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MATHEMATICAL MODELING OF ICE ACCRETION ON AIRFOILS*

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Aircraft icing, long recognized as a serious safety problem, has received renewed interest due to increased use of small aircraft and helicopters. As the number of operations of these low and medium altitude aircraft grows, encounters with the meterological conditions leading to ice accretion become more prevalent and consequently so does the safety risk. Commuter air services, helicopter transportation to off-shore oil platforms - especially on the Atlantic coast - and military needs for helicopter operations in Western Europe are all situations in which icing is of major concern. Helicopters are a particular problem since there are at present no helicopters certified for flight under icing conditions.

Two types of icing are generally recognized. Under colder conditions (air temperatures less than about -10C), ice accumulates in a rough yet fairly evenly distributed porous coating around the airfoil's leading edge. This "rime" icing can result in increased drag, loss of lift, and increased weight. Rime icing, however, seldom produces aerodynamic penalties that are severe enough to result in a loss of control. A more dangerous condition, "glaze" icing, occurs when cloud temperatures are in the range of 0 to -10C. Here the latent heat released by the droplets freezing on impact can result in some liquid water on the acceting surface, usually near the stagnation point. This water then runs back to colder regions away from the leading edge where it freezes, producing the characteristic horn-shaped growth formations. Such formations produce severe lift and drag

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penalties and can result in situations where it is impossible to maintain altitude. "Mixed" situations in which both glaze and rime ice exists are also common.

In the past the testing of icing systems could only be achieved using costly and sometimes questionable techniques. The most realistic technique has been the full scale flight testing of the icing system in the natural icing environment. However, in addition to the expense of flight testing, it has proven difficult to plan for and actually locate the severe icing environment often required for testing purposes. Other icing techniques include the artificial generation of icing conditions by water droplet release from a tanker, with the test aircraft closely following. This technique has difficulty reproducing the natural environment in terms of cloud liquid water content and droplet size spectrum. Stationary spray rigs have also been used for icing tests, especially for the rotor blades of helicopters. Finally, tests have been performed in full scale icing/wind tunnels. The NASA/Lewis icing tunnel, in operation since the early 1950's, has been used extensively for such tests. The expense of using the icing tunnel makes it impractical to cost-effectively analyze all of the icing problems of importance to the various segments of the aviation community.

MATHEMATICAL MODELING OF ICE ACCRETION

Computer simulation of the ice accretion process provides an attractive method for analyzing a wide range of icing problems at a low cost. An ice accretion model that accurately predicts growth shapes on an arbitrary airfoil is valuable for analysis of the sensitivity of airfoils to ice accretion, and for analysis of the influence of variables such as airspeed and angle of attack to the accretion process. Such a model could also be used to assess the energy requirements necessary to prevent and/or remove ice from an airfoil. Such a model would also have applications for the icing analysis of helicopter rotoblades. To confidently made use of a model to perform the analysis described above, the model must be thoroughly tested and evaluated over the applicable range of environmental conditions and airfoil shapes. Once a model has been validated, it will provide a cost effective means of performing most of the icing research studies which now rely upon experimental techniques.

The University of Dayton Research Institute (UDRI) has developed an ice accretion model, 'LEWICE', that solves a set of mathematical equations that describe the physical processes that occur in the accretion of ice on an airfoil. In many respects the UDRI model is an amalgamation and extension of the desirable features of previous research efforts. The two most widely known are the computer codes of Ackley and Templeton (1979), and Lozowski et al. (1979). Another, less well known model, has been developed by Hankey and Kirchner (1979). Each of these models represents a substantial contribution to the understanding of ice accretion. However, certain aspects of these models are open to improvements or generalizations.

The model of Lozowski et al. incorporates the runback or unfrozen water, and allows for a mixture of ice crystals and supercooled droplets in the cloud. However, no accounting is made for time dependent effects and the changing geometry necessary to handle a growing ice/airfoil surface. Further, the model is restricted to the simulation of the icing of a cylinder. The model of Ackley and Templeton incorporates time dependent effects due to the changing ice surface shape. These manifest themselves as a time dependent collection efficiencies and heat transfer coefficients. Neither of these factors, however, is made a function of position on the airfoil but is taken as an overall value representative of the entire airfoil. Other limitations include no account of runback and a restriction of geometry to ellipses. The Ackley and Templeton formulation does, however, allow the accreted ice density to be a function of temperature, freestream velocity, and average droplet radius.

Hankey and Kirchner's model is similar to that of Lozowski et al., except that calculations were made for a realistic airfoil rather than a cylinder. A similar formulation has also been propsoed by Cansdale and McNaughtan (1977).

