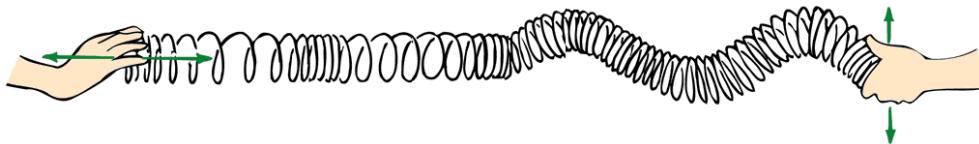


This is how radio waves encode a signal on top of a “carrier” wave. Your radio’s antenna receives (“picks up”) radio waves within a certain range of frequencies. Imagine that the bottom wave (the one with the shortest wavelength and highest frequency) is the “carrier” wave. If you tune your radio to its frequency, the radio will filter out other waves that don’t include the carrier frequency. Then your radio subtracts the carrier wave, and everything that is left is sent to the speakers.

Use this space for summary and/or additional notes.

Homework Problem

1. A Slinky is held at both ends. The person on the left creates a longitudinal wave, while at same time the person on the right creates a transverse wave with the same frequency. Both people stop moving their ends of the Slinky just as the waves are about to meet.

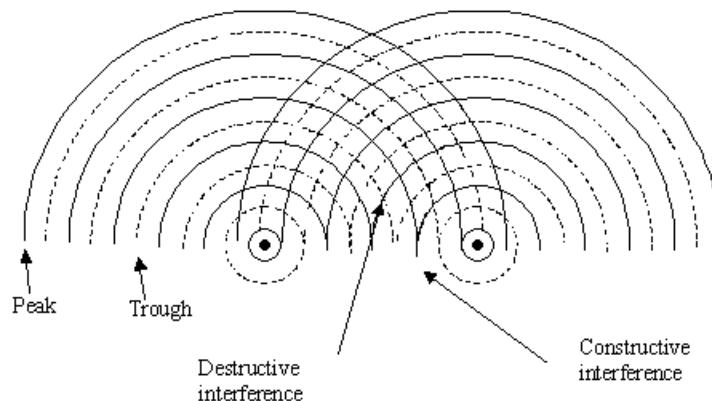


- a. Draw a picture of what the Slinky will look like when the waves completely overlap.
- b. Draw a picture of what the Slinky will look like just after the waves no longer overlap.

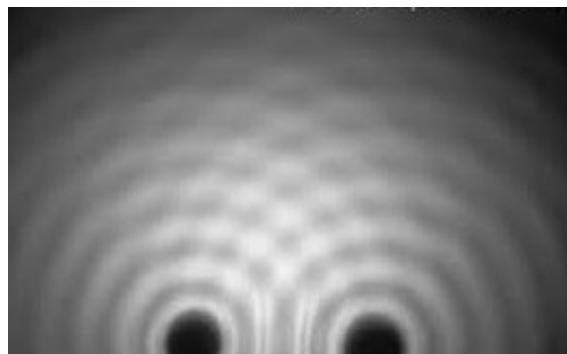
Use this space for summary and/or additional notes.

Interference Patterns

When two progressive waves propagate into each other's space, the waves produce interference patterns. This diagram shows how interference patterns form:



The resulting interference pattern looks like the following picture:



In this picture, the bright regions are wave peaks, and the dark regions are troughs. The brightest intersections are regions where the peaks interfere constructively, and the darkest intersections are regions where the troughs interfere constructively.

Use this space for summary and/or additional notes.

The following picture shows an interference pattern created by ocean waves that have been reflected off two points on the shore. (The island in the background is Jost Van Dyke, in the British Virgin Islands.) The wave at the left side of the picture is traveling toward the right, and the wave at the bottom right of the picture (which has just reflected off a point on the shore) is traveling toward the top of the picture. The interference pattern in the bottom center is highlighted by reflected light from the setting sun.



Use this space for summary and/or additional notes.

Sound & Music

Unit: Mechanical Waves & Sound

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding Goals:

- how musical notes are produced and perceived

Skills:

- calculate the frequency of the pitch produced by a string or pipe

Language Objectives:

- Understand and correctly use the terms “resonance,” “frequency,” and “harmonic series.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to the frequencies and pitches (notes) produced by musical instruments.

Labs, Activities & Demonstrations:

- Show & tell: violin, penny whistle, harmonica, boomwhackers.
- Helmholtz resonators—bottles of different sizes/air volumes, slapping your cheek with your mouth open.
- Frequency generator & speaker.
- Rubens tube (sonic flame tube).
- Measure the speed of sound in air using a resonance tube.

Notes:

Sound waves are caused by vibrations that create longitudinal (compressional) waves in the medium they travel through (such as air).

Use this space for summary and/or additional notes.

pitch: how “high” or “low” a musical note is. The pitch is determined by the frequency of the sound wave.

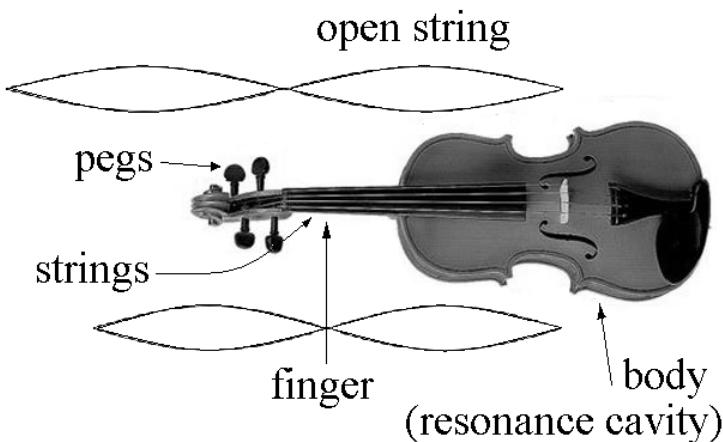


resonance: when the wavelength of a half-wave (or an integer number of half-waves) coincides with one of the dimensions of an object. This creates standing waves that reinforce and amplify each other. The body of a musical instrument is an example of an object that is designed to use resonance to amplify the sounds that the instrument produces.

Use this space for summary and/or additional notes.

String Instruments

A string instrument (such as a violin or guitar) typically has four or more strings. The lower strings (strings that sound with lower pitches) are thicker, and higher strings are thinner. Pegs are used to tune the instrument by increasing (tightening) or decreasing (loosening) the tension on each string.



The vibration of the string creates a half-wave, i.e., $\lambda = 2L$. The musician changes the half-wavelength by using a finger to shorten the length of the string that is able to vibrate. (A shorter wavelength produces a higher frequency = higher pitch.)

The velocity of the wave produced on a string (such as a violin string) is given by the equation:

$$v_{string} = \sqrt{\frac{F_T L}{m}}$$

where:

f = frequency (Hz)

F_T = tension (N)

m = mass of string (kg)

L = length of string (m) = $\frac{\lambda}{2}$

The frequency (pitch) is therefore:

$$f = \frac{v}{\lambda} = \frac{v}{2L} = \frac{1}{2L} \sqrt{\frac{F_T L}{m}} = \sqrt{\frac{F_T}{4mL}}$$

Use this space for summary and/or additional notes.

Pipes and Wind Instruments

A pipe (in the musical instrument sense) is a tube filled with air. Something in the design of the mouthpiece causes the air inside the instrument to vibrate. When air is blown through the instrument, the air molecules compress and spread out at regular intervals that correspond with the length of the instrument, which determines the wavelength.

Most wind instruments use one of three ways of causing the air to vibrate:

Brass Instruments

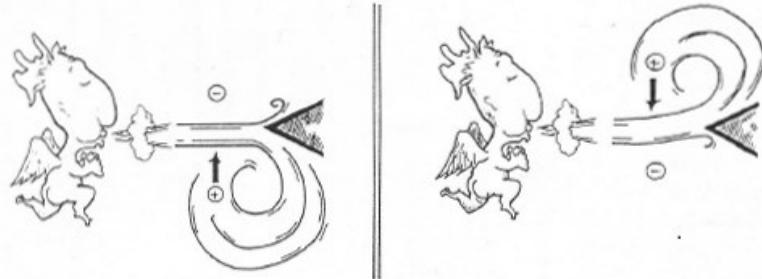
With brass instruments like trumpets, trombones, French horns, etc., the player presses his/her lips tightly against the mouthpiece, and the player's lips vibrate at the appropriate frequency.

Reed Instruments

With reed instruments, air is blown past a reed (a semi-stiff object) that vibrates back and forth. Clarinets and saxophones use a single reed made from a piece of cane (a semi-stiff plant similar to bamboo). Oboes and bassoons ("double-reed instruments") use two pieces of cane that vibrate against each other. Harmonicas and accordions use reeds made from a thin piece of metal.

Fipples

Instruments with fipples include recorders, whistles and flutes. A fipple is a sharp edge that air is blown past. The separation of the air going past the fipple causes a pressure difference on one side vs. the other. The pressure builds more on one side, which forces air past the sharp edge. Then the pressure builds on the other side and the air switches back:



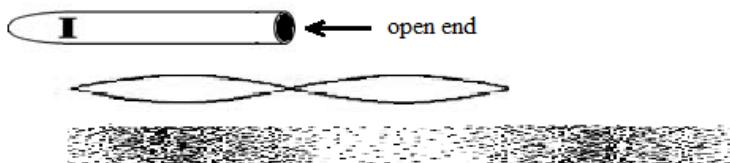
The frequency of this back-and-forth motion is what determines the pitch.

Use this space for summary and/or additional notes.

Open vs. Closed-Pipe Instruments

An open pipe has an opening at each end. A closed pipe has an opening at one end, and is closed at the other.

Examples of open pipes include uncapped organ pipes, whistles, recorders and flutes;



Notice that the two openings determine where the nodes are—these are the regions where the air pressure must be equal to atmospheric pressure (*i.e.*, the air is neither compressed nor expanded). Notice also that as with strings, the wavelength of the sound produced is twice the length of the pipe, *i.e.*, $\lambda = 2L$.

If the pipe is open to the atmosphere at only one end, such as a clarinet or brass instrument, there is only one node, at the mouthpiece. The opening, where the person is blowing into the instrument, is an antinode—a region of high pressure. This means that the body of the instrument is $\frac{1}{4}$ of a wave instead of $\frac{1}{2}$, *i.e.*, $\lambda = 4L$.



This is why a closed-pipe instrument (*e.g.*, a clarinet) sounds an octave lower than an open-pipe instrument of similar length (*e.g.*, a flute).

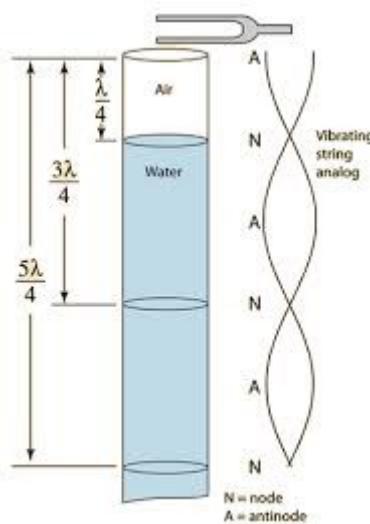
Use this space for summary and/or additional notes.

The principle of a closed-pipe instrument can be used in a lab experiment to determine the frequency of a tuning fork (or the speed of sound) using a resonance tube—an open tube filled with water to a specific depth.

A tuning fork generates sound waves of a precise frequency at the top of the tube. Because this is a closed pipe, the source (just above the tube) is an antinode (maximum amplitude).

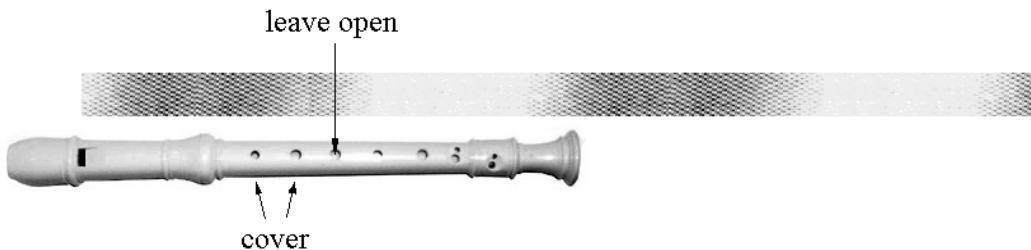
When the height of air above the water is exactly $\frac{1}{4}$ of a wavelength ($\frac{\lambda}{4}$), the waves that are reflected back have maximum constructive interference with the source wave, which causes the sound to be significantly amplified. This phenomenon is called resonance.

Resonance will occur at any integer plus $\frac{1}{4}$ of a wave—*i.e.*, any distance that results in an antinode exactly at the top of the tube ($\frac{\lambda}{4}, \frac{5\lambda}{4}, \frac{9\lambda}{4}$, etc.)



Use this space for summary and/or additional notes.

For an instrument with holes, like a flute or recorder, the first open hole creates a node at that point, which determines the half-wavelength (or quarter-wavelength):



The speed of sound in air is v_s ($343 \frac{\text{m}}{\text{s}}$ at 20°C and 1 atm), which means the frequency of the note (from the formula $v_s = \lambda f$) will be:

$$f = \frac{v_s}{2L} \text{ for an open-pipe instrument (flute, recorder, whistle), and}$$

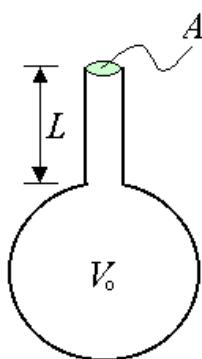
$$f = \frac{v_s}{4L} \text{ for a closed-pipe instrument (clarinet, brass instrument).}$$

Note that the speed of sound in air increases as the temperature increases. This means that as the air gets colder, the frequency gets lower, and as the air gets warmer, the frequency gets higher. This is why wind instruments go flat at colder temperatures and sharp at warmer temperatures. When this happens, it's not the instrument that is going out of tune, but the speed of sound!

Use this space for summary and/or additional notes.

Helmholtz Resonators (Bottles)

The resonant frequency of a bottle or similar container (called a Helmholtz resonator, named after the German physicist Hermann von Helmholtz) is more complicated to calculate, because it depends on the resonance frequencies of the air in the large cavity, the air in the neck of the bottle, and the cross-sectional area of the opening.



The formula works out to be:

$$f = \frac{v_s}{2\pi} \sqrt{\frac{A}{V_o L}}$$

where:

f = resonant frequency

v_s = speed of sound in air ($343 \frac{\text{m}}{\text{s}}$ at 20°C and 1 atm)

A = cross-sectional area of the neck of the bottle (m^2)

V_o = volume of the main cavity of the bottle (m^3)

L = length of the neck of the bottle (m)

(Note that it may be more convenient to use measurements in cm, cm^2 , and cm^3 , and use $v_s = 34,300 \frac{\text{cm}}{\text{s}}$.)

You can make your mouth into a Helmholtz resonator by tapping on your cheek with your mouth open. You change the pitch by changing the size of the opening.

Use this space for summary and/or additional notes.

Frequencies of Music Notes

The frequencies that correspond with the pitches of the Western equal temperament scale are:

pitch	frequency (Hz)	pitch	frequency (Hz)
	C 261.6		G 392.0
	D 293.7		A 440.0
	E 329.6		B 493.9
	F 349.2		C 523.2

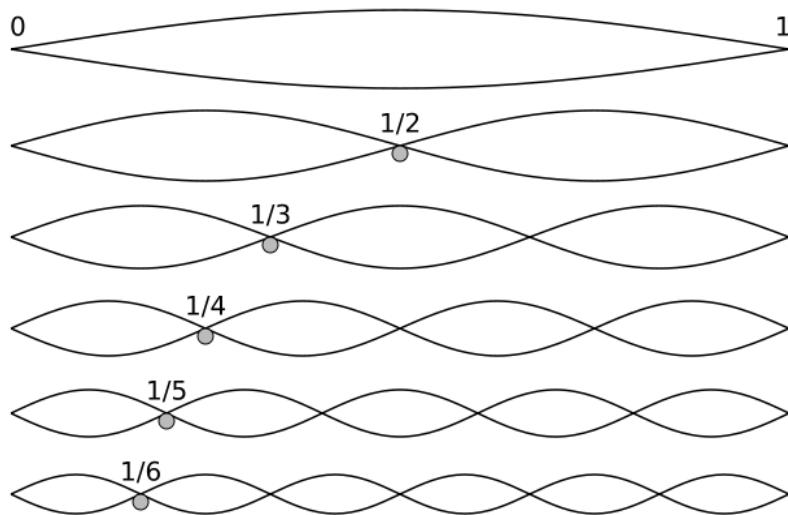
Note that a note that is an octave above another note has exactly twice the frequency of the lower note.

Harmonic Series

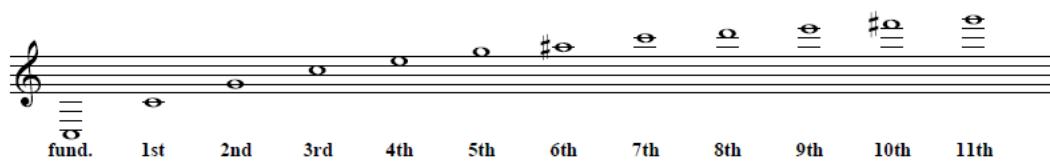
harmonic series: the additional, shorter standing waves that are generated by a vibrating string or column of air that correspond with integer numbers of half-waves. The natural frequency is called the fundamental frequency, and the harmonics above it are numbered—1st harmonic, 2nd harmonic, etc.) Any sound wave that is produced in a resonance chamber (such as a musical instrument) will produce the fundamental frequency plus all of the other waves of the harmonic series. The fundamental is the loudest, and each harmonic gets more quiet as you go up the harmonic series.

Use this space for summary and/or additional notes.

The following diagram shows the fundamental frequency and the first five harmonics produced by a pipe or a vibrating string:



Fraction of String	Wave-length	Harmonic	Frequency	Pitch (relative to fundamental)
1	$2L/1$	0	f_o	Fundamental.
$1/2$	$2L/2$	1 st	$2f_o$	One octave above.
$1/3$	$2L/3$	2 nd	$3f_o$	One octave + a fifth above.
$1/4$	$2L/4$	3 rd	$4f_o$	Two octaves above.
$1/5$	$2L/5$	4 th	$5f_o$	Two octaves + approximately a major third above.
$1/6$	$2L/6$	5 th	$6f_o$	Two octaves + a fifth above.
$1/n$	$2L/n$	(n-1) th	$n f_o$	etc.

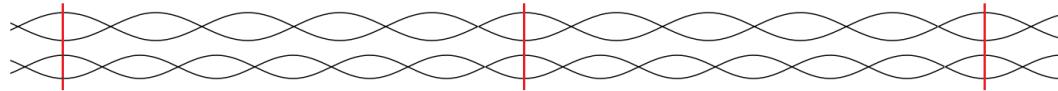


Use this space for summary and/or additional notes.

Beats

When two or more waves are close but not identical in frequency, their amplitudes reinforce each other at regular intervals.

For example, when the following pair of waves travels through the same medium, the amplitudes of the two waves have maximum constructive interference every five half-waves ($2\frac{1}{2}$ full waves) of the top wave *vs.* every six half-waves (3 full waves) of the bottom wave.



If this happened with sound waves, you would hear a pulse or “beat” every time the two maxima coïncided.

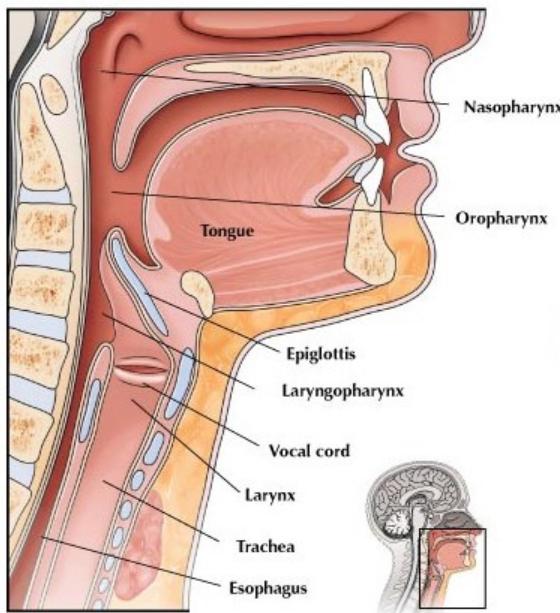
The closer the two wavelengths (and therefore also the two frequencies) are to each other, the more half-waves it takes before the amplitudes coïncide. This means that as the frequencies get closer, the time between the beats gets longer.

Piano tuners listen for these beats and adjust the tension of the string they are tuning until the time between beats gets longer and longer and finally disappears.

Use this space for summary and/or additional notes.

The Biophysics of Sound

When a person speaks, abdominal muscles force air from the lungs through the larynx.



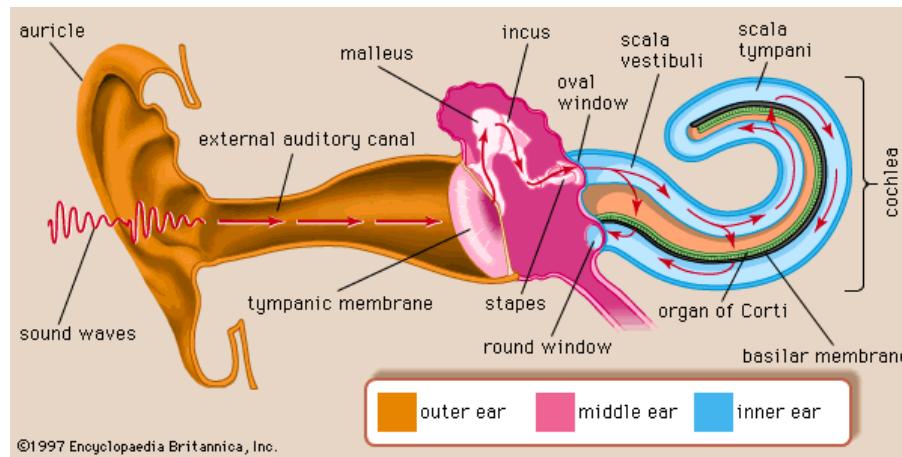
The vocal cord vibrates, and this vibration creates sound waves. Muscles tighten or loosen the vocal cord, which changes the frequency at which it vibrates. Just like with a string instrument, the change in tension changes the pitch.

Tightening the vocal cord increases the tension and produces a higher pitch, and relaxing the vocal cord decreases the tension and produces a lower pitch.

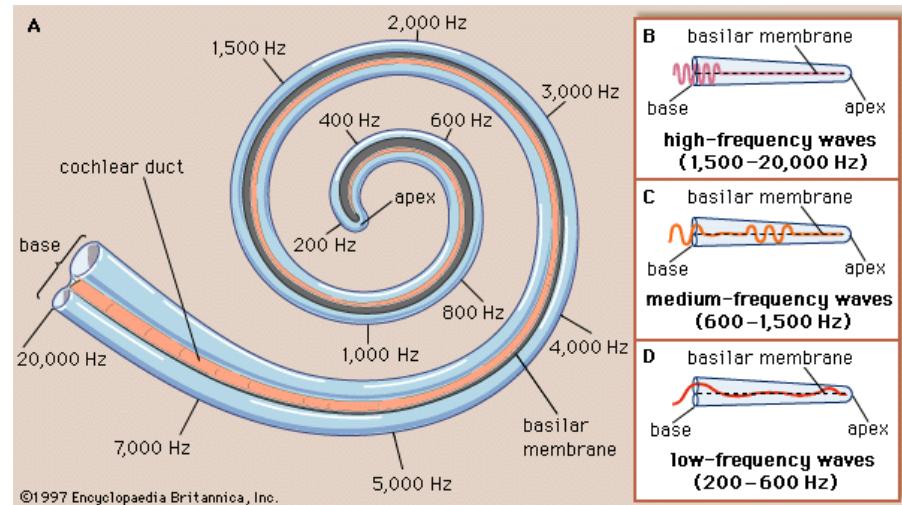
This process happens naturally when you sing. Amateur musicians who sing a lot of high notes can develop laryngitis from tightening their laryngeal muscles too much for too long. Professional musicians need to train themselves to keep their larynx muscles relaxed and use other techniques (such as breath support) to adjust their pitch.

