Physics 2A Lab Manual

Student Edition

San José State University

Spring 2016

Contents

List of FNTs

Discussion/Lab 1

1 A Introduction to the Course Format

This class is run in a way that is likely different from what you are used to. *Active* participation is *essential* for your success in the course.

To help you become a productive and successful member of our class community, we will start off with an introductory activity that will allow you to familiarize yourself with the course format. Since our philosophy about how you learn best in this course most likely differs from other instructors', we will do everything we can to help you get into the habit of participating actively and asking questions as soon as you have them.

1 B Pretest

Our physics department is very interested in finding out about the effectiveness of our courses. Therefore, in many courses, students are asked to complete a pre-test at the beginning of the semester (and a post-test at the end) that aims to measure how well a particular course helps students understand certain physics topics.

In the first discussion lab meeting, you will be taking a pretest to show what you know coming into the class. There is absolutely no need to study for this ahead of time. This test is ***not*** graded (however, we see taking the test as part of your active participation in this course). We may use the overall results to help us decide what to focus on teaching during the semester.

Unit I

The Three-Phase Model of Matter and The Energy-Interaction Model

Discussion/Lab 2

2 A Making Sense of Thermal Phenomena

Overview: We start our exploration of physics this semester by observing and reflecting on the phenomenon of "freezing" things.

2 A.1 Observing what happens when things freeze

1. Freezing a Heat Pack

As a group, trigger one of the heat packs (you may know them as "hand warmers") on your table by clicking the button that's submerged in the liquid. Observe what happens within the heat pack during the first few minutes after triggering. Describe what you observe (i.e., tell each other a *story* about what happened to the heat pack).

In a few sentences, *write on the whiteboard* your description (your story) of what you ***observed*** (***not*** what you *think* happened!). Please use words that everyone in your group is comfortable with. **Write clearly so that everybody in the room can read!**

2. Freezing liquid water

Based on your everyday experience, describe what you have to do to freeze water. After you have discussed this in your group for a few minutes, write your group's response on the board.

3. Does the behavior of a heat pack make sense?

Discuss in your group:

(a) Based on your everyday experience with freezing things, would you say that the heat pack "froze" when it was triggered?

(b) Why or why not?

(c) Does the behavior of the heat pack make sense to you? Why or why not?

Write your thoughts in response to (a), (b), and (c) on the whiteboard.

Whole Class Discussion

2 A.2 Using models to make sense of physical phenomena: Introducing the Three-Phase Model of Matter

Please read before moving on:

Before we can *make sense* of the process that occurs when the heat pack is triggered, we need to organize what we know about freezing and melting. You probably studied the processes of freezing and boiling in several previous science classes – in a physical science class or chemistry in high school and perhaps in a college chemistry class. You have also had a good deal of everyday experience with freezing and boiling. But it is likely that your knowledge about these processes and phases of matter is somewhat fragmented and not well organized. Pulling together what we know and getting it organized in a useful way is what we need to do.

Learning Science* means *Refining Intuitions* and *Organizing Knowledge

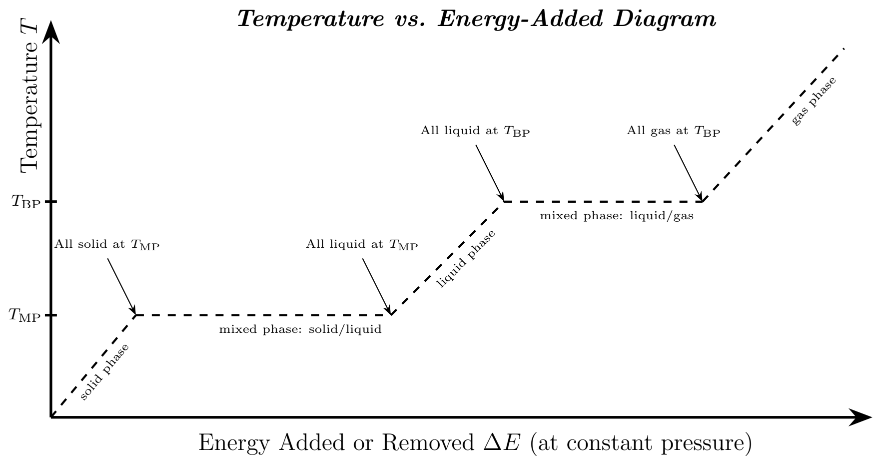
We all have everyday experience with lots of physical processes and phenomena. Sometimes, our intuitions seem to initially contradict the "accepted scientific understanding." This is true for beginning scientists and experienced researchers alike! To continually improve our understanding of the world, we often need to refine our intuitions. In fact, we do this every day, while interacting with people, things, and situations in our environment. We use what we know about the world to make sense of our surroundings. Sometimes, we have to reorganize what we know in order to understand a new situation, and sometimes we learn something new altogether – new knowledge that we have to somehow fit in with what we already know. When this new knowledge organization becomes "second nature" to us, we have refined our intuition.

Models help scientists organize knowledge

One tool scientists use to organize knowledge related to a class of phenomena is the *scientific model*. Models bring together in one place the ideas and data patterns we use to make sense of phenomena in a concrete and useful way. A model represents these ideas in such a way that we can readily access them and use them. That is, a model typically provides the representational tools needed to use the model to develop explanations, make predictions, and more generally to make sense of phenomena. Over time, the features of a model will become second nature to us; we will have refined our intuitions about the phenomena in question.

The Three-Phase Model of Matter

The **Three-Phase Model of Matter** provides a summary of the ideas needed to make sense of phase changes in matter (really, it only applies to so-called "pure substances," but we won't make that distinction yet). There are instances, however, when the "standard model" is not sufficient to understand certain phenomena that undergo a phase change. This is the case with the heat pack. We have to extend the model, which is what we will do together in the following activities.



1. **Examine the Three-Phase Model of Matter Summary**

   There should be several copies of the model summary sheet on your table. You can also find this summary in the online resources. Take about five minutes to look at what is on the summary sheet. Then focus on the diagrammatic representation: The *Temperature vs. Energy-Added Diagram*. Discuss with your group what the axes mean.

2. **Use the Three-Phase Model of Matter to construct a general explanation**

   (a) Put a sketch of the complete *Temperature vs. Energy-Added Diagram* on your whiteboard that shows all three phases. Put a large dot on the graph that would represent tap water at room temperature.

   (b) Now with the whole group participating, have someone put their finger on the dot and the rest of the group (take turns) describing what is happening to the water (tell another story!) as it is brought to a boil in the electric hotpot on your table. The person with their finger on the dot should move her/his finger along the graph to match the description. Imagine you leave the heating pot on for five or ten minutes, but not so long that all the water boils away. Now imagine putting the pot in your freezer for an hour or so and move your finger accordingly along the cooling curve.

   (c) Repeat (b) over and over until everyone in your group is ready to explain to the whole class what goes on as you heat a substance way up and when you cool it way down. Use the language of the **Three-Phase Model of Matter** in your description, and use the diagram to illustrate your explanation.

Whole Class Discussion

3. Prepare the experiment

Use the model (and your knowledge of freezing and boiling points of water) to make a prediction (i.e., **don't start boiling the water yet!**) about temperatures:

(a) The hotpot in front of you should be filled about two thirds full with water. What temperature range does the model predict for the liquid water? I.e., what temperatures can liquid water have before it freezes or boils, according to the model?

(b) Suppose you begin heating the water. Immediately after the water has begun to boil, what does the model predict for the temperature of the liquid water? What about after it has boiled for five minutes? Compare your initial intuition with what the model tells you the temperature absolutely must be (assuming the model is appropriate for this phenomenon).

(c) If the lid is kept tightly on the pot and you have one thermometer immersed in the liquid and another one just above the surface of the liquid, and the water has been boiling vigorously for several minutes, what does the model tell you about the two temperatures?

4. Do the experiment

(a) Check the model's predictions from Part 3 using the hot pot and the two thermometers you have at your table. On the board make a chart and compare your predictions with your experimental results.

(b) Are there issues in the design of the experiment you just performed that are not taken into account in the model? Describe these on the board and be prepared to discuss them with the class.

Remember: Make sure your responses are legible, and that your entire group agrees with what is written on the board.

Whole Class Discussion

5. What does the model say?

Use the **Three-Phase Model of Matter** to answer the following question.

When two phases of a substance are present and in thermal equilibrium (i.e., both phases have the same temperature), what do you know with certainty about certain properties or physical conditions as they relate to that substance?

Discuss in your group. How does the model representation help you answer this question?

Whole Class Discussion

2 B Analyzing a Heat Pack Using the Three-Phase Model of Matter

Overview: Our goal is to make sense of the heat pack behavior using both the **Three-Phase Model of Matter** and the **Energy-Interaction Model**. We'll start with a focus on the **Three-Phase Model of Matter**.

2 B.1 Macroscopic properties and energy transfers

When analyzing a physical phenomenon, a model greatly simplifies and restricts what we focus on. **The constructs that make up the model *limit* the aspects of the phenomenon that we must pay attention to.**

1. Using *only* the constructs of the **Three-Phase Model of Matter**, what are ***macroscopic properties*** of the heat pack that changed (and how did they change) during the first few minutes after triggering?

   Look at the Model Summary Sheet and try to use only those words.

   Write your new story on the board.

2. How does the amount of energy associated with clicking the small disk inside the heat pack compare to the other energy transfers that occur? Think, for example, about the amount of heat transferred from the heat pack to your hands or to the environment.

   Is it about the same amount of energy? Much larger? Much smaller?

   Write your group's response on the board.

Whole Class Discussion

2 B.2 Completing the cycle

Now, let's take the heat pack we had triggered before through the rest of its cycle.

1. Read and follow the instructions on the heat pack to get it back to being a liquid at room temperature.

2. Describe qualitatively and briefly how the properties of the heat pack identified in Part 2 B.1 changed during the rest of the cycle.

3. Write your new story on the board.

Attention: You will continue your analysis of the heat pack in your homework assignment. Therefore, please write down in your notes how the properties changed as the heat pack went through its complete cycle. You will need this!

Whole Class Discussion

2 B.3 Using our model

1. Think about your response to Question 5 in Activity 2 A.[1] When your heat pack is in a mixed solid/liquid state, what do you know about its temperature?

   Use this knowledge to experimentally determine the melting-point temperature of the heat pack. In order to ensure equilibrium conditions, you can minimize energy transfers to or from the heat pack by wrapping plastic bubble-wrap around the heat pack and thermometer. You can also fold your heat pack around the end of the thermometer.

   Does it matter how you attained the mixed state: melting when in the hot pot or freezing when triggered?

   Record your result on the board, indicating a "direction" on your diagram.

2. Two processes that you can readily observe with the heat pack are: i) changing from liquid to solid and ii) changing from solid to liquid.

   (a) For which of these two processes does the heat pack seem to follow the predictions of the **Three-Phase Model of Matter** and for which process does it ***not*** follow the model? Be brief, but specific. Put your response on the board. [Hint: Use your finger to trace the graph.]

   (b) For which of these two processes does the heat pack behavior pretty much agree with your intuition about phase changes? For which process does it not agree? Put your response on the board. Does your intuition now agree pretty much with the model? If not, what is different?

Whole Class Discussion

[1]Here's the question again, for reference, and so you don't need to flip back and forth: "When two phases of a substance are present and in thermal equilibrium (i.e., both phases have the same temperature), what do you know with certainty about certain properties or physical conditions as they relate to that substance?"

2 C Practicing the Three-Phase Model of Matter; Introducing the Energy-Interaction Model

Overview: We briefly interrupt our efforts of making sense of the heat pack to get a bit of practice using the **Three-Phase Model of Matter** and to introduce the *Energy-System Diagram*.

Your instructor will assign one or more of the following physical processes to your small group:

(a) Cooling a piece of solid copper (Cu) from 500 °C to 350 °C.

(b) Warming a piece of ice from -20 °C to the melting point.

(c) Condensing steam completely to liquid at 100 °C.

(d) Completely sublimating a chunk of dry ice at -79 °C.

(e) Partially melting 25% of ice initially at 0 °C.

(f) Heating a piece of copper initially at 300 °C until it is half melted.

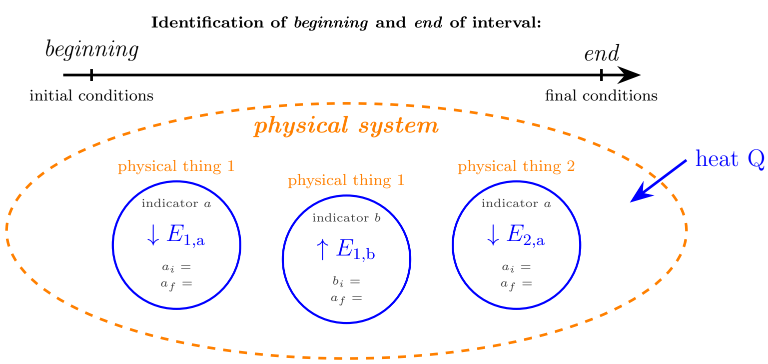
(g) Cooling and completely freezing H_2O initially at 80 °C.

For each of these processes, sketch the following on your whiteboard and be prepared to explain to the whole class:

(i) A complete *Temperature vs. Energy-Added Diagram* (remember, this is the graphical part of the **Three-Phase Model of Matter**) with the initial and final states clearly marked;

and

(ii) An *open Energy-System Diagram*. Refer to "Steps Involved in Using the *Energy-System Diagram*" on the **Energy-Interaction Model** summary on your table. Try your hand at translating your *Energy-System Diagram* into an algebraic expression of energy conservation.



$$\overset{(-)}{\Delta E_{1,a}} + \overset{(+)}{\Delta E_{1,b}} + \overset{(-)}{\Delta E_{2,a}} = \overset{(+)}{Q}$$

Again, *you need to be prepared* to explain and describe the process you have been assigned to the whole class using both models. When you use the **Three-Phase Model of Matter** trace the process with your finger as you describe it. Make sure you clearly show the beginning and ending points of the process in the diagram.

NOTE: You will have an opportunity to rework these examples in the homework. These are all very straightforward once you're familiar with the basic ideas of the two models. Working through these examples will also give you practice in applying the models and using the representations with a variety of phenomena. It is very important that you can do this quickly and correctly (i.e., it should become *second nature* to you). When applying these models to make sense of more complicated phenomena, e.g., on a quiz, it is crucial that you are not stumbling over the basics.

Whole Class Discussion

Discussion/Lab 3

3 A Practicing Our Two Models

Overview: We continue to practice using the **Three-Phase Model of Matter**, and we'll start to get more familiar with the *Energy System Diagrams* of the **Energy-Interaction Model**.

3 A.1 Basics of the Three-Phase Model of Matter

FNT 3-1

Scientists don't come up with "right answers."

Usually, when your instructor asks you to "do problems" for homework or on a test, they expect you to get a "right answer," right? Well, we don't. As a matter of fact, for many of the problems you will encounter in this course, there won't be one "right answer." Science just doesn't work that way. For every problem, there are a number of ways one can approach the problem, and each different solution path might result in different answers. But, that doesn't mean that one of them is "right" and the others are not.

Scientists come up with arguments.

Much more important than an answer to a question or problem in science is usually the method by which one arrived at this answer. So, our emphasis is on the method you use to solve a problem – the pathway to your solution. **We want to know what assumptions and decisions you're making as you solve a problem, and how you're using these assumptions and decisions to *make an argument* for *your* solution.** That's how science works! There is nobody who knows "the right answer." But there are people who can look at what you've done and tell you whether the assumptions you've made are appropriate in a particular situation and if the argument you've constructed is valid and convincing. In science, this is called "peer-review" because the people who look at your work are other scientists.

So, if we don't care so much about you "getting the right answer," how will we evaluate your progress in this course?

Just like in a scientific publication, **we ask you to be *very specific* about what you did and did not do, and *how* you arrived at a particular solution to a problem.** We'll practice this a lot, so that you know what we expect from you. This first problem is an extreme example of what problem-solving in this class looks like for you: *We'll actually give you some answers.* A little further down, you'll find a box with some answers that one might reasonably get when solving this problem. Note that we're not saying these are the exact answers you will get or that we expect you to get! But your solutions to this problem might be quite similar to the ones given below.

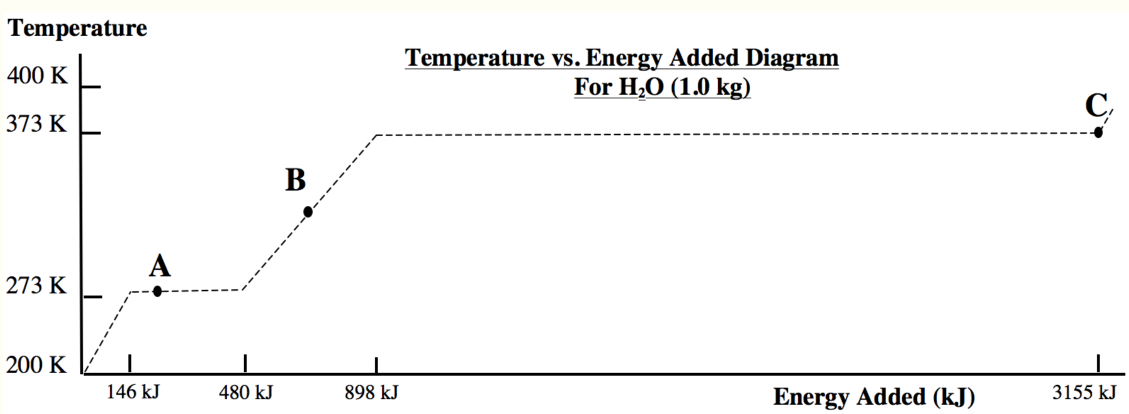
Now, if we give you the answers, what do we want you to do?

We want you to describe in detail how you are using the diagram below to respond to the prompts in the problem. Write a story about the diagram and about how you are reading the graph to get the values you get. While this is stated a few times below, it's worth repeating: *We don't want you to do any calculations for this problem!* Instead, use the diagram and tell us *how* you're using the diagram and *why* we should believe you that your answer to the problem is a reasonable one.

Yes, this might be a bit more work than what you're used to from other classes. Later in this course, we'll find shorter ways for you to show us your work and argument. But for now, ***please do write out a detailed story – including your assumptions, decisions, etc. – for each part of the problem.***

On to the actual problem:

Use the particular *Temperature vs. Energy-Added Diagram* of the **Three-Phase Model of Matter** shown below to respond to the prompts in this FNT.



The values along the x-axis (146 kJ, 480 kJ, etc.) are the amounts of energy that must be added to get the 1.0 kg of water from an initial temperature of 200 K to the next phase change. For example, 480 kJ is the total amount of energy that must be added to the water starting at the initial temperature of 200 K to completely melt all the ice.

