Imperial College London

EE4-13 Adaptive Signal Processing and Machine Intelligence (2018-2019)

Imperial College London

Department of Electrical and Electronic Engineering

Coursework

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Contents

1 Classical and Modern Spectrum Estimation

1.1 Properties of Power Spectral Density (PSD)

1.1.a Approximation in the Definition of PSD

Starting with the equation provided, (1), we: use the relationship between modulus and complex conjugate for complex numbers, we move out the summation terms from the expectation operator, we factor out the exponential terms - as they are independent of the random variable x and we finally use the property that the expectation of a complex conjugate is ....

$$\begin{aligned} P(\omega) &= \lim_{N\to\infty} \mathbb{E}\left\{\frac{1}{N}\left|\sum_{n=0}^{N-1} x(n)e^{-j\omega n}\right|^2\right\} && (1) \\ &= \lim_{N\to\infty} \mathbb{E}\left\{\frac{1}{N}\sum_{m=0}^{N-1} x(m)e^{-j\omega m}\sum_{k=0}^{N-1} x^*(k)e^{j\omega k}\right\} \\ &= \lim_{N\to\infty} \frac{1}{N}\sum_{m=0}^{N-1}\sum_{k=0}^{N-1} \mathbb{E}\left\{x(m)e^{-j\omega m}x^*(k)e^{j\omega k}\right\} \\ &= \lim_{N\to\infty} \frac{1}{N}\sum_{m=0}^{N-1}\sum_{k=0}^{N-1} \mathbb{E}\left\{x(m)x^*(k)\right\}e^{-j\omega(m-k)} \\ &= \lim_{N\to\infty} \frac{1}{N}\sum_{m=0}^{N-1}\sum_{k=0}^{N-1} r_{xx}(m-k)e^{-j\omega(m-k)} = \lim_{N\to\infty} \frac{1}{N}\sum_{m=0}^{N-1}\sum_{k=0}^{N-1} g(m-k) && (2) \end{aligned}$$

where $g(\tau) = r_{xx}(\tau)e^{-j\omega\tau}$

We can convert the double summation into a single summation using:

$$\sum_{m=-N}^{N}\sum_{k=-N}^{N} g(m-k) = \sum_{\tau=-2N}^{2N} (2N+1-|\tau|)g(\tau) \tag{3}$$

(2) can then be written as:

$$\begin{aligned} P(\omega) &= \lim_{N\to\infty} \frac{1}{N}\sum_{\tau=-(N-1)}^{N-1} (N-|\tau|)r_{xx}(\tau)e^{-j\omega\tau} \\ &= \lim_{N\to\infty} \sum_{\tau=-(N-1)}^{N-1} r_{xx}(\tau)e^{-j\omega\tau} - \lim_{N\to\infty}\frac{1}{N}|\tau| \sum_{\tau=-(N-1)}^{N-1} r_{xx}(\tau)e^{-j\omega\tau} \\ &\approx \sum_{\tau=-\infty}^{\infty} r_{xx}(\tau)e^{-j\omega\tau} && (4) \end{aligned}$$