Use this space for summary and/or additional notes.

When the sound reaches the ears, it travels through the auditory canal and causes the tympanic membrane (eardrum) to vibrate. The vibrations of the tympanic membrane cause pressure waves to travel through the middle ear and through the oval window into the cochlea.



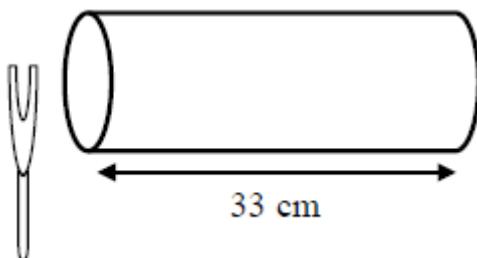
The basilar membrane in the cochlea is a membrane with cilia (small hairs) connected to it, which can detect very small movements of the membrane. As with a resonance tube, the wavelength determines exactly where the sound waves will vibrate the basilar membrane the most strongly, and the brain determines the pitch (frequency) of a sound based on the precise locations excited by these frequencies.



Use this space for summary and/or additional notes.

Homework Problem

A tuning fork is used to establish a standing wave in an open ended pipe filled with air at a temperature of 20°C, where the speed of sound is $343 \frac{\text{m}}{\text{s}}$, as shown below:



The sound wave resonates at the 3rd harmonic frequency of the pipe. The length of the pipe is 33 cm.

1. Sketch the standing wave inside the diagram of the pipe above. (For simplicity, you may sketch a transverse wave to represent the standing wave.)
2. Determine the wavelength of the resonating sound wave.

Answer: 22 cm

3. Determine the frequency of the tuning fork.

Answer: 1559 Hz

4. What is the next higher frequency that will resonate in this pipe?

Answer: 2079 Hz

Use this space for summary and/or additional notes.

Sound Levels

Unit: Mechanical Waves & Sound

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 4.6

Knowledge/Understanding Goals:

- sound levels (decibels)
- Lombard effect

Language Objectives:

- Understand and correctly use the terms “sound level” and “decibel”.
- Understand the use of the word “volume” to mean sound level instead of the space taken up by an object.
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Labs, Activities & Demonstrations:

- VU meter.

Notes:

sound level: the perceived intensity of a sound. Usually called “volume”.

Sound level is usually measured in decibels (dB). One decibel is one tenth of one bel.

Sound level is calculated based on the logarithm of the ratio of the power (energy per unit time) causing a sound vibration to the power that causes some reference sound level.

You will not be asked to calculate decibels from an equation, but you should understand that because the scale is logarithmic, a difference of one bel (10 dB) represents a tenfold increase or decrease in sound level.

Use this space for summary and/or additional notes:

The following table lists the approximate sound levels of various sounds:

sound level (dB)	Description
0	threshold of human hearing at 1 kHz
10	a single leaf falling to the ground
20	background in TV studio
30	quiet bedroom at night
36	whispering
40	quiet library or classroom
42	quiet voice
40–55	typical dishwasher
50–55	normal voice
60	TV from 1 m away
	normal conversation from 1 m away
60–65	raised voice
60–80	passenger car from 10 m away
70	typical vacuum cleaner from 1 m away
75	crowded restaurant at lunchtime
72–78	loud voice
85	hearing damage (long-term exposure)
84–90	shouting
80–90	busy traffic from 10 m away
100–110	rock concert, 1 m from speaker
110	chainsaw from 1 m away
110–140	jet engine from 100 m away
120	threshold of discomfort
	hearing damage (single exposure)
130	threshold of pain
140	jet engine from 50 m away
194	sound waves become shock waves

Use this space for summary and/or additional notes.

In crowds, people unconsciously adjust the sound levels of their speech in order to be heard above the ambient noise. This behavior is called the Lombard effect, named for Étienne Lombard, the French doctor who first described it.

The Lombard coefficient is the ratio of the increase in sound level of the speaker to the increase in sound level of the background noise:

$$L = \frac{\text{increase in speech level (dB)}}{\text{increase in background noise (dB)}}$$

Researchers have observed values of the Lombard coefficient ranging from 0.2 to 1.0, depending on the circumstances.

When you are working in groups in a classroom, as the noise level gets louder, each person has to talk louder to be heard, which in turn makes the noise level louder. The Lombard effect creates a feedback loop in which the sound gets progressively louder and louder until your teachers complain and everyone resets to a quieter volume.

Use this space for summary and/or additional notes.

The Doppler Effect

Unit: Mechanical Waves & Sound

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 4.6

Knowledge/Understanding Goals:

- Understand the Doppler effect

Skills:

- Calculate the apparent shift in wavelength/frequency due to a difference in velocity between the source and receiver.

Language Objectives:

- Understand and correctly use the term “Doppler effect.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving the Doppler effect.

Labs, Activities & Demonstrations:

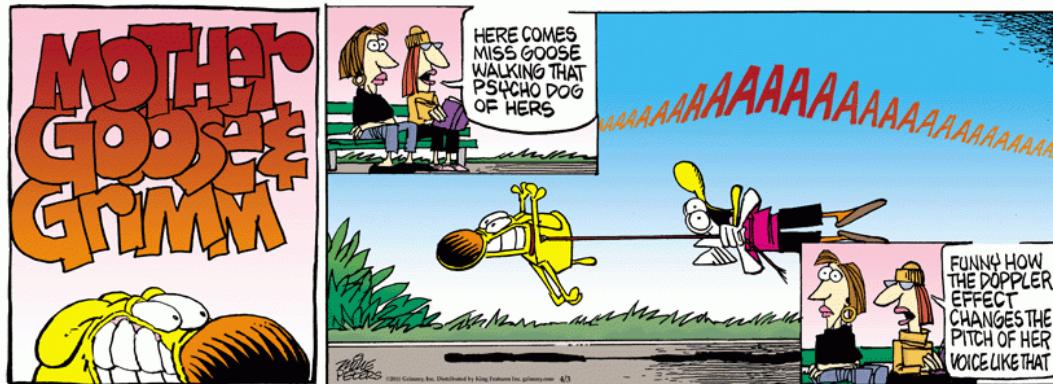
- Buzzer on a string.

Notes:

Doppler effect or Doppler shift: the apparent change in frequency/wavelength of a wave due to a difference in velocity between the source of the wave and the observer. The effect is named for the Austrian physicist Christian Doppler.

Use this space for summary and/or additional notes.

You have probably noticed the Doppler effect when an emergency vehicle with a siren drives by.

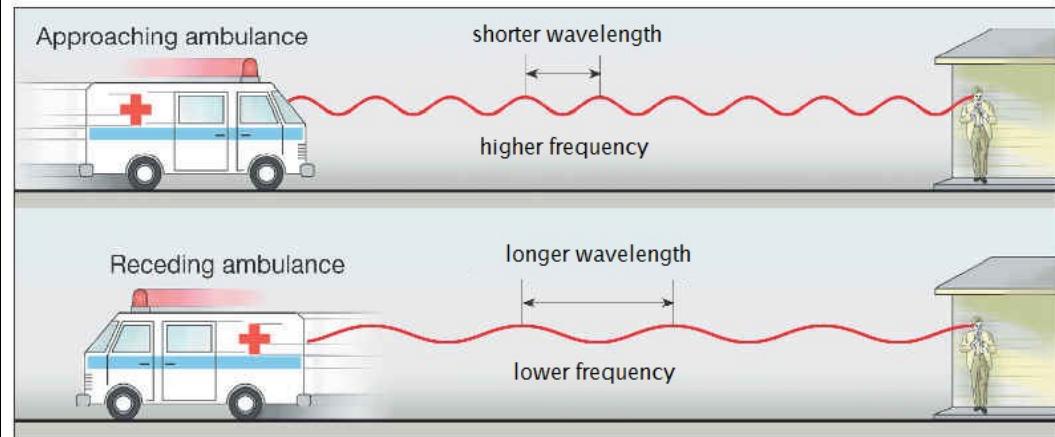


Why the Doppler Shift Happens

The Doppler shift occurs because a wave is created by a series of pulses at regular intervals, and the wave moves at a particular speed.

If the source is approaching, each pulse arrives sooner than it would have if the source had been stationary. Because frequency is the number of pulses that arrive in one second, the moving source results in an increase in the frequency observed by the receiver.

Similarly, if the source is moving away from the observer, each pulse arrives later, and the observed frequency is lower.



Use this space for summary and/or additional notes.

Calculating the Doppler Shift

The change in frequency is given by the equation:

$$f = f_o \left(\frac{v_w \pm v_r}{v_w \pm v_s} \right)$$

where:

f = observed frequency

f_o = frequency of the original wave

v_w = velocity of the wave

v_r = velocity of the receiver (you)

v_s = velocity of the source

The rule for adding or subtracting velocities is:

- The receiver's (your) velocity is in the numerator. If you are moving toward the sound, this makes the pulses arrive sooner, which makes the frequency higher. So if you are moving **toward** the sound, **add** your velocity. If you are moving **away** from the sound, **subtract** your velocity.
- The source's velocity is in the denominator. If the source is moving toward you, this makes the frequency higher, which means the denominator needs to be smaller. This means that if the source is moving **toward** you, **subtract** its velocity. If the source is moving **away** from you, **add** its velocity.

Don't try to memorize a rule for this—you will just confuse yourself. It's safer to reason through the equation. If something that's moving would make the frequency higher, that means you need to make the numerator larger or the denominator smaller. If it would make the frequency lower, that means you need to make the numerator smaller or the denominator larger.

Use this space for summary and/or additional notes.

Sample Problem:

Q: The horn on a fire truck sounds at a pitch of 350 Hz. What is the perceived frequency when the fire truck is moving toward you at $20 \frac{m}{s}$? What is the perceived frequency when the fire truck is moving away from you at $20 \frac{m}{s}$? Assume the speed of sound in air is $343 \frac{m}{s}$.

A: The observer is not moving, so $v_r = 0$.

The fire truck is the source, so its velocity appears in the denominator.

When the fire truck is moving toward you, that makes the frequency higher. This means we need to make the denominator smaller, which means we need to **subtract** v_s :

$$f = f_o \left(\frac{v_w}{v_w - v_s} \right)$$
$$f = 350 \left(\frac{343}{343 - 20} \right) = 350 (1.062) = 372 \text{ Hz}$$

When the fire truck is moving away, the frequency will be lower, which mean we need to make the denominator larger. This means we need to **add** v_s :

$$f = f_o \left(\frac{v_w}{v_w + v_s} \right)$$
$$f = 350 \left(\frac{343}{343 + 20} \right) = 350 (0.9449) = 331 \text{ Hz}$$

Note that the pitch shift in each direction corresponds with about one half-step on the musical scale.

Use this space for summary and/or additional notes.

Exceeding the Speed of Sound

Unit: Mechanical Waves & Sound

NGSS Standards: N/A

MA Curriculum Frameworks (2006): 4.6

Knowledge/Understanding Goals:

- Understand what happens when an object moves faster than sound.

Skills:

Calculate mach number.

Language Objectives:

- Understand and correctly use the term “mach number.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems involving mach numbers.

Labs, Activities & Demonstrations:

- Crack a bullwhip.

Notes:

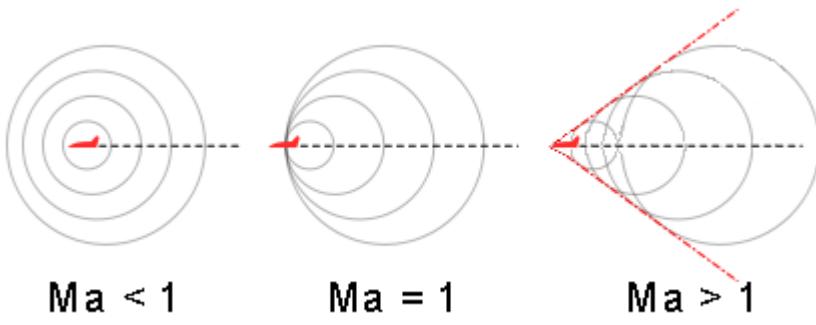
The speed of an object relative to the speed of sound in the same medium is called the Mach number (abbreviation Ma), named after the Austrian physicist Ernst Mach.

$$Ma = \frac{v_{object}}{v_{sound}}$$

Thus “Mach 1” or a speed of $Ma = 1$ is the speed of sound. An object such as an airplane that is moving at 1.5 times the speed of sound would be traveling at “Mach 1.5” or $Ma = 1.5$.

Use this space for summary and/or additional notes.

When an object such as an airplane is traveling slower than the speed of sound, the jet engine noise is Doppler shifted just like any other sound wave.



When the airplane's velocity reaches the speed of sound ($Ma = 1$), the leading edge of all of the sound waves produced by the plane coincides. These waves amplify each other, producing a loud shock wave called a "sonic boom".

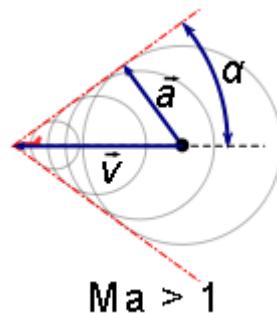


The shock wave temporarily increases the temperature of the air affected by it. If the air is humid enough, when it cools by returning to its normal pressure, the water vapor condenses forming a cloud, called a vapor cone.

The "crack" of a bullwhip is a small sonic boom—when a bullwhip is snapped sharply, the loop at the end of the bullwhip travels faster than sound.

Use this space for summary and/or additional notes.

When an airplane is traveling faster than sound, the sound waves coincide at points behind the airplane at a specific angle, α :



The angle α is given by the equation:

$$\sin(\alpha) = \frac{1}{Ma}$$

i.e., the faster the airplane is traveling, the smaller the angle α , and the narrower the cone.

Use this space for summary and/or additional notes.

Introduction: Thermal Physics (Heat)

Unit: Thermal Physics (Heat)

Topics covered in this chapter:

Heat & Temperature	426
Heat Transfer	430
Energy Conversion	435
Specific Heat Capacity & Calorimetry	438
Phase Changes & Heating Curves	447
Thermal Expansion.....	455

This chapter is about heat as a form of energy and the ways in which heat affects objects, including how it is stored and how it is transferred from one object to another.

- *Heat & Temperature* describes the concept of heat as a form of energy and how heat energy is different from temperature.
- *Heat Transfer, Energy Conversion* and *Efficiency* describe how to calculate the rate of the transfer of heat energy from one object to another.
- *Specific Heat Capacity & Calorimetry* describes different substances' and objects' abilities to store heat energy. *Phase Changes & Heating Curves* addresses the additional calculations that apply when a substance goes through a phase change (such as melting or boiling).
- *Thermal Expansion* describes the calculation of the change in size of an object caused by heating or cooling.

New challenges specific to this chapter include looking up and working with constants that are different for different substances.

Textbook:

- *Physics: Principles and Problems* Ch. 12: Thermal Energy (pp. 240–263)

Use this space for summary and/or additional notes:

Next Generation Science Standards (NGSS):

HS-PS2-6. Communicate scientific and technical information about why the molecular-level structure is important in the functioning of designed materials.

HS-PS3-1. Create a computational model to calculate the change in the energy of one component in a system when the change in energy of the other component(s) and energy flows in and out of the system are known.

HS-PS3-2. Develop and use models to illustrate that energy at the macroscopic scale can be accounted for as either motions of particles or energy stored in fields.

HS-PS3-4. Plan and conduct an investigation to provide evidence that the transfer of thermal energy when two components of different temperature are combined within a closed system results in a more uniform energy distribution among the components in the system (second law of thermodynamics).

Massachusetts Curriculum Frameworks (2006):

3.1 Explain how heat energy is transferred by convection, conduction, and radiation.

3.2 Explain how heat energy will move from a higher temperature to a lower temperature until equilibrium is reached.

3.3 Describe the relationship between average molecular kinetic energy and temperature. Recognize that energy is absorbed when a substance changes from a solid to a liquid to a gas, and that energy is released when a substance changes from a gas to a liquid to a solid. Explain the relationships among evaporation, condensation, cooling, and warming.

3.4 Explain the relationships among temperature changes in a substance, the amount of heat transferred, the amount (mass) of the substance, and the specific heat of the substance.

Use this space for summary and/or additional notes.

Topics from this chapter assessed on the SAT Physics Subject Test:

- **Thermal Properties**, such as temperature, heat transfer, specific and latent heats, and thermal expansion.
- **Laws of Thermodynamics**, such as first and second laws, internal energy, entropy, and heat engine efficiency.
 1. Heat and Temperature
 2. The Kinetic Theory of Gases & the Ideal Gas Law
 3. The Laws of Thermodynamics
 4. Heat Engines

Skills learned & applied in this chapter:

- Working with material-specific constants from a table.
- Working with more than one instance of the same quantity in a problem.
- Combining equations and graphs.

Use this space for summary and/or additional notes.

Heat & Temperature

Unit: Thermal Physics (Heat)

NGSS Standards: HS-PS3-2

MA Curriculum Frameworks (2006): 3.2, 3.3

Knowledge/Understanding Goals:

- the difference between heat and temperature
- thermal equilibrium

Language Objectives:

- Understand and correctly use the terms “heat,” “temperature,” “system,” “surroundings,” “enthalpy,” “entropy,” and “thermometer.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.

Labs, Activities & Demonstrations:

- Heat a small weight and large weight to slightly different temperatures.

Notes:

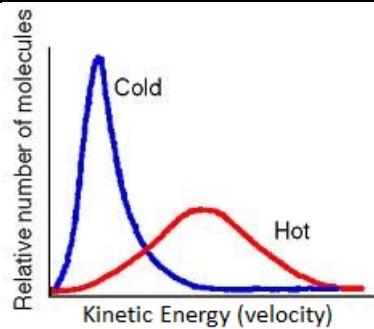
heat: energy that can be transferred by moving atoms or molecules via transfer of momentum.

temperature: a measure of the average kinetic energy of the particles (atoms or molecules) of a system.

thermometer: a device that measures temperature, most often via thermal expansion and contraction of a liquid or solid.

Note that heat is the energy itself, whereas temperature is a measure of the quality of the heat—the average of the kinetic energies of the individual molecules:

Use this space for summary and/or additional notes.



When objects are placed in contact, heat is transferred via the transfer of momentum that occurs when the individual molecules collide. Molecules that have more energy transfer more energy than they receive. Molecules that have less energy receive more energy than they transfer. This means two things:

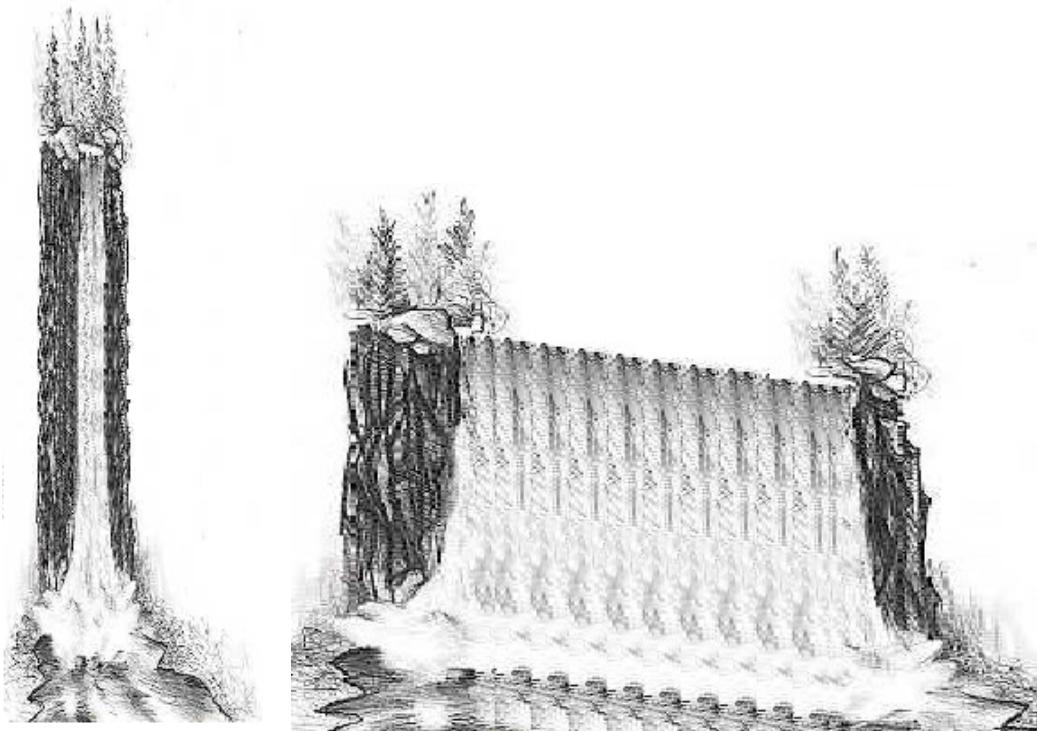
1. Heat always transfers from objects with a higher temperature (more kinetic energy) to objects with a lower temperature (less kinetic energy).
2. If you wait long enough, all of the molecules will have the same temperature (*i.e.*, the same average kinetic energy).

thermal equilibrium: when all of the particles in a system have the same average kinetic energy (temperature). When a system is at thermal equilibrium, no net heat is transferred. (*I.e.*, collisions between particles may still transfer energy, but the average temperature of the particles in the system—what we measure with a thermometer—is not changing.)

Use this space for summary and/or additional notes.

In other words, the temperature of one object relative to another determines which direction the heat will flow, much like the way the elevation of one location relative to another determines which direction water will flow.

However, the total heat (energy) contained in an object depends on the mass as well as the temperature, in the same way that the total energy of the water going over a waterfall depends on the amount of water as well as the height:



Use this space for summary and/or additional notes.

Heat Flow

system: the region being considered in a problem.

surroundings: everything that is outside of the system.

E.g., if a metal block is heated, we would most likely define the system to be the block, and the surroundings to be everything outside of the block.

We generally use the variable Q to represent heat in physics equations.

Heat flow is always represented in relation to the system.

Heat Flow	Sign of Q	System	Surroundings
from surroundings to system	+ (positive)	gains heat (gets warmer)	lose heat (get colder)
from system to surroundings	- (negative)	loses heat (gets colder)	gain heat (get hotter)

A positive value of Q means heat is flowing into the system. Because the heat is transferred from the molecules outside the system to the molecules in the system, the temperature of the system increases, and the temperature of the surroundings decreases.

A negative value of Q means heat is flowing out of the system. Because the heat is transferred from the molecules in the system to the molecules outside the system, the temperature of the system decreases, and the temperature of the surroundings increases.

This can be confusing. Suppose you set a glass of ice water on a table. When you pick up the glass, your hand gets colder because heat is flowing from your hand (which is part of the surroundings) into the system (the glass of ice water). This means the system (the glass of ice water) is gaining heat, and the surroundings (your hand, the table, etc.) are losing heat. The value of Q would be positive in this example.