(a) Describe what is happening at each of the "corners" on the graph. What phase is the water in at the letter A, B, C?

(b) Assume that you have one kilogram of water at an initial temperature of 200 K. If 795 kJ of energy is added, describe the final state (approximate temperature and phase) of the water. If the water is in a mixed phase, determine very approximately the percent in each phase.

You are not expected to do any calculations at this time. Simply tell us what the diagram tells you (and how you know that it does).

(c) Repeat Part b but with 146 kJ of energy added to the water initially at 200 K.

(d) Repeat Part b but with 1650 kJ of energy added to the water initially at 200 K.

(e) State the initial and final conditions of the process that takes the system from Point C to Point B and determine very approximately how much energy would have to be added or removed.

(f) Repeat Part e for water initially in State A and ending in State C.

Again: Do not make any calculations to respond to these prompts!

You've all done this FNT individually at home. Now you have an opportunity to practice creating scientific arguments together. Back up your claims with evidence and try to convince each other of *your* solution!

1. Discuss with your group and make sure *every* group member is confident that she/he can explain Question a on the FNT.

2. While there is not one single correct answer for Parts b – f of the FNT, you probably estimated the values somewhere in the vicinity of those in the box. Work out in your small groups any significant discrepancies you might have. EVERYONE in your group should be ready to explain how you obtained these results in the whole class discussion. Your instructor will tell your group which prompt you should respond to on the board.

Possible Answers

(a) liquid at ~350 K

(b) completely solid at 273 K

(c) ~1/3 gas, ~2/3 liquid at 373 K

(d) initial conditions: all gas at 373 K, final conditions: liquid at ~50 °C; $\Delta E \approx 2470$ kJ

(e) initial conditions: ~25% liquid at 0 °C, final conditions: gas at 100 °C; $\Delta E \approx 3000$ kJ

Whole Class Discussion

3 A.2 Basics of the Energy-Interaction Model

FNT 3-2

Neatly write out all of the *Energy System Diagrams*, with the accompanying *Temperature vs. Energy-Added Diagrams*, listed below (these are the same scenarios that were requested in Activity 2 C).

Remember that complete *Energy System Diagrams* always include algebraic expressions of energy conservation. Refer to the **Energy-Interaction Model** discussion in the online resources.

(a) Cooling a piece of solid copper (Cu) from 500 °C to 350 °C.

(b) Warming a piece of ice from -20 °C to the melting point.

(c) Condensing steam completely to liquid at 100 °C.

(d) Completely sublimating a chunk of dry ice at -79 °C.

(e) Partially melting 25% of ice initially at 0 °C.

(f) Heating a piece of copper initially at 300 °C until it is half melted.

(g) Cooling and completely freezing H_2O initially at 80 °C.

1. Compare your responses for the processes in Activity 2 C, both the *Temperature vs. Energy-Added Diagrams* and the *Energy System Diagrams*.

2. Your instructor will tell you which one of the scenarios (a) through (g) to put on the board.

   Put the *Temperature vs. Energy-Added Diagram* and the *Energy-System Diagram* near each other. Make sure the two diagrams are consistent with each other and make sure everyone in the group can explain precisely why both are drawn the way you have them.

Whole Class Discussion

3 B Making Sense of the Heat Pack: Exploring New Physical Phenomena

Overview: We return to our efforts of making sense of the heat pack by using both the **Three-Phase Model of Matter** and the **Energy-Interaction Model**. In the process, we'll explore some new, related thermal phenomena – *Super-Heating* and *Super-Cooling* – and we'll see that we have to modify the standard **Three-Phase Model of Matter** to describe and understand the heat pack.

3 B.1 Taking the Heat Pack through its Entire Cycle

FNT 3-3

When using the **Energy-Interaction Model** to make sense of the behavior of a heat pack (or really, any thermal cycle), it helps to divide the overall process (cycle) into multiple sub-processes. Use the following sub-processes:

	Initial conditions	Action	Final Conditions
(a)	liquid at 100 °C	taken out of boiler	liquid at 23 °C
(b)	liquid at 23 °C	triggered (and insulated)	solid/liquid at 54 °C shortly after triggering (in mixed state)
(c)	solid/liquid at 54 °C	sitting on table	solid at 23 °C
(d)	solid at 23 °C	placed in boiler	solid/liquid at 54 °C
(e)	solid/liquid at 54 °C	left in boiler	liquid at 100 °C

Make four *Temperature vs. Energy-Added Diagrams*, one for each sub-process (a), (c), (d), and (e), but NOT (b). Also, make an *Energy-System Diagram* for each process. Make sure you can describe each process *in your own words* using both of the representations.

Very important for the following: Make sure everyone in your group is ready to *explain* to the whole class ***how*** they know the answers to the following questions (detailed instructions below):

1. Does each energy system increase or decrease?
2. Is this an open or closed system with respect to energy, and if open, does energy come in or leave?
3. How are your responses consistent with your *Temperature vs. Energy-Added Diagram*?
4. And finally, how do both diagrams tell you about the sign of each term in the equation expressing energy conservation?

To make this task a bit more manageable, your instructor will assign one of the four heat pack processes listed in FNT 3-3 to your group: (a), (c), (d), or (e) – note that we're not concerned with Part (b) yet. For your assigned process, put on the board both

(i) a *Temperature vs. Energy-Added Diagram*,

and

(ii) an *Energy-System Diagram.*

Make sure that the two representations of the process are in agreement and that everyone in your group can explain why they are drawn the way they are. Refer to "Steps Involved in Using the *Energy-System Diagram*" on the model summary page.

Whole Class Discussion

3 B.2 The Clicking Mystery: Super-heating and super-cooling

As we'll find out here, we have to modify the standard **Three-Phase Model of Matter** to illustrate what happens in the heat pack. Let's start with some preparations in this FNT.

FNT 3-4

We want to analyze triggering step (b) in FNT 3-4 more closely using the *Temperature vs. Energy-Added Diagram* of the **Three-Phase Model of Matter**. This will help us get a deeper understanding of this part of the process and will enable us to extend the **Three-Phase Model of Matter** to make it explain actual phenomena more realistically. (It turns out that super cooling and super-heating are rather common. Usually, however, they occur over a very small temperature range and so go unnoticed.)

(a) Sketch a *Temperature vs. Energy-Added Diagram* for sodium acetate going from room temperature to approximately 150 °C, assuming the sodium acetate does not undergo any super-cooling, i.e., assuming that the heat pack is solid at room temperature, and changes phase at its normal phase change temperature, which you determined in Activity 2 B.3.

(b) On the same diagram, sketch the path representing the process of the liquid heat pack cooling down from 150 °C to room temperature with no phase change (now assuming there is supercooling). Check that the path you sketched makes sense by verifying that the changes in temperature and energy shown on the diagram as you move along the path you sketched match what you know about the actual changes in temperature and energy of the heat pack as it cools to room temperature without changing phase from liquid to solid.

1. Extending the *Temperature vs. Energy-Added Diagram*

Now you are going to take the two paths from FNT 3-4, and draw them "on top of each other." The trick is to figure out where they exactly overlap and where the two paths deviate from each other.

Erase your board and draw, fairly large, the *Temperature vs. Energy-Added Diagram* that you drew for process A in FNT 3-3: Cooling from 100 °C to 23 °C with no phase change.

Now, we are going to draw the combined heating process (d) plus (e) in the same diagram. **BUT WAIT!** The heating curve you draw will need to be "on top of one another" **where they describe the exact same state: namely warming or cooling the *liquid from 54 °C to 100 °C.***

They are in the same phase (liquid) when they are at 100 °C (and consequently in the same state), but they are in different phases when they are at 23 °C, so they are not in the same state. When they are in different states, the curves won't be on top of each other.

Make sure your heating curve and cooling curve overlap where they are supposed to. Make sure everyone in your group knows *why* the two curves overlap the way they do.

Whole Class Discussion

2. What happens when the clicker is clicked?

Now, to make the analysis a little simpler, let's assume this: When you activate your heat pack, it is thermally insulated – like it is when wrapped tightly with bubble wrap.

Sketch the path of the clicking process on the cooling curve you just made above – starting at the bottom end of the liquid cooling curve at 23 °C. You can determine this new path after the click from what you know from your experiments about the temperature change, and from the implications of *heat-in* or *heat-out* because it was insulated with the bubble wrap.

What state (temperature and phase) is the sodium acetate in *shortly after* the clicking has occurred and while still insulated? Use both what you know from your direct observation of the phenomenon and from your diagrams to draw this portion of the process (from 23 °C to 54 °C). Make sure the new part of the path you draw on the diagram (from before to shortly after clicking) describes this process.

Whole Class Discussion

3. The *Energy-System Diagram* for clicking

Now make an *Energy-System Diagram* for the clicking process. Open or closed? Which energy systems? Which energy system increased and which decreased?

What feature of your *Energy-System Diagram* tells you that the part of the graph in the *Temperature vs. Energy-Added Diagram* that represents the "clicking" is simply a vertical straight line?

Whole Class Discussion

4. Getting back to room temperature

If you now partially remove the bubble wrap so the heat pack can exchange energy with the environment, sketch the rest of the path that gets the heat pack back to a solid at room temperature. Should this path fall "on top of" the heating path?

5. Three-Phase Model of Matter with Super-Cooling

Discuss in your small group how you can summarize the difference in the **Three-Phase Model of Matter** as applied to situations when there is and when there isn't significant/noticeable supercooling. Put your summary on your board and be ready to explain to the whole class.

Whole Class Discussion

3 C Getting Quantitative: New Thermal Properties

Overview: So far, we've used qualitative descriptions and we've estimated numbers to describe the thermal phenomena occurring in the heat pack. However, we often need to be able to accurately quantify our observations and come up with mathematical models that can hopefully give us precise predictions for measurements. In this section, we'll familiarize ourselves with some mathematical models for thermal properties. In particular, we'll talk about *Heat Capacity and Specific Heat,* as well as *Heat of Vaporization* and *Heat of Melting.*

Your instructor will give you one or two minutes to discuss each of the following in the following Parts 3 C.1 and 3 C.2 in your Small Groups. Quickly come to a consensus and put your response on the board. Each question will be followed by a brief **Whole Class Discussion**.

3 C.1 Heat Capacity

1. You may have used the relationship $\boxed{Q = C\Delta T}$ in chemistry and physical science classes. **What does "heat capacity" mean *in your own words?*** You don't have to put this on the board, but make sure everyone in your group is ready to explain.

2. Imagine you have a big bucket of water and a small (like, European-sized) teacup of water, which would have the greater heat capacity – the water in the bucket or in the teacup? Which would have the greater specific heat?

3. Create an *open-system Energy-System Diagram* that illustrates the process of making a heat capacity measurement for the big bucket of water (adding heat to this water). **Note:** We are assuming that this measurement takes place at a *constant volume* (no water is lost during the measurement). The significance of this will make more sense later in the course.

4. Use your diagram from (3) and the definition of heat capacity to develop an algebraic relationship that relates change in thermal energy to temperature change and heat capacity.

5. Use the following definitions to determine simple algebraic relationships between the heat capacity and each of two specific heats (*mass* and *molar*):

   - Mass specific heat is the heat capacity per kilogram,

     and

   - Molar specific heat is the heat capacity per mole.

   In what situations would one or the other be more useful or convenient to use?

3 C.2 Heat of Vaporization and Heat of Melting

1. You may have used the relationship $\boxed{Q = \Delta H}$ in chemistry and physical science classes. **What does "heat of melting" – or "heat of vaporization" – mean *in your own words?*** As above, you don't have to put this on the board, but you should make sure everyone in your group is ready to explain.

2. Use the **Energy-Interaction Model** to illustrate a process in which $Q = \Delta E_{\text{bond}}$.

3. If $|\Delta H|$ were given in units of joules per kilogram ($^{\text{J}}/_{\text{kg}}$) or units of joules per mole ($^{\text{J}}/_{\text{mol}}$), instead of units of joules, how would the equation, $Q = \Delta H$, change?

4. Use Questions 1 through 3 above to describe a specific relationship between the change in bond energy in our **Energy-Interaction Model**, and the indicator of the bond energy system (i.e., the portion of mass of a substance that changed phase).

5. Rewrite the equation expressing energy conservation for process (b) in FNT 3-3, using the expressions for $\Delta E_{\text{thermal}}$ and ΔE_{bond} that were introduced in this activity in terms of C and H. Then substitute all known values for each variable in your expression.

Let's think about our heat pack again: What information do you need to determine how much mass will change phase when the button is clicked, assuming the heat pack is completely thermally insulated?

Whole Class Discussion

Discussion/Lab 4

4 A Refining Intuitions through Practice

Overview: Remember how we talked about "refining intuitions" and becoming so familiar with using our models that they become "second nature" to us? For that to happen, we need to practice using our models even more. This is what the following activities are all about. We'll attempt to make sense of more thermal phenomena by using the **Three-Phase Model of Matter** and the **Energy-Interaction Model**, so that we'll become able to intuitively use them in new situations.

4 A.1 Energy in ice and liquid water

FNT 4-1

Use the **Energy-Interaction Model** to explain whether the following statement is true or false.

> "A quantity of ice at 0 °C must contain less total energy than the same quantity of water at 0 °C."

Make an *Energy-System Diagram* that relates the initial and final states described in the FNT. Use the diagram, and the logic of the model to explain which state would have more total energy.

Whole Class Discussion

4 A.2 The physical concept of *Heat*

FNT 4-2

According to the definition of heat in your course notes (pages 8 & 16), can an energy system contain a certain amount of heat? Explain.

Discuss your response for this FNT with your group mates. Make sure everyone in your group is prepared to explain to the whole class. This is a group responsibility!

Whole Class Discussion

4 A.3 Equilibrating Copper and Water

FNT 4-3

Imagine that you place a piece of copper with an initial temperature of 20 °C in contact with an amount of liquid water with an initial temperature of 100 °C. Assume that the physical system consisting of the copper and the water is thermally isolated from everything else; i.e., water and copper can *only* exchange energy *with each other.*

(a) Using the **Three-Phase Model of Matter** as applied to each substance and your understanding of what "coming to thermal equilibrium" means:

   i. Sketch two *Temperature vs. Energy-Added Diagrams* – one for each substance – and indicate the initial state for each one.

   ii. Explain in a sentence or two how you can tell if either substance will undergo a phase change during the process of coming to thermal equilibrium. You are expected to use what you already know regarding the thermal properties of copper and water, but you do not need to do any calculations.

(b) Construct a complete *Energy-System Diagram* for the process that ends when the two substances are in thermal equilibrium. Don't forget that a complete diagram always includes an algebraic expression of energy conservation.

(c) Now consider a similar process. Use the same initial conditions for the copper, but assume that the H_2O is initially in the gas phase at 100 °C.

   i. Sketch two *Temperature vs. Energy-Added Diagrams*, one for each substance, and mark the initial state for each one.

   ii. How can you determine if the H_2O will undergo a temperature change? (What calculations and comparisons – think energy – would you need to make? Don't actually do the calculations; just explain what you would need to do.)

1. Put your *Temperature vs. Energy-Added Diagrams* on the board with the initial state marked on each one. Explain briefly how you can tell if either substance might undergo a phase change during this process.

2. Put your complete *Energy-System Diagram* on the board for this process.

   If $\Delta E_{\text{thermal(Cu)}} = 250\,\text{kJ}$, what is $\Delta E_{\text{thermal}(H_2O)}$? Explain why.

Whole Class Discussion

3. Redraw your *Temperature vs. Energy-Added Diagrams* for the new initial conditions. Explain briefly how you could determine if the H_2O will undergo a phase change or temperature change.

Whole Class Discussion

4 A.4 Thermal and Bond Energy Systems

FNT 4-4

Consider again the phenomenon of two substances at different temperatures exchanging energy as they come to thermal equilibrium: *Substance A* and *Substance B*, at initially different temperatures, are placed in contact. They are able to exchange energy via heating, and they can only exchange it with each other. *Substance A* has a greater initial temperature than *Substance B*. No phase changes occur during this process.

Use the **Energy-Interaction Model** to create a logical explanation that unequivocally shows that the following statement about the heat exchange described above for *Substances A* and B is true or that it is false:

> "The energy that is transferred as heat to or from the object with the larger heat capacity must be greater than the energy that is transferred as heat to or from the object with the smaller heat capacity."

Discuss this FNT in your group and be prepared to explain in the whole class discussion. You do not need to put anything on the board.

Whole Class Discussion

FNT 4-5

Consider the following example of two substances at different temperatures exchanging energy via heating as they come to thermal equilibrium.

A 3.50 kg piece of lead and a 1.5 kg piece of aluminum, at different temperatures, are placed in contact. They are able to exchange energy via heating and they can only exchange it with each other. The initial temperature of the lead is 48 °C, and the initial temperature of the aluminum is 35 °C. The table on Page 5 the course notes, and Table A.6 in Appendix A, has phase change temperatures and values of specific heats for these substances.

(a) If it is possible (How do you know this?), construct a single, closed *Energy-System Diagram* for the process of lead and aluminum coming to thermal equilibrium.

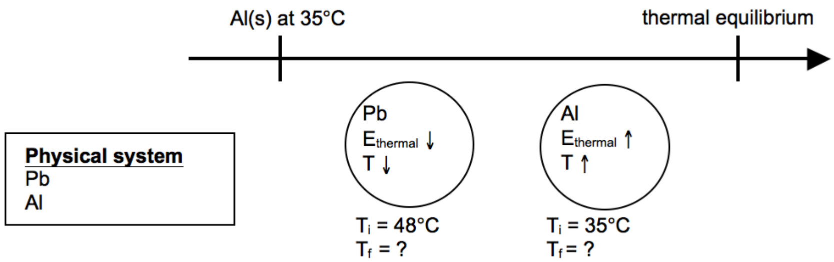
(b) Write an algebraic expression of energy conservation in terms of the ΔEs of the two energy systems, and then substitute in the appropriate expressions for the ΔEs.

(c) Substitute in all known numerical values. How many unknowns are there in your equation expressing energy conservation?

(d) At an *intermediate* point in the entire process described above the lead has reached a temperature of 43 °C.

Follow the instructions for part (a) for the interval that *ends when the lead is at a temperature of 43 °C*. What feature related to the process connects the final temperatures of lead and aluminum that appear in your diagram to each other?