1.1.b Simulation of the Limiting Case

1.2 Periodogram-based Methods Applied to Real–World Data

1.2.a The SunSpot Dataset

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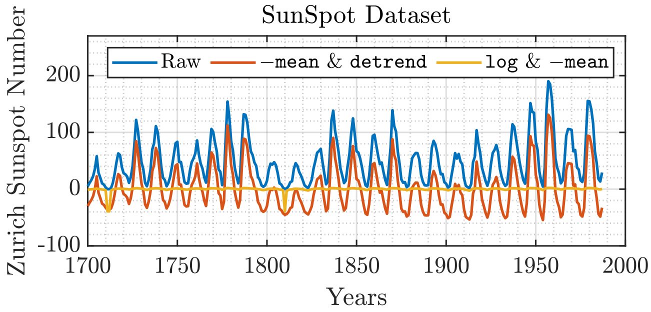
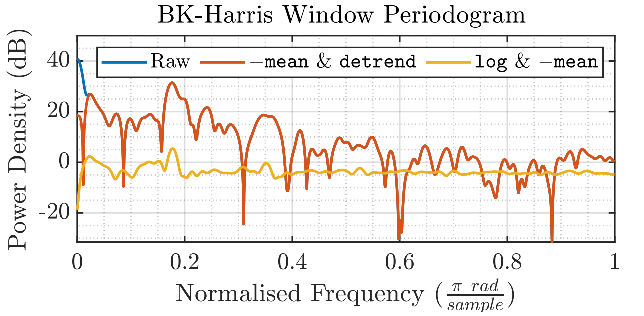
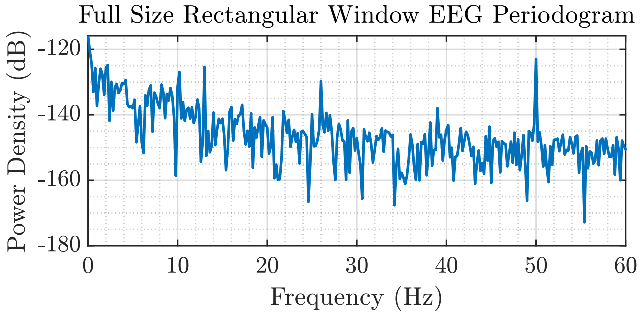