In simple terms, you need to remember that your hand is part of the surroundings, not part of the system.

Use this space for summary and/or additional notes.

Heat Transfer

Unit: Thermal Physics (Heat)

NGSS Standards: HS-PS2-6, HS-PS3-2

MA Curriculum Frameworks (2006): 3.1

Knowledge/Understanding:

- heat transfer via conduction, radiation & convection

Skills:

- calculate heat transfer using Fourier's Law of Heat Conduction

Language Objectives:

- Understand and correctly use the terms "conduction," "convection," "radiation," "conductor," and "insulator."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to Fourier's Law of Heat Conduction.

Labs, Activities & Demonstrations:

- Heat a piece of sheet metal against a block of wood.
- Radiometer & heat lamp.
- Almond & cheese stick.
- Flammable soap bubbles.

Notes:

Heat transfer is the flow of heat energy from one object to another. Heat transfer usually occurs through three distinct mechanisms: conduction, radiation, and convection.

conduction: transfer of heat through collisions of particles of objects that are in direct contact with each other. Conduction occurs when there is a net transfer of momentum from the molecules of an object with a higher temperature transfer to the molecules of an object with a lower temperature.

Use this space for summary and/or additional notes.

thermal conductivity (k): a measure of the amount of heat that a given length of a substance can conduct in a specific amount of time. Thermal conductivity is measured in units of $\frac{\text{J}}{\text{m}\cdot\text{s}\cdot^\circ\text{C}}$ or $\frac{\text{W}}{\text{m}\cdot^\circ\text{C}}$.

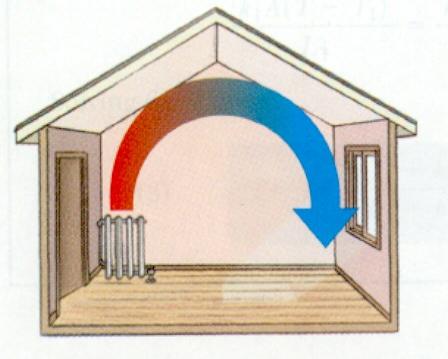
conductor: an object that allows heat to pass through itself easily; an object with high thermal conductivity.

insulator: an object that does not allow heat to pass through itself easily; a poor conductor of heat; an object with low thermal conductivity.

radiation: transfer of heat through space via electromagnetic waves (light, microwaves, etc.)

convection: transfer of heat by motion of particles that have a higher temperature exchanging places with particles that have a lower temperature. Convection usually occurs when air moves around a room.

Natural convection occurs when particles move because of differences in density. In a heated room, because cool air is more dense than warm air, the force of gravity is stronger on the cool air, and it is pulled harder toward the ground than the warm air. The cool air displaces the warm air, pushing it upwards out of the way.



In a room with a radiator, the radiator heats the air, which causes it to expand and be displaced upward by the cool air nearby. When the (less dense) warm air reaches the ceiling, it spreads out, and it continues to cool as it spreads. When the air reaches the opposite wall, it is forced downward toward the floor, across the floor, and back to the radiator.

Forced convection can be achieved by moving heated or cooled air using a fan. Examples of this include ceiling fans and convection ovens. If your radiator does not warm your room enough in winter, you can use a fan to speed up the process of convection.

Use this space for summary and/or additional notes.

Calculating Heat Transfer by Conduction

Heat transfer by conduction can be calculated using Fourier's Law of Heat Conduction:

$$\frac{Q}{t} = -kA \frac{\Delta T}{L}$$

where:

Q = heat transferred (J)

t = time (s)

k = coefficient of thermal conductivity ($\frac{W}{m \cdot ^\circ C}$)

A = cross-sectional area (m^2)

ΔT = temperature difference (K or $^\circ C$)

L = length (m)

The minus sign is because heat transfer is calculated assuming that the system is the heat source. (Heat is moving out of the system, so we use a negative number.)

For insulation (the kind you have in the walls and attic of your home), the effectiveness is measured by the "R value", where:

$$R_i = \frac{L}{k}$$

and therefore:

$$\frac{Q}{t} = -\frac{1}{R_i} A \Delta T$$

The industry uses this definition because most people think larger numbers are better. Therefore a larger "R value" means less heat is transferred (lost) through the insulation, which means the insulation is doing a better job of preventing the heat loss.

Use this space for summary and/or additional notes.

Sample Problems:

Q: A piece of brass is 5.0 mm (0.0050 m) thick and has a cross-sectional area of 0.010 m². If the temperature on one side of the metal is 65°C and the temperature on the other side is 25°C, how much heat will be conducted through the metal in 30. s? The coefficient of thermal conductivity for brass is 120 $\frac{W}{m \cdot ^\circ C}$.

A:
$$\frac{Q}{t} = -kA \frac{\Delta T}{L}$$

$$\frac{Q}{30} = -(120)(0.010) \left(\frac{65 - 25}{0.0050} \right)$$

$$\frac{Q}{30} = -9600$$

$$Q = -288000 \text{ J} = -288 \text{ kJ}$$

(Note that because the quantities of heat that we usually measure are large, values are often given in kilojoules or megajoules instead of joules.)

Q: Suppose your house has 15 cm-thick insulation, with an R value of 16, the temperature inside your house is 21°C and the temperature outside is 0.0°C. How much heat is lost through one square meter of insulation over an 8-hour (28800 s) period?

A: An R value of 16 means $\frac{L}{k} = 16$, which means $\frac{k}{L} = \frac{1}{16}$.

$$\frac{Q}{t} = -kA \frac{\Delta T}{L} = -\frac{k}{L} A \Delta T$$

$$\frac{Q}{28800} = -\frac{1}{16}(1)(21)$$

$$\frac{Q}{28800} = -1.3125$$

$$Q = -37800 \text{ J} = -37.8 \text{ kJ}$$

Use this space for summary and/or additional notes.

Homework Problems

You will need to look up coefficients of thermal conductivity in Table H of your Reference Tables on page 507.

1. The surface of a hot plate is made of 12.0 mm (0.012 m) thick aluminum and has an area of 64 cm^2 (which equals 0.0064 m^2). If the heating coils maintain a temperature of 80°C underneath the surface and the air temperature is 22°C , how much heat can be transferred through the plate in 60. s?

Answer: $-464\,000 \text{ J}$ or -464 kJ

2. A cast iron frying pan is 5.0 mm thick. If it contains boiling water (100°C), how much heat will be transferred into your hand if you place your hand against the bottom for two seconds? (Assume your hand has an area of 0.0040 m^2 , and that body temperature is 37°C .)

Answer: $-8\,064 \text{ J}$ or -8.064 kJ

3. Suppose the attic in your home is insulated with 27 cm of insulation with an R-value of 22, and the total surface area of the roof is 75 m^2 . During a 24-hour period, the temperature outside is -5.0°C , and the temperature inside is 21°C . How much heat is lost through the roof during that 24-hour period? (Note: $24 \text{ h} = 86\,400 \text{ s}$.)

Answer: $-7\,658\,181 \text{ J}$ or $-7\,658 \text{ kJ}$

Use this space for summary and/or additional notes.

Energy Conversion

Unit: Thermal Physics (Heat)

NGSS Standards: HS-PS3-1

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding:

- conversion of energy between forms

Language Objectives:

- Accurately describe the law of conservation of energy, using appropriate academic language.

Labs, Activities & Demonstrations:

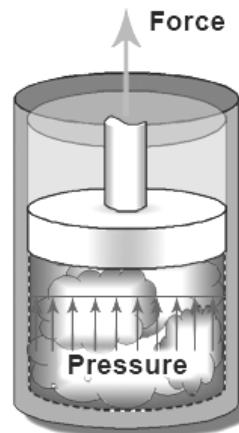
- Fire syringe.
- Steam engine.
- Incandescent light bulb in water.

Notes:

The law of conservation of energy states that total energy is always conserved, but that energy can be converted from one form to another.

We have already seen this in mechanics with the conversion between gravitational potential energy and kinetic energy.

Heat is energy. Like other forms of energy, it can do work. For example, in a steam engine, heat is used to boil water in a sealed container. As more water boils, there is more gas in the boiler, which makes the pressure increase. If the gas can only expand by pushing against something (like a piston), the force from the pressure can do work by moving the piston and whatever it's connected to. (We will revisit the concept of pressure as a force when we study fluid mechanics. For now, it's enough to understand that heat energy can be converted to kinetic energy.)

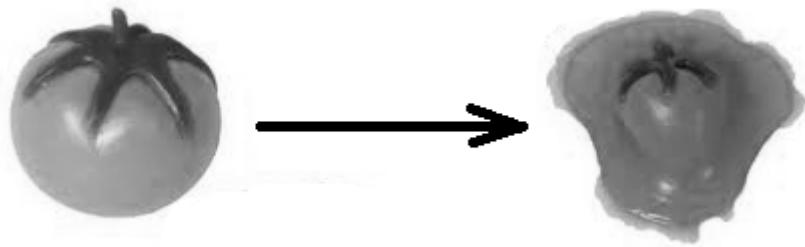


Use this space for summary and/or additional notes.

In mechanics, recall that collisions can be elastic or inelastic. In an elastic collision, kinetic energy is conserved; in an inelastic collision, some of the kinetic energy is converted to other forms, mostly heat.

We can use the law of conservation of energy to estimate the amount of energy converted to heat in a completely inelastic collision.

Consider a 0.150 kg “splat ball” hitting the wall at a velocity of $20.0 \frac{\text{m}}{\text{s}}$.



After the collision, the velocity of the ball and the wall are both zero. This means the kinetic energy of the ball after the collision is zero. Because energy must be conserved, this means all of the kinetic energy from the ball must have been converted to heat.

$$E_k = \frac{1}{2}mv^2$$

$$E_k = \left(\frac{1}{2}\right)(0.150)(20.0)^2 = 30.0 \text{ J}$$

Use this space for summary and/or additional notes.

Now consider the same splat ball with a mass of 0.150 kg and a velocity of $20.0 \frac{\text{m}}{\text{s}}$ hitting a 1.00 kg piece of wood that is initially at rest. This is still an inelastic collision, but now the wood is free to move, which means it has kinetic energy after the collision.

To solve this problem, we need to use conservation of momentum to find the velocity of the splat ball + wood after the collision, and then use the velocity before and after to calculate the change in kinetic energy.

Before the collision:

$$\vec{p} = m_{sb}\vec{v}_{sb} + m_w\vec{v}_w^0$$
$$\vec{p} = (0.150)(+20.0) + 0 = +3.00 \text{ N}\cdot\text{s}$$

$$E_k = \frac{1}{2}m_{sb}v_{sb}^2 + \frac{1}{2}m_wv_w^2$$
$$E_k = (\frac{1}{2})(0.150)(20.0)^2 + 0 = 30.0 \text{ J}$$

After the collision:

$$\vec{p} = (m_{sb} + m_w)\vec{v}$$
$$+ 3.00 = (0.150 + 1.00)\vec{v} = 1.15\vec{v}$$
$$\vec{v} = +2.61 \frac{\text{m}}{\text{s}}$$

$$E_k = \frac{1}{2}mv^2$$
$$E_k = (\frac{1}{2})(1.15)(2.61)^2 = 3.91 \text{ J}$$

This means there is $30.0 - 3.91 = 26.1 \text{ J}$ of kinetic energy that is “missing” after the collision. This “missing” energy is mostly converted to heat. If you could measure the temperature of the “splat ball” and the wood extremely accurately before and after the collision, you would find that both would be warmer as a result of the “missing” 26.1 J of energy.

Use this space for summary and/or additional notes.

Specific Heat Capacity & Calorimetry

Unit: Thermal Physics (Heat)

NGSS Standards: HS-PS2-6, HS-PS3-1

MA Curriculum Frameworks (2006): 3.4

Knowledge/Understanding:

- specific heat capacity
- calorimetry

Skills:

- solve calorimetry (specific heat) problems

Language Objectives:

- Understand and correctly use the terms “specific heat capacity” and “calorimetry.”
- Accurately describe and apply the concepts described in this section, using appropriate academic language.
- Set up and solve word problems relating to specific heat capacity and calorimetry.

Labs, Activities & Demonstrations:

- Calorimetry lab.

Notes:

Different objects have different abilities to hold heat. For example, if you enjoy pizza, you may have noticed that the sauce holds much more heat (and burns your mouth much more readily) than the cheese or the crust.

The amount of heat that a given mass of a substance can hold is based on its specific heat capacity.

specific heat capacity (C): a measure of the amount of heat required per gram of a substance to produce a specific temperature change in the substance.

C_p : specific heat capacity, measured at constant pressure. For gases, this means the measurement was taken allowing the gas to expand as it was heated.

Use this space for summary and/or additional notes.

C_v : specific heat capacity, measured at constant volume. For gases, this means the measurement was made in a sealed container, allowing the pressure to rise as the gas was heated.

For solids and liquids, $C_p \approx C_v$ because the pressure and volume change very little as they are heated. For gases, $C_p > C_v$ (always). For ideal gases, $C_p - C_v = R$, where R is a constant known as "the gas constant."

When there is a choice, C_p is more commonly used than C_v because it is easier to measure. When dealing with solids and liquids, most physicists just use C for specific heat capacity and don't worry about the distinction.

Calculating Heat from a Temperature Change

The amount of heat gained or lost when an object changes temperature is given by the equation:

$$Q = mC\Delta T$$

where:

Q = heat (J or kJ)

m = mass (g or kg)

C = specific heat capacity ($\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$)

ΔT = temperature change (K or $^{\circ}\text{C}$)

Because problems involving heat often involve large amounts of energy, specific heat capacity is often given in kilojoules per kilogram per degree Celsius.

Note that $1 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \equiv 1 \frac{\text{kJ}}{\text{kg}\cdot{}^{\circ}\text{C}} \equiv 1 \frac{\text{J}}{\text{g}\cdot{}^{\circ}\text{C}}$ and $1 \frac{\text{cal}}{\text{g}\cdot{}^{\circ}\text{C}} \equiv 1 \frac{\text{kcal}}{\text{kg}\cdot{}^{\circ}\text{C}} = 4.184 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$.

You need to be careful with the units. If the mass is given in kilograms (kg), your specific heat capacity will have units of $\frac{\text{kJ}}{\text{kg}\cdot{}^{\circ}\text{C}}$ and the heat energy will come out in kilojoules (kJ). If mass is given in grams, you will use units of $\frac{\text{J}}{\text{g}\cdot{}^{\circ}\text{C}}$ and the heat energy will come out in joules (J).

Use this space for summary and/or additional notes.

Specific Heat Capacities of Some Substances

Substance	Specific Heat Capacity ($\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$)	Substance	Specific Heat Capacity ($\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$)	Substance	Specific Heat Capacity ($\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$)
water at 20°C	4.181	vegetable oil	2.00	copper	0.385
ethylene glycol	2.460	air	1.012	brass	0.380
ice at -10°C	2.080	glass	0.84	silver	0.233
steam at 100°C	2.11	aluminum	0.897	lead	0.160
steam at 130°C	1.99	iron	0.450	gold	0.129

Calorimetry

calorimetry: the measurement of heat flow

In a calorimetry experiment, heat flow is calculated by measuring the mass and temperature change of an object and applying the specific heat capacity equation.

calorimeter: an insulated container for performing calorimetry experiments.

coffee cup calorimeter: a calorimeter that is only an insulated container—it does not include a thermal mass (such as a mass of water). It is usually made of styrofoam, and is often nothing more than a styrofoam coffee cup.

bomb calorimeter: a calorimeter for measuring the heat produced by a chemical reaction. A bomb calorimeter is a double-wall metal container with water between the layers of metal. The heat from the chemical reaction makes the temperature of the water increase. Because the mass and specific heat of the calorimeter (water and metal) are known, the heat produced by the reaction can be calculated from the increase in temperature of the water.

Use this space for summary and/or additional notes.

Solving Coffee Cup Calorimetry Problems

Most coffee cup calorimetry problems involve placing a hot object in contact with a colder one. Many of them involve placing a hot piece of metal into cold water.

To solve the problems, assume that both objects end up at the same temperature. The heat lost by the hot object (Q_h) equals the heat gained by the cold object (Q_c). (However, remember that Q_h will be negative because the hot object is losing heat.)

$$Q_c = m_c C_c \Delta T_c$$

$$Q_h = m_h C_h \Delta T_h$$

$$Q_c = -Q_h$$

$$m_c C_c \Delta T_c = -m_h C_h \Delta T_h$$

Notice that there are six quantities that you need: the two masses (m_h and m_c), the two specific heat capacities (C_h and C_c), and the two temperature changes (ΔT_h and ΔT_c). (You might be given initial and final temperatures for either or both, in which case you'll need to subtract.) The problem will give you all but one of these and you will need to find the missing one.

Don't fret about the negative sign. The value of ΔT_h will be negative (because it is cooling off), and the two minus signs will cancel.

Use this space for summary and/or additional notes.

Steps for Solving Coffee Cup Calorimetry Problems

1. Identify the variables for both the hot and cold substance. This can be tricky because you have the two masses, the two specific heat capacities, and the two temperature changes. For each quantity, you have to identify both the variable and which substance it applies to.
2. Look up the specific heat capacities of the substances involved.
3. Plug each set of numbers into the equation $Q = mC\Delta T$. (I.e., you'll have two separate $Q = mC\Delta T$ equations.)
 - a. Remember that for the substance that is cooling off, heat is going out of the system, which means the equation will be $Q = -mC\Delta T$.
 - b. Because ΔT will be negative for the substance that was cooling off, the two negative signs will cancel.
4. Use the fact that Q is the same for both equations to solve for the unknown quantity. This will involve doing one of the following:
 - a. Calculate the value of Q from one equation and use it in the other equation.
 - b. If you need to find the final temperature, set the two $mC\Delta T$ expressions (or $mC(T_f - T_i)$ expressions) equal to each other.

Use this space for summary and/or additional notes.

Sample Problems:

Q: An 0.050 kg block of aluminum is heated and placed in a calorimeter containing 0.100 kg of water at 20°C. If the final temperature of the water was 30°C, to what temperature was the aluminum heated?

A: The heat gained by the water equals the heat lost by the aluminum.

The heat gained by the water is:

$$Q = mC\Delta T$$

$$Q = (0.100 \text{ kg})(4.18 \frac{\text{kJ}}{\text{kg}\cdot\text{K}})(+10^\circ\text{C})$$

$$Q = 4.18 \text{ kJ}$$

The heat lost by the metal must therefore be 4.18 kJ.

$$Q = -mC\Delta T$$

$$4.18 \text{ kJ} = -(0.050 \text{ kg})(0.897 \frac{\text{kJ}}{\text{kg}\cdot\text{K}}) \Delta T$$

$$4.18 = -(0.0449)(\Delta T)$$

$$\Delta T = \frac{4.18}{-0.0449} = -93.2^\circ\text{C}$$

The temperature of the aluminum was -93°C (*i.e.*, it went down by 93°C)

$$\Delta T = T_f - T_i$$

$$-93.2 = 30 - T_i$$

$$T_i = 123.2^\circ\text{C}$$

This means the initial temperature must have been 123.2°C .

Use this space for summary and/or additional notes.

Q: An 0.025 kg block of copper at 95°C is dropped into a calorimeter containing 0.075 kg of water at 25°C. What is the final temperature?

A: Once again, the heat lost by the copper equals the heat gained by the water.

$$\begin{aligned} -Q_c &= Q_w \\ -m_c C_c \Delta T_c &= m_w C_w \Delta T_w \\ -(0.025)(0.385)(T_f - 95) &= (0.075)(4.18)(T_f - 25) \\ -(0.009625)(T_f - 95) &= (0.3138)(T_f - 25) \\ -(0.009625 T_f - 0.9144) &= 0.3138 T_f - 7.845 \\ -0.009625 T_f + 0.9144 &= 0.3138 T_f - 7.845 \\ +0.009625 T_f &= +0.009625 T_f \\ 0.9144 &= 0.3234 T_f - 7.845 \\ +7.845 &= +7.845 \\ 8.759 &= 0.3234 T_f \\ \frac{8.759}{0.3234} &= 27^\circ\text{C} = T_f \end{aligned}$$

Note that because the specific heat of the water is so much higher than that of copper, and because the mass of the water was larger than the mass of the copper, the final temperature ended up much closer to the initial water temperature.

Homework Problems

You will need to look up specific heat capacities in Table H of your Reference Tables on page 507.

1. 375 kJ of heat is added to a 25.0 kg granite rock. How much does the temperature increase?

Answer: 19.0°C

Use this space for summary and/or additional notes.

2. A 0.040 kg block of copper at 95°C is placed in 0.105 kg of water at an unknown temperature. After equilibrium is reached, the final temperature is 24°C. What was the initial temperature of the water?

Answer: 21.5°C

3. A sample of metal with a specific heat capacity of $0.50 \frac{\text{kJ}}{\text{kg}\cdot\text{°C}}$ is heated to 98°C and then placed in an 0.055 kg sample of water at 22°C. When equilibrium is reached, the final temperature is 35°C. What was the mass of the metal?

Answer: 0.0948 kg

4. A 0.280 kg sample of a metal with a specific heat capacity of $0.43 \frac{\text{kJ}}{\text{kg}\cdot\text{°C}}$ is heated to 97.5°C then placed in an 0.0452 kg sample of water at 31.2°C. What is the final temperature of the metal and the water?

Answer: 57°C

Use this space for summary and/or additional notes.

5. You want to do an experiment to measure the conversion of gravitational potential energy to kinetic energy to heat by dropping 2.0 kg of copper off the roof of LEHS, a height of 14 m. How much will the temperature of the copper increase?

(Hint: Remember that potential energy is measured in J but specific heat capacity problems usually use kJ.)

Answer: 0.356°C

6. Based on your answer to question #5 above, you decide to modify your experiment by dropping the 2.0 kg bag of copper from a height of 2.0 m to the floor multiple times. How many times would you need to drop the copper bag to get a temperature increase of 2°C?

(Hint: Remember that potential energy is measured in J but specific heat capacity problems usually use kJ.)

Answer: 39 times

Use this space for summary and/or additional notes.

Phase Changes & Heating Curves

Unit: Thermal Physics (Heat)

NGSS Standards: HS-PS3-1

MA Curriculum Frameworks (2006): 3.3

Knowledge/Understanding Goals:

- phases and phase changes
- how heat is transferred in a phase change
- why evaporation causes cooling

Skills:

- calculate the heat absorbed or produced during phase changes
- plot and make calculations from heating curves

Language Objectives:

- Understand and correctly use the terms “phase change,” “heat of fusion,” “heat of vaporization,” and “heating curve.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve heating curve problems.

Labs, Activities & Demonstrations:

- Evaporation from washcloth.
- Fire & ice.

Notes:

phase: a term that relates to how rigidly the atoms or molecules in a substance are connected.

solid: molecules are rigidly connected. A solid has a definite shape and volume.

liquid: molecules are loosely connected—bonds are continuously forming and breaking. A liquid has a definite volume, but not a definite shape.

gas: molecules are not connected. A gas has neither a definite shape nor a definite volume. Gases will expand to fill whatever space they occupy.

Use this space for summary and/or additional notes.

plasma: the system has enough heat to remove electrons from atoms, which means the system is comprised of particles with rapidly changing charges.

phase change: when an object or substance changes from one phase to another through gaining or losing heat.

Breaking bonds requires energy. Forming bonds releases energy. This is true for the bonds that hold a solid or liquid together as well as for chemical bonds (regardless of what your biology teacher may have told you!)