None of these models handle any time dependent aspects of the ice accretion process in a very realistic manner. Rates of ice accretion are calculated as a function of position on the airfoil and are merely projected at a constant rate to approximate a finite growth or a prescribed period of time. It is clear that any sensitivity in the thermodynamics of the ice formation process to the changing airfoil shape will not be included by this approach.

LEWICE is a time dependent model that takes into account all important heat transfer processes that occur when supercooled water droplets strike an airfoil and either freeze immediately or remain liquid to run back onto a colder region of the airfoil. The model requires as input values of cloud liquid water content, mean droplet diameter, ambient air temperature, air velocity, and relative humidity. Any airfoil, or other object, can be used for the body on which the ice accretes, provided that the airflow around this body can be calculated using a conventional two-dimensional potential flow code. Figure 1 shows the general structure of the model.

Starting with the appropriate airfoil and environmental data, the potential flow field around the airfoil is calculated. The potential flow program used is a generalized version of the Douglas Aircraft Company two-dimensional flow code supplied by NASA-Lewis Research Center. Next, droplet trajectories are calculated in a potential flow field using the appropriate mean cloud droplet diameter. From the location of impacts of the various trajectories on the airfoil, local values of water droplet collection efficiency are calculated around the airfoil. LEWICE uses a trajectory code developed by C. F. Shieh of FWG

Associates (Shieh, 1981) for the determination of local collection efficiencies. Using local collection efficiencies and the environmental conditions of free stream temperature, cloud liquid water content, and relative humidity, thermodynamic calculations are made which determine the rate of ice growth at each segment around the airfoil. The calculation of the ice growth is made by applying the steady state continuity and energy equations to a small control volume at each segment of the ice surface. The mass and energy flows for this control volume are shown in Figure 2. From the steady state analysis of the energy (heat) and mass fluxes through this volume a rate of ice formation may be determined. This rate is assumed to apply to a small time interval--usually on the order of one second--and thus the thickness of the ice layer formed during the time interval may be found. When the added layer thicknesses are found for all segments, the airfoil shape is updated and a new time step is begun. The integration of the continuity and energy equations is therefore performed under a quasi-steady state assumption.

In principle the flow field and collection efficiency computations should be made anew after each update of the airfoil shape. Practically, however, these computations are sufficiently time consuming so that they can be done only once for every ten or so shape changes. At present the user of the model has interactive control over the time step size and the frequency of recomputation of the flow field and collection efficiencies. Further experience with the model will help determine the optimum ratio of calculations of the flow field to calculations of the shape changes by accreting ice.

Because the governing equations allow incoming water droplets to freeze immediately, or liquify, runback, and possibly refreeze, the model reproduces both glaze and rime ice conditions. In running the model a direction may be specified which defines the direction of ice growth at each airfoil segment. The model, in its present form, has been run with ice growth occurring in the local direction of incoming water droplets in regions of rime icing conditions, i.e., when the droplet freezes immediately on impact. Where runback occurs the growth direction is defined as the local normal to the airfoil surface. The model, however, allows for other definitions of growth rules if model results do not satisfactorily compare with experimental data.

THE ENERGY EQUATION

The energy equation considers six sources of energy transport: the sensible heat contained in the incoming liquid droplets (assumed to be at the temperature of the ambient air), the latent heat release from freezing of the supercooled water droplets, the evaporative cooling at the ice or liquid surface, the convective cooling of the ice surface from the airflow above, the aerodynamic heating of the surface, and the energy added by the kinetic energy of droplet impacts. Under the quasi-steady assumption the net change in energy at each segment is zero during one integration step. Symbolically this is

$$\dot{E} = 0 = \dot{Q}_1 + \dot{Q}_2 + \dot{Q}_3 + \dot{Q}_4 + \dot{Q}_5 + \dot{Q}_6 \quad . \tag{1}$$

The six terms in (1) are

$$\dot{Q}_1 = HA(T_a - T) \qquad \text{Convective cooling} \tag{2}$$

$$\dot{Q}_2 = HAR_c V_\infty^2/(2C_a) \qquad \text{Aerodynamic heating} \tag{3}$$

$$\dot{Q}_3 = 0.7HAL_v(Rhe_a - e_{surf})/(P_a C_a) \qquad \text{Evaporative cooling at surface} \tag{4}$$

$$\dot{Q}_4 = \dot{M}_C C_w T_a \qquad \text{Heat content of collected mass} \tag{5}$$

$$\dot{Q}_5 = \dot{M}_c V_\infty^2/2 \qquad \text{Heat from drop impacts} \tag{6}$$

$$\dot{Q}_6 = \dot{M}_I L_f \qquad \text{Latent heat release during freezing} \tag{7}$$

where

H = convective heat transfer coefficient (W/m^2-K)