1. A student constructed the following *Energy-System Diagram* for Part (a) of this FNT:



$$\Delta E_{thermal}\,(Pb) + \Delta E_{thermal}\,(Al) = 0$$

$$[mc(T_f - T_i)]_{Al} + [mc(T_f - T_i)]_{Pb} = 0$$

$$(3.5\text{kg})(128\text{ joules/kg-K})(T_f - 48°\text{C}) + (1.5\text{kg})(900\text{ joules/kg-K})(T_f - 35°\text{C}) = 0$$

$$T_f = 38.2°\text{C}$$

Discuss the diagram briefly in your group, and identify any questions you have. You don't need to put anything on the board.[1]

2. Put a complete *Energy-System Diagram* on the board for the shorter interval described in Part (d) of this FNT. What specifically is happening in the process that "connects" the two final temperatures (of lead and aluminum) in your diagram? Illustrate this on the board with two separate *Temperature vs. Energy-Added Diagrams*, one for each substance.

Whole Class Discussion

FNT 4-6

Suppose you used a hot pot to convert a 150 g piece of ice that was initially at -15 °C into liquid water at 50 °C.

(a) Represent this process in two complete *Energy System Diagrams*, one covering the interval from -15 °C to 0 °C liquid, and the second covering the interval from 0 °C liquid to 50 °C.

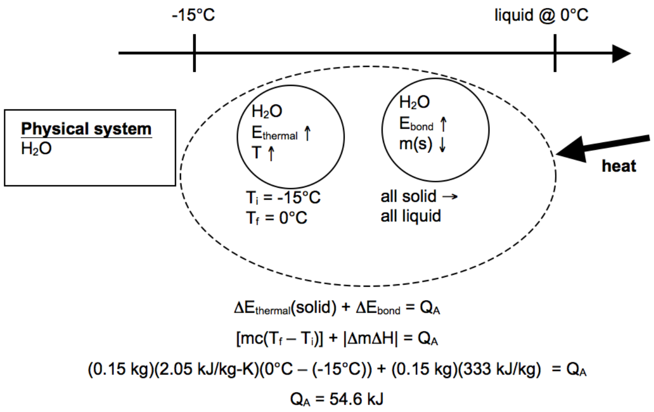
(b) Now represent the entire process in one complete *Energy-System Diagram* covering the entire interval.

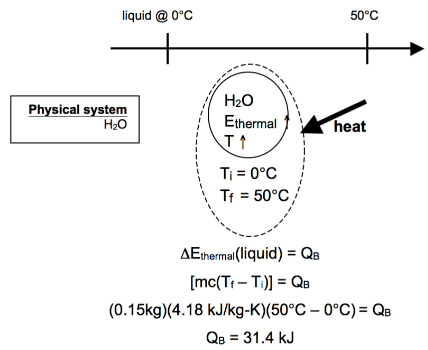
(c) If your hot pot has a power rating of 600 watts, show how to find how long it will

[1] Incidentally, this diagram illustrates how much algebra is useful to put down for the purpose of clear communication in whole class discussions. In general, it is not useful to show the details of solving the algebra on the board. Being able to construct the correct diagram and getting the first three lines of algebra (including verifying the signs of the terms in the third line) are worth most of the credit on an exam.

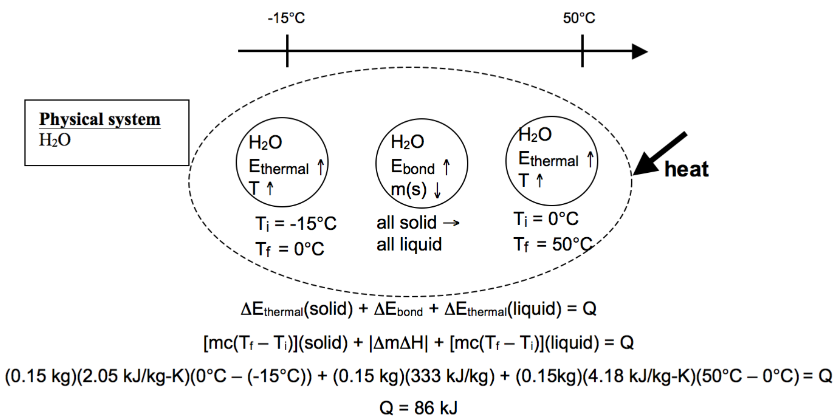
take to complete the process. (Make sure you are comfortable using standard units of energy, and that you understand the relation of power to energy.)

1. Again, we consider a student's response to this FNT. Discuss the diagrams for Part (a) below and identify any questions you have. As before, you don't have to put anything on the board for this part.





2. A student objects to the diagram for Part (b) of this FNT below because some of the indicators do not correspond to the beginning and end of the overall interval.



Another student says that's ok because each physical process in the interval happens sequentially and nothing has been left out. Come to a consensus about which of these two students you agree with and put a brief explanation on the board.

3. Discuss the question about power in Part (c) in your group and identify any questions you have. Make sure you have a clear understanding of the relationship of power to energy. You don't have to put anything on the board for Part (c), but be ready to explain the difference between energy and power if called upon in the whole class discussion.

Whole Class Discussion

4 A.5 Asking Questions and Determining Intervals of Interest

FNT 4-7

FNT 4-5 and FNT 4-6 emphasized the significance of the beginning and end of the interval – we've called them "initial" and "final" states so far. This FNT illustrates

(i) that the question you're asking determines the interval being analyzed with the **Energy-Interaction Model**; and

(ii) that sometimes it's necessary to adjust the size of the interval in order to be able to use the model.

The physical process we want to analyze with the **Three-Phase Model of Matter** and the **Energy-Interaction Model** is the following:

Imagine you are using your hot pot to gradually heat a 500 g block of ice that has just been removed from a freezer with an internal temperature of -25 °C. The hot pot is fairly well insulated, so it is reasonable to assume that all of the heat put into the pot from the electrical heater located in the bottom of the pot goes into the H_2O. We can also assume that the heat capacity of the pot is much smaller than the heat capacity of 500 g of H_2O (whether solid or liquid), so we can ignore the thermal energy system of the pot itself.

One question we could ask is, "What is the final state of the water (phase and temperature) after the addition of 252 kJ of heat?"

(a) Explain why – based *only* on the given information and without further analysis or calculation – it is ***not possible*** to construct a *single Energy-System Diagram* that could be used to determine the final state of the water.

[Hint: try to construct such a diagram. Another way to think about this is whether you can define the final state so that it depends on only one variable without making unjustified assumptions?]

(b) In cases like this, you must first use the model to analyze a *shorter interval*. Can you construct an *Energy-System Diagram* that could be used to determine how much energy is needed to increase the temperature of the ice to 0 °C?

Now explain in a few sentences how to proceed using the **Energy-Interaction Model** to find the final state of the H_2O.

1. Why is it *not* possible to apply the **Energy-Interaction Model** in a *straightforward* way to the interval of interest – the interval that ends when all of the 252 kJ has been added to the water? In other words, why can't the overall interval be diagrammed using *only* the given information prior to doing any further analysis? Draw a *Temperature vs. Energy-Added Diagram* to help you make sense of this and to use to explain it. In particular, be sure to make explicit which part of the graph in the diagram refers to which "energy bubble" on the *Energy-System Diagram*. [To save time, explanations on the board can be more abbreviated than they would be on an exam. In this case, anything that is unclear can be easily clarified in the whole class discussion.]

2. Explain how to use the **Energy-Interaction Model** to analyze shorter intervals in order to determine the final state (phase and temperature) of the water. (Note: "water," without a modifier, usually means H_2O in any one of its three phases.) Use a *Temperature vs. Energy-Added Diagram* in your analysis and to use in your explanation.

Whole Class Discussion

Discussion/Lab 5

5 A More Practice with the Energy-Interaction Model

Overview: We continue our efforts to practice using our models – particularly the **Energy-Interaction Model** – by investigating new phenomena.

5 A.1 Cooling Water Below its Freezing Point

FNT 5-1

Perhaps you recall that when table salt, NaCl, is added to water, the freezing point of water is lowered. Consider a system composed of a mixture of 2.5 kg of ice and 50 g of liquid water and a small, separate container of finely powdered salt. This physical system is contained in a fully insulated container that prevents all thermal interactions with the environment. Both the salt and the ice-water mixture are initially at the freezing point of water, 0 °C. The salt is then added to the ice-water mixture, and the system of ice-water and salt is allowed to come to thermal equilibrium. The final equilibrium temperature is less than 0 °C.

Use the **Energy-Interaction Model** to predict if there will be a greater or lesser amount of ice in the final equilibrium state than in the initial state before the salt was added. Your explanation should include a complete *Energy-System Diagram*.

[The simplest way to model this physical system is with one thermal energy system for everything and one bond energy system, i.e., in terms of the model, it is not useful to distinguish between the various chemical components in order to answer this particular question.]

On the board, construct the simplest *Energy-System Diagram* that represents a particular model of the process described in this FNT.

Make sure everyone in your group is ready to explain which elements of your completed diagram represent information that was given, or known, and which elements you had to infer, based on given or known information and the logic of the model?

Whole Class Discussion

FNT 5-2

This FNT is a ***thought experiment*** to make predictions about an actual experiment we will do in class, where you will get to observe the phenomenon described in FNT 5-1.

Imagine you fill an insulated cup almost full with chopped or crushed ice, and measure the temperature after a minute or two, once it's all come to thermal equilibrium. Since this ice is frozen water, the temperature should be at 0 °C or not more than a couple of degrees below. Then, imagine you add a bunch of salt and stir it around.

What do you think the lowest temperature you can attain will be? Why? What happens to the amount of liquid present if you keep stirring and adding salt? How can we understand this phenomenon in terms of thermal and bond energy systems?

Develop an explanation for the changes you would observe in this physical system (decrease in temperature and change of phase) in terms of the **Energy-Interaction Model**.

Now we'll actually do this experiment:

1. On your table, you have an insulated mug and a thermometer. Get some ice from the ice chest on the counter and fill your insulated mug. Measure the temperature inside the ice-filled cup and verify it is at around 0 °C.
2. Now, add some salt and stir it into the ice with a stirrer (don't use the thermometer; it might break!). Measure the temperature of the salt-ice mixture inside the cup. Add some more salt and repeat the stirring and temperature measurement. How far can you get the temperature to drop?
3. On the board, list the low temperatures attained by your group.

Would the following statement be something that would be good to memorize?

"When bond energy increases, thermal energy must decrease, and vice versa."

Explain on the board why it would or would not be good to memorize this.

Whole Class Discussion

5 A.2 Heating Water with Microwaves

FNT 5-3

This is an actual experiment that you should perform at home:

Put approximately 1 cup of cold water in a microwave safe dish (Tupperware, drinking glass, etc.), measure the temperature (you can use a thermometer you would use to measure your body temperature), then heat it for a set period of time (somewhere between 30 seconds and a minute will work well). When you take the water out, measure the temperature again.

(a) Draw an *Energy-System Diagram* for the water in your cup, and determine the change in thermal energy of the water. Convert this to Watts (a unit of power).

(b) Compare the number of Watts you determined in Part (a) to the power rating listed on the microwave information plate (usually located on the back of the oven, but you can always look up the model online). Which is greater? What does this mean?

(c) Where might the energy go if it is not heating up the water?

(d) Does the percentage of energy going to the water depend on the amount of water? You might choose a small amount, say half a cup, and a large amount, say two cups to investigate this question.

On the board, choose the data from a member of your group, and sketch a graph of cooking power vs. amount of water.

You'll notice that there appears to be some "missing power." That is, the difference in power actually used to heat the water is less than the power used by the unit as stated on a label on the back of the microwave unit.

[**Hint:** Can you feel or hear anything near the back of the microwave unit when it is turned on?]

Whole Class Discussion

5 A.3 Boiling Liquid Nitrogen with Ice

FNT 5-4

A 2.2 kg block of ice (H_2O) initially at a temperature of -20 °C is immersed in a *very large* amount of liquid nitrogen (N_2) at a temperature of -196 °C. The N_2 and H_2O are allowed to come to thermal equilibrium. [$T_{\mathrm{BP(N_2)}}$ = -196 °C]

Create a particular model of this process and use it to determine how much liquid N_2 is converted to gas (vapor).

[Hint: The emphasis on "very large" implies that there will still be liquid N_2 left when the two come to thermal equilibrium.]

1. There exists a definite correlation between each energy system in your *Energy-System Diagram* and each ΔE term in your energy conservation equation. In addition to ensuring that you include all the appropriate ΔE's (and no extra ones) in your algebraic equation expressing conservation of energy, what other very important detail can your *Energy-System Diagram* tell you about each ΔE term in your equation?

2. Construct a complete *Energy-System Diagram* (including an algebraic statement of energy conservation in terms of the ΔE's) for this process. Then rewrite the equation expressing energy conservation substituting in the appropriate expressions for changes in energy of the energy systems using variables only (no numbers). **Do not substitute any numbers for the variables, and do not solve for the unknown, but circle**

the unknown variable in the equation. Put the algebraic sign in parentheses above each algebraic term in the algebraic equation.

We know you know how to do simple algebra and arithmetic. However, the most common mistake students often make is in the algebraic sign of the whole term when subtracting *initial* values from *final* values of the indicators. By using the *Energy-System Diagram* to check that the sign is correct, you will avoid making this error.[1]

5 A.4 Warming Ice with Liquid Water

FNT 5-5

The 2.2 kg block of ice from the FNT 5-4 is eventually removed from the liquid N_2 (after reaching thermal equilibrium with the liquid N_2) and placed in a *very large* amount of liquid H_2O at 0 °C, where it comes to thermal equilibrium with the liquid H_2O. Create a particular model of this process and use it to determine the final mass of the ice.

1. Describe in words what is going on in this process, specifically: What physical system is changing phase? What system is changing temperature? What energy systems need to be included in the particular model you create for this process?

2. Construct a complete *Energy-System Diagram* (including an algebraic statement of energy conservation in terms of the ΔE's) for this process. Then rewrite the equation that expresses energy conservation, substituting in the appropriate expressions for changes in energy of the energy systems using variables only (no numbers). **Do not substitute any numbers for the variables, and do not solve for the unknown,** but circle the unknown variable in the equation.[2]

Whole Class Discussion

5 A.5 Melting Gold

FNT 5-6

2.0 kg of solid gold (Au) at an initial temperature of 1000 K is allowed to exchange energy with 1.5 kg of *liquid* gold at an initial temperature of 1336 K. The solid and liquid can only exchange energy with each other. When the two reach thermal equilibrium, will the mixture be entirely solid, or will there be a mixed solid/liquid phase? *Explain how you know.*

Discuss in your group and make sure everyone is prepared to explain your group's response in the whole class discussion.

Whole Class Discussion

[1] Just for reference: You likely found that ~4 kg of liquid N_2 was converted to vapor.

[2] Again, for reference: You likely got a final mass of ice of ~5 kg.

5 B Defining Appropriate Intervals for Analysis

Overview: Remember that we talked about how asking different questions makes us choose different intervals of interest for our investigations using the **Energy-Interaction Model**? Here, we'll explore this issue a bit further.

Consider the following phenomenon: A **big** chunk of ice (water) at an initial temperature of -65 °C is placed inside a well-insulated container with some tea at an initial temperature of 20 °C, and the two are allowed to come to thermal equilibrium. (The tea can be treated as water with respect to thermal properties.)

1. There are several possible final states of this process. We will be using and reusing the same **Three-Phase Model of Matter** to determine possible final states. Draw two *Temperature vs. Energy-Added Diagrams*, one for tea and one for H_2O.

   (a) Why can't either substance reach thermal equilibrium in the gaseous phase nor the mixed (liquid/gas) phase nor the gaseous phase?

   (b) Using the grid below, your instructor will assign your group one of the potentially possible final states. Trace your *Temperature vs. Energy-Added Diagram* to show how your assigned ending phase is or is not possible.

Table 5.1: Grid for 1b

			Tea	
		Solid	Mixed Phase (Solid/Liquid)	Liquid
	Solid	i.	ii.	iii.
Ice	Mixed Phase (Solid/Liquid)	iv.	v.	vi.
	Liquid	vii.	viii.	ix.

Whole Class Discussion

2. Now that we have a class consensus on which states are possible, your instructor will assign you one of the possible final states:

   (a) For your possible final state, draw an *Energy-System Diagram* (with energy conservation equation, as always).

   (b) Would your final state be possible if there were A LOT of ice? What about with A LOT of tea?

Whole Class Discussion

3. **Thinking about how to proceed: How do we determine the final state?**

   Put a *Temperature vs. Energy-Added Diagram* on the board for each of the two systems: The water that starts out as ice at -65 °C and the tea (liquid water) that starts out at 20 °C. Put a big dot on each graph at the initial state.

   (a) Have two separate group members place a finger on the two starting dots. Then a third group member should start explaining how energy is transferred from one physical system to the other, specifically naming the energy systems that are losing energy and that are gaining energy. Move the fingers along each graph in response to the explanation. Make sure every group member agrees with the explanation and the motion along the graphs. STOP when one substance gets to a phase change temperature.

   (b) We need to use quantitative skills to figure out **which substance gets to its phase change temperature first**. With your group members, think about ways that you might be able to determine this. Hint: Are there two values you can compare? Write in words what you need to do to figure this out.

   (c) After you have explained in words how you will figure out which substance gets to its phase change first, draw an *Energy-System Diagram*for each calculation you want to perform, and then perform the calculation.

Whole Class Discussion

   (d) Is it possible to determine the final state of the substances yet? Is it possible to eliminate any of the possible final states? Which one(s)?

   (e) We now need to repeat Part 3b to figure out which happens first: the tea decreases to 0 °C, or the ice goes through an entire phase change. Draw *NEW initial points* and determine which changes in energy you need to compare. Then draw the *Energy System Diagrams*, and do the calculations.

   (f) Are the two physical systems in thermal equilibrium at the end of this step? How do you know this?

   (g) Can you now determine the final state of the ice and tea system? Draw an appropriate *Energy-System Diagram*on the board for this. If you finish early, you may solve for the final temperature or state.

Whole Class Discussion

Unit II

Mechanical Energy Systems

Discussion/Lab 6

6 A Modeling with Mechanical Energy Systems

Overview: After spending the first few weeks of the semester discussing and making sense of various *thermal* phenomena, we now shift our focus to *mechanical* phenomena. We'll start with some new definitions and then immediately dive into some examples.

6 A.1 Internal Energy and Mechanical Energy

Please read before moving on:

At this point, it is useful to define and distinguish two different *types* of energy systems. Make sure the definitions make sense to everybody in your group!

Internal Energy: Internal energy systems are closely related to the *motions* of and *relationships* between *individual atoms and molecules*. Thermal Energy and Bond Energy are examples of internal energy systems.