I.e., you need to add energy to turn a solid to a liquid (melt it), or to turn a liquid to a gas (boil it). Energy is released when a gas condenses or a liquid freezes. (*E.g.*, ice in your ice tray needs to give off heat in order to freeze. Your freezer needs to remove that heat in order to make this happen.)

The reason evaporation causes cooling is because the system (the water) needs to absorb heat from its surroundings (*e.g.*, your body) in order to make the change from a liquid to a gas (vapor). When the water absorbs heat from you and evaporates, you have less heat, which means you have cooled off.

Calculating the Heat of Phase Changes

heat of fusion (ΔH_{fus}) (sometimes called “latent heat” or “latent heat of fusion”): the amount of heat required to melt one kilogram of a substance. This is also the heat released when one kilogram of a liquid substance freezes. For example, the heat of fusion of water is $334 \frac{\text{kJ}}{\text{kg}}$ (which also equals $334 \frac{\text{J}}{\text{g}}$).

The heat required to melt a sample of water is therefore:

$$Q = m\Delta H_{fus} = m(334 \frac{\text{kJ}}{\text{kg}})$$

(Note that by convention, heat is in kilojoules rather than Joules.)

heat of vaporization (ΔH_{vap}): the amount of heat required to vaporize (boil) one kilogram of a substance. This is also the heat released when one kilogram of a gas condenses. For example, the heat of vaporization of water is $2260 \frac{\text{kJ}}{\text{kg}}$.

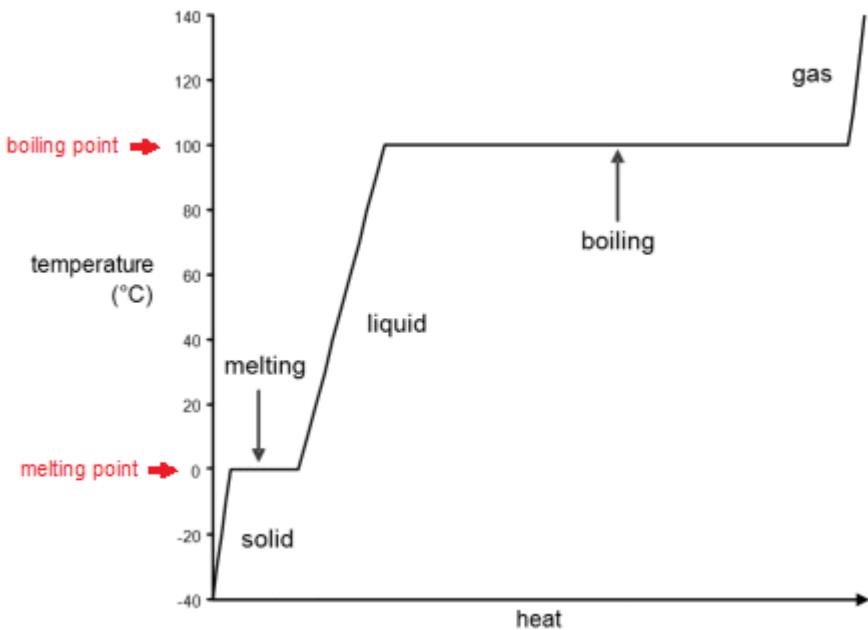
The heat required to boil a sample of water is therefore:

$$Q = m\Delta H_{vap} = m(2260 \frac{\text{kJ}}{\text{kg}})$$

(Again, note that heat is in kilojoules rather than Joules.)

Use this space for summary and/or additional notes.

heating curve: a graph of temperature vs. heat added. The following is a heating curve for water:



In the “solid” portion of the curve, the sample is solid water (ice). As heat is added, the temperature increases. The specific heat capacity of ice is $2.11 \frac{\text{kJ}}{\text{kg}\cdot\text{°C}}$, so the heat required is:

$$Q_{solid} = mC\Delta T = m(2.11 \frac{\text{kJ}}{\text{kg}\cdot\text{°C}})(\Delta T)$$

In the “melting” portion of the curve, the sample is a mixture of ice and water. As heat is added, the ice melts, but the temperature remains at 0°C until all of the ice is melted. The heat of fusion of ice is $334 \frac{\text{kJ}}{\text{kg}}$, so the heat required is:

$$Q_{melt} = m\Delta H_{fus} = m(334 \frac{\text{kJ}}{\text{kg}})$$

In the “liquid” portion of the curve, the sample is liquid water. As heat is added, the temperature increases. The specific heat capacity of liquid water is $4.184 \frac{\text{kJ}}{\text{kg}\cdot\text{°C}}$, so the heat required is:

$$Q_{liquid} = mC\Delta T = m(4.184 \frac{\text{kJ}}{\text{kg}\cdot\text{°C}})(\Delta T)$$

Use this space for summary and/or additional notes.

In the “boiling” portion of the curve, the sample is a mixture of water and water vapor (steam). As heat is added, the water boils, but the temperature remains at 100°C until all of the water has boiled. The heat of vaporization of water is 2260 $\frac{\text{kJ}}{\text{kg}}$, so the heat required is:

$$Q_{boil} = m\Delta H_{vap} = m(2260 \frac{\text{kJ}}{\text{kg}})$$

In the “gas” portion of the curve, the sample is water vapor (steam). As heat is added, the temperature increases. The specific heat capacity of steam is approximately 2.08 $\frac{\text{kJ}}{\text{kg}\cdot^\circ\text{C}}$ (at 100°C; the specific heat capacity of steam decreases as the temperature increases), so the heat required is:

$$Q_{gas} = mC\Delta T = m(2.08 \frac{\text{kJ}}{\text{kg}\cdot^\circ\text{C}})(\Delta T)$$

Use this space for summary and/or additional notes.

Steps for Solving Heating Curve Problems

A heating curve problem is a problem in which a substance is heated across a temperature range that passes through the melting and/or boiling point of the substance, which means the problem includes heating or cooling steps and melting/freezing or boiling/condensing steps.

1. Sketch the heating curve for the substance over the temperature range in question. Be sure to include the melting and boiling steps as well as the heating steps.
2. From your sketch, determine whether the temperature range in the problem passes through the melting and/or boiling point of the substance.
3. Split the problem into:
 - a. Heating (or cooling) steps within each temperature range.
 - b. Melting or boiling (or freezing or condensing) steps.
4. Find the heat required for each step.
 - a. For the heating/cooling steps, use the equation $Q = mC\Delta T$.
 - b. For melting/freezing steps, use the equation $Q = m\Delta H_{fus}$.
 - c. For boiling/condensing steps, use the equation $Q = m\Delta H_{vap}$.
5. Add the values of Q from each step to find the total.

Use this space for summary and/or additional notes.

Sample Problem

Q: How much heat would it take to raise the temperature of 0.0150 kg of H₂O from -25.0°C to +130.0°C?

A: The H₂O starts out as ice. We need to:

1. Heat the ice from -25.0°C to its melting point (0°C).
2. Melt it.
3. Heat the water up to its boiling point (from 0°C to 100°C).
4. Boil it.
5. Heat the steam from 100°C to 130°C.
6. Add up the heat for each step to find the total.

$$\text{heat solid: } Q_1 = mC\Delta T = (0.0150 \text{ kg})(2.11 \frac{\text{kJ}}{\text{kg}\cdot^\circ\text{C}})(25^\circ\text{C}) = 0.79125 \text{ kJ}$$

$$\text{melt: } Q_2 = m\Delta H_{fus} = (0.0150 \text{ g})(334 \frac{\text{kJ}}{\text{kg}}) = 5.010 \text{ kJ}$$

$$\text{heat liquid: } Q_3 = mC\Delta T = (0.0150 \text{ kg})(4.18 \frac{\text{kJ}}{\text{kg}\cdot^\circ\text{C}})(100^\circ\text{C}) = 6.27 \text{ kJ}$$

$$\text{boil: } Q_4 = m\Delta H_{vap} = (0.0150 \text{ kg})(2260 \frac{\text{kJ}}{\text{kg}}) = 33.90 \text{ kJ}$$

$$\text{heat gas: } Q_5 = mC\Delta T = (0.0150 \text{ kg})(2.08 \frac{\text{kJ}}{\text{kg}\cdot^\circ\text{C}})(30^\circ\text{C}) = 0.936 \text{ kJ}$$

$$Q_{total} = Q_1 + Q_2 + Q_3 + Q_4 + Q_5$$

$$Q_{total} = 0.791 + 5.010 + 6.27 + 33.90 + 0.936 = 46.91 \text{ kJ}$$

Use this space for summary and/or additional notes.

Homework Problems

For the following problems, use data from the following table:

	C (sol.) ($\frac{\text{kJ}}{\text{kg}\cdot\text{C}}$)	M.P. (°C)	ΔH_{fus} ($\frac{\text{kJ}}{\text{kg}}$)	C (liq) ($\frac{\text{kJ}}{\text{kg}\cdot\text{C}}$)	B.P. (°C)	ΔH_{vap} ($\frac{\text{kJ}}{\text{kg}}$)	C _p (gas) ($\frac{\text{kJ}}{\text{kg}\cdot\text{C}}$)
water	2.11	0	334	4.18	100	2260	2.08*
potassium	0.560	62	61.4	1.070	760	2025	0.671
mercury	0.142	-39	11.3	0.140	357	293	0.104
silver	0.217	962	111	0.318	2212	2360	—

*Note that because of the volume change from heating, the specific heat capacity of gases, C_p, increases with increasing temperature.

1. A 0.0250 kg sample of water is heated from -40.0°C to 150. °C.
 - a. Sketch the heating curve for the above process. Label the starting temperature, melting point, boiling point, and final temperature on the y-axis.
 - b. Calculate the heat required for each step of the heating curve, and the total heat required.

Answer: 80.01 kJ

Use this space for summary and/or additional notes.

2. A 0.085 kg sample of mercury is heated from 25°C to 500. °C.
- Sketch the heating curve for the above process. Label the starting temperature, melting point, boiling point, and final temperature on the y-axis.
 - Calculate the heat required for each step of the heating curve, and the total heat required.

Answer: 30.12 kJ

3. A 0.045 kg block of silver at a temperature of 22°C is heated with 20.0 kJ of energy. Calculate the total heat required by calculating the heat for each step until the entire 20.0 kJ is accounted for.

What is the final temperature and what is the physical state (solid, liquid, gas) of the silver at that temperature?

Answers: liquid, 1369°C

Use this space for summary and/or additional notes.

Thermal Expansion

Unit: Thermal Physics (Heat)

NGSS Standards: HS-PS2-6

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding:

- thermal expansion in solids, liquids and gases

Skills:

- calculate changes in length and volume for solids and liquids
- calculate changes in volume for gases

Language Objectives:

- Understand and correctly use the term “thermal expansion.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to thermal expansion of solids, liquids and gases.

Labs, Activities & Demonstrations:

- Balloon with string & heat gun.
- Brass ball & ring.
- Bi-metal strip.

Notes:

expand: to become larger

contract: to become smaller

thermal expansion: an increase in the length and/or volume of an object caused by a change in temperature.

When a substance is heated, the particles it is made of move farther and faster. This causes the particles to move farther apart, which causes the substance to expand.

Use this space for summary and/or additional notes.

Solids tend to keep their shape when they expand. (Liquids and gases do not have definite shape to begin with.)

A few materials are known to contract with increasing temperature over specific temperature ranges. One well-known example is liquid water, which contracts as it heats from 0°C to 4°C. (Water expands as the temperature increases above 4°C.)

Thermal Expansion of Solids and Liquids

Thermal expansion is quantified in solids and liquids by defining a coefficient of thermal expansion. The changes in length or volume are given by the following equations:

$$\text{Length: } \Delta L = \alpha L_i \Delta T$$

$$\text{Volume: } \Delta V = \beta V_i \Delta T$$

where:

$$\Delta L = \text{change in length (m)}$$

$$\Delta V = \text{change in volume (m}^3\text{)}$$

$$L_i = \text{initial length (m)}$$

$$V_i = \text{initial volume (m}^3\text{)}$$

$$\alpha = \text{linear coefficient of thermal expansion (}\text{^\circ C}^{-1}\text{)}$$

$$\beta = \text{volumetric coefficient of thermal expansion (}\text{^\circ C}^{-1}\text{)}$$

$$\Delta T = \text{temperature change}$$

Values of α and β at 20°C for some solids and liquids:

Substance	$\alpha (\text{ }^\circ \text{C}^{-1})$	$\beta (\text{ }^\circ \text{C}^{-1})$	Substance	$\alpha (\text{ }^\circ \text{C}^{-1})$	$\beta (\text{ }^\circ \text{C}^{-1})$
aluminum	2.3×10^{-5}	6.9×10^{-5}	gold	1.4×10^{-5}	4.2×10^{-5}
copper	1.7×10^{-5}	5.1×10^{-5}	iron	1.18×10^{-5}	3.33×10^{-5}
brass	1.9×10^{-5}	5.6×10^{-5}	lead	2.9×10^{-5}	8.7×10^{-5}
diamond	1×10^{-6}	3×10^{-6}	mercury	6.1×10^{-5}	1.82×10^{-4}
ethanol	2.5×10^{-4}	7.5×10^{-4}	silver	1.8×10^{-5}	5.4×10^{-5}
glass	8.5×10^{-6}	2.55×10^{-5}	water (liq.)	6.9×10^{-5}	2.07×10^{-4}

Use this space for summary and/or additional notes.

expansion joint: a space deliberately placed between two objects to allow room for the objects to expand without coming into contact with each other.

Bridges often have expansion joints in order to leave room for sections of the bridge to expand or contract without damaging the bridge or the roadway:

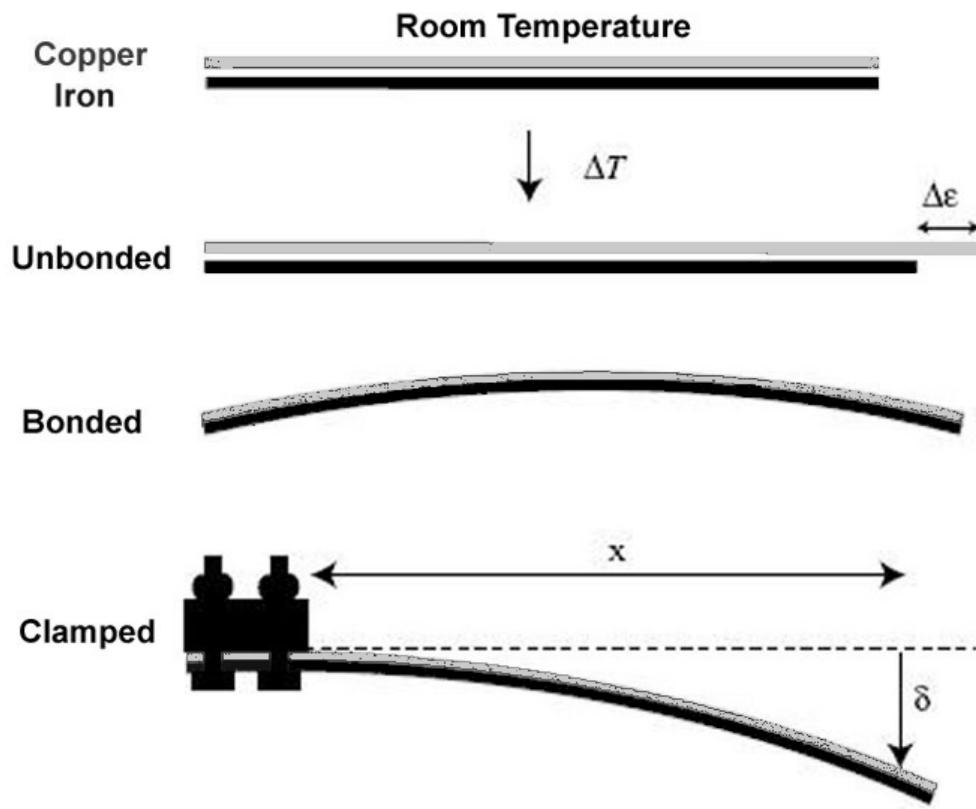


Railroad rails are sometimes welded together in order to create a smoother ride, which enables high-speed trains to use them. Unfortunately, if expansion joints are not placed at frequent enough intervals, thermal expansion can cause the rails to bend and buckle, resulting in derailments:



Use this space for summary and/or additional notes.

bimetal strip: a strip made from two metals with different coefficients of thermal expansion that are bonded together. When the strip is heated or cooled, the two metals expand or contract different amounts, which causes the strip to bend. When the strip is returned to room temperature, the metals revert back to their original lengths.



Use this space for summary and/or additional notes.

Sample Problems:

Q: Find the change in length of an 0.40 m brass rod that is heated from 25°C to 980°C.

A: For brass, $\alpha = 1.9 \times 10^{-5} \text{ }^{\circ}\text{C}^{-1}$.

$$\Delta L = \alpha L_i \Delta T$$

$$\Delta L = (1.9 \times 10^{-5})(0.40)(955)$$

$$\Delta L = 0.0073 \text{ m}$$

Q: A typical mercury thermometer contains about 0.22 cm³ (about 3.0 g) of mercury. Find the change in volume of the mercury in a thermometer when it is heated from 25°C to 50.°C.

A: For mercury, $\beta = 1.82 \times 10^{-4} \text{ }^{\circ}\text{C}^{-1}$.

$$\Delta V = \beta V_i \Delta T$$

$$\Delta V = (1.82 \times 10^{-4})(0.22)(25)$$

$$\Delta V = 0.00091 \text{ cm}^3$$

If the distance from the 25°C to the 50°C mark is about 3.0 cm, we could use this information to figure out the bore of the thermometer:

$$V = \pi r^2 h$$

$$0.00091 = (3.14)r^2(3.0)$$

$$r^2 = 9.66 \times 10^{-5}$$

$$r = \sqrt{9.66 \times 10^{-5}} = 0.0098 \text{ cm}$$

The bore is the diameter, which is twice the radius, so the bore of the thermometer is 0.020 cm or about 0.20 mm.

Use this space for summary and/or additional notes.

Homework Problems

You will need to look up coefficients of thermal expansion in Table H of your Reference Tables on page 507.

1. A brass rod is 27.50 cm long at 25°C. How long would the rod be if it were heated to 750.°C in a flame?

Answer: 27.88 cm

2. A steel bridge is 625 m long when the temperature is 0°C.
 - a. If the bridge did not have any expansion joints, how much longer would the bridge be on a hot summer day when the temperature is 35°C? (Use the linear coefficient of expansion for iron.)

Answer: 0.258m

- b. Why do bridges need expansion joints?

Use this space for summary and/or additional notes.

3. A 15.00 cm long bimetal strip is aluminum on one side and copper on the other. If the two metals are the same length at 20.0°C, how long will each be at 800.°C?

Answers: aluminum: 15.269 cm; copper: 15.199 cm

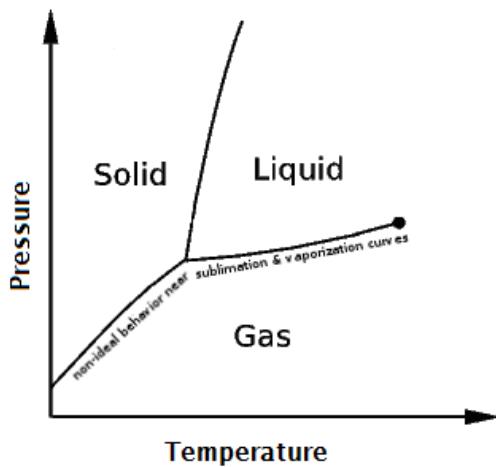
4. A glass volumetric flask is filled with water exactly to the 250.00 mL line at 50.°C. What volume will the water occupy after it cools down to 20.°C?

Answer: 248.45 mL

Use this space for summary and/or additional notes.

Thermal Expansion of Gases

ideal gas: a gas that behaves as if each molecule acts independently, according to kinetic-molecular theory. Most gases behave ideally except at temperatures and pressures near the vaporization curve on a phase diagram. (I.e., gases stop behaving ideally when conditions are close to those that would cause the gas to condense to a liquid or solid.)



For an ideal gas, the change in volume for a change in temperature (provided that the pressure and number of molecules are kept constant) is given by Charles' Law:

$$\frac{V_i}{T_i} = \frac{V_f}{T_f}$$

where volume can be any volume unit (as long as it is the same on both sides), but temperature must be in Kelvin.

Use this space for summary and/or additional notes.

Sample Problem:

Q: If a 250 mL container of air is heated from 25°C to 95°C, what is the new volume?

A: Temperatures must be in Kelvin, so we need to convert first.

$$25^{\circ}\text{C} + 273 = 298\text{K}$$

$$95^{\circ}\text{C} + 273 = 368\text{K}$$

$$\frac{V_i}{T_i} = \frac{V_f}{T_f}$$

$$\frac{250}{298} = \frac{V_f}{368}$$

$$V_f = 308.7 = \boxed{310\text{ mL}}$$

Because we used mL for V_1 , the value of V_2 is therefore also in mL.

Homework Problems

1. A sample of argon gas was cooled, and its volume went from 380. mL to 250. mL. If its final temperature was -45.0°C , what was its original temperature?

Answer: 347 K or 74°C

2. A balloon contains 250. mL of air at 50°C . If the air in the balloon is cooled to 20.0°C , what will be the new volume of the air?

Answer: 226.8mL

Use this space for summary and/or additional notes.

Introduction: Pressure & Fluid Mechanics

Unit: Pressure & Fluid Mechanics

Topics covered in this chapter:

Pressure	467
Hydraulic Pressure	472
Hydrostatic Pressure.....	475
Buoyancy.....	479
Gas Laws.....	486
Fluid Motion & Bernoulli's Law.....	496

Fluid mechanics is the study of behaviors that are specific to fluids (liquids and gases).

- *Pressure* is the property that is central to the topic of fluid mechanics.
- *Hydrostatics* and *Buoyancy* describe and give equations for the effects of gravity on pressure.
- *Gas Laws* describes behaviors and equations involving temperature, pressure and volume, as related to gases.
- *Fluid Motion & Bernoulli's Law* describes the effects of fluid motion on pressure.

This chapter focuses on real-world applications of fluids and pressure, including more demonstrations than most other topics. One of the challenges in this chapter is relating the equations to the behaviors seen in the demonstrations.

Textbook:

- *Physics: Principles and Problems* Ch. 13: States of Matter (pp. 264–285)

Standards addressed in this chapter:

Next Generation Science Standards (NGSS):

No NGSS standards are addressed in this chapter.

Use this space for summary and/or additional notes:

Massachusetts Curriculum Frameworks (2006):

No MA curriculum standards are addressed in this chapter.

Skills learned & applied in this chapter:

- Before & after problems.

Use this space for summary and/or additional notes.

Pressure

Unit: Pressure & Fluid Mechanics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding:

- pressure

Skills:

- calculate pressure as an applied force

Language Objectives:

- Understand and correctly use the term “pressure” as it applies to situations in physics.
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to pressure and hydraulics.

Labs, Activities & Demonstrations:

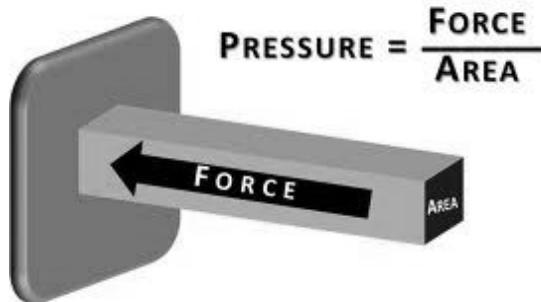
- Balloon.
- Pinscreen (pin art) toy.
- Balloon & weights on small bed of nails.
- Full-size bed of nails.