T_a = air temperature (K)

T = ice/airfoil surface temperature (K)

Δt = time interval (sec)

A = area of airfoil segment (exposed to the air) (m^2)

R_c = recovery factor (-)

V_∞ = free stream velocity (m/s)

C_a = specific heat of air (J/kg)

L_v = latent heat of vaporization (J/kg)

e_a = saturation vapor pressure of water in the ambient air (Pa)

e_{surf} = saturation vapor pressure at the surface (Pa)

Rh = relative humidity

P = air pressure (Pa)

$\dot{M}_C$ = mass of incoming water droplets in time interval Δt

C_w = specific heat of water (J/kg-k)

$\dot{M}_I$ = mass rate of freezing (kg/s)

L_f = latent heat of fusion (J/kg)

Equations 2 through 7 give the form of each of the transfer terms used in the energy equation. An order of magnitude calculation shows that the most significant terms are the convective cooling term, Q_1, and the latent heat term, Q_6. The convective heat term requires an accurate determination of the local heat transfer coefficient. The heat transfer coefficient is found by a standard method for situations in which the free stream velocity varies along the body (Kays and Crawford, 1981.) Both laminar and turbulent cases are considered, the transition point being recomputed with each new determination of the flow field. Transition is predicted from the value of the local momentum-thickness Reynolds number; the critical value being taken as R_{e_δ} (crit.) = 360. The momentum thickness is in turn computed by Thwaites' method (see White, 1974) for flows with varying free stream velocity.

In performing the ice growth calculations, an airfoil cross section is divided into a large number of linear segments of varying lengths. Considerably more airfoil segments are defined near the leading edge where ice accretion is anticipated. For each segment around the airfoil, the energy equation is solved in one of two ways. If the temperature is cold enough for all incoming water droplets to freeze, then the equation is solved for the temperature of the ice so that the sum of all heat fluxes Q_1 through Q_6 equals 0 over that segment of the airfoil. If on the other hand, the latent heat release from the incoming droplets raises the temperature of the surface to freezing, then only part of the incoming droplets may freeze while the others remain in the liquid form. In this case the temperature of the water/ice surface is set to 0C and the equation is solved for that fraction, f, of the total incoming liquid water mass that freezes. The airfoil coordinates are then changed to reflect the added ice masse. If liquid water is present, it is allowed to propagate downstream to other airfoil segments. The treatment of this runback water is discussed in the following section.

THE CONSERVATION OF MASS

Under the quasi-steady state assumption the net of the mass fluxes through the segment control volume shown in Figure 2 is zero,

$$\dot{M} = 0 = \dot{M}_C + \dot{M}_I + \dot{M}_{RI} + \dot{M}_{RO} \tag{8}$$

where

$$\dot{M}_C = \rho_{1w} \beta V_\infty A_p \qquad \text{Collected mass flux} \tag{9}$$

$$\dot{M}_I = f\dot{M}_T \qquad \text{Freezing rate} \qquad (10)$$

and

$\dot{M}_{RI}$ = Runback liquid into the C.V.

$\dot{M}_{RO}$ = Runback liquid out of the C.V.

β = Local collection efficiency (-)

ρ_{lw} = Liquid water content of the ambient air (kg/m^3)

A_p = Presented area of an airfoil segment (m^2)

$\dot{M}_T$ = Total incoming mass (kg/sec)

= $\dot{M}_{RI} + \dot{M}_c$

f = Fraction of incoming mass that freezes (-)

For the case where all incoming drops freeze at every surface segment $\dot{M}_c = \dot{M}_I$ and the runback terms are everywhere zero. When runback occurs at one or more of the segments the steady-state assumption requires that the liquid flow into a segment equals the flow out of the neighboring upwind segment. For the two segments on either side of the stagnation point the runback into these segments is zero, so, starting at the stagnation point, the runback flow into each downwind segment can be found if the values of $\dot{M}_c$ and f are known.

SOLUTION PROCEDURE

The conservation equations (1) and (8) are solved by an implicit, iterative technique. At the start of a time step initial estimates of the freezing fractions, f, for each segment are made. From these the values of $\dot{M}_c$ and $\dot{M}_{RI}$ are found moving step-by-step away from the stagnation point on both the upper and lower airfoil surfaces. When all the mass fluxes for either the frozen fraction (when runback occurs at a segment) or for the surface temperature (for segments where f=1.)

Equation (8) can then be used again to update the mass flux terms. This complete procedure amounts to finding new values of f from the old values. If two successive iterations in the calculations agree for all freezing fractions to within a preset tolerance, a consistent solution to (1) and (8) has been obtained for this time step. The amount of ice formed for each segment is then used to determine the added thickness at each segment and therefore the new ice mass shape.