Mechanical Energy: Mechanical energy systems have *indicators* involving the *position* or the *speed* of the *object as a whole*. The object can be anything from an airplane to an atom.

Important: Some physical phenomena (such as the thermal phenomena we've dealt with, so far) involve changes *only* in internal energy systems, while some phenomena involve changes *only* in mechanical energy systems. Other phenomena involve changes in *both* mechanical and internal energy systems.

Whole Class Discussion

6 A.2 Phenomenon: Dropping a golf ball

Drop a golf ball from about one meter above the floor and observe its motion and position. Focus on the motion and position between the **beginning** and **end** states defined below. If you do not have names of energy systems associated with the changes you observed, you can just make up names for these energy systems for now.

beginning → *end*

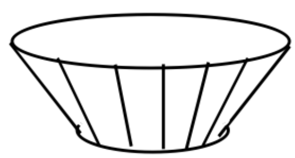
just as ball is released — *just before* ball hits the floor

1. Write on the board: What properties of the physical system (indicators) do you think change **significantly** between the initial and final state? What energy systems are associated with those indicators? Are these *internal* or *mechanical* energy systems, or both?
2. Put a complete *Energy-System Diagram* on the board. Show with an arrow if each energy system increases or decreases. Include initial and final values of the indicators.
3. Write an algebraic representation of your *Energy-System Diagram* that expresses energy conservation in terms of the changes of energy of the relevant energy systems.

Whole Class Discussion

6 A.3 Phenomenon: Dropping a coffee filter

Drop a basket type coffee filter from a height of about two meters. Release it oriented as shown in the figure.



beginning → *end*

one second *after* filter is released — *just before* filter hits the floor

1. Repeat part 1 above. Don't forget the last part of the question!
2. Repeat parts 2 and 3 above.

Whole Class Discussion

Discuss in your group:

3. Which of the energy systems you have identified depend on only the *magnitude* of the indicator and which depend on *both* the *magnitude* and the *sign* associated with that indicator?
4. How are your *particular models* for the golf ball and coffee filter different from each other? Be specific. What physical properties of the objects are responsible for this difference?

Whole Class Discussion

6 B A New Construct and A New Model

Overview: A third mechanical energy system – in addition to *translational kinetic* energy $KE_{\text{translational}}$ and *gravitational potential* energy PE_{gravity} – is the *spring-mass* energy $PE_{\text{spring-mass}}$, introduced here in the context of the **Spring-Mass Model**.

6 B.1 Oscillating Spring-Mass

Many physical systems oscillate. The spring-mass system that you are going to examine is one example of many such systems that can be modeled using the **Spring-Mass Model**.

1. Hang a 200 g mass on the spring. The equilibrium position of the mass is the location of the mass when the system is at rest. Use the two-meter stick with the pointer to mark the equilibrium position of the mass. Pull the mass down or push it up **a few centimeters** and release it. Describe the resulting motion of the mass (e.g., Where is it moving fastest? Where is its speed zero?).

2. Does the resulting motion of the mass depend on whether you started the motion by *lifting the mass up* a certain small distance (try ~1 cm) or by *pulling it down* that same distance from its equilibrium (resting) position?

Whole Class Discussion

6 B.2 Applying the Energy-Interaction Model

Work out your responses to the questions below with your small group at the board. Remember that in order to identify an energy system, we need to have an indicator that tells us that the energy system is changing.

Start with the mass pulled down 5 cm below its equilibrium position.

beginning — *just as* mass is released

end — *the instant* the mass passes through equilibrium position for the *first* time

1. What *properties* of the physical system (indicators) changed significantly between the initial and final state? What energy systems are associated with those indicators?

2. Make a complete *Energy-System Diagram*:

   Show the increases and decreases in the energy systems and show the initial and final values of the indicator associated with each energy system – be as explicit as possible.

3. Write down an algebraic representation of your *Energy-System Diagram*, expressing energy conservation.

Whole Class Discussion

6 B.3 Reflecting on the Spring-Mass Model

1. Do changes in the energy systems you identified depend on the direction that the indicator changed (the sign of the indicator), or only on the magnitude of the indicator?
2. Would the way you modeled this physical system be any different from what you did in your response to 6 B.2#2 if the **final state** were the instant the mass returned to its equilibrium position the second time, instead of the first? How about if it were the 200th time?

Whole Class Discussion

Discussion/Lab 7

7 A Creating Particular Models with Mechanical Energies

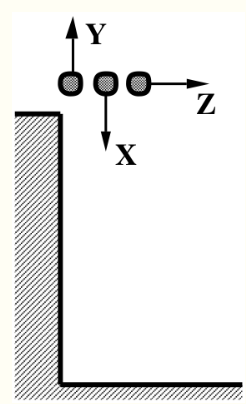
Overview: The problems in this section illustrate how we can use the **Energy-Interaction Model** to answer questions and make predictions about the allowed behavior of interacting objects that are difficult to approach in any other way.

7 A.1 Three Thrown Rocks

FNT 7-1

Before we start with this problem, let's review what the **Energy-Interaction Model** does for us. As we have said before, conservation of energy relates values of certain physical parameters at the beginning of a process to the values of those parameters at the end of the process. The parameters are typically the indicators of the energy systems that change during the process or interaction. If we have a question about – or want to predict values for – some parameter and this parameter happens to be an indicator of an energy system, or a coefficient in an expression for an energy system, then we can proceed to construct a particular model and see if it gets us what we want.

The Phenomena: Three rocks of equal mass are thrown with identical speeds from the top of the same building. (1) Rock X is thrown vertically downward, (2) Rock Y is thrown vertically upward, and (3) Rock Z is thrown horizontally.



The Question: Which rock has the greatest speed just before it hits the ground? Assume air resistance is negligible.

How can we determine this? We could take a guess, but it helps our argument if it is possible to apply the **Energy-Interaction Model**. The prompts below will guide you through the process.

(a) Let's start by making a prediction based on your prior experience. Don't waste a lot of time on this right now, but it's useful to give it some thought. We will definitely come back to this at the end in Part (f). Your first, intuitive ideas might be very useful!

(b) Does the question involve a parameter that you know to be an indicator of the change in an energy system? Which energy system and what is the indicator?

(c) **Rock X**: Construct an *Energy-System Diagram* for Rock X, which is thrown straight down. Write out the expression for energy conservation, based on your *Energy-System Diagram*, as ΔE's; then substitute in algebraic representations for the different energy systems. We are *not* asking to go any further, but if you "just have to," go ahead and try solving it for the final speed, v_f (but you *really* don't need to).

(d) **Rock Y**: Now, without actually writing anything down, consider what would be different in your *Energy-System Diagram* for Rock Y. How about the algebraic representation – anything different? Go back and re-read the "The Phenomena" description at the beginning of the FNT, if you are not sure. Is there anything different in terms of what goes into the model? In terms of the **Energy-Interaction Model** are there any differences? Yes or No? Are you 100% sure? Why?

(e) **Rock Z**: Repeat for Rock Z. Any differences in the model? Yes or No?

(f) Do you believe that conservation of energy holds true for these three phenomena? Another way to ask this: Does the *particular energy model* you developed apply to all three cases? If yes, what does conservation of energy tell you about the final ***speeds*** of the three rocks with 100% confidence?

(g) How do you reconcile your result in Part (f) with your initial ideas from Part (a) above? They are not crazy, because there are indeed very obvious differences in the three situations. Why don't these differences matter to energy conservation? Try to be as explicit here as you can be. This is what we will focus on in the FNT follow-up in DL.

Using the prompts below, **briefly** compare your group members' responses to FNT 7-1 and put a short answer on your board (each letter corresponds to the same letter in the FNT):

(b) Does the ***question*** involve a parameter that you know to be an indicator of the change in an energy system? Which energy system and what is the indicator?

(c) Draw an *Energy-System Diagram* for Rock X with an algebraic representation consisting of the following two lines.

  (i) 1^{st} line, using ΔE's with subscripts.

  (ii) 2^{nd} line, substitute in algebraic expressions for each ΔE.

  Do **NOT** solve for anything!

(d) & (e) Are there ANY differences in the *Energy System Diagrams* for Rocks X, Y, and Z?

(f) (i) In terms of the *particular* model you have created for these phenomena (determined by which energy systems you included), what does your model predict with 100% certainty about the final speed of each rock just before it hits the ground?

(ii) In light of the dropping golf ball and coffee filter activity (Activity 6 A), what aspect of your model might have to be changed that could change your prediction regarding the final speeds of the ball?

(iii) So, taking into account what you just determined in the previous two steps, what *is* different about the final states of the balls?

(g) Discuss any intuitive ideas that different members of your group might've had that "bug you the most," because they seem to contradict the **Energy-Interaction Model**'s prediction that all rocks have the same final speed.

Whole Class Discussion

Check-In: *Does this all make sense to you?* In the context of this FNT, we discussed some very fundamental issues about energy conservation. If you're not sure about any of the above, please follow up with other students and/or your instructor during office hours. But before you do, carefully work through the entire FNT again!

7 A.2 Pulling up a Bucket

FNT 7-2

A person pulls a bucket of water up from a well using a rope. Assume that the initial and final speeds of the bucket are zero ($v_i = v_f = 0$), and that the person lifted the bucket a vertical distance h. By looking at energy changes in the bucket-Earth physical system, we can make sense of the force the person must exert to pull the bucket up and determine the amount of work the person does.

(a) Is the system open or closed? What energy systems undergo a change during this process? Construct an *Energy-System Diagram* and include the algebraic expression of energy conservation (in terms of ΔE's and, if open, any heat or work).

(b) Substitute the algebraic expression we use for the change in gravitational potential energy and solve algebraically for the work done by the person on the rope/bucket.

(c) Use the definition of work in terms of force and distance (refer to the **Energy-Interaction Model** Summary) to find the average force exerted by the person on the rope and bucket while they are lifting it up.

1. Put on the board the algebraic expression you have for work in terms of energy changes. Be prepared to give an explanation that is both succinct and logically complete – based on the **Energy-Interaction Model** – for how you know this expression is true.

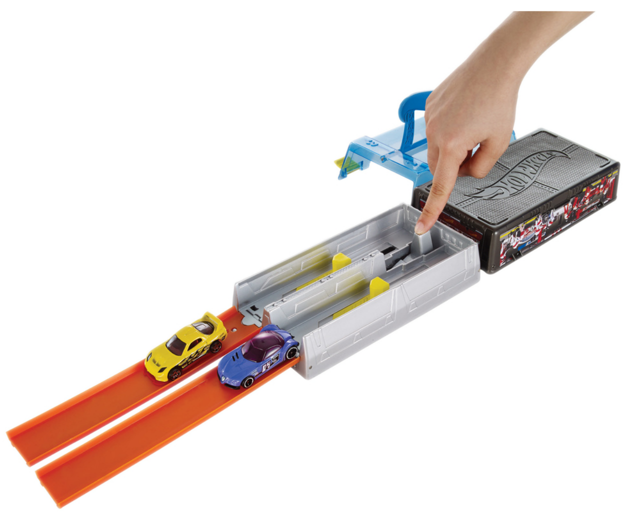
2. Substitute the expression for work in terms of force and distance moved (see **Energy-Interaction Model** Summary), and solve for the force.

Whole Class Discussion

7 A.3 Including the Potential Energy of a Spring

1. The Phenomenon

A toy car launcher works as follows: There is a compressible spring that is attached to a horizontal rail, on which a toy car can roll. The toy car can be pushed back against the spring, compressing it a certain distance. There is a little hook that holds the car against the compressed spring. When the hook is released, the spring pushes the car out of the launcher, down the horizontal track.



We want to ask some questions and make some predictions about different cars as they come out of the launcher. Upon close examination of this launcher, it is noted that the spring is always compressed the same amount each time a car is loaded into the launcher. The spring stays put inside the launcher as the car is pushed out.

2. What we want to know or predict

- Do all cars come out of the launcher with the same speed?
- If they don't all have the same speed when they come out, what parameters cause the difference?
- What do we need to do to get our car to have a higher speed than our friend's cars?

3. Create a Particular Model and use it to answer the questions

Plan what you need to do in your group and how you will proceed BEFORE you start writing on the board. Be systematic. Then put your response on the board.

4. Suggestions:

- Use the **Energy-Interaction Model** and the **Spring-Mass Model**.
- Consider what energy systems are involved.
- **Note:** The car is in contact with the spring while it is compressed and separates from the spring just as the spring reaches its equilibrium length. During the time that the car and spring are together, they behave as a spring-mass system.
- To simplify the notation, use "d" for the distance the spring is compressed instead of "Δx".

5. Be prepared to present a precise, logical, explanation that "gets at" the answers to the questions.

Whole Class Discussion

7 B How We Use the Energy-Interaction Model

Overview: In this activity, we reflect on what we did when using the **Energy-Interaction Model**. We'll generate some general guidelines for successful application of the model to make sense of physical phenomena.

7 B.1 Reflecting on Using the Energy-Interaction Model

In the previous activities we have been applying the **Energy-Interaction Model** to a wide range of physical phenomena. Previously, we had applied it to thermal and chemical interactions. Here is a list of what we've just done with mechanical phenomena:

- Activity 6 A Dropped golf ball and dropped coffee filter
- Activity 6 B Oscillating mass attached to a spring
- Activity 7 A Three thrown rocks, Pulling up a bucket, and Toy car launcher

In all of these cases we used the **Energy-Interaction Model** to make sense of the phenomena, answer questions, make predictions, or develop explanations regarding the phenomena. But the **Energy-Interaction Model** is a very general model that needs to be refined to tell us anything about the particular phenomena.

1. What did you have to do and what decisions did you have to make in order to use/apply the **Energy-Interaction Model**? There are **three** major decisions you must initially make that determine how you are modeling the process or phenomenon. The process of making an *Energy-System Diagram* helps you with this. List the most important three things you can think of here and on your board:

   (a) ______________________

   (b) ______________________

   (c) ______________________

2. Which steps on the two-page blue model summary of the **Energy-Interaction Model** do the above three steps correspond to? ______________________

The process of making these decisions/choices is the ***first step*** in what we have and will continue to refer to as creating or developing a ***particular model***. This is the model that can be directly applied to a particular phenomenon.

Whole Class Discussion

7 B.2 The Whole Process

1. To complete the creation of the particular model, you must do some more things, perhaps make some decisions. When first encountering a phenomenon, an *Energy-System Diagram* can be particularly useful because as you draw the diagram, you are forced to make these decisions.

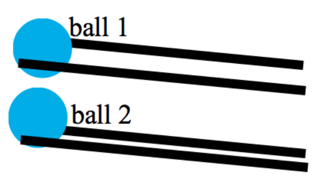
2. Create a particular model for the case of a mass hanging on a spring that is pulled down and released at a distance "*d*" below the point where the mass is hanging stationary. After the mass has completed a total of 10 complete oscillations, it is noticed that it didn't go quite as far down as the distance d. Put your particular model, in the form of a complete *Energy-System Diagram*, on the board. Be prepared to discuss in detail the decisions you had to make when forming this diagram.

3. What are some of the questions that you could ask/answer or predictions you could make with this particular model? You would likely need to have more specific information that you could obtain from the phenomenon. Put these questions on the board with any additional information you would need.

Whole Class Discussion

7 B.3 A New Phenomenon

Imagine this situation: Two *identical* balls roll down two sets of slanted channels. The slope and length of the channels are the *same*, so the change in height is the same for both balls. However, the channels have different widths: one is wider than the other.



Your instructor will give you two Pool balls. Let them roll down the channels – starting them at the same time – and observe what happens. Do this several times, carefully observing what is different about the way the balls roll. Could the difference be an indication that there is a new energy system that we need to take into account?

A maybe obvious question is, ***"Why does one ball go faster and get to the bottom before the other ball?"*** Use the **Energy-Interaction Model** to make sense of what is going on, and develop a succinct explanation with an appropriate *Energy-System Diagram* for why one of the balls gets to the bottom of the rails before the other ball.

Note: From careful measurements made on this apparatus, friction plays only a very small roll in the difference in speeds of the balls. You can safely assume that all frictional effects cause negligible energy changes compared to the changes in mechanical energies.

Whole Class Discussion

7 C Quantitative Modeling of Mechanical Energies

Overview: There are interesting features of some of the Mechanical Energy systems that only become apparent in the algebraic expression for the change in energy. These features can help us tremendously in making sense of physical phenomena. To remind you, the energy systems we are currently working with are $KE_{\text{translational}}$, $KE_{\text{rotational}}$, PE_{gravity}, $PE_{\text{spring-mass}}$, and PE_{spring}. In this activity, we will explore some quantitative features of these energy systems.

7 C.1 Relationships between Energy Systems and Indicators

In your small group, take a few minutes to think about and discuss *each* of the questions below. Each question will be followed immediately by a short **Whole Class Discussion**

1. What are the indicators of change in each of the Mechanical Energy systems?
2. (a) Which of these energies depend on the square of the indicator and which depend on the first power of the indicator?
   (b) Which of these energies depend on which direction something moves?
   (c) What is the connection between the two previous questions?
3. Which of these energy systems depend on the mass of the object and which depend on the weight? Why does this matter?

The fact that some indicators are linear and some are quadratic has additional consequences as the next two questions illustrate:

4. Consider a heavy ball. Would it take the same amount of work to vertically lift it from 25 cm to 30 cm as it does to lift it from 30 cm to 35 cm? If not, which situation requires more work? Why?
5. Does it take the same amount of work to speed your car up from 25 m/s to 30 m/s as it does to speed it up from 30 m/s to 35 m/s? If not, situation requires more work? Why?

7 C.2 Keeping Track of Directionality

We are about to begin using the **Energy-Interaction Model** to predict quantitative properties of various phenomena. A potential pitfall of quantitative modeling is that we sometimes tend to lose track of why we are doing something once we start calculating numbers. It is especially easy to lose track of *directionality*, which is mathematically expressed in *algebraic signs*. However, our *Energy System Diagrams* tell us the signs of the various terms in the conservation of energy equation most of the time. Therefore, don't forget to use the *Energy System Diagrams* to check the signs as you begin to model scenarios quantitatively!

The Phenomenon: Christine throws a ball straight up, letting go of the ball at a height of y_i above the ground. When she lets go, the ball has an initial speed v_i. The ball travels straight up to its maximum height (y_{max}) and falls back down. Assume the frictional effects from air resistance are insignificant.

Constructing a particular model to find the maximum height:

1. Construct a particular model of this phenomenon that can be used to determine the maximum height of the ball. ($y_f = y_{max}$) Put a sketch of the path of the ball and a complete *Energy-System Diagram* (with accompanying energy conservation equation) on the board [leave half the board free for the following questions]. What do you know about the ball's speed at its maximum height when it's thrown straight up?