Notes:

pressure: the exertion of force upon a surface by an object, fluid, etc. that is in contact with it.

Use this space for summary and/or additional notes.

Mathematically, pressure is defined as force divided by area:



$$P = \frac{F}{A}$$

The S.I. unit for pressure is the pascal (Pa).

$$1 \text{ Pa} \equiv 1 \frac{\text{N}}{\text{m}^2} \equiv 1 \frac{\text{kg}}{\text{ms}^2}$$

(Note that Pa is a two-letter symbol.)

Some other common pressure units are:

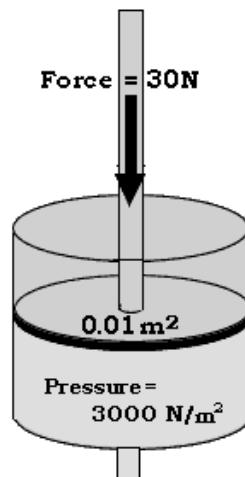
- bar: $1 \text{ bar} = 100000 \text{ Pa} = 10^5 \text{ Pa}$
- pound per square inch (psi)
- atmosphere (atm): the average atmospheric pressure on Earth at sea level.

$$1 \text{ atm} \equiv 101325 \text{ Pa} \equiv 101.325 \text{ kPa} \equiv 1.01325 \text{ bar} = 14.696 \text{ psi}$$

We will use the approximation that $1 \text{ atm} \approx 1 \text{ bar} = 100000 \text{ Pa}$

Air pressure can be described relative to a total vacuum (absolute pressure), but is more commonly described relative to atmospheric pressure (gauge pressure):

- absolute pressure: the total pressure on a surface. An absolute pressure of zero means there is zero force on the surface.



Use this space for summary and/or additional notes.

- gauge pressure: the difference between the pressure on a surface and atmospheric pressure. A gauge pressure of zero means the same as atmospheric pressure. The pressure in car tires is measured as gauge pressure. For example, a tire pressure of 30 psi (30 pounds per square inch, or $30 \frac{\text{lb.}}{\text{in.}^2}$) would mean that the air inside the tires is pushing against the air outside the tires with a pressure of 30 psi. A flat tire would have a gauge pressure of zero.

Sample Problem

Q: What is the pressure caused by a force of 25 N acting on a piston with an area of 0.05 m^2 ?

A: $P = \frac{F}{A} = \frac{25 \text{ N}}{0.05 \text{ m}^2} = 500 \text{ Pa}$

Homework Problems

1. A person wearing snow shoes does not sink into the snow of the tundra, whereas the same person without snow shoes sinks into the snow. Explain.
2. A balloon is inflated to a pressure of 0.2 bar. A 5.0 kg book is balanced on top of the balloon. With what surface area does the balloon contact the book? (*Hint: Remember that 1 bar = 100 000 Pa.*)

Answer: 0.0025 m^2

Use this space for summary and/or additional notes.

3. A carton of paper has a mass of 22.7 kg. The area of the bottom is 0.119 m^2 . What is the pressure between the carton and the floor?

Answer: 1908 Pa

4. A 1000 kg car rests on four tires, each inflated to 2.2 bar. What surface area does *each* tire have in contact with the ground? (Assume the weight is evenly distributed on each wheel.)

Answer: 0.0114 m^2

5. A student with a mass of 75.0 kg is sitting on 4-legged lab stool that has a mass of 3.0 kg. Each leg of the stool is circular and has a diameter of 2.50 cm. Find the pressure under each leg of the stool.

(*Hints: (1) Remember to convert cm^2 to m^2 for the area of the legs of the stool. (2) Remember that the stool has four legs. (3) Note that the problem gives the diameter of the legs of the stool, not the radius.*)

Answer: 397250 Pa

Use this space for summary and/or additional notes.

6. A student with a mass of 50. kg is lying on the floor of the classroom. The area of the student that is in contact with the floor is 0.6 m^2 . What is the pressure between the student and the floor? Express your answer both in pascals and in bar.

Answer: 833 Pa or 0.00833 bar

7. The same student, with a mass of 50 kg, is lying on a single nail has a cross-sectional area of $0.1 \text{ mm}^2 = 1 \times 10^{-7} \text{ m}^2$. What is the pressure (in bar) that the student exerts on the head of the nail?

Answer: $5 \times 10^9 \text{ Pa} = 50000 \text{ bar}$

8. The same student, with a mass of 50 kg, is lying on a bed of nails that contains 2500 nails evenly spread over an area of 0.84 m^2 . If the student is in contact with 1800 of the nails, what is the pressure (in bar) between the student and each nail?

Answer: 28 bar

Use this space for summary and/or additional notes.

Hydraulic Pressure

Unit: Pressure & Fluid Mechanics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding:

- hydraulic pressure

Skills:

- calculate the force applied by a piston given the force on another piston and areas of both in a hydraulic system

Language Objectives:

- Understand and correctly use the term “hydraulic pressure.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to hydraulic pressure.

Labs, Activities & Demonstrations:

- Syringe (squirter) & flow rate
- Hovercraft.

Notes:

Pascal’s Principle, which was discovered by the French mathematician Blaise Pascal, states that any pressure applied to a fluid is transmitted uniformly throughout the fluid.

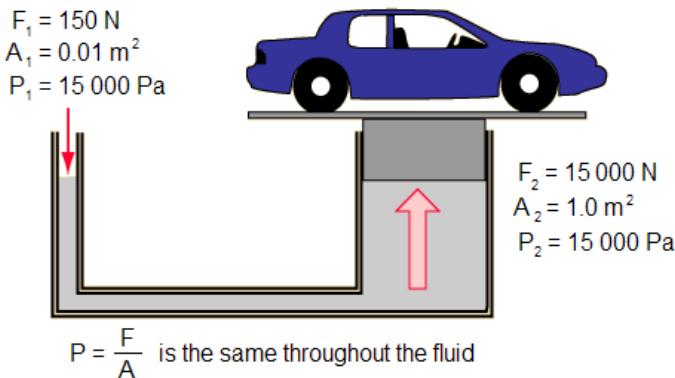
Because $P = \frac{F}{A}$, if the pressure is the same everywhere in the fluid, then $\frac{F}{A}$ must be the same everywhere in the fluid.

Use this space for summary and/or additional notes.

If you have two pistons whose cylinders are connected, the pressure is the same throughout the fluid, which means the force on each piston is proportional to its own area. Thus:

$$P_1 = P_2 \text{ which means } \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

This principle is called “hydraulics.” If you have a lift that has two pistons, one that is 100 times larger than the other, the larger one can supply 100 times as much force.



Conservation of energy tells us that the work done by F_1 must equal the work done by F_2 , which means F_1 must act over a considerably larger distance than F_2 . This also makes sense when you consider the volume of fluid transferred as a fraction of both the smaller and larger cylinders.

This is how hydraulic brakes work in cars. When you step on the brake pedal, the hydraulic pressure is transmitted to the master cylinder and then to the slave cylinders. The master cylinder is much smaller in diameter than the slave cylinders, which means the force applied to the brake pads is considerably greater than the force from your foot.

Sample Problem

Q: In a hydraulic system, a force of 25 N will be applied to a piston with an area of 0.50 m^2 . If the force needs to lift a weight of 500. N, what must be the area of the piston supporting the 500. N weight?

A:	$\frac{F_1}{A_1} = \frac{F_2}{A_2}$	$\frac{25}{0.50} = \frac{500}{A_2}$	$25A_2 = (500)(0.50)$
			$25A_2 = 250$
			$A_2 = 10. \text{ m}^2$

Use this space for summary and/or additional notes.

Homework Problems

1. A student who weighs 700. N stands on a hydraulic lift. The lift has a lever, which you push down in order to lift the student. The cross-sectional area of the piston pressing on the fluid under the student is 1 m^2 , and the cross-sectional area of the piston pressing on the fluid under the lever is 0.1 m^2 . How much force is needed to lift the student?

Answer: 70 N

2. Mr. Bigler's hovercraft is made from a circle of plywood and a wet/dry vacuum cleaner. The vacuum cleaner supplies a pressure of 15 000 Pa, and the radius of the plywood base is 0.6 m. How much weight (in newtons) can the hovercraft theoretically lift?

Answer: 16 965 N, which is approximately 3 800 lbs.

Use this space for summary and/or additional notes:

Hydrostatic Pressure

Unit: Pressure & Fluid Mechanics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding:

- hydrostatic pressure

Skills:

- calculate pressure exerted by a column of fluid

Language Objectives:

- Understand and correctly use the term “hydrostatic pressure.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to hydrostatic pressure.

Labs, Activities & Demonstrations:

- Bottle with hole (feel suction, pressure at exit)
- Buret & funnel manometer
- Syphon hose
- Cup of water & index card
- Magdeburg hemispheres
- Shrink-wrap students

Notes:

Hydrostatic Pressure

hydrostatic pressure: the pressure caused by the weight of a column of fluid

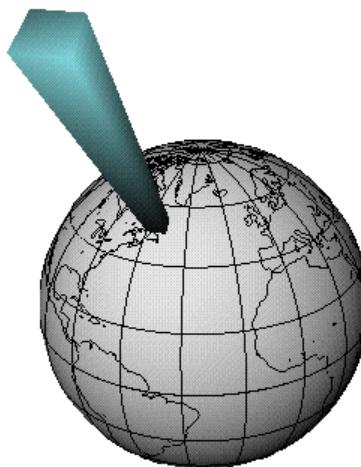
The force of gravity pulling down on the molecules in a fluid creates pressure. The more fluid there is above a point, the higher the pressure at that point.

Use this space for summary and/or additional notes.

The atmospheric pressure that we measure at the surface of the Earth is caused by the air above us, all the way to the highest point in the atmosphere, as shown in the picture at right:

Assuming the density of the fluid is constant, the pressure in a column of fluid is caused by the weight (force of gravity) acting on an area. Because the force of gravity is mg (where $g = 10 \frac{\text{m}}{\text{s}^2}$), this means:

$$P = \frac{F_g}{A} = \frac{mg}{A}$$



where:

P = pressure

g = acceleration due to gravity ($10 \frac{\text{m}}{\text{s}^2}$ on Earth)

A = area of the surface the fluid is pushing on

We can cleverly multiply and divide our equation by volume:

$$P = \frac{mg}{A} = \frac{mg \cdot V}{A \cdot V} = \frac{m}{V} \cdot \frac{gV}{A}$$

Then, recognizing that density, ρ^* , is mass divided by volume, we can substitute:

$$P = \rho \cdot \frac{gV}{A}$$

Finally, if the volume of an object is the area of the base times the height (h), we can rewrite the equation as:

$$P = \frac{\rho g A h}{A} = \rho gh$$

* Note that physicists use the Greek letter ρ ("rho") for density. You need to pay careful attention to the difference between the Greek letter ρ and the Roman letter "p".

Use this space for summary and/or additional notes.

Finally, if there is an external pressure, P_o , above the fluid, we have to add it to the pressure from the fluid itself, which gives us the equation:

$$P = P_o + \rho gh$$

where:

P_o = pressure above the fluid (if relevant)

ρ = density of the fluid (this is the Greek letter "rho")

g = acceleration due to gravity ($10 \frac{\text{m}}{\text{s}^2}$ on Earth)

h = height of the fluid above the point of interest

Sample Problem

Q: What is the water pressure in the ocean at a depth of 25 m? The density of sea water is $1025 \frac{\text{kg}}{\text{m}^3}$.

A: $P = \rho gh = (1025 \frac{\text{kg}}{\text{m}^3})(10 \frac{\text{m}}{\text{s}^2})(25 \text{m}) = 256250 \text{ Pa} = 2.56 \text{ bar}$

Use this space for summary and/or additional notes.

Homework Problems

For all problems, assume that the density of fresh water is $1000 \frac{\text{kg}}{\text{m}^3}$.

1. A diver dives into a swimming pool and descends to a maximum depth of 3.0 m. What is the pressure on the diver due to the water at this depth? Give your answer in both pascals (Pa) and in bar.

Answer: 30 000 Pa or 0.3 bar

- The specifications for the wet/dry vacuum cleaner that Mr. Bigler used for his hovercraft state that it is capable of creating enough of a pressure difference to lift a column of water to a height of 1.5 m. How much pressure can the vacuum cleaner apply?

Answer: 15 000 Pa

- A standard water tower is 40 m above the ground. What is the resulting water pressure at ground level? Express your answer in pascals, bar, and pounds per square inch. (1 bar \approx 14.7 psi)

Answer: 400 000 Pa, 4 bar, or 58.8 psi

Use this space for summary and/or additional notes.

Buoyancy

Unit: Pressure & Fluid Mechanics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding:

- Buoyancy & Archimedes' Principle

Skills:

- Calculate the buoyant force on an object

Language Objectives:

- Understand and correctly use the terms “displace” and “buoyant” or “buoyancy.”
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems relating to buoyancy.

Labs, Activities & Demonstrations:

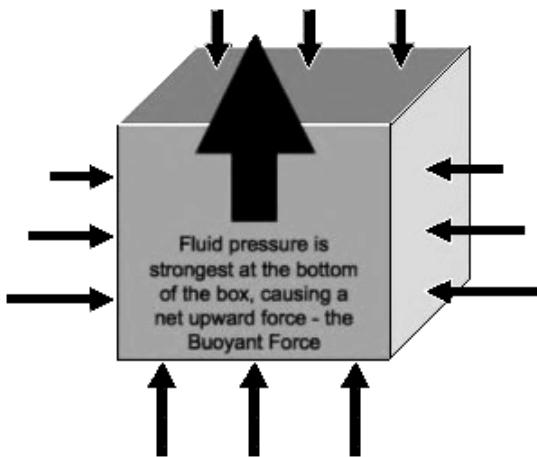
- Upside-down beaker with tissue
- Ping-pong ball or balloon under water
- beaker floating in water
 - right-side-up with weights
 - upside-down with trapped air
- Spring scale with mass in & out of water on a balance
- Cartesian diver
- Aluminum foil & weights
- Cardboard & duct tape canoes

Notes:

displace: to push out of the way

Use this space for summary and/or additional notes.

buoyancy: a net upward force caused by the differences in hydrostatic pressure at different levels within a fluid:



The hydrostatic pressure is stronger at the bottom of the object than at the top, which causes a net upward force on the object.

When an object displaces a fluid:

1. The volume of the fluid displaced equals the volume of the submerged part of the object: $V_d = V_{submerged}$
2. The weight of the fluid displaced equals the buoyant force (F_B).
3. The net force on the object, if any, is the difference between its weight and the buoyant force: $F_{net} = F_g - F_B$.

The buoyant force is caused by the difference in hydrostatic pressure from the bottom of the object to the top, *i.e.*, $F_B = \rho A_d h$. However, the hydrostatic pressure is a function of the depth and the area is a function of the shape of the object. By manipulating the equations and recognizing that area times height equals volume, we get the equation for the buoyant force:

$$F_B = \rho V_d g$$

Where F_B is the buoyant force, ρ is the density of the fluid that the object is submerged in, V_d is the volume of the fluid that has been displaced, and g is the strength of the gravitational field (which is the same as the acceleration due to gravity, $10 \frac{m}{s^2}$).

Use this space for summary and/or additional notes.

If the object floats, there is no net force, which means the weight of the object is equal to the buoyant force. This means:

$$F_g = F_B$$
$$mg = \rho V_d g$$

Cancelling g from both sides gives $m = \rho V_d$, which can be rearranged to give the equation for density:

$$\rho = \frac{m}{V_d}$$

If the object sinks, the weight of the object is greater than the buoyant force.

This means $F_B = \rho V_d g$, $F_g = mg$, and the weight of the submerged object is $F_{net} = F_g - F_B$.



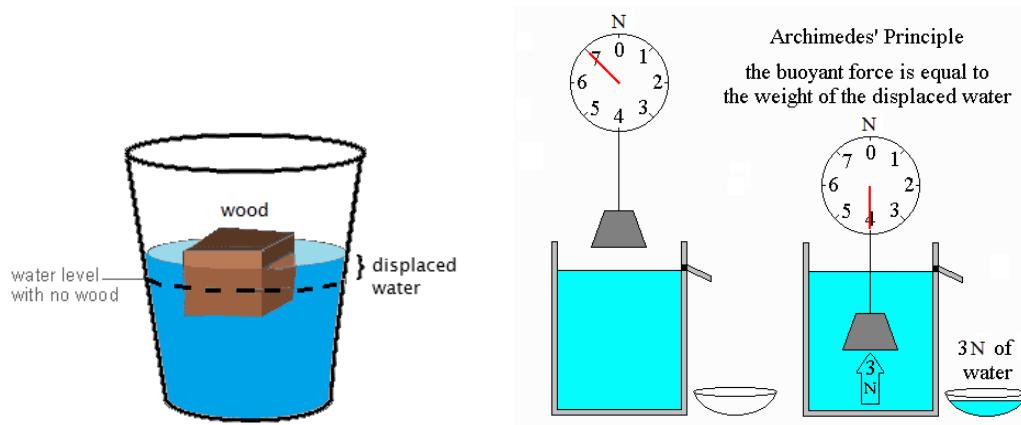
Note that if the object is resting on the bottom of the container, the net force must be zero, which means the normal force and the buoyant force combine to supply the total upward force. I.e., for an object resting on the bottom:

$$F_{net} = 0 = F_g - (F_B + F_N)$$

which means:

$$F_g = F_B + F_N$$

Use this space for summary and/or additional notes.



This concept is known as Archimedes' Principle, named for the ancient Greek scientist who discovered it.

The buoyant force can be calculated from the following equation:

$$F_B = m_d g = \rho V_d g$$

where:

- F_B = buoyant force
- m_d = mass of fluid displaced by the object
- g = acceleration due to gravity ($10 \frac{m}{s^2}$ on Earth)
- ρ = density of the fluid applying the buoyant force (e.g., water, air)
- V_d = volume of fluid displaced by the object

Use this space for summary and/or additional notes:

Sample Problems:

Q: A cruise ship displaces 35 000 tonnes of water when it is floating.

(1 tonne = 1 000 kg) If sea water has a density of $1\ 025 \frac{\text{kg}}{\text{m}^3}$, what volume of water does the ship displace? What is the buoyant force on the ship?

A: $\rho = \frac{m}{V}$

$$1\ 025 \frac{\text{kg}}{\text{m}^3} = \frac{35\ 000\ 000 \text{ kg}}{V}$$

$$V = 34\ 146 \text{ m}^3$$

$$F_B = \rho V_d g$$

$$F_B = (1\ 025 \frac{\text{kg}}{\text{m}^3})(34\ 146 \text{ m}^3)(10 \frac{\text{m}}{\text{s}^2})$$

$$F_B = [3.5 \times 10^8 \text{ N}]$$

Use this space for summary and/or additional notes.

Homework Problems

1. 1. A block is 0.12 m wide, 0.07 m long and 0.09 m tall and has a mass of 0.50 kg. The block is floating in water with a density of $1000 \frac{\text{kg}}{\text{m}^3}$.
 - a. What volume of the block is below the surface of the water?

Answer: $5 \times 10^{-4} \text{ m}^3$

- b. If the entire block were pushed under water, what volume of water would it displace?

Answer: $7.56 \times 10^{-4} \text{ m}^3$

- c. How much additional mass could be piled on top of the block before it sinks?

Answer: 0.256 kg

Use this space for summary and/or additional notes.

2. The SS United Victory was a cargo ship launched in 1944. The ship had a mass of 15 200 tonnes fully loaded. (1 tonne = 1 000 kg). The density of sea water is $1025 \frac{\text{kg}}{\text{m}^3}$. What volume of sea water did the SS United Victory displace when fully loaded?

Answer: $14\ 829\ \text{m}^3$

3. An empty box is 0.11 m per side. It will slowly be filled with sand that has a density of $3500 \frac{\text{kg}}{\text{m}^3}$. What volume of sand will cause the box to sink in water? (Assume water has a density of $1000 \frac{\text{kg}}{\text{m}^3}$. You may neglect the weight of the box.)

Answer: $3.80 \times 10^{-4}\ \text{m}^3$

Use this space for summary and/or additional notes.

Gas Laws

Unit: Pressure & Fluid Mechanics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding:

- Boyle's Law (pressure vs. volume)
- Amontons' Law (pressure vs. temperature)
- Charles' Law (temperature vs. volume)
- Avogadro's Principle (number of particles vs. volume)
- ideal gas law
- combined gas law

Skills:

- Solve problems using the gas laws

Language Objectives:

- Understand and correctly use the terms "pressure," "volume," and "temperature," and "ideal gas."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems using the combined gas law and the ideal gas law.

Labs, Activities & Demonstrations:

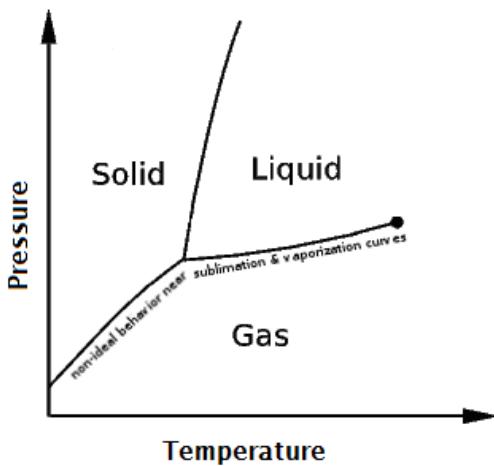
- Vacuum pump (pressure & volume) with:
 - balloon (air vs. water)
 - shaving cream
- Absolute zero apparatus (pressure & temperature)
- Can crush (pressure, volume & temperature)

Use this space for summary and/or additional notes.

Notes:

ideal gas: a gas that behaves as if each molecule acts independently, according to kinetic-molecular theory. Specifically, this means the molecules are far apart, and move freely in straight lines at constant speeds. When the molecules collide, the collisions are perfectly elastic, which means they bounce off each other with no energy or momentum lost.

Most gases behave ideally except at temperatures and pressures near the vaporization curve on a phase diagram. (*I.e.*, gases stop behaving ideally when conditions are close to those that would cause the gas to condense to a liquid or solid.)

**Boyle's Law**

In 1662, British physicist and chemist Robert Boyle published his findings that the pressure and volume of a gas were inversely proportional. If temperature and the number of particles of gas are constant, then for an ideal gas:

$$P_1 V_1 = P_2 V_2$$

(Note that by convention, gas laws use subscripts “1” and “2” instead of “i” and “f”.)

Use this space for summary and/or additional notes.

Charles' Law

In the 1780s, French physicist Jacques Charles discovered that the volume and temperature of a gas were directly proportional. If pressure and the number of particles are constant, then for an ideal gas:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Amontons' Law

In 1702, French physicist Guillaume Amontons discovered that the pressure and temperature of a gas were directly proportional. If volume and the number of particles are constant, then for an ideal gas:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

(This law is often erroneously attributed to the French chemist Joseph Louis Gay-Lussac.)

Avogadro's Principle

In 1811, Italian physicist Amedeo Avogadro (whose full name was Lorenzo Romano Amedeo Carlo Avogadro di Quaregna e di Cerreto) published the principle that equal volumes of an ideal gas at the same temperature and pressure must contain equal numbers of molecules:

$$\frac{V_1}{N_1} = \frac{V_2}{N_2}$$

Use this space for summary and/or additional notes.