RESULTS

At this writing LEWICE is in continuing test and development. Some preliminary results are presented here to illustrate the model's performance.

Figure 3 shows the prediction by an early version of the model for the icing of a 2.54 cm diameter circular cylinder. A comparison to the experimental results of Stallabrass and Hearty (1979) is also given. In this situation, glaze-like conditions exist near the stagnation point on the left. Runback water moves over the ice surface to freeze at angles of about ± 30° from the stagnation point forming the characteristic horn shapes which are also seen in the experiment. In this and a number of other cases for which data on cylinders is available LEWICE has done well in predicting the qualitative features of the ice shape and the quantitative amount of accreted mass.

Figures 4 and 5 show the predictions of the most recent version of LEWICE for the icing of a NACA 0012 airfoil at 0° angle of attack. Figure 4 is the accretion after one minute of exposure and Figure 5 is the shape after 3 minutes. To this point the icing is completely rime, that is, no liquid water persists on the surface although the surface temperature rises toward OC for the segments around the stagnation point as the simulation progresses.

As mentioned above, LEWICE is at this time still under active development. Many refinements and extensions will be made as the model is exercised for a wider range of conditions and compared to experimental results.

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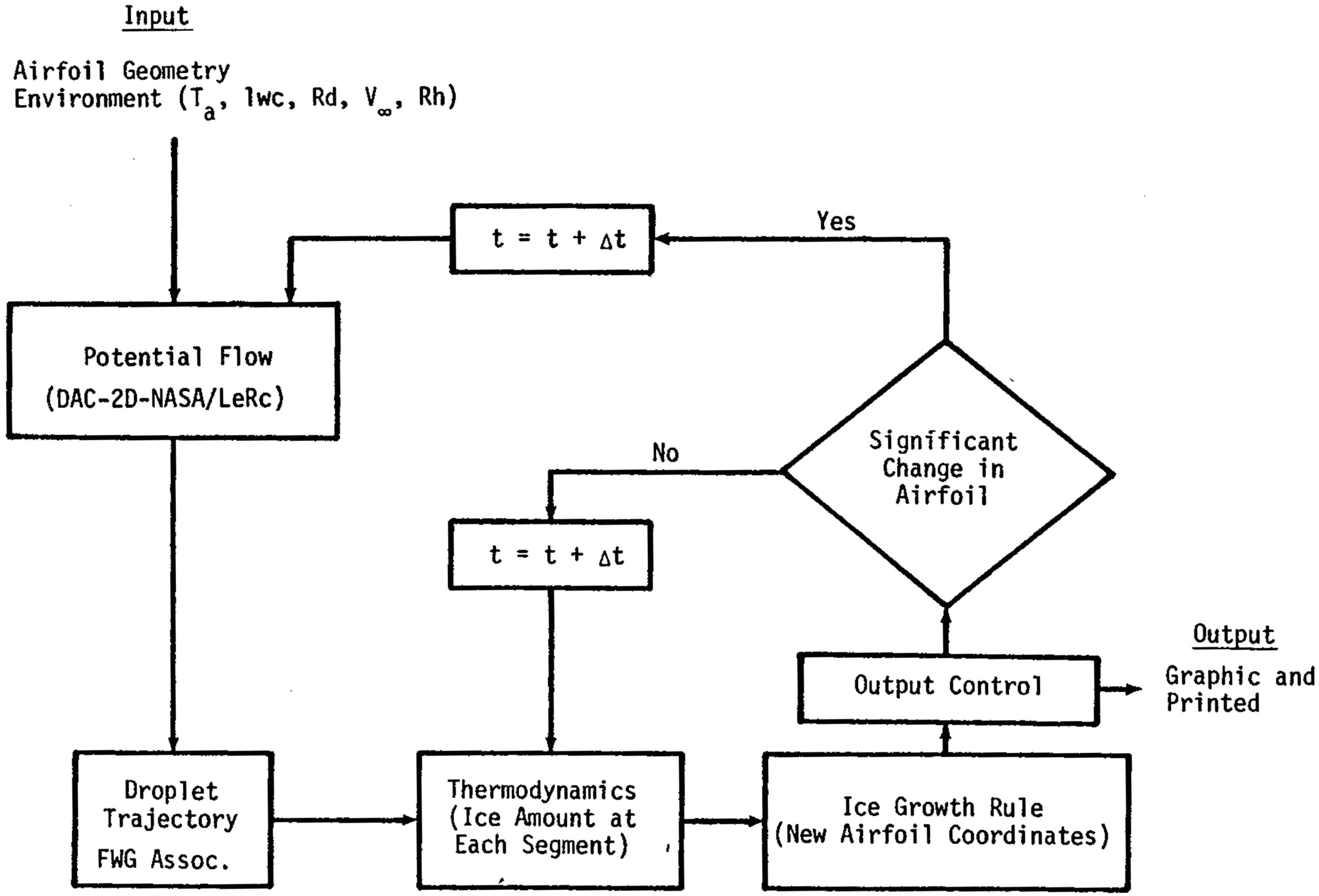

Figure 1. Schematic of UDRI "LEWICE" Ice Accretion Model.