(a) Raw and its preprocessed datas

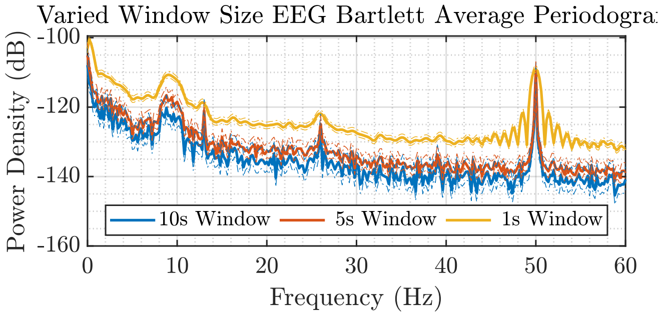


(b) Periodograms

1.2.b The ECG Dataset



(a) Standard Periodogram



(b) Bartlett Average Periodograms

1.3 Correlation Estimation

1.3.a Unbiased and Biased ACF Estimates

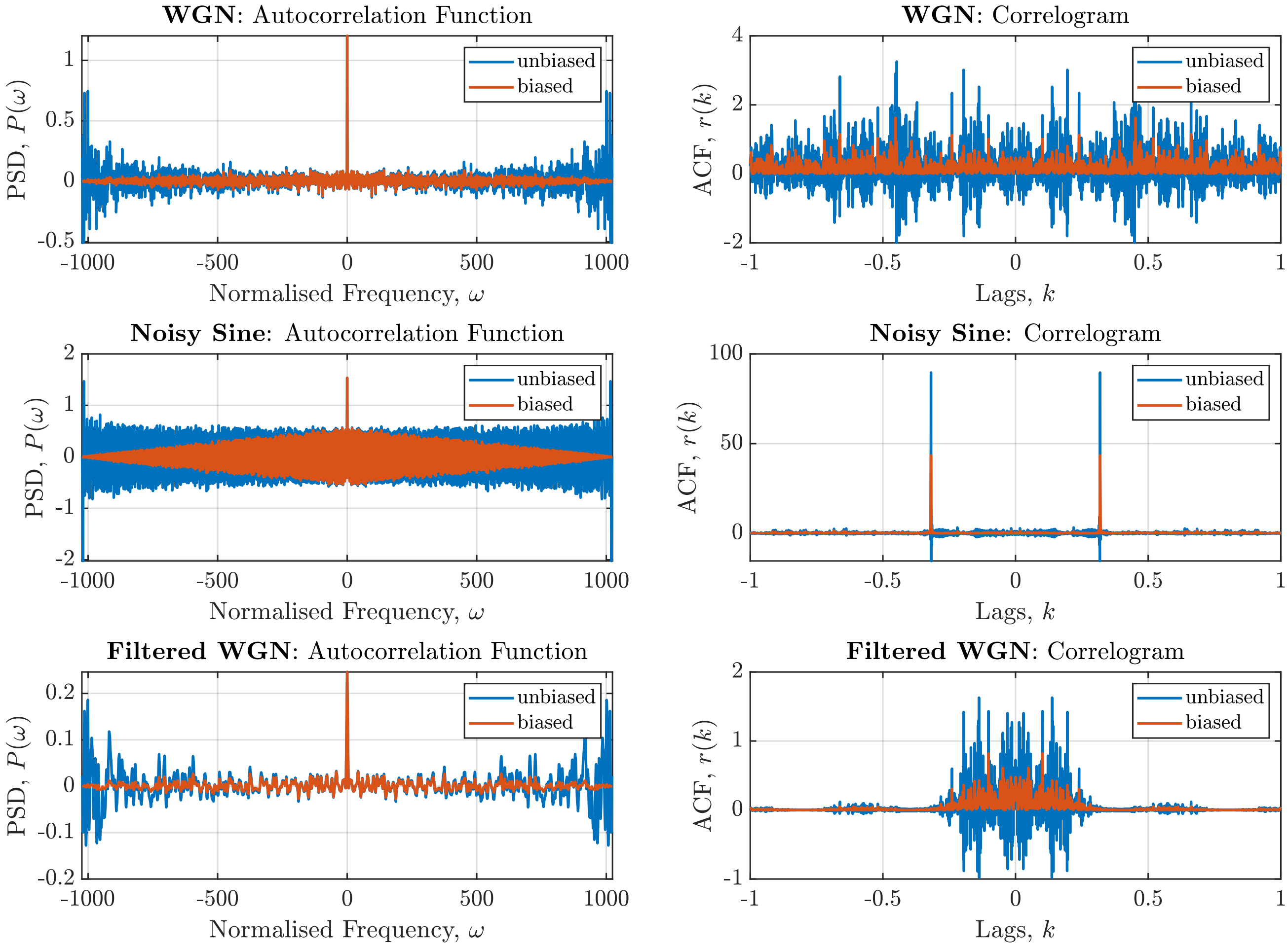
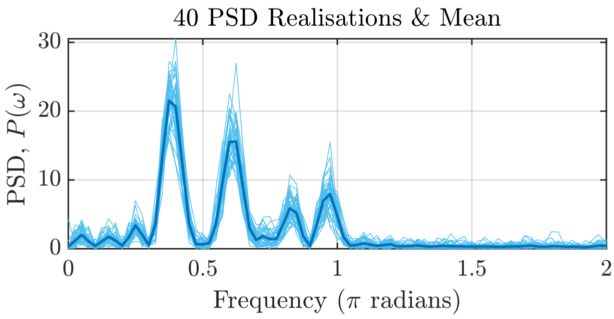
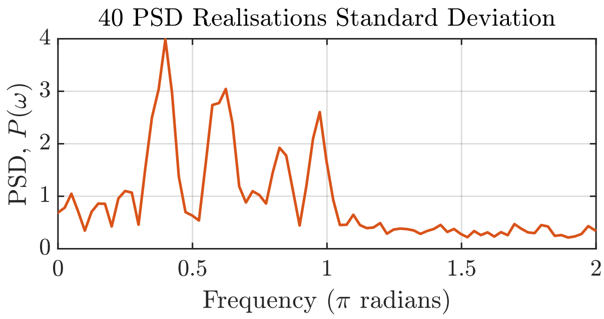