The Combined Gas Law

If each of the above principles is combined, the following relationship is found for an ideal gas:

$$\frac{P_1 V_1}{N_1 T_1} = \frac{P_2 V_2}{N_2 T_2} = \text{constant}$$

If pressure is in Pa, volume is in m³, and temperature is in Kelvin, the value of the constant turns out to be $1.38 \times 10^{-23} \frac{\text{J}}{\text{K}}$. This number is called Boltzmann's constant, named for the German physicist Ludwig Boltzmann. Physicists usually use the variable k_B to represent Boltzmann's constant.

Note, however, that in most problems, the number of particles of gas doesn't change. This means $N_1 = N_2$ and we can cancel it from the equation, which gives:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

This equation is called the "combined gas law", which is used to solve most "before/after" problems involving ideal gases.

When using the combined gas law, any quantity that is not changing may therefore be cancelled out of the equation. (If a quantity is not mentioned in the problem, you can assume that it is constant and may be cancelled.) For example, suppose a problem doesn't mention anything about temperature. That means T is constant and you can cancel it. When you cancel T from both sides of the combined gas law, you get:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \text{ which simplifies to } P_1 V_1 = P_2 V_2 \text{ (Boyle's Law)}$$

Use this space for summary and/or additional notes.

Solving Problems Using the Combined Gas Law

You can use this method to solve any “before/after” gas law problem:

1. Determine which variables you have
2. Determine which values are initial (#1) vs. final (#2).
3. Start with the combined gas law and cancel any variables that are explicitly not changing or omitted (assumed not to be changing).
4. Substitute your numbers into the resulting equation and solve. (Make sure all initial and final quantities have the same units, and don’t forget that temperatures must be in Kelvin!)

The Ideal Gas Law

The definition of Boltzmann’s constant is called the ideal gas law:

$$\frac{PV}{NT} = k_B$$

Multiplying both sides of this equation by NT gives the ideal gas law in its most common form:

$$PV = Nk_B T$$

where:

P = absolute pressure (Pa)

V = volume (m^3)

N = number of particles (or molecules)

k_B = Boltzmann’s constant ($1.38 \times 10^{-23} \frac{J}{K}$).

T = absolute temperature (K)

Note that the units on both sides of this equation are equivalent to Newton-meters, which are equivalent to joules (energy). This means that the ideal gas law is really just a mathematical statement of the work-energy theorem.

Use this space for summary and/or additional notes.

The left side of the equation, PV , represents the work that the gas can do on its surroundings. This is because:

$$P = \frac{F}{A}, \text{ which means } PV = \frac{FV}{A} = Fd = W$$

The right side of the equation, $Nk_B T$, represents the number of particles (N) times the average kinetic energy of each particle (T), which equals the total kinetic energy of the particles. (Boltzmann's constant, k_B , is just the number that makes the units work out correctly.)

In other words, the work that a gas can do equals the kinetic energy of its particles.

Chemists more commonly work with moles rather than particles. In chemistry, the ideal gas law is more commonly written:

$$PV = nRT$$

where:

n = moles of gas

R = gas constant ($8.31 \frac{\text{J}}{\text{mol}\cdot\text{K}}$)

P , V , and T are as above

The gas constant, R , is equal to Boltzmann's constant times the conversion from moles to particles (Avogadro's constant):

$$R = k_B N_A = (1.38 \times 10^{-23} \frac{\text{J}}{\text{K}})(6.02 \times 10^{23} \text{ mol}^{-1}) = 8.31 \frac{\text{J}}{\text{mol}\cdot\text{K}}$$

Solving Problems Using the Ideal Gas Law

If a gas behaves according to the ideal gas law, simply substitute the values for pressure, volume, number of particles (or number of moles), and temperature into the equation. Be sure your units are correct (especially that temperature is in Kelvin), and that you use the correct constant, depending on whether you know the number of particles or the number of moles of the gas.

Use this space for summary and/or additional notes.

Sample problems:

Q: A sample of an ideal gas contains 2.10×10^{24} particles, has a pressure of 1.20 bar, and a temperature of 35°C. What is its volume?

A: First, we need to declare our variables.

Note that for the units to work out correctly (because of the units of Boltzmann's constant), pressure must be in Pa, volume must be in m³, and temperature must be in Kelvin.

$$P = 1.20 \text{ bar} \times \frac{100000 \text{ Pa}}{1 \text{ bar}} = 120000 \text{ Pa}$$

$V = V$ (because we don't know it yet)

$$N = 2.1 \times 10^{24}$$

$$k_B = 1.38 \times 10^{-23} \frac{\text{m}^2 \cdot \text{kg}}{\text{s}^2 \cdot \text{K}}$$

$$T = 35^\circ\text{C} + 273 = 308 \text{ K}$$

Then we substitute these numbers into the ideal gas law and solve:

$$\begin{aligned} PV &= Nk_B T \\ (120000 \text{ Pa})V &= (2.10 \times 10^{24})(1.38 \times 10^{-23} \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2 \cdot \text{K}})(308 \text{ K}) \\ 120000 V &= 8926 \\ V &= 0.0744 \text{ m}^3 \end{aligned}$$

Use this space for summary and/or additional notes.

Q: A gas has a temperature of 25°C and a pressure of 1.5 bar. If the gas is heated to 35°C, what will the new pressure be?

A: 1. Find which variables we have.

We have two temperatures (25°C and 35°C), and two pressures (1.5 bar and the new pressure that we're looking for).

1. Find the action being done on the gas ("heated"). Anything that was true about the gas *before* the action is time "1", and anything that is true about the gas *after* the action is time "2".

Time 1 ("before"):

$$P_1 = 1.5 \text{ bar}$$

$$T_1 = 25^\circ\text{C} + 273 = 298 \text{ K}$$

Time 2 ("after"):

$$P_2 = P_2$$

$$T_2 = 35^\circ\text{C} + 273 = 308 \text{ K}$$

2. Set up the formula. We can cancel volume (V), because the problem doesn't mention it:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \text{ which gives us } \frac{P_1}{T_1} = \frac{P_2}{T_2} \text{ (Amontons' Law)}$$

3. Plug in our values and solve:

$$\frac{1.5 \text{ bar}}{298 \text{ K}} = \frac{P_2}{308 \text{ K}} \rightarrow P_2 = 1.55 \text{ bar}$$

Use this space for summary and/or additional notes.

Homework Problems

For these problems, you may either convert kilograms to grams to moles (as you did in chemistry) and use $PV = nRT$, or you may convert kilograms to molecules directly using the atomic mass, and use $PV = Nk_B T$. ($1 \text{ amu} = 1.66 \times 10^{-27} \text{ kg}$)

1. A sample of 0.032 kg of oxygen (atomic mass 16.0 amu; formula O₂) at 50.0°C and 98 600 Pa occupies what volume?

Answer: 0.0272 m³

- When the pressure in a certain gas cylinder with a volume of 0.0045 m³ reaches 500. bar, the cylinder is likely to explode. What would the pressure in this cylinder be if it contained 3.2 kg of argon gas (atomic mass 40.0 amu) at 25.0°C? Would it explode?

Answer: 440 bar; No.

- A sample of oxygen gas occupies a volume of 0.250 m³ at a pressure of 1.75 bar. What volume will it occupy at 2.50 bar?

Answer: 0.175 m³

- H₂ gas was cooled from 150.°C to 50.°C. Its new pressure is 75 000 Pa. What was its original pressure?

Answer: 98 220 Pa

Use this space for summary and/or additional notes.

5. A sample of air has a volume of 60.0 mL at 1.00 bar and 0.0°C. What volume will the sample have at 55.0°C and 0.95 bar?

Answer: 75.88 mL

6. A scuba diver's 10. L air tank is filled to a pressure of 210 bar at a dockside temperature of 32.0°C. The water temperature is 8.0°C, the diver is swimming at a depth of 11.3 m, and the barometric (atmospheric) pressure is 1.013 bar. The density of sea water is $1025 \frac{\text{kg}}{\text{m}^3}$.
- a. What is the total pressure on the diver?

214 808 Pa or 2.14 bar

- b. How many liters of air does the diver use?

899.9 L

- c. If the diver uses air at the rate of 8.0 L/min, how long will the diver's air last?

112.5 min

Use this space for summary and/or additional notes.

Fluid Motion & Bernoulli's Law

Unit: Pressure & Fluid Mechanics

NGSS Standards: N/A

MA Curriculum Frameworks (2006): N/A

Knowledge/Understanding:

- Bernoulli's Law/Equation

Language Objectives:

- Understand and correctly use the term "flow."
- Accurately describe and apply the concepts described in this section using appropriate academic language.
- Set up and solve word problems using Bernoulli's equation.

Labs, Activities & Demonstrations:

- Blow across paper (unfolded & folded)
- Blow between two empty cans.
- Ping-pong ball and air blower (without & with funnel)
- Venturi tube
- Compressed air blowing across Styrofoam
- Leaf blower & large ball

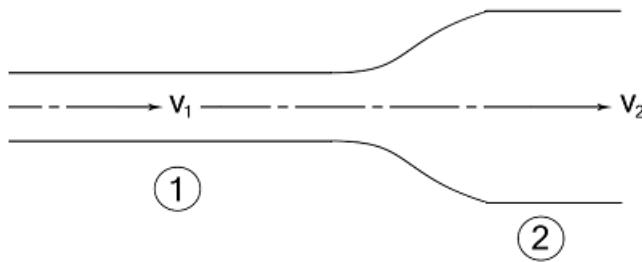
Notes:

flow: the net movement of a fluid

velocity of a fluid: the average velocity of a particle of fluid as the fluid flows past a reference point.

Use this space for summary and/or additional notes.

When a fluid flows through a pipe, the total volume of fluid per unit of time ($\frac{m^3}{s}$) is equal to the velocity ($\frac{m}{s}$) times the cross-sectional area (m^2). If the total flow remains the same, but the diameter of the pipe changes:



then the cross-sectional area (A) times the fluid velocity (v) at point 1 equals the cross-sectional area times the fluid velocity at point 2:

$$A_1v_1 = A_2v_2$$

According to the Dutch-Swiss mathematician Daniel Bernoulli, the pressures in a moving fluid are caused by:

- The pressure exerted by the fluid, P (This is the pressure we would measure with a pressure gauge.)
- The hydrostatic pressure, ρgh .
- The dynamic pressure, $\frac{1}{2}\rho v^2$, which results from the force that the moving fluid particles exert on the other fluid particles around them. (Note that $\frac{1}{2}\rho v^2$ has units of pressure: $\left(\frac{kg}{m^3}\right) \cdot \left(\frac{m}{s}\right)^2 = \frac{kg}{m \cdot s^2} = Pa$.)

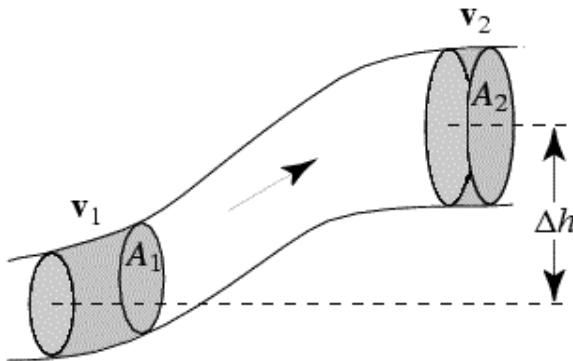
A change in any of these pressures affects the others, which means:

$$P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$$

This equation is Bernoulli's equation.

Use this space for summary and/or additional notes.

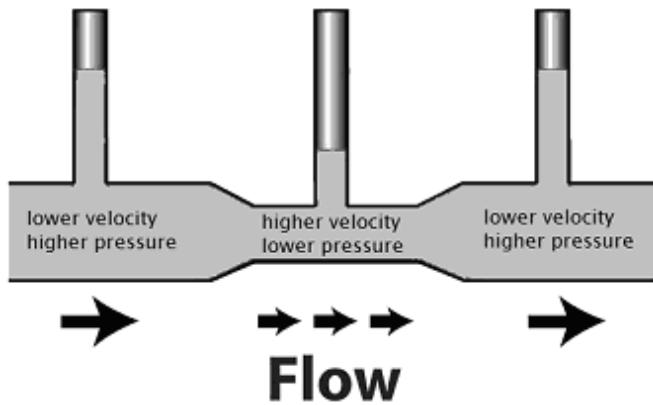
If the velocity and height of the fluid are changing, as in the following diagram, then the pressure must also change as a result:



In this situation, Bernoulli's equation becomes:

$$P_1 + \rho gh_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho gh_2 + \frac{1}{2} \rho v_2^2$$

In Bernoulli's equation, increasing the fluid velocity (v) increases the $\frac{1}{2} \rho v^2$ term. Consider the following example:

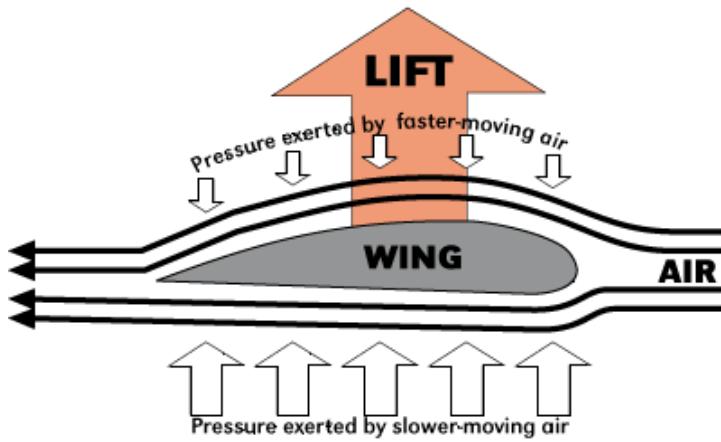


This pipe is horizontal, which means h is constant; therefore ρgh is constant.

This means that if $\frac{1}{2} \rho v^2$ increases, then pressure (P) must decrease so that $P + \rho gh + \frac{1}{2} \rho v^2$ remains constant.

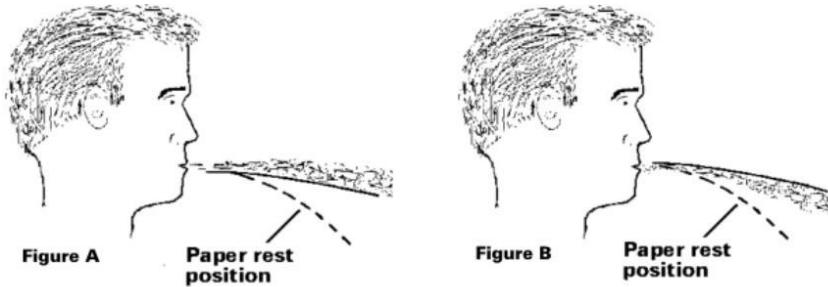
Use this space for summary and/or additional notes.

This decrease in pressure caused by an increase in fluid velocity explains one of the ways in which an airplane wing provides lift:



Of course, most of an airplane's lift comes from the fact that the wing is inclined with an angle of attack relative to its direction of motion.

A common demonstration of Bernoulli's Law is to blow across a piece of paper:



The air moving across the top of the paper causes a decrease in pressure, which causes the paper to lift.

Use this space for summary and/or additional notes.

Sample Problems:

Q: A fluid in a pipe with a diameter of 0.40 m is moving with a velocity of $0.30 \frac{\text{m}}{\text{s}}$. If the fluid moves into a second pipe with half the diameter, what will the new fluid velocity be?

A: The cross-sectional area of the first pipe is:

$$A_1 = \pi r^2 = (3.14)(0.20)^2 = 0.126 \text{ m}^2$$

The cross-sectional area of the second pipe is:

$$A_2 = \pi r^2 = (3.14)(0.10)^2 = 0.0314 \text{ m}^2$$

$$A_1 v_1 = A_2 v_2$$

$$(0.126)(0.30) = (0.0314)v_2$$

$$v_2 = \boxed{1.2 \frac{\text{m}}{\text{s}}}.$$

Q: A fluid with a density of $1250 \frac{\text{kg}}{\text{m}^3}$ has a pressure of 45 000 Pa as it flows at $1.5 \frac{\text{m}}{\text{s}}$ through a pipe. The pipe rises to a height of 2.5 m, where it connects to a second, smaller pipe. What is the pressure in the smaller pipe if the fluid flows at a rate of $3.4 \frac{\text{m}}{\text{s}}$ through it?

A:

$$P_1 + \rho gh_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho gh_2 + \frac{1}{2} \rho v_2^2$$

$$45\,000 + (1250)(10)(0) + (\frac{1}{2})(1250)(1.5)^2 =$$

$$P_2 + (1250)(10)(2.5) + (\frac{1}{2})(1250)(3.4)^2$$

$$45\,000 + 1406 = P_2 + 31\,250 + 7225$$

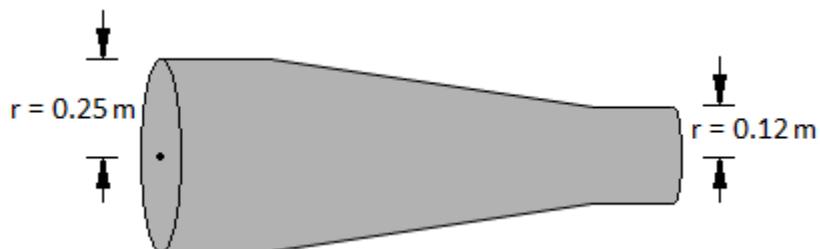
$$46\,406 = P_2 + 38\,475$$

$$P_2 = \boxed{7931 \text{ Pa}}$$

Use this space for summary and/or additional notes.

Homework Problems

1. A pipe has a radius of 0.25 m at the entrance and a radius of 0.12 m at the exit, as shown in the figure below:



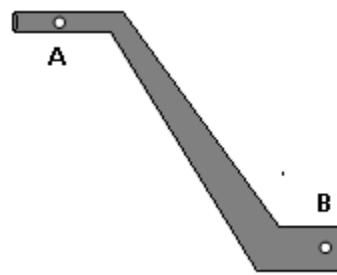
If the fluid in the pipe is flowing at $5.2 \frac{\text{m}}{\text{s}}$ at the inlet, then how fast is it flowing at the outlet?

Answer: $22.6 \frac{\text{m}}{\text{s}}$

Use this space for summary and/or additional notes.

2. At point A on the pipe to the right, the water's speed is $4.8 \frac{\text{m}}{\text{s}}$ and the pressure is 52.0 kPa. The water drops 14.8 m to point B, where the pipe's cross sectional area is twice that at point A.

- a. Calculate the velocity of the water at point B.



Answer: $2.4 \frac{\text{m}}{\text{s}}$

- b. Calculate the pressure at point B.

Answer: 50.6 kPa

Use this space for summary and/or additional notes.

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Table A. Metric Prefixes

Factor	Prefix	Symbol
1 000 000 000 000 000 000 000 000	10^{24}	yotta
1 000 000 000 000 000 000 000	10^{21}	zeta
1 000 000 000 000 000 000	10^{18}	exa
1 000 000 000 000 000 000 000	10^{15}	peta
1 000 000 000 000 000	10^{12}	tera
1 000 000 000	10^9	giga
1 000 000	10^6	mega
1 000	10^3	kilo
100	10^2	hecto
10	10^1	deca
1	10^0	—
0.1	10^{-1}	deci
0.01	10^{-2}	centi
0.001	10^{-3}	milli
0.000 001	10^{-6}	micro
0.000 000 001	10^{-9}	nano
0.000 000 000 001	10^{-12}	pico
0.000 000 000 000 001	10^{-15}	femto
0.000 000 000 000 000 001	10^{-18}	atto
0.000 000 000 000 000 000 001	10^{-21}	zepto
0.000 000 000 000 000 000 000 001	10^{-24}	yocto

Table B. Physical Constants

Description	Symbol	Precise Value	Common Approximation
universal gravitational constant	G	$6.67384(80) \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$	$6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$
acceleration due to gravity on Earth's surface	g	9.7639 $\frac{\text{m}}{\text{s}^2}$ to 9.8337 $\frac{\text{m}}{\text{s}^2}$ average value at sea level is 9.80665 $\frac{\text{m}}{\text{s}^2}$	9.8 $\frac{\text{m}}{\text{s}^2}$ or 10 $\frac{\text{m}}{\text{s}^2}$
speed of light in a vacuum	c	$299792458 \frac{\text{m}}{\text{s}}$ *	$3.00 \times 10^8 \frac{\text{m}}{\text{s}}$
elementary charge (proton or electron)	e	$\pm 1.602176565(35) \times 10^{-19} \text{ C}$	$\pm 1.6 \times 10^{-19} \text{ C}$
1 coulomb (C)		$6.24150965(16) \times 10^{18}$ elementary charges	6.24×10^{18} elementary charges
(electric) permittivity of a vacuum	ϵ_0	$8.85418782 \times 10^{-12} \frac{\text{A}^2 \cdot \text{s}^4}{\text{kg} \cdot \text{m}^3}$	$8.85 \times 10^{-12} \frac{\text{A}^2 \cdot \text{s}^4}{\text{kg} \cdot \text{m}^3}$
(magnetic) permeability of a vacuum	μ_0	$4\pi \times 10^{-7} = 1.25663706 \times 10^{-6} \frac{\text{T}\cdot\text{m}}{\text{A}}$	$1.26 \times 10^{-6} \frac{\text{T}\cdot\text{m}}{\text{A}}$
electrostatic constant	k	$\frac{1}{4\pi\epsilon_0} = 8.9875517873681764 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$ *	$9.0 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$
1 electron volt (eV)		$1.602176565(35) \times 10^{-19} \text{ J}$	$1.6 \times 10^{-19} \text{ J}$
Planck's constant	h	$6.62606957(29) \times 10^{-34} \text{ J}\cdot\text{s}$	$6.6 \times 10^{-34} \text{ J}\cdot\text{s}$
1 universal (atomic) mass unit (u)		$931494061(21) \text{ MeV}/c^2$ $1.660538921(73) \times 10^{-27} \text{ kg}$	$931 \text{ MeV}/c^2$ $1.66 \times 10^{-27} \text{ kg}$
Avogadro's constant	N_A	$6.02214129(27) \times 10^{23} \text{ mol}^{-1}$	$6.02 \times 10^{23} \text{ mol}^{-1}$
Boltzmann constant	k_B	$1.3806488(13) \times 10^{-23} \frac{\text{J}}{\text{K}}$	$1.38 \times 10^{-23} \frac{\text{J}}{\text{K}}$
universal gas constant	R	$8.3144621(75) \frac{\text{J}}{\text{mol}\cdot\text{K}}$	$8.31 \frac{\text{J}}{\text{mol}\cdot\text{K}}$
Rydberg constant	R_H	$\frac{m_e e^4}{8\epsilon_0^2 h^3 c} = 10973731.6 \frac{1}{\text{m}}$	$1.1 \times 10^7 \text{ m}^{-1}$
standard atmospheric pressure at sea level		101 325 Pa \equiv 1.01325 bar*	100 000 Pa \equiv 1.0 bar
rest mass of an electron	m_e	$9.10938215(45) \times 10^{-31} \text{ kg}$	$9.11 \times 10^{-31} \text{ kg}$
mass of a proton	m_p	$1.672621777(74) \times 10^{-27} \text{ kg}$	$1.67 \times 10^{-27} \text{ kg}$
mass of a neutron	m_n	$1.674927351(74) \times 10^{-27} \text{ kg}$	$1.67 \times 10^{-27} \text{ kg}$

* denotes an exact value (by definition)