2. Indicate the initial and final positions of the ball on your sketch that correspond to the initial and final conditions in your *Energy-System Diagram*.

3. On the board, rewrite the energy conservation equation by replacing the two terms with algebraic expressions for the changes in energy of the energy systems. Does the resulting algebraic sign of each term in your energy conservation equation agree with the increases and decreases in energy systems in your diagrams?

4. Solve the equation for $(y_f - y_i)$. Does the sign of $(y_f - y_i)$ make sense for this particular physical situation?

5. If you knew the numerical value of v_i, could you calculate the maximum height?

Whole Class Discussion

Discussion/Lab 8

8 More Modeling of Scenarios involving Mechanical Energy Systems

Overview: In this activity, we will explore additional scenarios that are related to phenomena we've already discussed. Here, we add some variations to the scenarios and answer some different questions we haven't asked yet.

Note: Remember Christine throwing a ball in the air in Activity 7 C.2?

FNT 8-1

If the initial upward speed of the ball in Activity 7 C.2 is $10\,\mathrm{m/s}$, and the ball is released at a height of $1.5\,\mathrm{m}$ above the floor, what is *the maximum height above the floor* that the ball reaches? How far *above Christine's hand* is the ball when it reaches its maximum height, and how is this value related to your answer to 4 in Activity 7 C.2?

FNT 8-2

With the same initial conditions as in FNT 8-1, use the **Energy-Interaction Model *in two different ways*** to determine the speed of the ball when it is 4 meters above the floor, *headed down*:

(a) Construct a particular model of *the entire physical process*, with the initial time when the ball leaves Christine's hand, and the final time when the ball is 4 meters above the floor, *headed down*.

(b) Divide the overall process into *two physical processes* by constructing two *Energy System Diagrams* and applying energy conservation for each: one diagram for the interval corresponding to the ball traveling from Christine's hand to the maximum height; and one diagram corresponding to the interval for the ball traveling from the maximum height to 4 meters above the floor, *headed down*.

(c) Did you get different answers in parts (a) and (b) for the speed of the ball when it is 4 meters above the floor, *headed down*?

FNT 8-3

Use the **Energy-Interaction Model** to show that an object thrown vertically upward will have the same speed as it comes down through any point that it had going up through that same point.

One way to do this is to construct two *Energy System Diagrams*: One diagram should be from the point of release of the ball to some *intermediate* height as the ball is traveling upward, less than the *maximum* height; the second diagram should be from the point of release of the ball to that same intermediate height as the ball is on its way down. Then, compare the two diagrams.

FNT 8-4

In Activity 7 C.2 we assumed that $y = 0$ at the level of the floor. If, instead, we assume that $y = 0$ where Christine releases the ball – still 1.5 meters above the floor – will this change the maximum height above the floor attained by the ball? Use the **Energy-Interaction Model** to answer this question.

After throwing her ball in the air a few times, Christine is tired of playing with the ball and instead moves on to water balloons (much more fun, right?). She stands on top of the Science Building, ready to launch her water balloon.

FNT 8-5

Christine drops a water balloon from the top of the Science Building. Let's assume that the balloon does not break when it strikes the ground. There are many questions we could ask about this situation. To answer any of them, it makes sense to model the Energy dynamics of the scenario first. Let's do that and then answer some particular questions!

(a) Create a particular model for this phenomenon by making an *Energy-System Diagram* for the process that takes place from the time the water balloon is dropped until it is motionless on the ground. Consider the indicators to determine what energy systems must be present/can be excluded. Have you included enough systems?

(b) If the water balloon falls a distance of 21 m, what is the maximum temperature rise of the water balloon due to its being dropped? (Does you answer seem reasonable? Why or why not? It may help to check your units.)

(c) Is there anything that prohibits the water balloon from suddenly cooling off to its original temperature and leaping 21 m into the air? From the random nature of thermal energy, why do you think we never see this happen? Respond *briefly*.

Eventually, Christine grows tired of throwing and dropping things. She goes back into the Science Building to play with a mass that's hanging on a spring in one of the laboratory rooms.

FNT 8-6

Christine's mass-spring system consists of a 250 g mass hanging from a spring with a spring constant $k = 0.182\,{}^{\mathrm{J}}/_{\mathrm{m}^2}$. She pulls the mass down 7.1 cm from its equilibrium position and then releases it from rest.

(a) How much work did Christine do when she pulled the spring down from its equilibrium position? Assume that the mass was at rest before she pulled it down, and before it was released. (Use the **Energy-Interaction Model**, *not* the expression $W = F_{\text{avg}} \cdot \Delta x$, to determine the work.)

(b) Create a particular model (construct an *Energy-System Diagram*) for each of the following final conditions to predict the speed of the mass after it is released, and when it is:

(i) moving up through the equilibrium position,

(ii) moving down through equilibrium, and

(iii) 5.0 cm below the equilibrium position, moving down.

Each group is responsible for putting one of the following on the board. Write so that all other groups can easily follow your presentation!

Group 1

FNT 8-1 & FNT 8-4 (<5 min)

Read: You should already have used the principle of energy conservation to develop an equation for the change in height of a ball thrown straight up in the air (from Activity 7 C.2).

Do: Write a sentence or two explaining what fundamental feature of the **Energy-Interaction Model** "accounts for" why changing the location of $y = 0$ does not change your calculation of the ball's maximum height above the floor. [**Hint:** think about the general form of the algebraic expression of energy conservation.]

FNT 8-2 (<5 min)

Read: When you model the process described in FNT 8-2 in two different ways, i.e., using an interval corresponding to the overall process in versus splitting the overall process into two contiguous pieces with separate intervals corresponding to each part, you should get the same result for the speed of the ball at 4 m above the floor.

Check with your instructor for the correct numerical answer.

Do: In terms of the **Energy-Interaction Model**, why is it not necessary to divide the overall process into two pieces in order to find the speed of the ball at 4 m above the floor falling down?

Group 2

FNT 8-3 (<10 min)

Construct one or two *Energy System Diagrams* that you can use to explain how you know that an object thrown vertically upward will have the same speed as it comes down through any point that it had going up through that same point.

Group 3

FNT 8-5 (<10 min)

Construct a complete *Energy-System Diagram* for the process described in Part (a) of this FNT.

Group 4

FNT 8-6, Part (a) (<10 min)

Construct a complete *Energy-System Diagram* for the process described in Part (a) of this FNT.

Group 5

FNT 8-6, Parts (b)-(i) & (b)-(ii) (<10 min)

Construct a complete *Energy-System Diagram* for the process described in Parts (b)-(i) and (b)-(ii) of this FNT.

Group 6

FNT 8-6, Part (b)-(iii) (<10 min)

Construct a complete *Energy-System Diagram* for the process described in Part (b)-(iii) of this FNT.

Whole Class Discussion

Discussion/Lab 9

9 A Graphically Representing Energy Relationships

Overview: By now, you are very familiar with an *algebraic* representation of the **energy conservation principle**. In this activity, we are restating this algebraic representation and introducing a *graphical* representation of this principle as a useful tool for understanding certain physical phenomena, for example a **falling ball**.

9 A.1 Rethinking and Restating Energy Conservation

Situation 1: A dropped ball at any time between when it is dropped from rest to *just* before it hits ground.

Our standard expression of energy conservation for this situation is $\Delta PE_{\text{gravity}} + \Delta KE = 0$. However, there are times when it is more useful to have a different expression of energy conservation (e.g., when graphically representing energy amounts). Your instructor will use the definition of the *difference operator* "Δ" to algebraically show how the expression:

$$E_{\text{total}}(\text{at any time}) = \text{const.} = PE_{\text{gravity}} + KE$$

is equivalent to:

$$\Delta PE_{\text{gravity}} + \Delta KE = 0.$$

9 A.2 Using Energy Conservation to Graphically Represent Energy Amounts for the Falling Ball

In order to plot the different amounts of energy (PE_{gravity}, KE, and E_{total}) of the dropped ball (Situation 1 above) as a function of height, it is necessary to decide where to set $y = 0$. This means, we need to decide where in space to put the origin of the y-coordinate, which measures the height of the ball above the surface of the earth.

For example, you could choose $y = 0$ to be at the floor level, or at 3 m below the floor. Since we use $PE_{\text{gravity}} = mgy$ as our standard expression for gravitational potential energy, the place where $PE_{\text{gravity}} = 0$ depends on where we put $y = 0$. For any given physical situation you can set $y = 0$ *wherever you want*.

Your instructor will demonstrate how to create the graphical representation of energy amounts with $y = 0$ set at 3 m *below* the floor. Then, each small group will construct an energy graph for a dropped ball following the directions given below for where to set the origin of the y-coordinate system.

In your group: Assume the ball is being dropped from a height of 2 meters above the floor (**Note:** This physical situation is exactly the same for each group). Graph the three energy amounts in Situation 1 using the $y = 0$ location listed here:

Group	$y = 0$ at:	Group	$y = 0$ at:
1	2 m below the floor	4	2 m above the floor
2	1 m below the floor	5	3 m above the floor
3	1 m above the floor	6	4 m above the floor

(a) First, create a small, simple sketch of the physical scenario, indicating where $y = 0$, the ball's initial position, and the floor. Label each of these places with its respective y-value and a physical description.

(b) Plot PE_{gravity}, E_{total}, and KE of the ball (all on the same graph) as functions of height using coordinate axes as described above. Why does it make sense to graph PE_{gravity} *first*, E_{total} *second*, and only then KE? Be ready to explain how you determined E_{total}.[1] Be prepared to explain how you constructed your graph.

(c) Choose an arbitrary value of y on your graph. For this value, indicate on your graph how $E_{\text{total}} = PE_{\text{gravity}} + KE$. Then, choose two different values of y on your graph. For those values, indicate on your graph how $\Delta PE_{\text{gravity}} + \Delta KE = 0$.

Whole Class Discussion

9 A.3 Using Energy Conservation to Graphically Represent Energy Amounts for a Mass-Spring System

Situation 2: A mass hanging on a spring is pulled down and released. In your plot, consider all times between when the mass-spring is released and the first time it is momentarily at rest.

Repeat (a), (b), and (c) from Part 9 A.2 using Situation 2. The three energy amounts are now plotted as functions of "distance from equilibrium." In this situation, **everyone** should have $y = 0$ at the same place: The *equilibrium position* of the mass-spring system.

Whole Class Discussion

[1] Remember: It is standard convention to plot the *dependent* variable on the vertical axis and the *independent* variable on the horizontal axis. Consequently, the energy is plotted on the vertical axis.

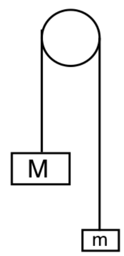
9 B Explicit Reasoning Using Models

Overview: Now that we've discussed many different physical scenarios, our models probably have become second nature to you. Maybe so much so that we may have to revisit how to explicitly model a phenomenon. We'll do that with a new phenomenon.

9 B.1 Two Masses Over a Pulley – Atwood's Machine

1. Think and Discuss

Imagine this situation: A string connecting a smaller mass, m, and a larger mass, M, passes over a small pulley that is almost frictionless. The masses are initially held at rest; then are released and allowed to move:



Based on your intuition which set of attached masses, each differing by 20 g, will move faster when released?

Set A: $M_A = 220\,\text{g}, \quad m_A = 200\,\text{g}$
Set B: $M_B = 70\,\text{g}, \quad m_B = 50\,\text{g}$

Circle your choice here: (a) Set A moves faster
(b) Set B moves faster
(c) Both move with the same speed

2. Put your group's choice on the board

How sure are you about your reason(s) and predictions? What is your intuition based on?

3. Try it out

Pair up with an adjacent table. One table will attach the heavier set of masses, the other table should attach the lighter set. Try it out and see which set moves faster.

What do you observe?

Whole Class Discussion

9 B.2 Applying the Energy-Interaction Model

Apply the **Energy-Interaction Model** to the two-masses-over-a-pulley situation. Model this system as if the pulley is massless and frictionless, so you won't have to worry about energy systems associated with the pulley.

Take the **initial state** to be just as you release the masses (what is v_i? what is Δv?).

Take the **final state** to be when the masses have moved a distance d and have speed v, but before they hit anything or run out of string.[2]

1. Create a particular model of the phenomenon described above by constructing a complete *Energy-System Diagram* for **each** of the mass sets using the initial and final states described above.

2. When you substitute algebraic expressions for changes in individual energy systems in the algebraic representations of your particular **Energy-Interaction Models**, you will find the symbols m, M, g, v, and d useful. Watch your "minus signs!"

3. One important practical use of the *Energy-System Diagram* is in the interpretation of algebraic expressing of energy conservation. For example, what do each of the terms in the equation mean, and what should their sign be? Check, and be ready to show, that the signs of the terms in the equation from (2) are consistent with the increases and decreases in energy of the energy systems in your diagram.

4. What energy systems have you ignored by your assumptions about the pulley?

   [You don't have to put (4) on the board.]

Whole Class Discussion

9 B.3 Reasoning and Explaining with Particular Models

You have constructed two particular **Energy-Interaction Models**, one for each of the different sets of masses. You are now going to use these models to make sense of and explain why one set of the masses (the set with the smaller masses) moved faster than the other set.

Comparing the changes in energy of the various energy systems

1. During the process (from beginning to end), does the total KE of the two masses (the sum of the KEs) increase, decrease, or stay the same? How do you know? Is your response the same for both the heavier and the lighter pair of masses?

2. During the process, does the total PE of both masses (sum of the PEs) increase, decrease, or stay the same? How do you know? Is your response the same for both the heavier and the lighter pair of masses?

[2] Since d denotes a distance, it is a positive number; so, $\Delta y = \pm d$, as appropriate.

3. Compare the magnitudes of the total change in PE of both masses that occurs during the process across the two systems (the heavier pair and the lighter pair). Are the total changes the same or different? Explain.

4. Compare the magnitudes of the total change in KE of both masses that occurs during the process across the two systems (the heavier pair and the lighter pair). Are the total changes the same or different? Explain.

5. Did the original question regarding the two sets of masses have to do with the KE of the masses or something else? Was it related to the KE? Compare this across the two systems (the heavy pair and the lighter pair). Is this the same or different? Explain.

9 B.4 Creating a Convincing Scientific Explanation

Turn your responses to the questions (1) through (5) from 9 B.3 into a set of statements that constitute a logical argument based on your particular **Energy-Interaction Models** that explains why the set of masses with smaller total mass move faster than the set with greater total mass, as long as the difference between the two masses in each set is the same.

Whole Class Discussion

Discussion/Lab 10

10 A Practicing Graphical and Quantitative Modeling of Mechanical Energy Systems

Overview: Before we introduce a new class of physical phenomena, ***forces***, we want to make sure you get a bit more practice modeling phenomena using the tools we have accumulated until now, especially the method of graphing energy amounts introduced in the last activities and the carefully crafted model-based, scientific arguments we've been practicing for a while.

We've done a practice activity like this before:

At home, please work through all the FNTs listed in this activity. In class, each group will work on a portion of these FNTs to come to a consensus before we discuss the scenarios with the whole class.

FNT 10-1

This FNT is excellent practice in constructing a scientific argument based on logic that follows directly from the physical situation and the relationships of the models you use to, well, model the situation. This is what science is all about!

Please refer back to Activity 9 B:

(a) Finish any of the prompts from 9 B.2 and 9 B.3 that you were unable to finish during the last discussion lab.

(b) Respond to the prompt in 9 B.4, which asks you to turn your responses to prompts from 9 B.2 and 9 B.3 into a set of statements that constitute a logical argument based on the **Energy-Interaction Model**.

With your argument, explain why the set of masses with smaller total mass moves faster than the set with greater total mass even though the difference between the two masses in each set is the same.

FNT 10-2

A 0.4 kg mass is attached to a spring that can compress as well as stretch (spring constant 50 N/m). The mass and spring are resting on a horizontal tabletop. The mass is pulled, stretching the spring 48 cm. When it is released, the system begins to oscillate.

(a) Assuming the transfer of energy to thermal energy systems is negligible, construct a complete *Energy-System Diagram* that could be used to predict the speed of the mass as it passes a point that is a distance of 39 cm from its equilibrium point on the other side of the equilibrium position (spring is compressed).

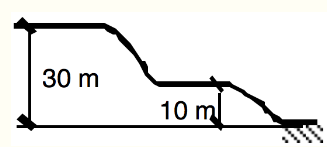
Substitute all known values of constants and variables into the algebraic expression of energy conservation, and identify any unknown(s). Do you have enough information to find the speed of the mass?

(b) Now assume that the effects of friction are not negligible. When pulled back and released as before, the mass now reaches its furthest distance from equilibrium at 40 cm on the compressed side (before bouncing back again). Construct a complete *Energy-System Diagram* that could be used to determine the amount of energy transferred to thermal systems when going from the initial stretched position to where it first momentarily stops.

Proceed as in Part a: Substitute all known values and identify any unknown(s). Can you determine the increase in thermal energy?

FNT 10-3

A skier (of mass 55 kg) skies down the smooth (frictionless) ski slope illustrated in the cross-sectional diagram. She pushes off at the top with a speed of 10 m/s. At the bottom (0 m), she comes to a stop by digging her skis in sideways.



(a) Construct a complete *Energy-System Diagram* that can be used to predict the speed of the skier when she is on the middle flat part (at 10 m). Substitute all known values of constants and variables, and identify any unknown(s). Do you have enough information to determine the speed at this part of the hill?

(b) Construct a complete *Energy-System Diagram* that can be used to predict the skier's maximum speed just before digging in her skis at the bottom. Substitute for constants and variables and identify any unknown(s) as in Part (a).

(c) Assuming that the snow at the point where she comes to a stop is at a temperature of 0 °C and that all of the kinetic energy of the skier goes into melting the snow, construct a complete *Energy-System Diagram* that can be used to predict the amount of snow melted by the skier while stopping. Substitute for constants and variables and identify any unknown(s) as in Part (a).

FNT 10-4

Christine (from Activity 7 C.2) throws a 300 g ball straight up into the air. The ball is exactly 2 m above the ground when Christine lets go of it. It reaches a height 12 m above the height from which it was released, and then falls straight back down.

(a) Assume that $y = 0$ at the point of release of the ball. Find the maximum and minimum values of y and PE_{gravity}.

What is the total energy of the system? Find the maximum and minimum values of KE.

(b) Repeat Part (a), with $y = 0$ at the ground.

FNT 10-5

A 200 g mass is attached to a spring, just like in Activity 9 A.3. The mass is lifted up 5 cm and released so that it begins to oscillate about the equilibrium point. The spring has a spring constant $k = 500\,{}^{\text{N}}/_{\text{m}}$ ($= 500\,{}^{\text{J}}/_{\text{m}^2}$).