Figure 3: Set of Auto-Correlation Functions (ACFs) and their Correlograms

1.3.b Biased ACF Estimator PSDs

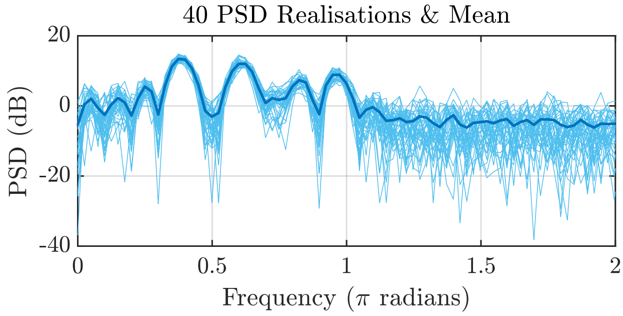


(a) Periodogram

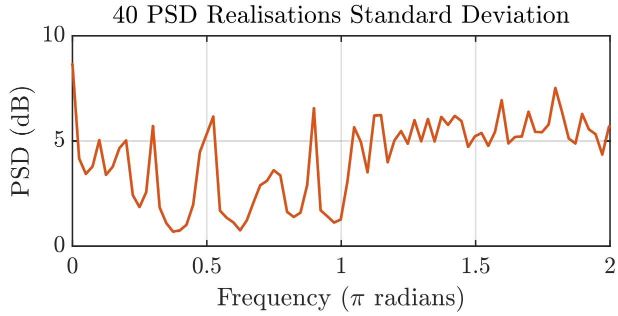


(b) Standard Deviation

1.3.c Biased ACF Estimator PSDs on the dB Scale



(a) Periodogram



(b) Standard Deviation

1.3.d Influence of Data Samples on the PSD

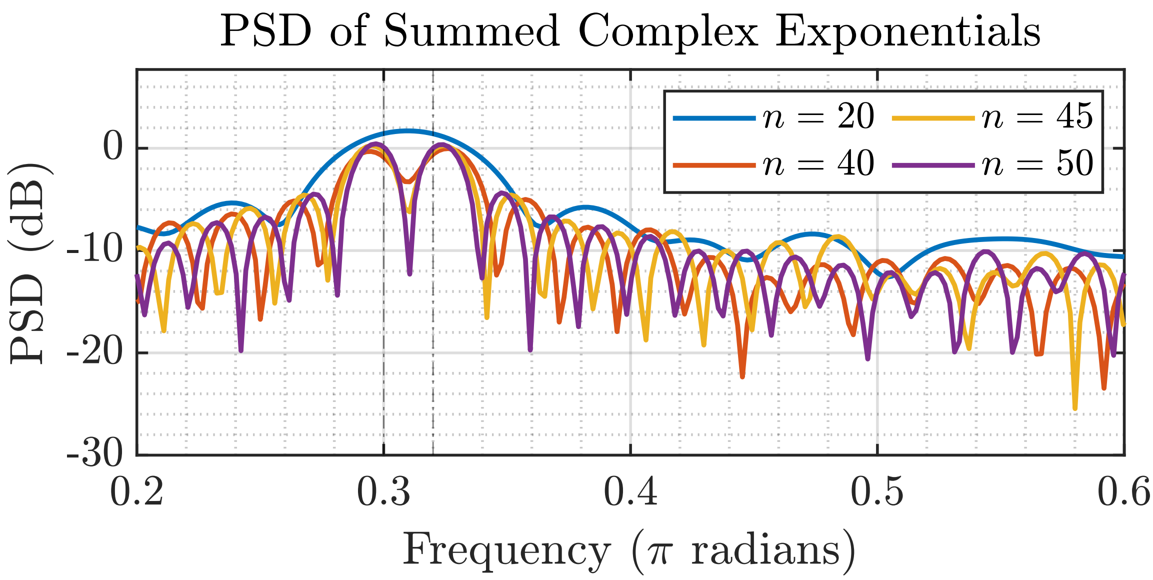