Table C. Approximate Coefficients of Friction

Substance	Static (μ_s)	Kinetic (μ_k)	Substance	Static (μ_s)	Kinetic (μ_k)
rubber on concrete (dry)	0.90	0.68	wood on wood (dry)	0.42	0.30
rubber on concrete (wet)		0.58	wood on wood (wet)	0.2	
rubber on asphalt (dry)	0.85	0.67	wood on metal	0.3	
rubber on asphalt (wet)		0.53	wood on brick	0.6	
rubber on ice		0.15	wood on concrete	0.62	
steel on ice	0.03	0.01	Teflon on Teflon	0.04	0.04
waxed ski on snow	0.14	0.05	Teflon on steel	0.04	0.04
aluminum on aluminum	1.2	1.4	graphite on steel	0.1	
cast iron on cast iron	1.1	0.15	leather on wood	0.3–0.4	
steel on steel	0.74	0.57	leather on metal (dry)	0.6	
copper on steel	0.53	0.36	leather on metal (wet)	0.4	
diamond on diamond	0.1		glass on glass	0.9–1.0	
diamond on metal	0.1–0.15		metal on glass	0.5–0.7	0.4

Appendix: Reference Tables

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Table D. Quantities, Variables and Units

Quantity	Variable	MKS Unit Name	MKS Unit Symbol	S.I. Base Unit
position	\vec{x}	meter*	m	m
distance/displacement, (length, height)	$d, \vec{d}, (\ell, h)$	meter*	m	m
angle	ϑ	radian, degree	$\text{—}, {}^\circ$	—
area	A	square meter	m^2	m^2
volume	V	cubic meter, liter	$\text{m}^3, \ell, \text{L}$	m^3
time	t	second*	s	s
velocity	\vec{v}	meter/second	$\frac{\text{m}}{\text{s}}$	$\frac{\text{m}}{\text{s}}$
speed of light	c			
angular velocity	$\vec{\omega}$	radians/second	$\frac{1}{\text{s}}$	$\frac{1}{\text{s}}$
acceleration	\vec{a}	meter/second ²	$\frac{\text{m}}{\text{s}^2}$	$\frac{\text{m}}{\text{s}^2}$
acceleration due to gravity	\vec{g}			
mass	m	kilogram*	kg	kg
force	\vec{F}	newton	N	$\frac{\text{kg}\cdot\text{m}}{\text{s}^2}$
pressure	P	pascal	Pa	$\frac{\text{kg}}{\text{m}\cdot\text{s}^2}$
energy	E			
potential energy	U	joule	J	$\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$
heat	Q			
work	W	newton-meter	N·m	$\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$
torque	$\vec{\tau}$	newton-meter	N·m	$\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$
power	P	watt	W	$\frac{\text{kg}\cdot\text{m}^2}{\text{s}^3}$
momentum	\vec{p}	newton-second	N·s	$\frac{\text{kg}\cdot\text{m}}{\text{s}}$
impulse	\vec{j}			
moment of inertia	I	kilogram-meter ²	$\text{kg}\cdot\text{m}^2$	$\text{kg}\cdot\text{m}^2$
angular momentum	\vec{L}	newton-meter-second	N·m·s	$\frac{\text{kg}\cdot\text{m}^2}{\text{s}}$
frequency	f	hertz	Hz	s^{-1}
wavelength	λ	meter	m	m
period	T	second	s	s
index of refraction	n	—	—	—
electric current	\vec{I}	ampere*	A	A
electric charge	q	coulomb	C	A·s
potential difference (voltage)	V	volt	V	$\frac{\text{kg}\cdot\text{m}^2}{\text{A}\cdot\text{s}^3}$
electromotive force (emf)	ϵ			
electrical resistance	R	ohm	Ω	$\frac{\text{kg}\cdot\text{m}^2}{\text{A}^2\cdot\text{s}^3}$
capacitance	C	farad	F	$\frac{\text{A}^2\cdot\text{s}^4}{\text{m}^2\cdot\text{kg}}$
electric field	\vec{E}	netwon/coulomb volt/meter	$\frac{\text{N}}{\text{C}}, \frac{\text{V}}{\text{m}}$	$\frac{\text{kg}\cdot\text{m}}{\text{A}\cdot\text{s}^3}$
magnetic field	\vec{B}	tesla	T	$\frac{\text{kg}}{\text{A}\cdot\text{s}^2}$
temperature	T	kelvin*	K	K
amount of substance	n	mole*	mol	mol
luminous intensity	I_v	candela*	cd	cd

Variables representing vector quantities are typeset in ***bold italics***. * = S.I. base unit

Table E. Mechanics Formulas and Equations

Kinematics (Distance, Velocity & Acceleration)	$\vec{d} = \Delta\vec{x} = \vec{x} - \vec{x}_o$	$\Delta = \text{change, difference}$ $\Sigma = \text{sum}$ $d = \text{distance (m)}$ $\vec{d} = \text{displacement (m)}$ $\vec{x} = \text{position (m)}$ $s = \text{arc length (m)}$ $t = \text{time (s)}$ $\vec{v} = \text{velocity } (\frac{\text{m}}{\text{s}})$ $\bar{v} = \text{average velocity } (\frac{\text{m}}{\text{s}})$ $\vec{a} = \text{acceleration } (\frac{\text{m}}{\text{s}^2})$ $f = \text{frequency } (\text{Hz} = \frac{1}{\text{s}})$ $a_c = \text{centripetal acceleration } (\frac{\text{m}}{\text{s}^2})$ $\vec{F} = \text{force (N)}$ $F_f = \text{force due to friction (N)}$ $\vec{F}_g = \text{force due to gravity (N)}$ $F_N = \text{normal force (N)}$ $F_c = \text{centripetal force (N)}$ $m = \text{mass (kg)}$ $\vec{g} = \text{acceleration due to gravity } (\frac{\text{m}}{\text{s}^2})$
	$\bar{v} = \frac{\vec{d}}{t} = \frac{\Delta\vec{x}}{\Delta t} = \frac{\vec{v}_o + \vec{v}}{2}$	
	$\Delta\vec{v} = \vec{v} - \vec{v}_o = \vec{a}t$	
	$\vec{x} - \vec{x}_o = \vec{d} = \vec{v}_o t + \frac{1}{2}\vec{a}t^2$	
Circular Motion	$\vec{v}_T = r\vec{\omega}$	$\vec{v}^2 - \vec{v}_o^2 = 2\vec{a}d$ $s = r\Delta\theta$ $a_c = \frac{v^2}{r} = \omega^2 r$ $\theta - \theta_0 = \vec{\omega}_0 t + \frac{1}{2}\vec{a}t^2$
	$F_f \leq \mu_s F_N$	
	$\vec{F}_g = \frac{Gm_1 m_2}{r^2}$	
Forces & Dynamics	$\sum \vec{F} = \vec{F}_{net} = m\vec{a}$	$\vec{F}_g = m\vec{g}$ $F_f = \mu_k F_N$ $F_c = ma_c = \frac{mv^2}{r}$ $\vec{r} = \vec{r} \times \vec{F}$ $\tau = rF \sin\theta = r_\perp F$ $\sum \vec{\tau} = \vec{\tau}_{net} = I\vec{\alpha}$
	$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$	
	$I = \int_0^m r^2 dm = mr^2$	
	$F_c = ma_c = \frac{mv^2}{r}$	
	$\vec{r} = \vec{r} \times \vec{F}$	
	$\tau = rF \sin\theta = r_\perp F$	
Rotational Dynamics	$T = \frac{2\pi}{\omega} = \frac{1}{f}$	$T_s = 2\pi\sqrt{\frac{m}{k}}$ $T_p = 2\pi\sqrt{\frac{L}{g}}$ $\vec{F}_s = -k\vec{x}$ $U_s = \frac{1}{2}kx^2$
	$G = \text{gravitational constant } (\frac{\text{Nm}^2}{\text{kg}^2})$	
	$r = \text{radius (m)}$	
	$\vec{r} = \text{radius (vector)}$	
	$\mu = \text{coefficient of friction (dimensionless)}$	
	$\theta = \text{angle } (^{\circ}, \text{rad})$	
Simple Harmonic Motion	$\omega = \text{angular velocity } (\frac{\text{rad}}{\text{s}})$	$k = \text{spring constant } (\frac{\text{N}}{\text{m}})$ $\vec{x} = \text{displacement of spring (m)}$ $L = \text{length of pendulum (m)}$ $\vec{\tau} = \text{torque (N·m)}$ $K = \text{kinetic energy (J)}$ $U = \text{potential energy (J)}$
	$\vec{p} = m\vec{v}$	
	$\sum m_i \vec{v}_i = \sum m_f \vec{v}_f$	
	$\vec{J} = \Delta\vec{p} = \vec{F}_{net} \Delta t$	
	$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$	
	$L = rp \sin\theta = I\omega$	
Momentum	$\Delta\vec{L} = \vec{\tau} \Delta t$	$W = \vec{F} \bullet \vec{d} = Fd \cos\theta = F_{ } d$ $W = \tau \Delta\theta$ $U_g = mgh = \frac{Gm_1 m_2}{r}$ $E_k = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ $E_k = \frac{1}{2}I\omega^2$ $E_{total} = U + E_k + Q$ $W = \Delta K + \Delta U$ $P = \frac{W}{t} = \vec{F} \bullet \vec{v} = Fv \cos\theta = \tau \omega$
	$W = \vec{F} \bullet \vec{d} = Fd \cos\theta = F_{ } d$	
	$W = \tau \Delta\theta$	
	$U_g = mgh = \frac{Gm_1 m_2}{r}$	
	$E_k = \frac{1}{2}mv^2 = \frac{p^2}{2m}$	
	$E_k = \frac{1}{2}I\omega^2$	
Energy, Work & Power	$E_{total} = U + E_k + Q$	$W = \Delta K + \Delta U$ $P = \frac{W}{t} = \vec{F} \bullet \vec{v} = Fv \cos\theta = \tau \omega$
	$W = \Delta K + \Delta U$	
	$P = \frac{W}{t} = \vec{F} \bullet \vec{v} = Fv \cos\theta = \tau \omega$	
	$W = \Delta K + \Delta U$	
	$P = \frac{W}{t} = \vec{F} \bullet \vec{v} = Fv \cos\theta = \tau \omega$	
	$W = \Delta K + \Delta U$	

Table F. Moments of Inertia

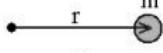
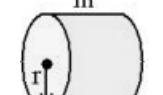
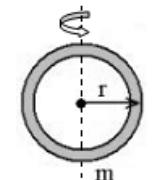
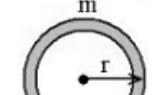
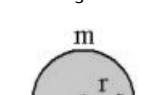
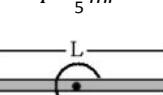
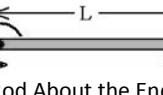
	Point Mass: $I = mr^2$
	Hollow Cylinder: $I = mr^2$
	Solid Cylinder: $I = \frac{1}{2}mr^2$
	Hoop About Diameter: $I = \frac{1}{2}mr^2$
	Hollow Sphere: $I = \frac{2}{3}mr^2$
	Solid Sphere: $I = \frac{2}{5}mr^2$
	Rod About the Middle: $I = \frac{1}{12}mL^2$
	Rod About the End: $I = \frac{1}{3}mL^2$

Table G. Heat and Thermal Physics Formulas and Equations

Temperature	$^{\circ}\text{F} = 1.8(^{\circ}\text{C}) + 32$ $\text{K} = ^{\circ}\text{C} + 273.15$	Δ = change $^{\circ}\text{F}$ = Fahrenheit temperature ($^{\circ}\text{F}$) $^{\circ}\text{C}$ = Celsius temperature ($^{\circ}\text{C}$) K = Kelvin temperature (K) Q = heat (J, kJ) m = mass (kg) C = specific heat capacity $(\frac{\text{kJ}}{\text{kg}\cdot^{\circ}\text{C}})$ (C_p = const. pressure; C_v = const. volume) T = temperature (K) t = time (s) L = length (m) k = coefficient of thermal conductivity $(\frac{\text{J}}{\text{m}\cdot\text{s}\cdot^{\circ}\text{C}}, \frac{\text{W}}{\text{m}\cdot^{\circ}\text{C}})$ V = volume (m^3) α = linear coefficient of thermal expansion ($^{\circ}\text{C}^{-1}$) β = volumetric coefficient of thermal expansion ($^{\circ}\text{C}^{-1}$) R_i = "R" value of insulation R = gas constant $(\frac{\text{J}}{\text{mol}\cdot\text{K}})$ U = internal energy (J) W = work ($\text{N}\cdot\text{m}$)
Heat	$Q = mC \Delta T$ $Q_{melt} = m \Delta H_{fus}$ $Q_{boil} = m \Delta H_{vap}$ $C_p - C_v = R$ $\Delta L = \alpha L_i \Delta T$ $\Delta V = \beta V_i \Delta T$ $\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad \frac{Q}{\Delta t} = kA \frac{\Delta T}{L}$ $\frac{Q}{t} = -\frac{1}{R_i} A \Delta T$	
Thermodynamics	$\Delta U = \Delta Q + \Delta W$ $K = \frac{3}{2} k_B T$ $\Delta U = \frac{3}{2} N k_B \Delta T = \frac{3}{2} n R \Delta T$ $W = -\Delta(PV)$	

Table H. Thermal Properties of Selected Materials

Substance	Melting Point ($^{\circ}\text{C}$)	Boiling Point ($^{\circ}\text{C}$)	Heat of Fusion ΔH_{fus} ($\frac{\text{kJ}}{\text{kg}}$)	Heat of Vaporization ΔH_{vap} ($\frac{\text{kJ}}{\text{kg}}$)	Specific Heat Capacity C_p ($\frac{\text{kJ}}{\text{kg}\cdot^{\circ}\text{C}}$) at 25°C	Thermal Conductivity k ($\frac{\text{J}}{\text{m}\cdot\text{s}\cdot^{\circ}\text{C}}$) at 25°C	Coefficients of Expansion at 20°C	
							Linear $\alpha (^{\circ}\text{C}^{-1})$	Volumetric $\beta (^{\circ}\text{C}^{-1})$
air (gas)	—	—	—	—	1.012	0.024	—	—
aluminum (solid)	659	2467	395	10460	0.897	250	2.3×10^{-5}	6.9×10^{-5}
ammonia (gas)	-75	-33.3	339	1369	4.7	0.024	—	—
argon (gas)	-189	-186	29.5	161	0.520	0.016	—	—
carbon dioxide (gas)	—	-78	—	574	0.839	0.0146	—	—
copper (solid)	1086	1187	134	5063	0.385	401	1.7×10^{-5}	5.1×10^{-5}
brass (solid)	—	—	—	—	0.380	120	1.9×10^{-5}	5.6×10^{-5}
diamond (solid)	3550	4827	10 000	30 000	0.509	2200	1×10^{-6}	3×10^{-6}
ethanol (liquid)	-117	78	104	858	2.44	0.171	2.5×10^{-4}	7.5×10^{-4}
glass (solid)	—	—	—	—	0.84	0.96–1.05	8.5×10^{-6}	2.55×10^{-5}
gold (solid)	1063	2660	64.4	1577	0.129	310	1.4×10^{-5}	4.2×10^{-5}
granite (solid)	1240	—	—	—	0.790	1.7–4.0	—	—
helium (gas)	—	-269	—	21	5.193	0.142	—	—
hydrogen (gas)	-259	-253	58.6	452	14.30	0.168	—	—
iron (solid)	1535	2750	289	6360	0.450	80	1.18×10^{-5}	3.33×10^{-5}
lead (solid)	327	1750	24.7	870	0.160	35	2.9×10^{-5}	8.7×10^{-5}
mercury (liquid)	-39	357	11.3	293	0.140	8	6.1×10^{-5}	1.82×10^{-4}
paraffin wax (solid)	46–68	~300	~210	—	2.5	0.25	—	—
silver (solid)	962	2212	111	2360	0.233	429	1.8×10^{-5}	5.4×10^{-5}
steam (gas) @ water (liq.) @ 25°C	0	100	—	2260	2.080	0.016	—	—
ice (solid) @ -10°C			334	—	4.181	0.58	6.9×10^{-5}	2.07×10^{-4}
				—	2.11	2.18	—	—

Table I. Electricity Formulas & Equations

Electrostatic Charges & Electric Fields	$\vec{F}_e = \frac{kq_1 q_2}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$ $\vec{E} = \frac{\vec{F}_e}{q} = \frac{Q}{\epsilon_0 A} \quad \vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} = \frac{\Delta V}{\Delta r}$ $W = q \vec{E} \bullet \vec{d} = qEd \cos\theta$ $V = \frac{W}{q} = \vec{E} \bullet \vec{d} = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$ $\Delta U_E = q\Delta V \quad U_E = \frac{kq_1 q_2}{r}$	Δ = change \vec{F}_e = force due to electric field (N) k = electrostatic constant $\left(\frac{N\cdot m^2}{C^2}\right)$ q = point charge (C) Q = charge (C) \vec{E} = electric field $\left(\frac{N}{C}, \frac{V}{m}\right)$ V = voltage = electric potential difference (V) W = work (N·m) d = distance (m) r = radius (m) \vec{I} = current (A) t = time (s) R = resistance (Ω) P = power (W) Q_H = heat (J) ρ = resistivity ($\Omega \cdot m$) ℓ = length (m) A = cross-sectional area (m^2) ϵ_0 = electric permittivity of free space U = potential energy (J) C = capacitance (F) \vec{v} = velocity (of moving charge or wire) $\left(\frac{m}{s}\right)$ \vec{B} = magnetic field (T) μ_0 = magnetic permeability of free space r = radius (distance) from wire
Circuits	$I = \frac{\Delta Q}{\Delta t} = \frac{V}{R} \quad V = IR$ $P = VI = I^2 R = \frac{V^2}{R}$ $W = Q_H = Pt = VIt = I^2 Rt = \frac{V^2 t}{R}$ $R = \frac{\rho \ell}{A}$ $V = \frac{Q}{C}$ $C = k\epsilon_0 \frac{A}{d}$ $U_{capacitor} = \frac{1}{2} QV = \frac{1}{2} CV^2$	
Series Circuits	$I = I_1 = I_2 = I_3 = \dots$ $V = V_1 + V_2 + V_3 + \dots = \sum V_i$ $R_{eq} = R_1 + R_2 + R_3 + \dots = \sum R_i$ $\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots = \sum \frac{1}{C_i}$ $P_{total} = P_1 + P_2 + P_3 + \dots = \sum P_i$	
Parallel Circuits	$I = I_1 + I_2 + I_3 + \dots = \sum I_i$ $V = V_1 = V_2 = V_3 = \dots$ $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots = \sum \frac{1}{R_i}$ $C_{total} = C_1 + C_2 + C_3 + \dots = \sum C_i$ $P_{total} = P_1 + P_2 + P_3 + \dots = \sum P_i$	

Table J. Electricity & Magnetism Formulas & Equations

Magnetism	$\vec{F}_M = q(\vec{v} \times \vec{B})$	$F_M = q v B \sin\theta$	Δ = change
	$\vec{F}_M = \ell(\vec{I} \times \vec{B})$	$F_M = \ell I B \sin\theta$	\vec{F}_M = force due to magnetic field (N)
	$V = \ell(\vec{v} \times \vec{B})$	$V = \ell v B \sin\theta$	k = electrostatic constant ($\frac{N \cdot m^2}{C^2}$)
		$B = \frac{\mu_0 I}{2\pi r}$	q = point charge (C)
		$\Phi_B = \vec{B} \bullet \vec{A} = BA \cos\theta$	V = voltage = electric potential difference (V)
		$\mathcal{E} = \frac{\Delta \Phi_B}{\Delta t} = B\ell v$	\mathcal{E} = emf = electromotive force(V)
			r = radius (m)
			\vec{I} = current (A)
			ℓ = length (m)
			t = time (s)

Electromagnetic Induction	$\frac{\# \text{turns}_{in}}{\# \text{turns}_{out}} = \frac{V_{in}}{V_{out}} = \frac{I_{out}}{I_{in}}$	A = cross-sectional area (m^2)
	$P_{in} = P_{out}$	\vec{v} = velocity (of moving charge or wire) ($\frac{m}{s}$)
		\vec{B} = magnetic field (T)

μ_0 = magnetic permeability of free space
Φ_B = magnetic flux

Table K. Resistor Color Code

Color	Digit	Multiplier
black	0	$\times 10^0$
brown	1	$\times 10^1$
red	2	$\times 10^2$
orange	3	$\times 10^3$
yellow	4	$\times 10^4$
green	5	$\times 10^5$
blue	6	$\times 10^6$
violet	7	$\times 10^7$
gray	8	$\times 10^8$
white	9	$\times 10^9$
gold		$\pm 5\%$
silver		$\pm 10\%$

Table L. Symbols Used in Electrical Circuit Diagrams

Component	Symbol	Component	Symbol
wire	—	battery	+ -
switch	-••-	ground	<u> </u> GND
fuse	-○○○-	resistor	—WW—
voltmeter	-○V-	variable resistor (rheostat, potentiometer, dimmer)	—VVV—
ammeter	-○A-	lamp (light bulb)	—RL— —XL—
ohmmeter	-○R-	capacitor	- -
		diode	→

Table M. Resistivities at 20°C

Conductors		Semiconductors		Insulators	
Substance	Resistivity ($\Omega \cdot m$)	Substance	Resistivity ($\Omega \cdot m$)	Substance	Resistivity ($\Omega \cdot m$)
silver	1.59×10^{-8}	germanium	0.001 to 0.5	deionized water	1.8×10^5
copper	1.72×10^{-8}	silicon	0.1 to 60	glass	1×10^9 to 1×10^{13}
gold	2.44×10^{-8}	sea water	0.2	rubber, hard	1×10^{13} to 1×10^{13}
aluminum	2.82×10^{-8}	drinking water	20 to 2 000	paraffin (wax)	1×10^{13} to 1×10^{17}
tungsten	5.60×10^{-8}			air	1.3×10^{16} to 3.3×10^{16}
iron	9.71×10^{-8}			quartz, fused	7.5×10^{17}
nichrome	1.50×10^{-6}				
graphite	3×10^{-5} to 6×10^{-4}				

Table N. Waves & Optics

Waves $\lambda = \frac{v}{f}$ $f = \frac{1}{T}$ $v_{\text{wave on a string}} = \sqrt{\frac{F_T}{\mu}}$ $f_{\text{dopplershifted}} = f \left(\frac{v_{\text{wave}} + v_{\text{detector}}}{v_{\text{wave}} + v_{\text{source}}} \right)$	v = velocity of wave ($\frac{\text{m}}{\text{s}}$) f = frequency (Hz) λ = wavelength (m) T = period (of time) (s) F_T = tension(force)onstring (N) μ = elastic modulus of string ($\frac{\text{kg}}{\text{m}}$) θ_i = angle of incidence ($^{\circ}$, rad) θ_r = angle of reflection ($^{\circ}$, rad) θ_c = critical angle ($^{\circ}$, rad) n = index of refraction (dimensionless) c = speed of light in a vacuum ($\frac{\text{m}}{\text{s}}$) s_f = distance to the focus of a mirror or lens (m) r_c = radius of curvature of a spherical mirror (m) s_i = distance from the mirror or lens to the image (m) s_o = distance from the mirror or lens to the object (m) h_i = height of the image (m) h_o = height of the object (m) M = magnification (dimensionless) d = separation (m) L = distance from the opening (m) m = an integer
Reflection, Refraction & Diffraction $\theta_i = \theta_r$ $n = \frac{c}{v}$ $n_1 \sin \theta_1 = n_2 \sin \theta_2$ $\theta_c = \sin^{-1} \left(\frac{n_2}{n_1} \right)$ $\frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$ $\Delta L = m\lambda = d \sin \theta$	
Mirrors & Lenses $s_f = \frac{r_c}{2}$ $\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{s_f}$ $M = \frac{h_i}{h_o} = -\frac{s_i}{s_o}$	