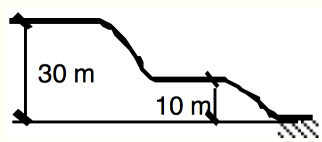
(a) Calculate and accurately plot on a letter size (8.5×11 in) sheet of graph paper $PE_{\text{spring-mass}}$, E_{total}, and KE. The vertical axis of the graph should be energy (in Joules). The horizontal axis is "distance from equilibrium" (in meters).

(b) On the same graph, quickly sketch (without calculating values) the $PE_{\text{spring-mass}}$, KE and E_{total} of the system if the mass were initially pulled back (stretched) 2.5 cm from its equilibrium point, instead of lifted up (compressed) 5 cm.

FNT 10-6

Remember the physical situation described in FNT 10-3? To refresh your memory:

A skier (of mass 55 kg) skies down the smooth (frictionless) ski slope illustrated in the cross-sectional diagram. She pushes off at the top with a speed of $10\,{}^{\text{m}}/_{\text{s}}$. At the bottom (0 m), she comes to a stop by digging her skis in sideways.



Use the algebraic expression of energy conservation that shows that the sum of all the energies at all points in time is constant and equal to the total energy for the following:

(a) Pick a location to set $y = 0$ and determine the total energy of the system.

(b) Write an algebraic representation expressing energy conservation that you could use to find the speed of the skier when she is on the middle flat part (at the height 10 m).

(c) Substitute all known values of constants and variables, and identify any unknown(s).

Each group is responsible for putting one of the following on the board. Write so that all other groups can easily follow your presentation!

Group 1

FNT 10-4

For each of Parts (a) and (b) of this FNT, make a simple sketch (not a graph) showing the origin (where $y = 0$), the ball's initial position, positions y_{max} and y_{min}, and the floor. Then, for each of your two sketches, show how to find the maximum and minimum values of gravitational potential energy, and the total energy.

Be ready to explain how changing the location of where $y = 0$ will affect PE_{gravity}.

Group 2

FNT 10-5

(a) Draw $PE_{\text{spring-mass}}$ and E_{total} (but not KE) on a graph of Energy vs. "Distance (x) from equilibrium" for the case of the mass displaced 5 cm from equilibrium and released. Clearly label the maximum and minimum values of $PE_{\text{spring-mass}}$.

(b) Demonstrate how to use point by point plotting to graph the KE by doing the following: For at least two different values of x, use energy conservation in the form: "$KE = E_{\text{total}} - PE$" to plot a point representing the KE. Label the graph clearly showing PE, E_{total}, and KE at each point. In the Whole Class Discussion you can show how to then fill in the KE by sketching a dashed line connecting the points with a smooth curve.

(c) On the same graph, quickly draw $PE_{\text{spring-mass}}$ and E_{total} (but not KE) for the case of the mass displaced 2.5 cm from equilibrium.

Group 3

FNT 10-2 Part a

Construct a complete *Energy-System Diagram* with accompanying equations as directed in Part (a) of this FNT, and be ready to explain if you have enough information to determine the speed of the mass.

FNT 10-2 Part b

Construct a complete *Energy-System Diagram* with accompanying equations as directed in Part (b) of this FNT, and be ready to explain if you have enough information to determine the increase in thermal energy.

Group 4

FNT 10-3 Part a

Construct a complete *Energy-System Diagram* with accompanying equations as directed in Part a of this FNT, and be ready to explain if you have enough information to determine the speed of the skier on the middle part of the hill.

Group 5

FNT 10-3 Part b

Construct a complete *Energy-System Diagram* with accompanying equations as directed in Part (b) of this FNT, and be ready to explain if you have enough information to determine the speed of the skier at the bottom of the hill.

Could you have used different initial values of energy system indicators to answer the same question? Briefly show/explain on the board.

Group 6

FNT 10-6

Put your response for this FNT on the board. Be sure to show how you determined the total energy of the physical system.

Is this the only correct value of total energy for the system? Briefly show/explain.

All Groups

FNT 10-1

Put your response for this FNT on the board.

Whole Class Discussion

10 B Connecting Potential Energy with Forces

Overview: We have studied two different potential energy systems so far, PE_{gravity} and $PE_{\text{mass-spring}}$. Our goal in this section is to understand how the forces associated with these potential energy systems are related to the graph of Potential Energy as a function of position.

10 B.1 Graphs for Gravitational Potential Energy

1. Sketch a graph of PE_{gravity} vs. the vertical position y of a ball that is thrown upward from the surface of the Earth. Let $y = 0$ be at the ground.

   Note: Be sure to plot PE on the vertical axis and y on the horizontal axis.

2. Next to your graph, write a mathematical expression for PE_{gravity} as a function of y.

3. Consider a particular value of y that is greater than y_{min} and less than y_{max}. At your value of y, what is the direction of the force of gravity on the ball (in the direction of *decreasing* y or *increasing* y)?

   Put an arrow on your graph showing the direction of the force. Repeat for a different value of y. How do the magnitudes of the forces at these two points compare?

4. What, in general, can you say about the magnitude and direction of the force at different y values?

10 B.2 Graphs for Mass-Spring Potential Energy

1. Sketch a graph of $PE_{\text{mass-spring}}$ vs. the distance x of a mass from its equilibrium position in an oscillating mass-spring system. Remember that for a mass-spring system, we always set the origin of the coordinate system at the equilibrium point.

2. Next to your graph, write an algebraic expression for $PE_{\text{mass-spring}}$ as a function of x.

3. Consider a particular value of x that is greater than zero and less than x_{max}. At your value of x, what is the direction of the force on the mass (*away* from equilibrium or *toward* equilibrium), and is that in the direction of *increasing* PE or *decreasing* PE?

   Note: You can use the mass-spring system at your table to test this. Pull the mass down ~2 cm from equilibrium, and then push it up ~2 cm from equilibrium.

   Put an arrow on your graph showing the direction of the force. Repeat for a different value of x. How do the magnitudes of the forces at the two points compare?

4. What, in general, can you say about the magnitude and direction of the force at different x values?

10 B.3 The Big Idea

Remember our goal: We want to relate our observations about the gravitational forces in 10 B.1 and the mass-spring forces in 10 B.2 to features of the graphs of the potential energy systems associated with these forces. That is, we want to find a relationship that **holds true for both forces**.

This relationship will need to relate **both** the **magnitude** of the force and the **direction** of the force to **specific features** of the **graphs** of the respective potential energies. The two features of the graphs we will focus on are the **instantaneous slope** of the graph of $PE(y)$ or $PE(x)$, and the **change in** PE as the position changes.

1. Can you find any correlation between the magnitude of the force and the magnitude of the instantaneous slope of the PE for these two forces?
2. Can you find any correlation between the direction of the force and the direction of decreasing PE for these two forces?
3. Come to a group consensus about how to state this relationship most succinctly and clearly in words. **Write this statement on the board.**

"Physicality Check"

When you use the feature you found above to predict a force from the potential energy vs. distance graphs, do your results make sense physically? For example, do the directions and magnitudes of the forces found at various points on the graph agree with what you know about the force? Try this with various points on both sets of graphs. Be ready to demonstrate this for the whole class.

Whole Class Discussion

Follow-up of Module 2.4 FNTs

Groups 1 & 2 discuss and respond to FNT 2.4.1-1 as directed below. Groups 3 & 4 discuss and respond to FNT 2.4.2-1 as directed below. Groups 5 & 6 discuss and respond to FNT 2.4.2-2 as directed below.

FNT 2.4.1-1 (<10 min)) Come to a consensus in your SG about how the changes in PE_{grav} compare for the two situations.) Come to a consensus in your SG about how the maximum heights compare.

All members of your group must now go to the board and work on this together

) After you reach consensus (and check with your instructor) about () and (), sketch the two plots of $PE_{\text{gravitational}}$ on the same graph of Energy vs. Height.) You know that the force of gravity is constant near the surface of the earth and near the surface of the moon, and that it is approximately six times stronger for the earth than for the moon. Make sure everyone in your SG is ready to explain how your graph conveys this information.

All members of your group must now go to the board and work on this together

FNT 2.4.2-1 (<10 min)) For two masses connected by a spring the potential energy is given by: $PE_{\text{spring}} = \frac{1}{2}k(r - r_0)^2$ How does doubling the force constant affect PE_{spring} at any particular value of r?) On one graph, sketch two plots of PE for the two different springs. Pick some particular value of r, and show explicitly on your graph how doubling the spring constant affects the potential energy of the system. How do these two systems differ physically?) On a second graph, sketch two plots of PE for the two different values of r_0. How do these two systems differ physically?

All members of your group must now go to the board and work on this together

FNT 2.4.2-2 (<10 min)) Why do we define the PE between atoms or molecules to be zero when r is very large (i.e., when the atoms are far apart instead of when they are close together)?) Sketch three separate graphs of PE vs. separation distance on the board: one for a pair of positively charged particles, one for a pair of negatively charged particles, and one for two particles with opposite charges. Make sure everyone in your SG is ready to explain why the graphs are drawn as they are.

Whole Class Discussion

Unit III

Forces and Motion: The Momentum Conservation Model

Discussion/Lab 11

11 A Adding and Subtracting Vectors

Overview: After studying the energy dynamics of mechanical systems, we're now moving on to take a look at some of the underlying mechanisms and some of the effects of energy transfers and transformations: ***forces and motion***. Since we will spend some time and effort on quantitatively describing both, we need to get familiar with mathematical entities that will make this quantitative treatment possible: ***vectors***.

11 A.1 Vectors

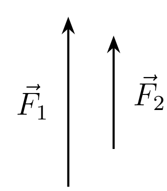
Often, a physical quantity cannot be fully described without giving both a magnitude and a direction for it. For instance, consider two physical quantities, mass and force. Suppose I have an object with a mass of 5.0 kg and another with a mass of 1.5 kg. What is the range of possible values for the total mass of these two objects (i.e. how big might the total mass be and how small might it be)?

Now suppose I push on some object with a force of 2.5 N and you push on it with a force of 1.5 N. What is the range of values for the total force on the object? Explain. (**Hint:** the forces might be in the same directions or in opposite directions.)

Whole Class Discussion

11 A.2 Vector Addition

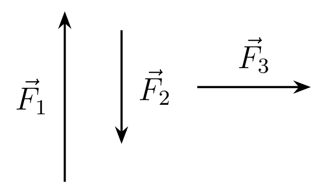
1. The picture to the right shows two force vectors. $\vec{F}_1$ represents a force of 2.5 N pushing straight up the page and $\vec{F}_2$ a force of 1.5 N pushing straight up the page. Decide in your groups what the magnitude and direction of the total force acting on an object would be if both $\vec{F}_1$ and $\vec{F}_2$ were acting on it at the same time. Then decide how you should arrange the two vectors that are shown to represent the addition of these vectors showing how you get the total vector that you expect.



   Hint: You can easily move the vectors around so that either:
   (i) their tails touch,
   (ii) their heads touch, or
   (iii) the tail of one touches the head of the other.

Which of these three methods best demonstrates addition of these two vectors to find the proper total vector? Sketch this method on the board and also sketch the total force (also called the "net force" or the "sum of the forces", $\vec{F}_{\text{net}} = \sum \vec{F}$).

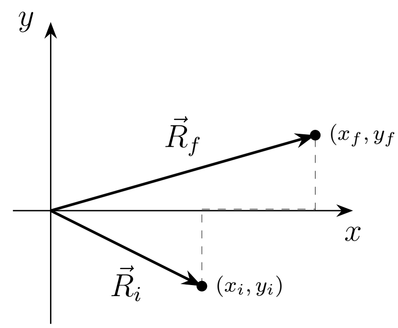
2. The picture to the right shows three force vectors. Show that your method from Part 1 gives a sensible result if $\vec{F}_1$ and $\vec{F}_2$ are the two forces acting on an object. Show that your method works if $\vec{F}_1$ and $\vec{F}_3$ are the two forces acting on an object.



Whole Class Discussion

11 A.3 Vector Subtraction

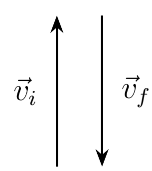
1. An object starts at the initial location $(x_i,\, y_i)$ as shown in the x-y graph to the right and then follows the dashed line path shown to a final location (x_f,y_f). The picture also shows the initial and final position vectors (position is always measured with respect to some axis origin so all position vectors are drawn from the origin to the position of the object). Draw this figure on the board. Then decide in your groups what arrow you would draw to represent the change in the position, $\Delta\vec{R}$ (both the distance moved as well as the direction moved) and then draw this vector on the board.



2. Like before, we define the change in a quantity to be the final value minus the initial value: $\Delta\vec{R} = \vec{R}_f - \vec{R}_i$. Decide in your groups whether $\vec{R}_i$ and $\vec{R}_f$ should be arranged tail-to-head, tail-to-tail, etc. to represent the subtraction of these vectors and show how you get the change $\Delta\vec{R}$ that you expected.

   (**Hint:** This might be easier than you think.)

3. Does the method used above give you a reasonable $\Delta\vec{v}$ for the two velocity vectors shown below?



Whole Class Discussion

11 B Working with Position and Displacement Vectors

Overview: Now that we know the basic properties of vectors, let's talk about two similar, yet very different kinds of vectors: ***position*** and ***displacement vectors***.

Phenomenon: You are in a strange city that has streets that are laid out in a perfectly square grid. Your job will be to move around to different locations as instructed and record your progress.

Make sure everyone in your group fully understands the ideas behind each question or part in these activities before going on to the next part.

1. Make a drawing on the board showing the streets in the central city (this should be a square grid with at least 10 streets going in each direction). Decide as a group how you are going to label the streets and use your labeling system. Make sure your diagram is large enough for everyone in the room to clearly see it.

2. Near one of the edges of the diagram, label a street corner "o" for origin of your coordinate system. Your origin should be different from the origins in other groups' drawings. Sketch a coordinate system so that the y-axis points North and the x-axis points East.

   You start walking somewhere in the city, so choose an intersection near the lower middle part of your diagram, and call that location your "initial" position. Draw a position vector on your diagram that shows this initial position. Label this vector "$\vec{R}_i$."

   (a) Write $\vec{R}_i$, in terms of its x- and y-components, as $(R_{i,x}, R_{i,y})$. Determine the length of $\vec{R}_i$. What units does it have? How would you describe the direction of the position vector?

Whole Class Discussion

   (b) Starting from your initial location, imagine walking a distance equal to 4 blocks North and then 1 block West and then 1 block South to a "final" location. Show the *position vector*, $\vec{R}_f$, for this new location and write $\vec{R}_f$ in terms of its components as $(R_{f,x}, R_{f,y})$.

   (c) As you might expect, we define the "change in position" to be $\Delta\vec{R} = \vec{R}_f - \vec{R}_i$, so that $\vec{R}_i + \Delta\vec{R} = \vec{R}_f$.

   Using the tail-to-head method shown on Page 38 of the course notes, redraw the vectors $\vec{R}_i$ and $\vec{R}_f$ off to the side of your map. Then show how you can **graphically obtain** $\Delta\vec{R}$ from $\vec{R}_i$ and $\vec{R}_f$. Make sure you draw these vectors with the same lengths and directions that they have on your map.

   Using the same picture that you used to show $\Delta\vec{R} = \vec{R}_f - \vec{R}_i$, show that $\vec{R}_i + \Delta\vec{R} = \vec{R}_f$ is also true. Now transfer your $\Delta\vec{R}$ over to your street diagram.

(d) In physics, we are interested in describing motion. If you could choose only one vector from $\vec{R}_i$, $\vec{R}_f$, and $\Delta\vec{R}$ to describe your motion, which one would it be? Why? What do either of the other two vectors, by themselves, tell you about your motion?

(e) How are the x-components of $\vec{R}_i$ and $\vec{R}_f$ related to the x-component of $\Delta\vec{R}$? How about the y-components?

Whole Class Discussion

(f) Suppose you walked the 4 blocks North and 1 block West and 1 block South in 60 seconds. Draw the *average velocity* vector for this situation. Write this average velocity vector in component form, (v_x, v_y). What is the "length" of this vector? Include the units. Draw another velocity vector for a situation where you took 180 seconds for this 6 block walk. Which arrow is longer? Why?

(g) In which direction(s) would your *instantaneous velocity* be?

(h) If you now walked back to your initial point and assuming the total journey took eight hours what is your average velocity for the entire trip?

Whole Class Discussion

3. Look around the room at the different street diagrams. What is similar in each one? What is different? Discuss in your group how you would summarize the meaning of everything you did in this activity. Be prepared to share with the whole class.

Discussion/Lab 12

12 A Check for Understanding: Vector Operations

Overview: In this section, we'll apply our newly-gained knowledge about vectors.

FNT 12-1

Three force vectors are added together. One has a magnitude of 9 N, the second one a magnitude of 18 N, and the third a magnitude of 15 N. What can we conclude about the magnitude of the net force vector? Explain.

I. It must equal 42 N.

II. It cannot be 0 N.

III. Anything from -42 N to +42 N.

IV. It can be anything from 0 N to 42 N.

V. None of the above can be concluded.

Whole Class Discussion

FNT 12-2

Alice, Bob and Chuck are three friends standing around, talking. We know that Alice is standing 9 m away from Bob, and that Bob is standing 3 m away from Chuck. Let $\Delta\vec{R}_{\mathrm{AB}}$ be the vector that starts at Alice and ends at Bob, and $\Delta\vec{R}_{\mathrm{BC}}$ be the vector that starts at Bob and ends at Chuck.

(a) Write an equation to find the displacement between Alice and Chuck.

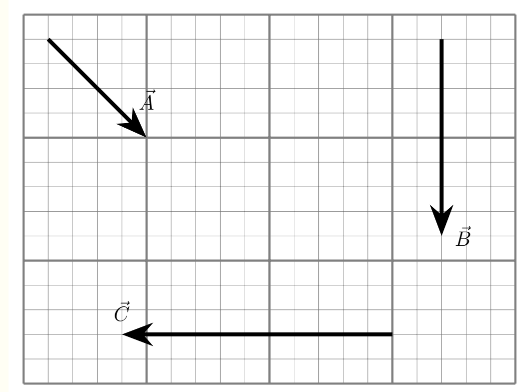
(b) What is the furthest distance Chuck can be from Alice, given the information in the question? What is the closest they can be?

(c) Draw a picture where Chuck is neither as close as he can be or as far as he can be from Alice. Draw and label the vectors $\Delta\vec{R}_{\mathrm{AB}}$ and $\Delta\vec{R}_{\mathrm{BC}}$.