Figure 6: PSD while varying n, the number of Data Samples used

1.3.e MUltiple SIgnal Classification (MUSIC) Estimator

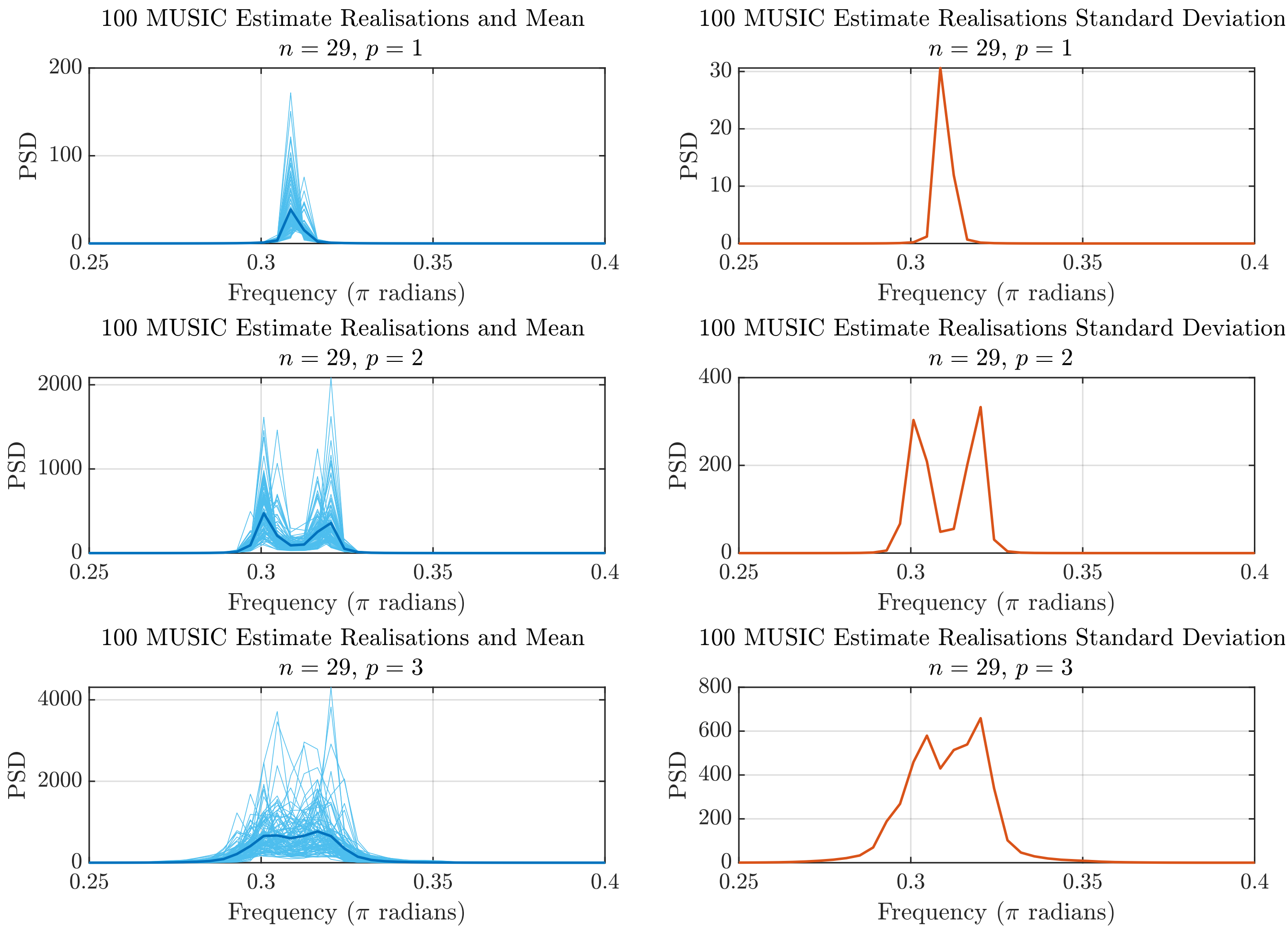
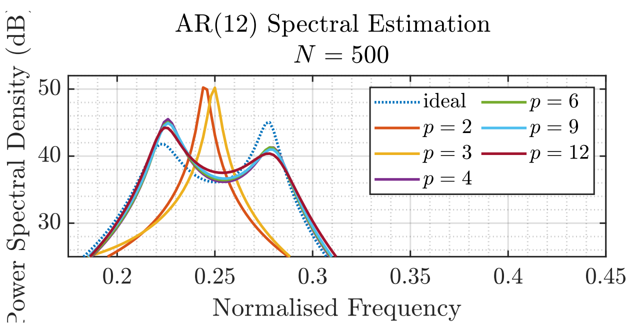


Figure 7: Set of Auto-Correlation Functions (ACFs) and their Correlograms. n is the number of samples used, p is the Signal Space Dimensionality

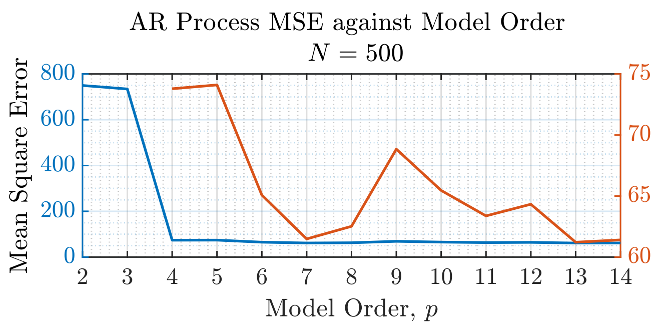
1.4 Spectrum of Autoregressive (AR) Processes