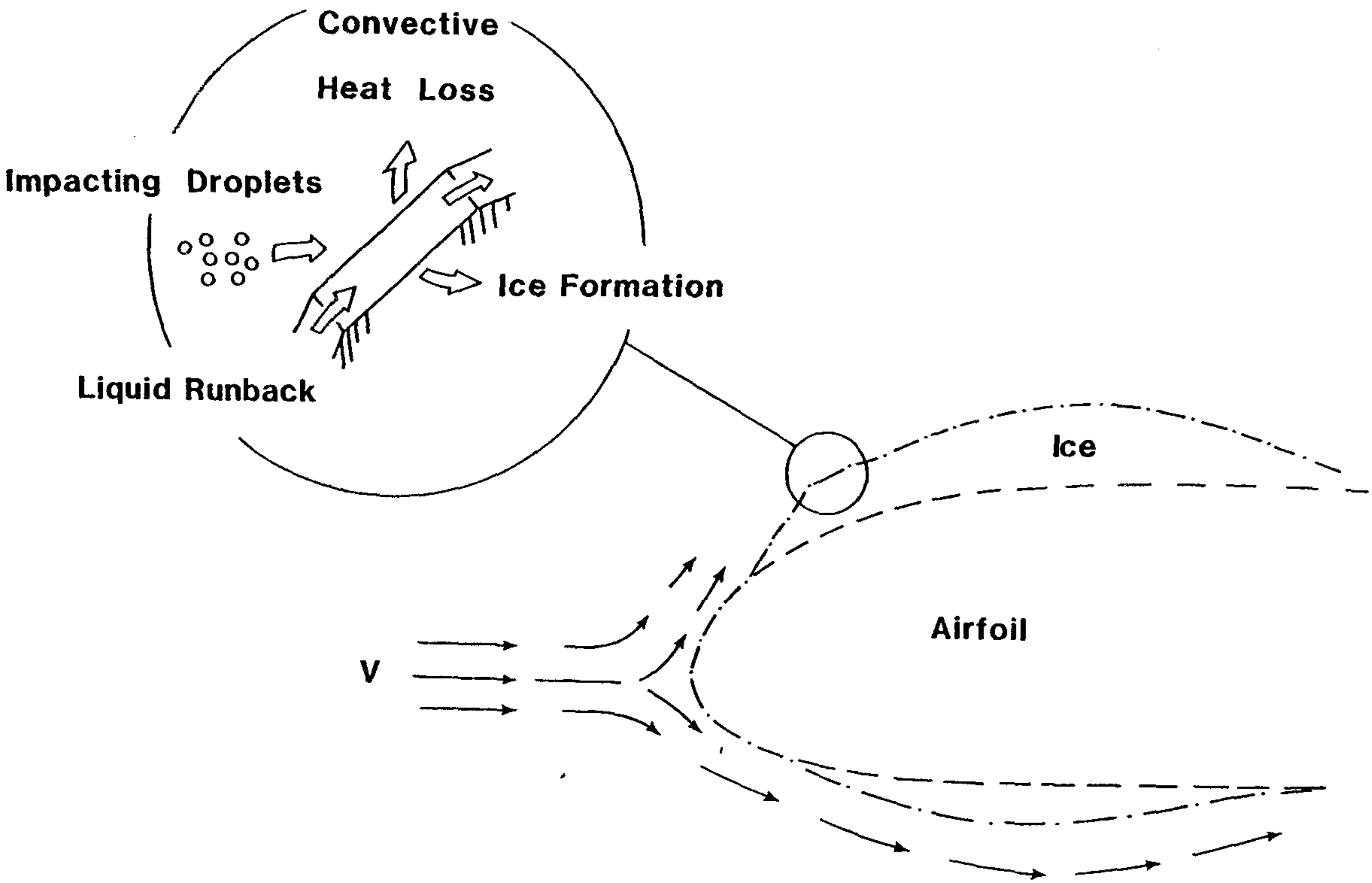

Figure 2. Ice Accretion Mass and Energy Flows.