Whole Class Discussion

FNT 12-3

Using a piece of graph paper, carry out the operations listed below on the vectors shown at right. Label all vectors.



(a) $\vec{A} + \vec{B}$

(b) $\vec{B} + \vec{A}$

(c) $\vec{A}$ - $\vec{B}$

(d) $\vec{A} + \vec{B} + \vec{C}$

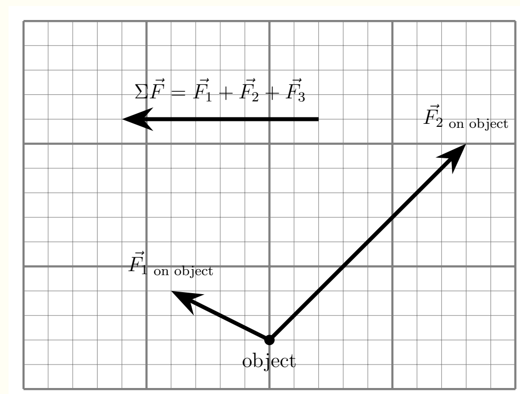
(e) $\vec{B}$ - $\vec{A}$

(f) -$\vec{C}$

Whole Class Discussion

FNT 12-4

Vectors $\vec{F}_{1\text{ on object}}$, $\vec{F}_{2\text{ on object}}$, and $\vec{F}_{3\text{ on object}}$ are all exerted on an object, adding together to form a net force vector, $\Sigma\vec{F}$, as shown in the graph to the right. However, only vectors $\vec{F}_{1\text{ on object}}$, $\vec{F}_{2\text{ on object}}$, and $\Sigma\vec{F}$ are known.



On a separate piece of graph paper, use the properties of vector addition to graphically determine the vector $\vec{F}_{3\text{ on object}}$.

Whole Class Discussion

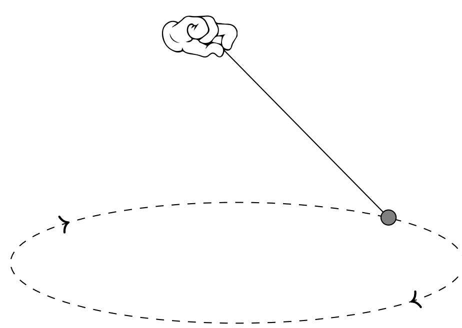
12 B Using Vectors to Represent the Motion of a Mass Moving in a Circle

Overview: So far, we've only talked about linear motion – an object moving along a straight path. However, many phenomena have objects moving along curved paths. As an example, we'll take a closer look at an object moving along a circle.

Phenomenon: You are going to swing a ball in a circle and represent its velocity in several different ways using vectors.

Make sure everyone in your group fully understands the ideas behind each question or part in these activities before going on to the next part.

1. (a) Get the ball swinging in a *large* horizontal *circle*, going *clockwise* when viewed from above:



Imagine looking down on the ball. Pick a position in space that you will identify as the "12 o'clock" position. Draw the circular path of the ball on the board with the 12 o'clock position toward the top of the board.

Choose a coordinate system and sketch it on the board.

(b) Draw a position vector identifying the position of the ball when it is in the 4 o'clock position. Show the x- and y-components of this vector on your diagram. Discuss in your group how to do this. Be prepared to share with the whole class.

(c) Draw another position vector when the ball is in the 5 o'clock position. To the side of your diagram, graphically subtract the position vectors **as accurately as possible** to find the displacement vector; label it $\Delta\vec{r}$.

Describe in a sentence what the delta means here.

Brief **Whole Class Discussion**

2. (a) Describe in words the **velocity** of the ball as it moves in its circle. Be prepared to share.

(b) What do you think is an *instantaneous velocity vector*? How might it be different from a position vector? Does it always point in the same direction? If you are having trouble answering these questions, apply them to a specific situation (e.g. driving northwest on I-5 at 65 mph and then curving toward the north).

3. (a) Imagine that you are sitting on the ball and "driving" it in a circle. Which way are you moving when you are at the 4 o'clock position? Draw a velocity vector (put the tail of the vector at the ball's location) showing the velocity of the ball at the 4 o'clock position. Show the x- and y-components of this velocity vector on your diagram. Discuss in your group how you do this.

   (b) Looking at the vectors you have drawn, is a *position* vector or a *displacement* vector more closely related to a velocity vector? Make sure this agrees with your knowledge of the definition of velocity.

   (c) Which vectors on your diagram would be different if you changed the origin?

Whole Class Discussion

4. Draw the velocity vector for the ball at the 6 o'clock position. Off to the side, graphically determine the change in velocity, $\Delta\vec{v}$, between the 4 o'clock position and the 6 o'clock position.

5. (a) Find the x- and y-components of the 6 o'clock velocity vector.

   (b) Find the x- and y-components of the vector $\Delta\vec{v}$.

6. How can you use the x- and y-components of the 4 o'clock and the 6 o'clock positions to get the change in velocity, $\Delta\vec{v}$? Summarize your method and be prepared to share it with the class.

Whole Class Discussion

Discussion/Lab 13

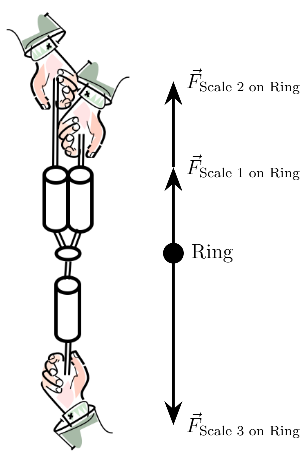
13 A Force Model: Net Force and Force Diagrams[1]

Overview: Now that we can represent linear and curved motion with vectors, let's see how vectors are tremendously useful to describe *forces*, as well.

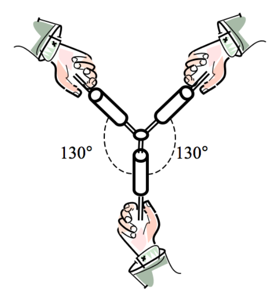
Phenomenon: You are going to pull on a metal ring with three ropes attached. A spring is attached to each rope to determine the tension in the rope.

- Before you start, make sure each scale is set to zero by twisting the knob.
- Work together as a group. No, really: This takes multiple hands!
- Attach the ropes with the scales turned so you can read force values in Newtons.

Note: To get the angles to be 130°, trace the ring on a sheet of paper and draw a vertical line pointing down from it. Use a protractor to mark the 130° angles from the vertical line. Line up the ropes along the lines you drew, pull and then note the force scale readings.



Scenario 1: Person 1 and Person 2 pull directly opposite to Person 3 so that the yellow spring scale for the third person reads 30 N. Record the values on all three scales.



Scenario 2: Persons 1 and 2 change the direction they are pulling until their ropes make an angle of about 130° with respect to Person 3. Record the readings of all scales.

[1]See Pages 41-51 of course notes and Force Model Summary.

1. Use appropriately scaled force vectors from the sheet of paper you've used as a guide, and graphically add the three vectors while preserving their angles on the board. What do you get? Why is that? Draw and label all vector components on your graph.

2. A properly labeled and scaled force diagram for the ring is shown for Scenario 1 (on the left, above). **Note:** Two forces acting on the same object and going in the same direction can be drawn as shown here, or both arrows can have their tails attached to the dot.

   Draw a properly labeled and scaled force diagrams for the ring as the object that the three forces act on in Scenario 2. Be sure to use the labeling conventions spelled out on Page 43 of the textbook on any force diagram you make.

3. On the board, off to the side of each force diagram, repeat the method of vector addition from (1). What is the net force on the ring in each case? Check: Why does the resultant net force make sense?

Brief **Whole Class Discussion**

4. Use trigonometry (sine and cosine, as appropriate) to find the magnitude of the components (x and y) of the two forces, $\vec{F}_{\text{Scale 1 on ring}}$ and $\vec{F}_{\text{Scale 2 on Ring}}$. Put responses to the following on the board:

   (a) What is the relationship between the components of these two forces and the components of the third force?

   (b) Develop an explanation for this relationship in your small group and be ready to share.

   (c) Explain why the scale readings changed when the two people pulling parallel separated to form 130° angles with the third force scale.

Whole Class Discussion

Follow-up Question: Could two people pull their ropes all the way to 180° apart with the third person still pulling at 30 N? Include a force diagram in your response. **Hint:** Think about the components.

Whole Class Discussion

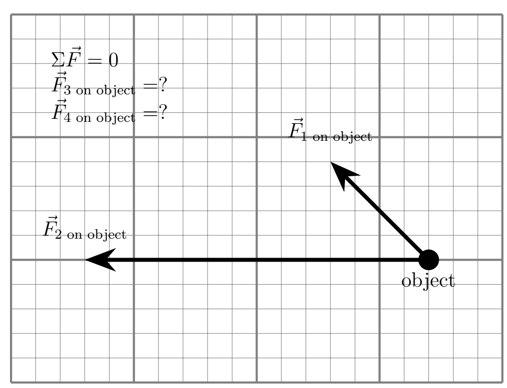
13 B Check for understanding: Force, Velocity, and Momentum Vectors

Overview: In this section, we'll practice what we've learned about force and velocity vectors, so far.

FNT 13-1

Vectors $\vec{F}_{1 \text{ on object}}$, $\vec{F}_{2 \text{ on object}}$, $\vec{F}_{3 \text{ on object}}$, and $\vec{F}_{4 \text{ on object}}$ are all exerted on an object, adding together to form a net force vector $\Sigma\vec{F} = 0$, as shown to the right.

However, only vectors $\vec{F}_{1 \text{ on object}}$, $\vec{F}_{2 \text{ on object}}$, and $\Sigma\vec{F}$ (which is zero) are known. It is known that $\vec{F}_{3 \text{ on object}}$ is completely vertical, and $\vec{F}_{4 \text{ on object}}$ is completely horizontal.



On a separate piece of graph paper, use the properties of vector addition to determine the magnitudes of the vertical vector $\vec{F}_{3 \text{ on object}}$, and the horizontal vector $\vec{F}_{4 \text{ on object}}$.

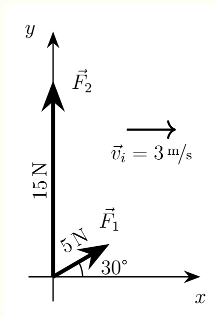
Whole Class Discussion

FNT 13-2

Two force vectors ($\vec{F}_1$ and $\vec{F}_2$, as shown to the right) act on a 2 kg object that has an initial velocity $\vec{v}_i$ of 3 m/s in the $+x$-direction.

(a) Use trigonometry to find the x- and y-components of the net force.

(b) Find the magnitude and direction of the net force.



Whole Class Discussion

FNT 13-3

Two rolling carts are moving toward each other at the **same speed**. Cart 1 has a mass $m_1 = 200\,\mathrm{g}$ and Cart 2 has a mass $m_2 = 400\,\mathrm{g}$.

(a) Draw a velocity vector $\vec{v}$ for each cart.

(b) Momentum $\vec{p}$ is a vector defined as $\vec{p} = m\vec{v}$. Draw a momentum vector for each cart.

(c) Add the two momentum vectors together to find the total momentum, $\vec{p}_{\text{total}} = \vec{p}_1 + \vec{p}_2$.

Whole Class Discussion

FNT 13-4

Rework the parts of Activity 13 A that you still have questions about. Bring any remaining questions to the next DL meeting.

Whole Class Discussion

Discussion/Lab 14

14 A Momentum and Change in Momentum: One-Dimensional Cases

Overview: We've previously discussed that moving objects have kinetic energy. In this section, we'll see that moving objects have another property, *momentum*. Colloquially, you may be familiar with this property as "Oomph."

14 A.1 Getting a Feel for Momentum

Phenomenon: Collisions of a cart with another object.

In your small group, you are going to observe, talk about, and analyze simple collisions in one dimension using carts that slide almost frictionlessly on a long aluminum track.

Please be gentle with the carts and track. Thanks!

Make sure everyone in your group fully understands the ideas behind each question or part in these activities before going on to the next part.

1. Arrange a collision so that a cart sticks to the bumper of the track. Observe the collision several times (observe the motion). Analyze the collision using the **Momentum Conservation Model**.

   (a) Use vectors to represent the various momenta. Draw on the board a *Momentum Chart*. Each column should have appropriately scaled and labeled vectors for either the momentum or the *change* in momentum $\Delta\vec{p}$ during the collision.

   (b) Write a vector equation for the momentum, off to the side of your chart, with appropriate subscripts on the symbols, to express what you observed during this collision.

   (c) Describe in words what physically happened (tell a story!) and how conservation of momentum applies in this situation. Draw a force diagram for the cart for the time when its momentum was changing. Put your diagrams, equation, and story on the board, but leave enough space so you can write the responses to (2) for comparison.

2. Arrange a collision so that a cart bounces off the bumper. Observe the collision several times. Analyze the collision using the **Momentum Conservation Model**, and repeat Steps (a) to (c), as above.

3. For each of the two collisions above, is the total kinetic energy conserved?

4. Does a case in which total kinetic energy is not conserved violate the *conservation of energy* law? If not, which other energy systems could the kinetic energy have gone to?

5. Is momentum conserved in the case where total kinetic energy is conserved and in the case where total kinetic energy is not conserved? Where in your *Momentum Chart* is this shown?

Whole Class Discussion

14 A.2 Change in Momentum and Impulse in One Dimension

A change in momentum of an object is caused by a net force acting on the object over a certain amount of time. We define the *Net Impulse* $\Sigma\vec{I}$ as the product of the net force $\Sigma\vec{F}_{\text{on object}}$ and the amount of time Δt during which the net force is acting on the object:

$$\text{Net Impulse} = \Sigma\vec{I} = \Sigma\vec{F}_{\text{on object}} \cdot \Delta t$$

Because a net impulse $\Sigma\vec{I}$ causes a change in momentum $\vec{p}$ of the object, we can also write:

$$\text{Net }\overset{\text{cause}}{\text{Impulse}} = \Sigma\vec{I} = \Sigma\vec{F}_{\text{on object}} \cdot \Delta t = \vec{p}_f - \vec{p}_i = \Delta\vec{p} = \text{Change in }\overset{\text{effect}}{\text{Momentum}}$$

Remember the law of Energy Conservation? We observed that when there is no energy coming into or going out of a given physical system, the total energy of that system is conserved, which means the total change of energy $\sum \Delta E$ is equal to zero. There is a similar phenomenon with momentum, the *Conservation of Momentum.* In this case, the total momentum of a system of objects remains unchanged:

$$\Delta\vec{p}_{\text{system}} = \vec{p}_{f,\text{sys}} - \vec{p}_{i,\text{sys}} = 0 \text{ when } \Sigma\vec{I} = 0 \text{ or } \Sigma\vec{F} = 0$$

1. In your small group, describe an example of an impulse. Identify two ways you can change this impulse. For instance, how could you make the impulse greater? Describe both

   (a) a system that involves one object and

   (b) a system that involves two interacting objects.

   Explain what $\Delta\vec{p}_{\text{system}}$ is for each of your physical systems. Put this on the board.

2. In your small group, develop a statement in your own words of what ***conservation of momentum*** means for your two systems in (1).

Whole Class Discussion

14 B Representing Momentum Conservation with Vectors

Overview: Just as *Energy System Diagrams* are useful in helping us work through conservation of energy questions/problems, *Momentum Charts* are useful for questions/problems involving conservation of momentum. The *Momentum Chart*, like an *Energy-System Diagram*, helps us keep track of what we know about the interaction, and it also helps us see what we do not know.

All *Momentum Charts* are to be filled in with *scaled* arrows representing momentum vectors.[1]

Closed System
Typically used for collisions/interactions involving two or more objects:

Closed $\vec{p}$ system	$\vec{p}_i$	$\Delta\vec{p}$	$\vec{p}_f$
Object 1			
Object 2			
Total System		0	

For total system: $\Delta\vec{p} = 0$
For each object: $\vec{p}_i + \Delta\vec{p} = \vec{p}_f$

Open System
Typically used when the phenomenon involves a **net impulse** acting on the system:

Open $\vec{p}$ system	$\vec{p}_i$	$\Delta\vec{p}$	$\vec{p}_f$
Total System			

For total system: $\Delta\vec{p} = \Sigma\vec{I}$
$\vec{p}_i + \Delta\vec{p} = \vec{p}_f$

To identify any forces that cause the object's change in momentum (change in motion), it helps to draw a force diagram for each object in the *Momentum Chart*.

1. On your boards, complete the appropriate *Momentum Chart* for each of your examples from Activity 14 A.2.
2. Each row and each column represent a separate vector equation. Check that every row equation and every column equation are added correctly and are consistent.
3. Which column is significant for showing whether momentum is conserved?
4. Determine what parts of the *Momentum Charts* are analogous to *Energy System Diagrams*. List the analogous parts on the board. In what fundamental way do these diagrams differ?

Whole Class Discussion

[1] The algebraic expression for a momentum vector is $\vec{p} = m\vec{v}$, where $\vec{v}$ is the velocity vector. This means that velocity and momentum point in the same direction at any instance in time!

14 C Check for understanding: 1-D Momentum

Overview: In this section, we'll practice what we've learned so far about momentum, change in momentum, and momentum conservation.

14 C.1 Collisions and Other Interactions between Objects

FNT 14-1

You're playing with two of the carts (each with mass m) that you used in Activity 14 A.1. Initially, these two carts are moving toward each other with the same initial speed v_i along the track. The carts collide and the result is one of these final states:

(a) Assume that the carts hit each other and stop so that the final state of the system has both carts just sitting still (not moving). Draw a *Momentum Chart* for this situation. Make a separate row for each cart.

(b) Assume that the carts bounce off each other so that the final state of the system has each cart moving opposite to its initial motion but with the same speed. Draw a *Momentum Chart* for this situation.

(c) As in (b), assume that the carts bounce off each other but now assume that the final speeds are smaller than the initial speeds, equal and in opposite directions. Draw a *Momentum Chart*.

(d) For each case above, does the total momentum of the system that contains the two carts change? How do the *Momentum Charts* help you answer this question?

(e) Is the total kinetic energy constant for all three cases? How do you know?

FNT 14-2

A rocket expels gas at a high speed out of its back for a short period of time. We are going to treat the rocket as being far away from any gravitational objects.

(a) Draw a *Momentum Chart* for the rocket expelling gas in space. Take the initial time before expelling gas and the final time after the rocket has finished expelling gas. The rocket has an initial constant speed in the horizontal direction. Put the rocket and the expelled gas on separate rows.

(b) Use your chart to explain why the rocket speed increases.

(c) Does the rocket have to keep expelling gas to stay at a constant speed? Explain.