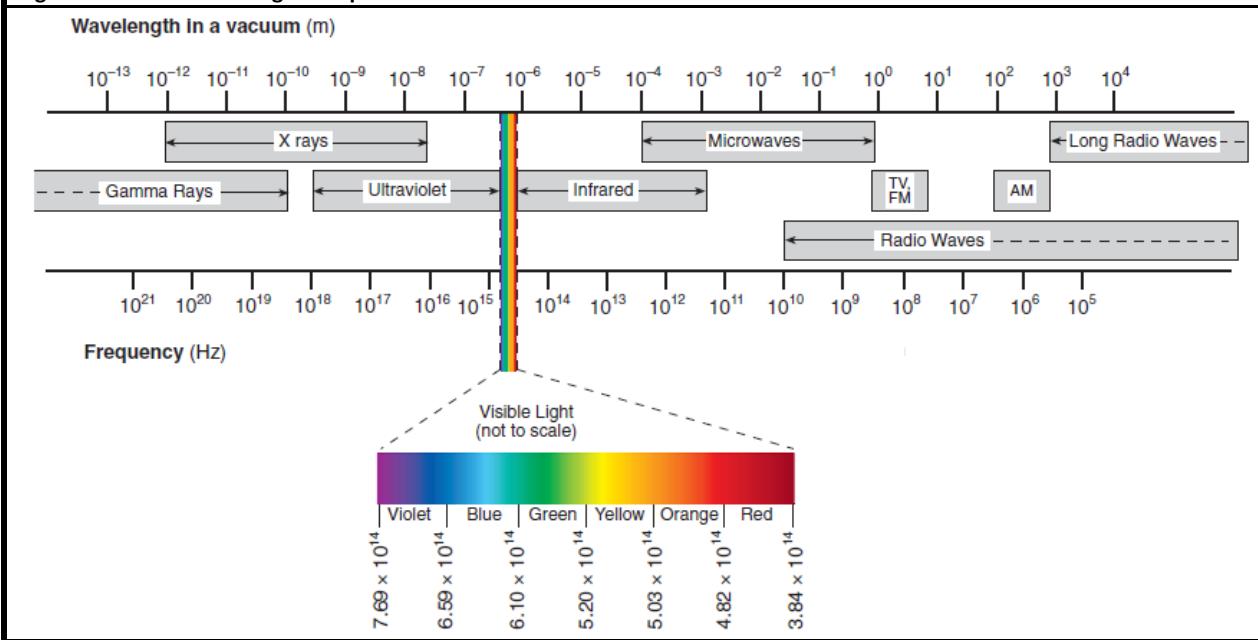
Figure O. The Electromagnetic Spectrum

Table P. Properties of Water and Air					
Temp. (°C)	Water			Air	
	Density (kg/m³)	Speed of Sound (m/s)	Vapor Pressure (Pa)	Density (kg/m³)	Speed of Sound (m/s)
0	999.78	1 403	611.73	1.288	331.30
5	999.94	1 427	872.60	1.265	334.32
10	999.69	1 447	1 228.1	1.243	337.31
20	998.19	1 481	2 338.8	1.200	343.22
25	997.02	1 496	3 169.1	1.180	346.13
30	995.61	1 507	4 245.5	1.161	349.02
40	992.17	1 526	7 381.4	1.124	354.73
50	990.17	1 541	9 589.8	1.089	360.35
60	983.16	1 552	19 932	1.056	365.88
70	980.53	1 555	25 022	1.025	371.33
80	971.79	1 555	47 373	0.996	376.71
90	965.33	1 550	70 117	0.969	382.00
100	954.75	1 543	101 325	0.943	387.23

Table Q. Absolute Indices of Refraction

Measured at $f = 5.09 \times 10^{14}$ Hz (yellow light)

Substance	Index of Refraction	Substance	Index of Refraction
air	1.000293	silica (quartz), fused	1.459
ice	1.309	plexiglass	1.488
water	1.3330	Lucite	1.495
ethyl alcohol	1.36	glass, borosilicate (Pyrex)	1.474
human eye, cornea	1.38	glass, crown	1.50–1.54
human eye, lens	1.41	glass, flint	1.569–1.805
safflower oil	1.466	sodium chloride, solid	1.516
corn oil	1.47	PET (#1 plastic)	1.575
glycerol	1.473	zircon	1.777–1.987
honey	1.484–1.504	cubic zirconia	2.173–2.21
silicone oil	1.52	diamond	2.417
carbon disulfide	1.628	silicon	3.96

Table R. Fluid Mechanics Formulas and Equations

Density & Pressure	$\rho = \frac{m}{V}$	$\Delta = \text{change}$
	$P = \frac{F}{A}$	$\rho = \text{density } \left(\frac{\text{kg}}{\text{m}^3} \right)$
	$\frac{F_1}{A_1} = \frac{F_2}{A_2}$	$m = \text{mass (kg)}$
	$P = P_o + \rho gh$	$V = \text{volume (m}^3\text{)}$
	$A_1 v_1 = A_2 v_2$	$P = \text{pressure (Pa)}$
	$P_1 + \rho g h_1 + \frac{1}{2} \rho v_1^2 =$	$g = \text{acceleration due to gravity } \left(\frac{\text{m}}{\text{s}^2} \right)$
	$P_2 + \rho g h_2 + \frac{1}{2} \rho v_2^2$	$h = \text{height or depth (m)}$
Forces, Work & Energy	$F_B = \rho V_d g$	$A = \text{area (m}^2\text{)}$
	$PV = Nk_B T = nRT$	$v = \text{velocity (of fluid) } \left(\frac{\text{m}}{\text{s}} \right)$
	$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$	$F = \text{force (N)}$
	$E_k (\text{molecular}) = \frac{3}{2} k_B T$	$n = \text{number of moles (mol)}$
	$v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3k_B T}{\mu}}$	$R = \text{gas constant } \left(\frac{\text{J}}{\text{mol K}} \right)$
	$W = -P\Delta V$	$N = \text{number of molecules}$
		$k_B = \text{Boltzmann's constant } \left(\frac{\text{J}}{\text{K}} \right)$
		$T = \text{temperature (K)}$
		$M = \text{molar mass } \left(\frac{\text{g}}{\text{mol}} \right)$

Table S. Planetary Data

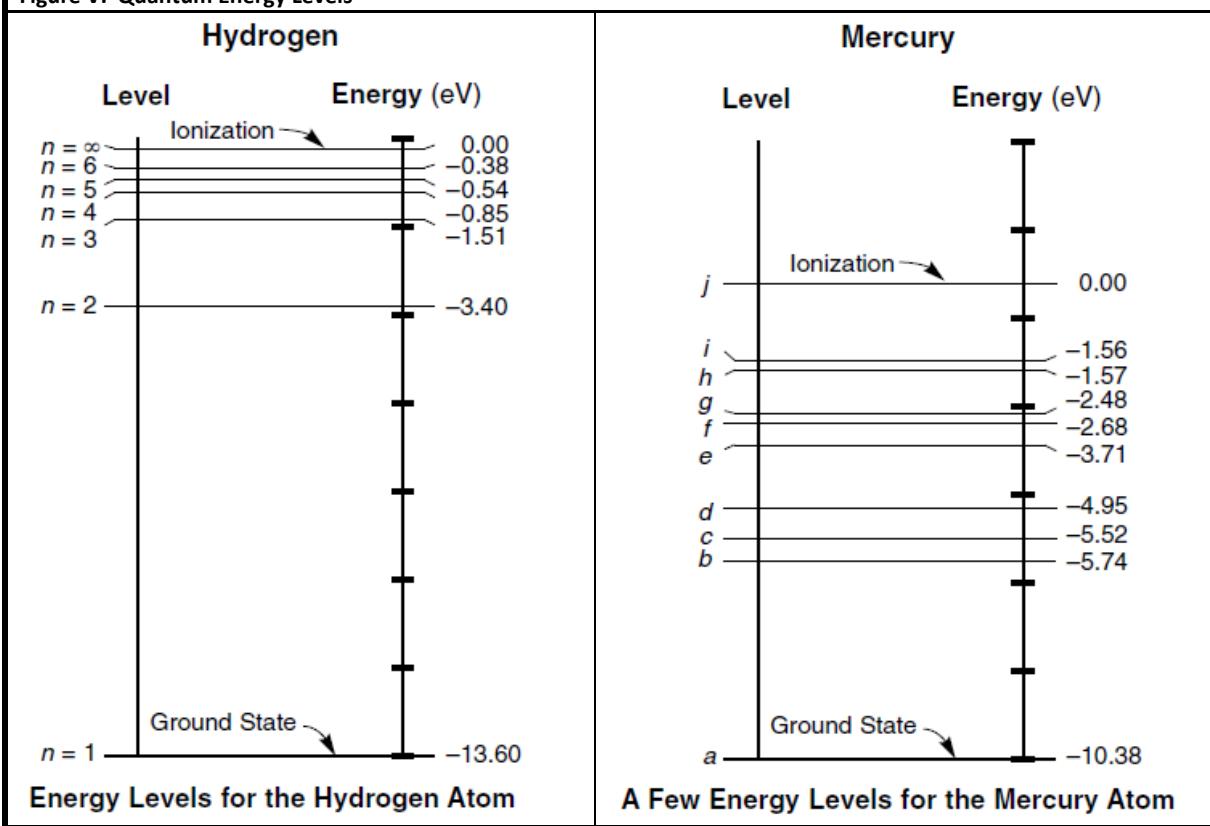
	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
Distance from Sun (m)	5.79×10^{10}	1.08×10^{11}	1.50×10^{11}	2.28×10^{11}	7.78×10^{11}	1.43×10^{12}	2.87×10^{12}	4.50×10^{12}
Radius (m)	2.44×10^6	6.05×10^6	6.37×10^6	3.39×10^6	6.99×10^7	5.82×10^7	2.54×10^7	2.46×10^7
Mass (kg)	3.30×10^{23}	4.87×10^{24}	5.97×10^{24}	6.42×10^{23}	1.90×10^{27}	5.68×10^{26}	8.68×10^{25}	1.02×10^{26}
Density $\left(\frac{\text{kg}}{\text{m}^3} \right)$	5430	5250	5520	3950	1330	690	1290	1640
Orbit (years)	0.24	0.62	1.00	1.88	11.86	84.01	164.79	248.54
Rotation Period (hours)	1408	5832	23.9	24.6	9.9	10.7	17.2	16.1
Tilt of axis	2°	177.3°	23.5°	25.2°	3.1°	26.7°	97.9°	29.6°
# of observed satellites	0	0	1	2	67	62	27	13

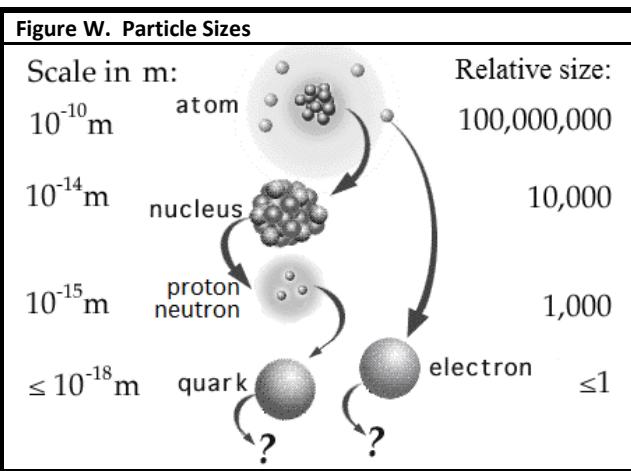
Table T. Sun & Moon Data

Radius of the sun (m)	6.96×10^8
Mass of the sun (kg)	1.99×10^{30}
Radius of the moon (m)	1.74×10^6
Mass of the moon (kg)	7.35×10^{22}
Distance of moon from Earth (m)	3.84×10^8

Table U. Atomic & Particle Physics (Modern Physics)

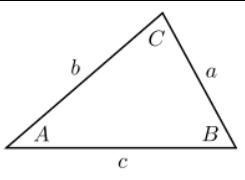
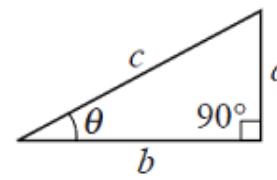
Energy $E_{\text{photon}} = hf = \frac{hc}{\lambda} = pc = \hbar\omega$ $E_{k,\max} = hf - \phi$ $\lambda = \frac{h}{p}$ $E_{\text{photon}} = E_i - E_f$ $E^2 = (pc)^2 + (mc^2)^2$ $\frac{1}{\lambda} = R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$	E = energy (J) h = Planck's constant (J·s) \hbar = reduced Planck's constant = $\frac{h}{2\pi}$ (J·s) f = frequency (Hz) c = speed of light ($\frac{\text{m}}{\text{s}}$) λ = wavelength (m) p = momentum (N·s) m = mass (kg) E_k = kinetic energy (J) ϕ = work function R_H = Rydberg constant ($\frac{1}{\text{m}}$)
Special Relativity $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$ $\gamma = \frac{L_o}{L}$ $\gamma = \frac{\Delta t'}{\Delta t}$ $\gamma = \frac{m_{\text{rel}}}{m_o}$	γ = Lorentz factor (dimensionless) L = length in moving reference frame (m) L_o = length in stationary reference frame (m) $\Delta t'$ = time in stationary reference frame (s) Δt = time in moving reference frame (s) m_o = mass in stationary reference frame (kg) m_{rel} = apparent mass in moving reference frame (kg) v = velocity ($\frac{\text{m}}{\text{s}}$)

Figure V. Quantum Energy Levels

**Table X. The Standard Model**

Generation				gauge bosons	
quarks	I	II	III		
	mass → 2.4 MeV/c ²	1.27 GeV/c ²	171.2 GeV/c ²	0 MeV/c ²	125.3 GeV/c ²
	charge → +2/3	+2/3	+2/3	0	0
	spin → 1/2 up quark	1/2 charm quark	1/2 top quark	γ photon	H ⁰ Higgs boson
leptons	mass → 4.8 MeV/c ²	104 MeV/c ²	4.2 GeV/c ²	0 MeV/c ²	0 MeV/c ²
	charge → -1/3	-1/3	-1/3	0	0
	spin → 1/2 down quark	1/2 strange quark	1/2 bottom quark	g gluon	Z ⁰ Z boson
	mass → < 2.2 eV/c ²	< 0.17 MeV/c ²	< 15.5 MeV/c ²	91.2 GeV/c ²	W [±] W boson
	charge → 0	0	0	0	
	spin → 1/2 electron neutrino	1/2 muon neutrino	1/2 tau neutrino		
	mass → 0.511 MeV/c ²	105.7 MeV/c ²	1.777 GeV/c ²		
	charge → -1	-1	-1		
	spin → 1/2 electron	1/2 muon	1/2 tau		

Table Y. Geometry & Trigonometry Formulas

Triangles	$A = \frac{1}{2}bh$ $c^2 = a^2 + b^2 - 2ab\cos C$ $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	
Right Triangles	$c^2 = a^2 + b^2$ $\sin \theta = \frac{a}{c} = \frac{\text{opposite}}{\text{hypotenuse}}$ $\cos \theta = \frac{b}{c} = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{a}{b} = \frac{\text{opposite}}{\text{adjacent}}$ $b = c \cos \theta$ $a = c \sin \theta$	
Rectangles, Parallelograms and Trapezoids	$A = \bar{b}h$	
Rectangular Solids	$V = \ell wh$	
Circles	$C = 2\pi r$ $A = \pi r^2$	
Cylinders	$S = 2\pi rl + 2\pi r^2 = 2\pi r(l+r)$ $V = \pi r^2 l$	
Spheres	$S = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$	

 a, b, c = length of a side of a triangle θ = angle A = area C = circumference S = surface area V = volume b = base h = height ℓ = length w = width r = radius

Table Z. Values of Trigonometric Functions

degree	radian	sine	cosine	tangent	degree	radian	sine	cosine	tangent
0°	0.000	0.000	1.000	0.000					
1°	0.017	0.017	1.000	0.017	46°	0.803	0.719	0.695	1.036
2°	0.035	0.035	0.999	0.035	47°	0.820	0.731	0.682	1.072
3°	0.052	0.052	0.999	0.052	48°	0.838	0.743	0.669	1.111
4°	0.070	0.070	0.998	0.070	49°	0.855	0.755	0.656	1.150
5°	0.087	0.087	0.996	0.087	50°	0.873	0.766	0.643	1.192
6°	0.105	0.105	0.995	0.105	51°	0.890	0.777	0.629	1.235
7°	0.122	0.122	0.993	0.123	52°	0.908	0.788	0.616	1.280
8°	0.140	0.139	0.990	0.141	53°	0.925	0.799	0.602	1.327
9°	0.157	0.156	0.988	0.158	54°	0.942	0.809	0.588	1.376
10°	0.175	0.174	0.985	0.176	55°	0.960	0.819	0.574	1.428
11°	0.192	0.191	0.982	0.194	56°	0.977	0.829	0.559	1.483
12°	0.209	0.208	0.978	0.213	57°	0.995	0.839	0.545	1.540
13°	0.227	0.225	0.974	0.231	58°	1.012	0.848	0.530	1.600
14°	0.244	0.242	0.970	0.249	59°	1.030	0.857	0.515	1.664
15°	0.262	0.259	0.966	0.268	60°	1.047	0.866	0.500	1.732
16°	0.279	0.276	0.961	0.287	61°	1.065	0.875	0.485	1.804
17°	0.297	0.292	0.956	0.306	62°	1.082	0.883	0.469	1.881
18°	0.314	0.309	0.951	0.325	63°	1.100	0.891	0.454	1.963
19°	0.332	0.326	0.946	0.344	64°	1.117	0.899	0.438	2.050
20°	0.349	0.342	0.940	0.364	65°	1.134	0.906	0.423	2.145
21°	0.367	0.358	0.934	0.384	66°	1.152	0.914	0.407	2.246
22°	0.384	0.375	0.927	0.404	67°	1.169	0.921	0.391	2.356
23°	0.401	0.391	0.921	0.424	68°	1.187	0.927	0.375	2.475
24°	0.419	0.407	0.914	0.445	69°	1.204	0.934	0.358	2.605
25°	0.436	0.423	0.906	0.466	70°	1.222	0.940	0.342	2.747
26°	0.454	0.438	0.899	0.488	71°	1.239	0.946	0.326	2.904
27°	0.471	0.454	0.891	0.510	72°	1.257	0.951	0.309	3.078
28°	0.489	0.469	0.883	0.532	73°	1.274	0.956	0.292	3.271
29°	0.506	0.485	0.875	0.554	74°	1.292	0.961	0.276	3.487
30°	0.524	0.500	0.866	0.577	75°	1.309	0.966	0.259	3.732
31°	0.541	0.515	0.857	0.601	76°	1.326	0.970	0.242	4.011
32°	0.559	0.530	0.848	0.625	77°	1.344	0.974	0.225	4.331
33°	0.576	0.545	0.839	0.649	78°	1.361	0.978	0.208	4.705
34°	0.593	0.559	0.829	0.675	79°	1.379	0.982	0.191	5.145
35°	0.611	0.574	0.819	0.700	80°	1.396	0.985	0.174	5.671
36°	0.628	0.588	0.809	0.727	81°	1.414	0.988	0.156	6.314
37°	0.646	0.602	0.799	0.754	82°	1.431	0.990	0.139	7.115
38°	0.663	0.616	0.788	0.781	83°	1.449	0.993	0.122	8.144
39°	0.681	0.629	0.777	0.810	84°	1.466	0.995	0.105	9.514
40°	0.698	0.643	0.766	0.839	85°	1.484	0.996	0.087	11.430
41°	0.716	0.656	0.755	0.869	86°	1.501	0.998	0.070	14.301
42°	0.733	0.669	0.743	0.900	87°	1.518	0.999	0.052	19.081
43°	0.750	0.682	0.731	0.933	88°	1.536	0.999	0.035	28.636
44°	0.768	0.695	0.719	0.966	89°	1.553	1.000	0.017	57.290
45°	0.785	0.707	0.707	1.000	90°	1.571	1.000	0.000	∞

Table AA. Some Exact and Approximate Conversions

Length	1 cm	\approx	width of a small paper clip
	1 inch (in.)	\equiv	2.54 cm
	length of a US dollar bill	=	6.14 in. = 15.6 cm
	12 in.	\equiv	1 foot (ft.) \approx 30 cm
	3 ft.	\equiv	1 yard (yd.) \approx 1 m
	1 m	=	0.3048 ft. = 39.37 in.
	1 km	\approx	0.6 mi.
	5,280 ft.	\equiv	1 mile (mi.) \approx 1.6 km
Mass/ Weight	1 small paper clip	\approx	0.5 gram (g)
	US 1¢ coin (1983–present)	=	2.5 g
	US 5¢ coin	=	5 g
	1 oz.	\approx	30 g
	one medium-sized apple	\approx	1 N \approx 3.6 oz.
	1 pound (lb.)	\equiv	16 oz. \approx 454 g
	1 pound (lb.)	\approx	4.45 N
	1 ton	\equiv	2000 lb. \approx 0.9 tonne
	1 tonne	\equiv	1000 kg \approx 1.1 ton
Volume	1 pinch	=	$\leq \frac{1}{8}$ teaspoon (tsp.)
	1 mL	\approx	10 drops
	1 tsp.	\approx	5 mL \approx 60 drops
	3 tsp.	\equiv	1 tablespoon (Tbsp.) \approx 15 mL
	2 Tbsp.	\equiv	1 fluid ounce (fl. oz.) \approx 30 mL
	8 fl. oz.	\equiv	1 cup (C) \approx 250 mL
	16 fl. oz.	\equiv	1 U.S. pint (pt.) \approx 500 mL
	20 fl. oz.	\equiv	1 Imperial pint (UK) \approx 600 mL
	2 pt.	\equiv	1 U.S. quart (qt.) \approx 1 L
	4 qt. (U.S.)	\equiv	1 U.S. gallon (gal.) \approx 3.8 L
	4 qt. (UK) \equiv 5 qt. (U.S.)	\equiv	1 Imperial gal. (UK) \approx 4.7 L
Speed	1 m/s	\approx	2.24 mi./h
	60 mi./h	\approx	100 km/h \approx 27 m/s
Energy	1 cal	\approx	4.18 J
	1 Calorie (food)	\equiv	1 kcal \approx 4.18 kJ
	1 BTU	\approx	1.05 kJ
Power	1 hp	\approx	746 W
	1 kW	\approx	1.34 hp
Temperature	0 K	\equiv	-273.15°C = absolute zero
	0°F	\approx	-18°C
	32°F	=	0°C = 273.15 K = water freezes
	70°F	\approx	21°C \approx room temp.
	212°F	=	100°C = water boils
Speed of light	$300\,000\,000 \text{ m/s}$	\approx	$186\,000 \text{ mi./s}$ \approx 1 ft./ns

Table BB. Greek Alphabet

A	α	alpha
B	β	beta
Γ	γ	gamma
Δ	δ	delta
E	ε, ϵ	epsilon
Z	ζ	zeta
H	η	eta
Θ	θ	theta
I	ι	iota
K	κ	kappa
Λ	λ	lambda
M	μ	mu
N	ν	nu
Ξ	ξ	xi
O	\circ	omicron
Π	π	pi
P	ρ	rho
Σ	σ	sigma
T	τ	tau
Υ	υ	upsilon
Φ	ϕ, φ	phi
X	χ	chi
Ψ	ψ	psi
Ω	ω	omega

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