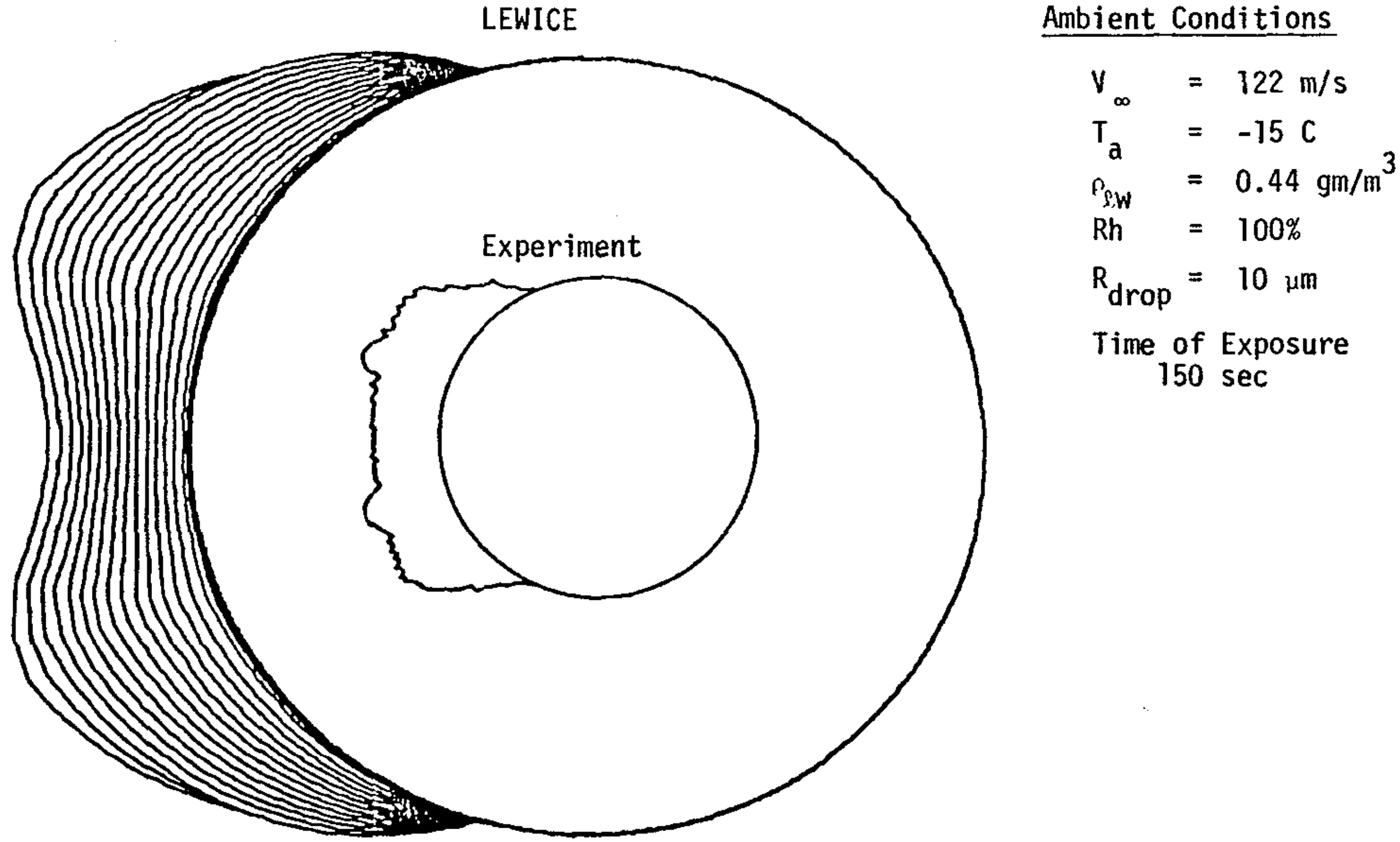

Figure 3. LEWICE prediction of icing of a 2.54 cm diameter circular cylinder. The inset shows an experimental result for these conditions by Stallabrass and Hearty (1979).