1.4.a Shortcomings of the Unbiased ACF in finding AR Parameters

1.4.b Error of the AR PSD Estimate

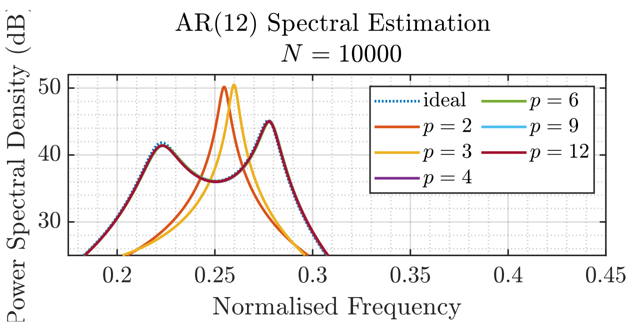


(a) AR Periodogram and its p Order Estimates

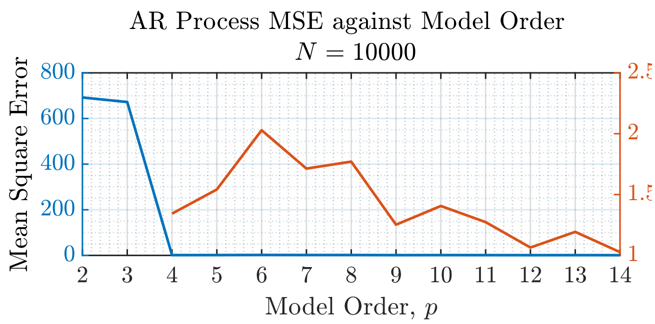


(b) Mean Squared Error

1.4.c Error of the AR PSD Estimate with more Samples



(a) AR Periodogram and its p Order Estimates



(b) Mean Squared Error