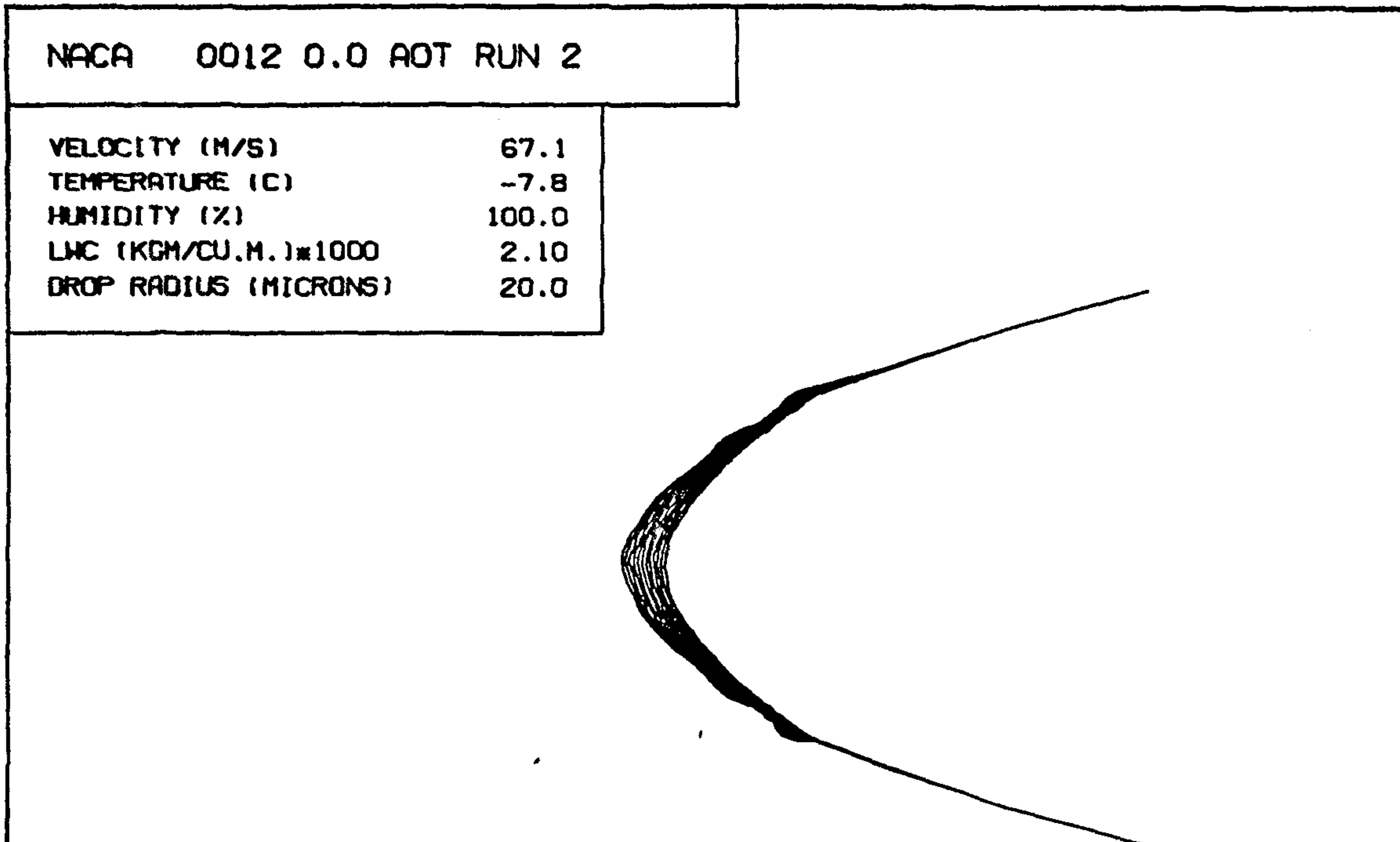

Figure 4. LEWICE prediction of the icing of a NACA 0012 airfoil (chord length 1 m) after 1 minute. Angle of attack is 0°.