1.5 Real World Signals: Respiratory Sinus Arrhythmia from RR-Intervals

1.5.a Standard & Average PSDs of the RRI Dataset

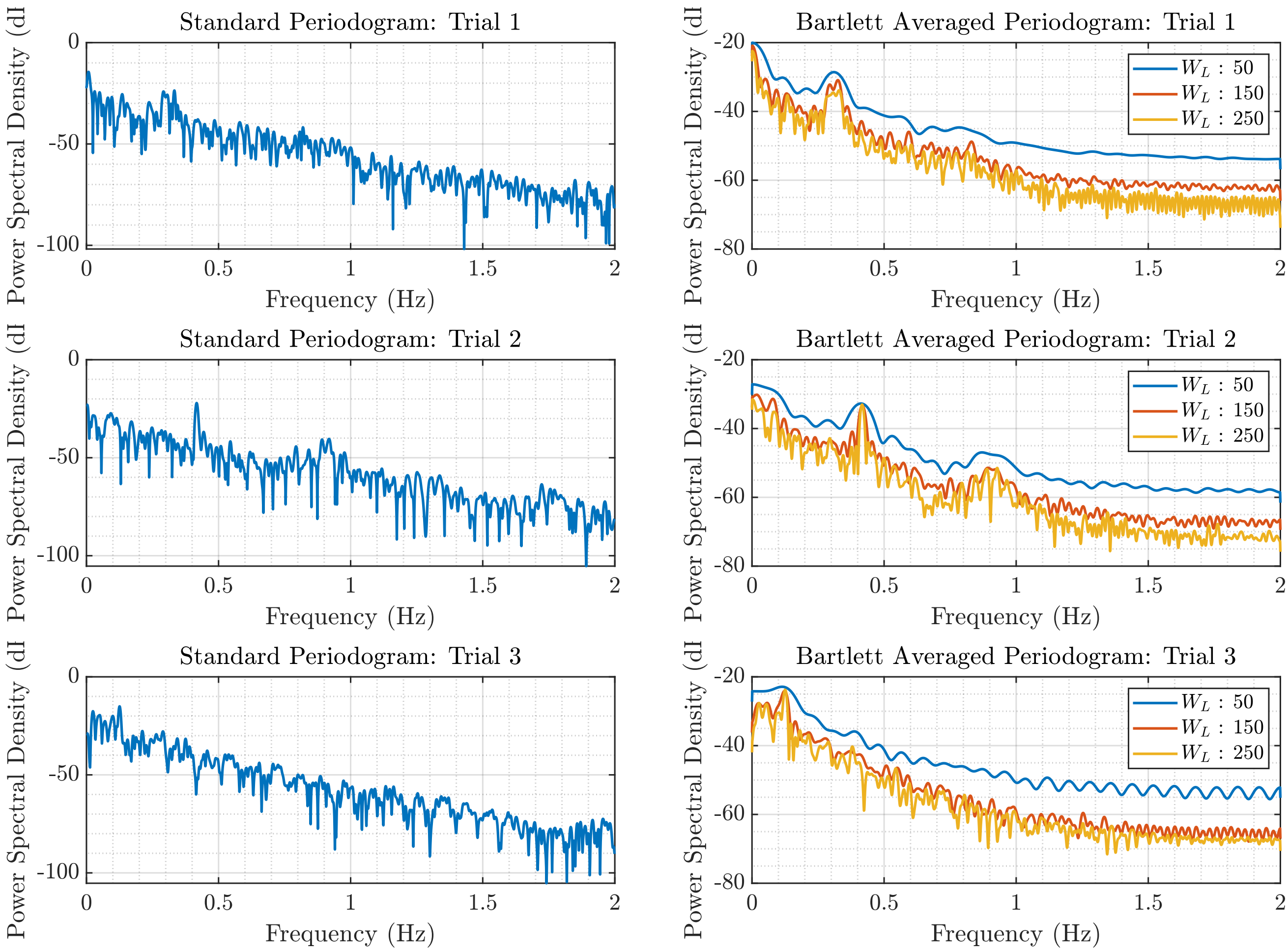


Figure 10: Standard and Bartlett Average Periodograms.
W_L is the Window Length used

1.5.b Analysis of the RRI PSD Estimates

1.5.c AR PSD Estimate for the RRI Dataset

Test text

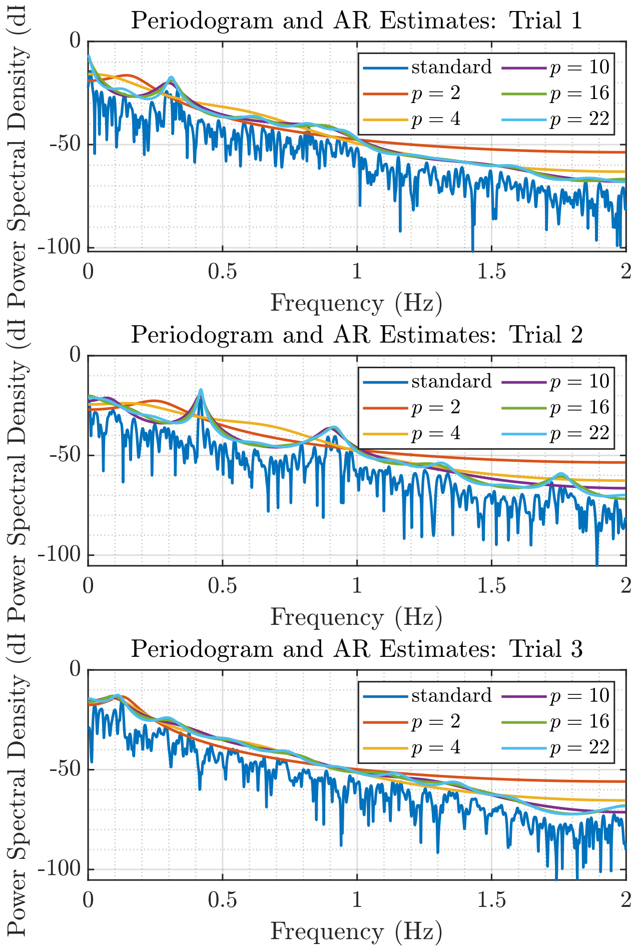
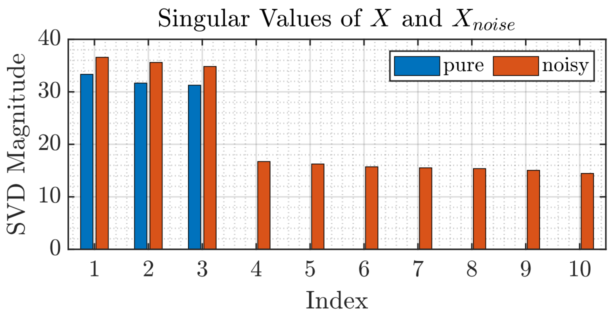


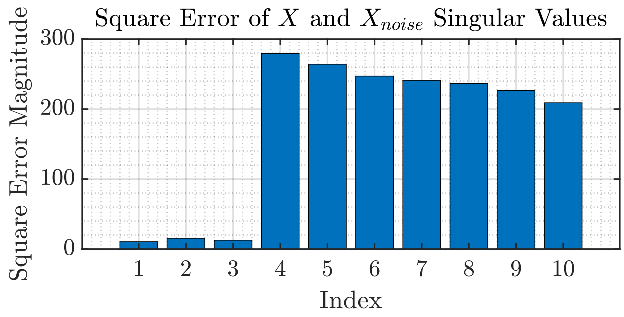
Figure 11: AR Estimate Periodograms. p is the model order.

1.6 Robust Regression

1.6.a Single Value Decomposition (SVD)

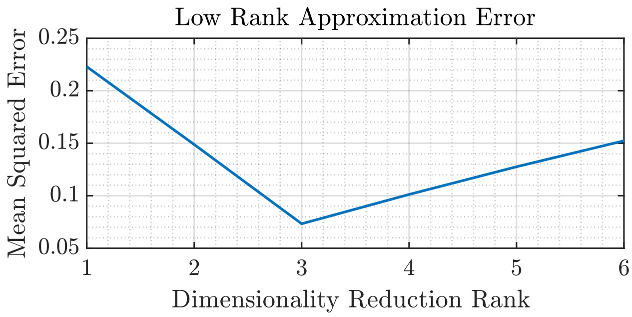


(a) SVD



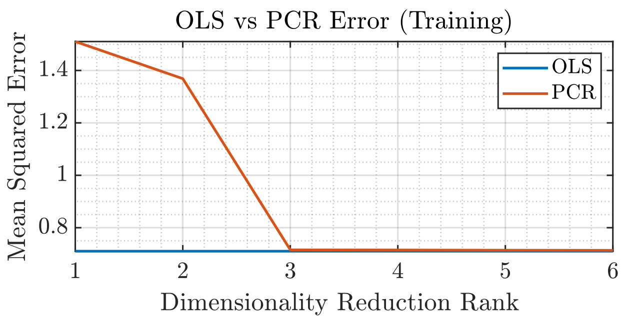
(b) Square Error

1.6.b Low Rank Approximation Error

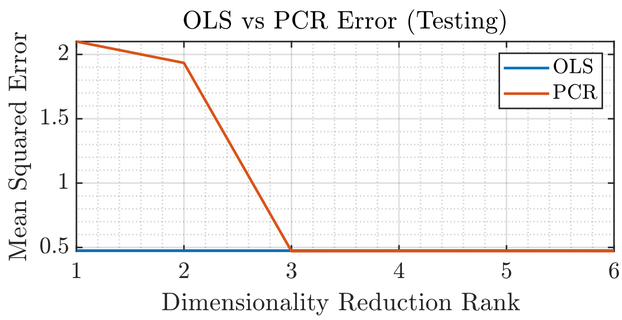


(a) Effect of Lower Ranks on the Approximation Error

1.6.c Ordinary Least Squares (OLS) & Principle Component Regression (PCR) Estimate Errors

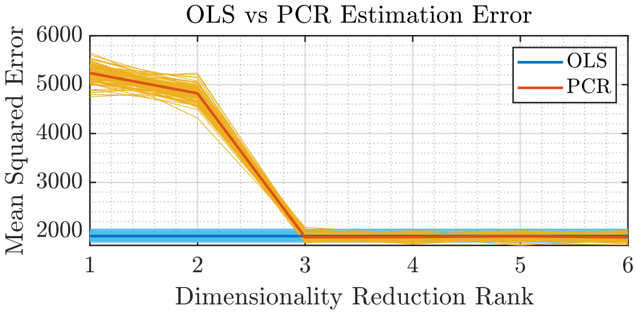


(a) Training Dataset Error



(b) Testing Dataset Error

1.6.d Ordinary Least Squares (OLS) & Principle Component Regression (PCR) Estimate Errors - Part 2



(a) Mean Sqaure Error over several Realisations