NACA 0012 0.0 AOT RUN 2

VELOCITY (M/S)	67.1
TEMPERATURE (C)	-7.8
HUMIDITY (%)	100.0
LWC (KGM/CU.M.)*1000	2.10
DROP RADIUS (MICRONS)	20.0

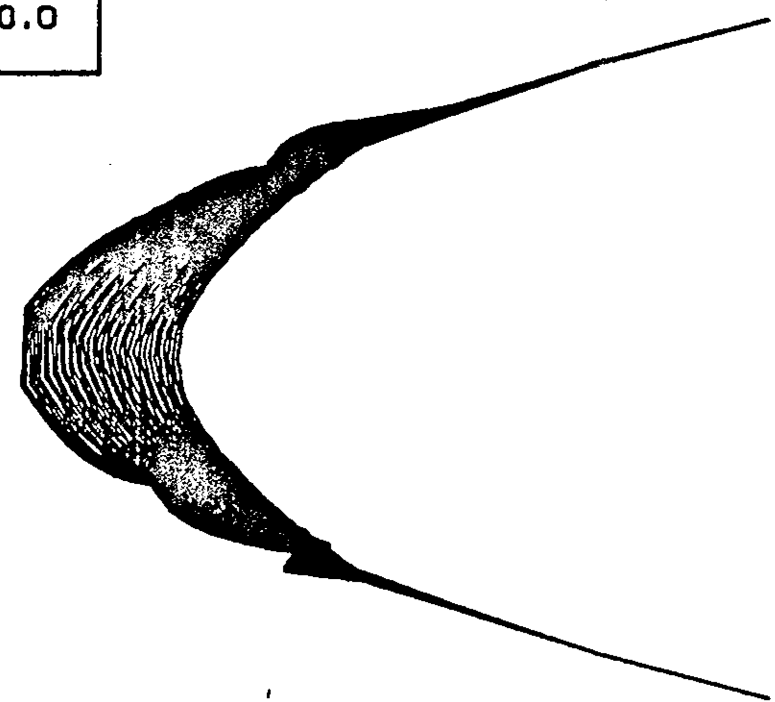

Figure 5. Icing of the airfoil of Figure 4 after 3 minutes of exposure.

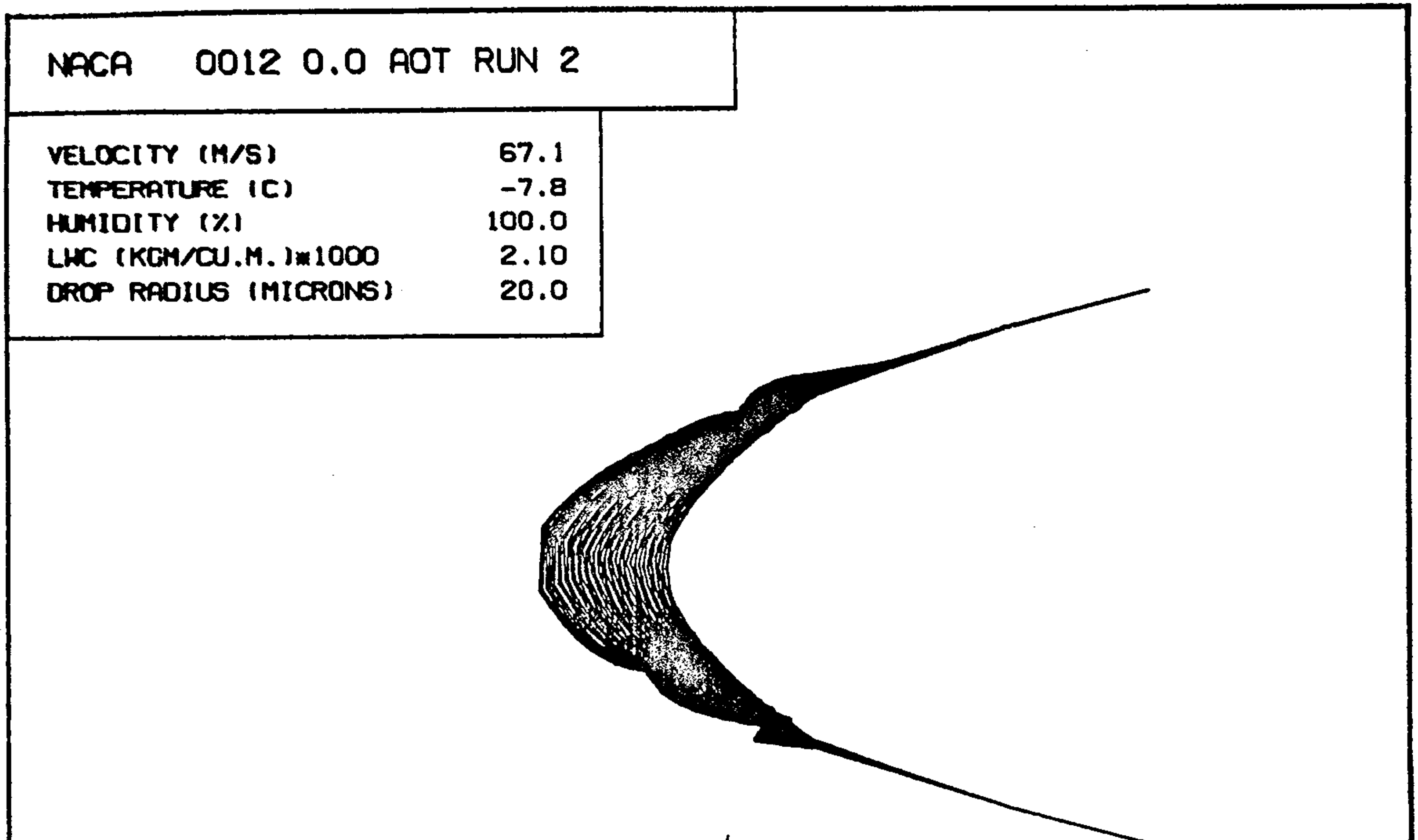

Figure 5. Icing of the airfoil of Figure 4 after 3 minutes of exposure.