FNT 14-3

Victoria is standing on a boat, during a perfectly calm day. Initially, both Victoria and the boat are not moving. Then Victoria walks from one end of the boat to the other. Take the initial time to be before she walks and the final time at some point while she is still walking.

(a) Draw a *Momentum Chart* for this situation. Does the boat move, and if so, in which direction?

(b) Compare the speed of the boat with Victoria's speed. Are they the same or different? Why?

Compare your responses to FNT 14-1, FNT 14-2, and FNT 14-3 with the other members of your small group. Come to a consensus on the appropriate *Momentum Charts* and answers, and put these on your board.

Added problem: Draw an appropriately scaled force diagram (for each object) that shows all the forces acting during the interaction (when the impulse occurs). Do this for each FNT.

Whole Class Discussion

14 C.2 More Collisions of Two Carts

Your instructor will assign each group a situation from 1–3 below and one from 4–5. Use the **Momentum Conservation Model** to analyze each of the collisions between two carts in the Situations 1–3 below. You may treat the system made up of the two carts as a closed physical system because there is no net external impulse imparted on the system during the collision.

Situations:

1. Use two carts of equal mass, physically arranged so the carts will bounce off one another. Start with ***one cart stationary***.

2. Use two carts of equal mass, physically arranged so the collision ends with the ***carts locked (stuck together)***.

3. Use two carts of equal mass initially ***moving toward each other*** with equal speed and ***ending with the carts locked (stuck together)***.

4. Place two carts of ***unequal mass*** on the track, turned so the collision ***ends with the carts locked (stuck together)***. Start with one cart stationary, and have the other move to collide with it. Make the stationary cart have more mass. Repeat, switching carts.

5. Place two carts of ***unequal mass*** on the track, turned so the collision ***ends with the carts locked (stuck together)***. Start with both carts ***moving toward each other*** with the same speed.

For each Situation:

(a) Draw and fill in a *Momentum Chart* to help you describe momentum conservation in this closed physical system.

(b) For each line of the *Momentum Chart*, write an algebraic vector equation, with appropriate subscripts on the symbols, to express what that line tells you about the collision.

(c) Draw a force diagram for each cart that shows the forces *during* the collision.

(d) Describe in words what physically happened and then how conservation of momentum applies to each cart and to the system as a whole. Put your equation and word statement on the board.

(e) Compare the total kinetic energy before the collision to the total kinetic energy after the collision.

Observe what is similar and what is different in your *Momentum Charts*. What patterns can you observe? Discuss in your group any rules you come up with for the patterns.

Whole Class Discussion

Discussion/Lab 15

15 A The Significance of Δt

Overview: The examples and activities in this section illustrate how the time duration Δt of an impulse imparted on an object (or system) relates to the net force ΣF and the change in momentum of the object (or system) $\Delta\vec{p}$.

15 A.1 The Tablecloth Trick

Phenomenon: If a tablecloth (or a large piece of paper) is pulled quickly enough from under some objects sitting on it, those objects slide only a very short distance on the table top after the tablecloth has been pulled out from under them. This implies that they acquired only a small velocity from the tablecloth moving out from under them. If the tablecloth is pulled a little less quickly, the objects slide a little further on the table top, implying they acquired a slightly greater velocity.

Try the trick: Use a "hanging mass" or another object and a fairly large piece of paper. Try pulling the piece of paper at different rates, so that the object

- moves a lot (but does not fall off the table); and
- moves very little.

In both cases, make sure you are pulling sufficiently fast so that the objects are continually sliding on the paper as it is being pulled. That is, you need to pull sufficiently fast so that the objects do not move with the paper.

Our goal is to make sense of this phenomenon using the relation of impulse to a force and the time interval over which it acts.

Establishing which part of the phenomenon we need to focus on:

1. The momentum of an object is first increased as the paper is pulled out from under it. Then the momentum is decreased back to zero as it slides to a stop on the table. How far it slides on the table after the paper is pulled out from under it is a qualitative measure of the speed it acquired when the paper was sliding under it: the greater the distance it slid, the greater the speed it had acquired.

So, thinking in terms of impulse and momentum, **which force** acting through **what time interval** determined the maximum speed the object acquired before sliding to a stop on the table top?

Brief **Whole Class Discussion**

Express the friction force, which is the horizontal component of the force the paper exerts on the object, $F_{||\text{ paper on object}}$, as a constant (coefficient of friction μ) times the object's weight mg.[1]

The important point for the analysis here is that the ***friction force*** *is* ***only proportional to the weight of the object mg****. It does* not *depend on how fast you pull the paper.*

2. In your small group, make two complete *Momentum Charts* – including force diagrams – for one of the objects on your table (such as keys, small bottles, etc.) that is sitting on a piece of paper, which is pulled out from under it. One *Momentum Chart* should show the process when the paper is pulled quickly, and the second *Momentum Chart* should show the process when it is pulled less quickly. Consider the interval to be just before the pull until immediately after the paper is no longer under the object.

3. Make sure all forces in the force diagrams are appropriately labeled. Does how fast you pull the tablecloth affect the net force acting on the object? Identify exactly what things are exerting forces on your object!

4. Develop an explanation of these phenomena using the *Momentum Charts* you have prepared. Start by writing out an expression for impulse.

   Extra question: Try using different objects for your performance of the tablecloth trick. Do all the objects appear to move about the same distance for a given pull? How can you explain this?

5. If you know the initial and the final momentum, you know the impulse. In each of your scenarios, did you have the same or a different impulse? So what is the effect of changing Δt?

Be prepared to explain what Δt is and why it is important in a momentum problem!

Be ready to illustrate and give your explanations to the whole class.

Whole Class Discussion

[1] Note that the coefficient will depend on the surfaces of the two materials in contact, but for reasonably smooth surfaces like paper and metal, it generally has a value in the range of 0.5 to 1.0.

15 A.2 Automobile crash

Phenomenon: You are riding in a car that crashes into a solid wall. The car comes to a complete stop without bouncing back. The car has a mass of 1500 kg and has a speed of 30 $^{\mathrm{m}}/_{\mathrm{s}}$ before the crash (this is about 65 $^{\mathrm{mi}}/_{\mathrm{hr}}$).

FNT 15-1

(a) What is the car's initial momentum?

(b) What is your initial momentum? Recall that the weight of one kilogram is 2.2 lbs.

(c) Draw separate *Momentum Charts* for the car and the person. Treat both as open systems with a net impulse.

(d) What is the change in the momentum of the car?

(e) What is the change in your momentum?

FNT 15-2

(a) What is the net impulse that acts on the car to bring it to a stop?

(b) What is the net impulse that acts on you to bring you to a stop?

FNT 15-3

Refer to your *Momentum Charts* from FNT 15-1. Consider the following two situations.

I. You remain buckled into the seat and the seat remains attached to the center of the car.

II. You are not buckled into your seat and you fly through the windshield and hit the wall.

(a) Is the impulse the same in both cases?

(b) Are the forces acting on you the same?

(c) Using the words impulse, force, time, and momentum explain why one scenario is safer for you. Hint: Think about how the shape of the car changes when it hits the wall.

1. In your group, decide on two specific scenarios (FNT 15-3) and use these scenarios in Parts 2, 3, and 4 below. For these two scenarios, think about how much time passes between the time the force is first applied by the object and the time when you have zero momentum. In which scenario will this time difference Δt be larger and why?

2. On the board, put up complete *Momentum Charts* (with force diagrams and equations worked through to numerical values) for the car and for the person in each of the two scenarios you have chosen. Describe the scenario above each of the *Momentum Charts* for the person.

3. Use your *Momentum Charts* to explain why the net force acting on the person will not be the same in both scenarios.

4. For each of the scenarios that you described in FNT 15-3, estimate the magnitude of the average force that would have been acting on you to bring you to a stop.

   (a) You will have to determine the time during which the impulse acts for the different scenarios. To do this you must first decide on the initial and final momentum for the cases you are describing and over *what distance* the impulse acts.

   (b) Now you need to *calculate* the time duration of the impulse using your knowledge of how distance and time are related. If you assume that you slow down at a constant rate, then your average speed during this time is one-half your initial speed.

   (c) Make an estimation of the average force for the two situations. If your answers differ for the two situations, explain what factor is causing this difference.

5. If you know the initial and the final momentum, you know the impulse. In each of your scenarios did you have the same or a different impulse? What then is the effect of changing Δt (with the given constraints in the case of this automobile crash)?

   How does this compare to your response to Question 5 for the tablecloth trick? Which parameters are variable and which are constrained in the scenarios for each phenomenon?

Be prepared to explain what Δt is and why it is important in a momentum problem!

Whole Class Discussion

15 B Check for Understanding: The Egg Throw

> **Overview:** In this section, we'll attempt to explain a new phenomenon (another party trick, if you will) with the models we've accumulated, so far.

Consider the following two phenomena:

1. Egg thrown at velocity $\vec{v}$ at a wall that stands motionless with respect to the earth.
2. Egg thrown at velocity $\vec{v}$ at a bed sheet that is held by two people standing motionless with respect to the earth.

For both cases, consider the following interval:

beginning — the egg flying at top speed

end — $\vec{v} = 0$ in horizontal direction

Make sure everyone in your group fully understands the ideas behind each question or part below before going on to the next part.

1. In your small groups, predict if the egg will or will not break in either case.
2. Use your intuition to think about what factors determine whether the egg will break or not. List these factors on your group's board.
3. Use ideas, models, language, and/or equations from this class to explain why these factors are important.

Whole Class Discussion

Now, go do it! You will investigate the egg throw cases together as a class.

- Come up with a logical argument for why the egg does or does not break in either case.
- Write out your argument as you might on a quiz.
- Use physics ideas, models, words, and/or equations from class to explain your reasoning.

Whole Class Discussion

Discussion/Lab 16

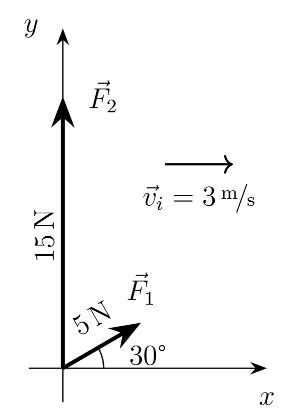
16 A Momentum and Change in Momentum in Two and Three Dimensions

Overview: So far, we have only concerned ourselves with momentum in one dimension – which means we've only looked at movement in the "forward/backward" directions. However, as you know, we can also move left and right, as well as up and down. In order to consider most real-life phenomena, we have to expand our ideas about momentum to those additional dimensions.

16 A.1 Impulse in Two Dimensions

FNT 16-1

Note: This is an extension of FNT 13-2. Two force vectors ($\vec{F}_1$ and $\vec{F}_2$, as shown to the right) act on a 2 kg object that has an initial velocity $\vec{v}_i$ of 3 m/s in the $+x$-direction.



(c) Use the x- and y-components of the force you found in (a) to determine the x- and y-components of impulse that would act on the 2 kg object if the forces were applied for a time interval of 0.50 s. Always include units with your answers. Start with the relationship of impulse and force. Find each component of impulse separately.

(d) Find separately, for each component, the change in velocity of the 2 kg object, due to the impulse from (c).

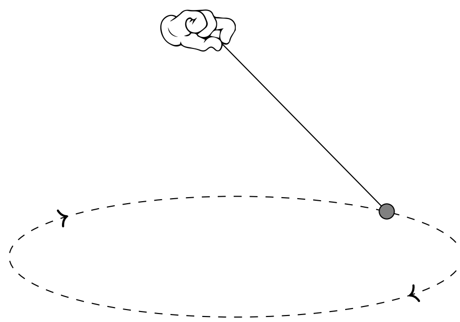
(e) Find the magnitude of the velocity of the object after the impulse has acted and the direction the velocity vector makes with the positive x-axis.

Compare with your group your responses to FNT 16-1. Come to a consensus and put your response on the board.

Whole Class Discussion

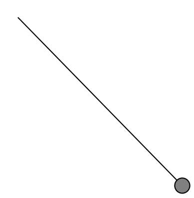
16 A.2 A Mass Swinging in a Horizontal Circle

Phenomenon: You are going to observe, talk about, and analyze a three dimensional problem. This will give you practice working with vectors and the concept of impulse in more than one dimension and practice determining the net force.



Hold the ball and swing it so it revolves in a horizontal circle as in Activity 12 A. Focus on a small section of the arc of the circle. Put your responses to the following parts on the board and be prepared to discuss them with the whole class.

1. If we treat the moving mass (just the ball) as our physical system, is there a net impulse on this physical system? How do you know? (Hint: Is the momentum changing?)
2. What must be the direction of the net impulse acting on the ball? How do you know this from the motion? Explicitly show the vectors $\vec{v}_i$, $\vec{v}_f$, and $\Delta\vec{v}$ on a diagram, drawn from above (looking down on the motion).
3. In which direction must the net force $\Sigma\vec{F}$ be? How do you know this from the motion?
4. Analyze the forces acting on the revolving ball. What objects exert forces on the ball? Remember, these could be contact forces or long-range forces. Now think about a side view of the revolving ball (see figure at right). Use what you know about the directions of these forces and the direction of $\Sigma\vec{F}$ to draw a force diagram for the ball in this side view. Be sure to label all forces with two subscripts and make their lengths appropriately scaled with respect to each other. Show the net force separately with a double arrow.



5. Write a few sentences explaining why the ball revolves in a horizontal circle in terms of momentum, impulse and net force.
6. Still treating our physical system as the moving ball, is mechanical energy conserved – that is, does the mechanical energy of the system remain constant? How do you know? You should answer this in terms of energy systems changing or not changing, as well as whether energy is transferred as work.
7. Is the net force more closely related to $\vec{v}_i$, $\vec{v}_f$, or $\Delta\vec{v}$?

Whole Class Discussion

16 A.3 Heavy Ball Swung in a Horizontal Circle – String Breaks

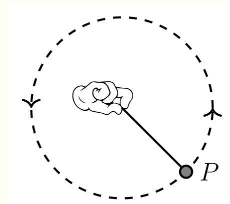
FNT 16-2

Turn to the **Momentum Conservation Model** summary page and do the following:

1. Use the model relationships found there to analyze each physical situation, and
2. Make logical arguments that would convince another physics student of your response to the following prompts.

Remember that a momentum conservation law requires you to compare a quantity at two times so you must always consider an initial time and a final time.

A heavy ball is attached to a string and swung in a circular path counter-clockwise in a horizontal plane as illustrated in the diagram to the right. At point P indicated in the diagram, the string suddenly breaks and the ball is released. If these events were observed from directly above, draw the path the ball takes immediately after the string breaks.

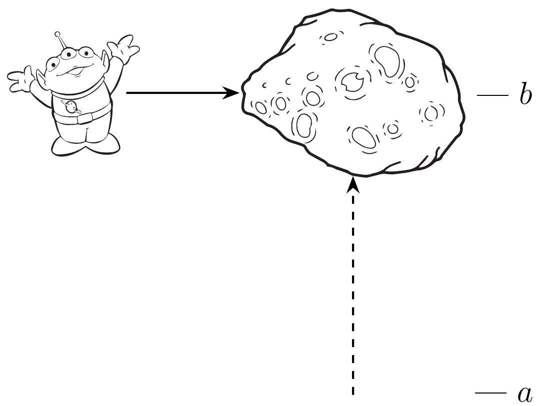


1. On the board, draw a physical picture of the situation. Make sure to include from 10° before the string breaks until after the string breaks.
2. On the board, make a *Momentum Chart* for this phenomenon, taking the initial point to be about 10° before the string breaks and the final point to be immediately before the string breaks.
3. On the board, make a second *Momentum Chart* for this phenomenon, taking the initial point to be immediately after the string breaks and the final point to be after the ball has gone a short distance.
4. Use your charts to explain the path of the heavy ball, before and after the string breaks.

Whole Class Discussion

16 A.4 An Asteroid Moving in a Straight Line Receives a Push at Right Angles

The diagram below depicts an asteroid, moving with a **constant velocity**, from point a to point b in space where there is no air resistance. When the asteroid reaches point b, an alien (who decides the asteroid is in her way) applies a force for an instant in the direction of the solid arrow.



FNT 16-3

Draw the path of the asteroid **after** the alien applies the force.

1. On the board, make a *Momentum Chart* for this phenomenon taking the initial point to be just before the push and the final point to be just after the push.
2. On the board, make a second *Momentum Chart* for this phenomenon taking the initial point to be the same point as the final point in (1) and the final point to be after the asteroid has traveled some distance.
3. Use your *Momentum Charts* to explain the path of the asteroid.
4. Considering your work above in (1) what is the predominant effect of the push?

Whole Class Discussion

FNT 16-4

Along the path you chose in FNT 16-3, how does the speed of the asteroid vary **after** receiving the "kick"? Is the velocity changing in direction? Is it increasing, decreasing or remaining the same in magnitude? Describe the motion.

5. On the board, use the appropriate *Momentum Chart* from (1) and (2) to justify your answer to this FNT.

Whole Class Discussion

FNT 16-5

Identify the forces acting on the asteroid, **after** the alien is finished applying the force.

6. On the board, use the appropriate *Momentum Chart* from (1) and (2) to make a scaled force diagram to answer this FNT.

Whole Class Discussion

16 A.5 A Ball Is Dropped and Bounces

FNT 16-6

When a rubber ball dropped from rest bounces off the floor, its direction of motion is reversed because

(a) energy of the ball is conserved;

(b) momentum of the ball is conserved;

(c) the floor exerts a force on the ball that stops its fall and then drives it upward;

(d) the floor is in the way and the ball has to keep moving;

(e) none of the above.

On the board, make a *Momentum Chart* (include a force diagram) that helps make sense of this situation and helps you to answer the question in this FNT.

Whole Class Discussion

16 B Check for Understanding: Momentum

Overview: In this section, we'll pull everything we've discussed about momentum together to apply our understanding to a few new situations.

16 B.1 Collisions in One and Two Dimensions

FNT 16-7

Two asteroids collide head-on and stick together. Before the collision, asteroid A (mass 1,000 kg) moved at 100 $^{\mathrm{m}}/_{\mathrm{s}}$, and asteroid B (mass 2,000 kg) moved at 80 $^{\mathrm{m}}/_{\mathrm{s}}$ in the opposite direction. Use momentum conservation (make a complete *Momentum Chart*) to find the velocity of the asteroids after the collision.

FNT 16-8

Two asteroids identical to those in FNT 16-7 collide at right angles and stick together. "Collide at right angles" means that their initial velocities were perpendicular to each other. You can assume that Asteroid A initially moved to the right and Asteroid B initially moved up.

Use the **Momentum Conservation Model** (make a complete *Momentum Chart*) to find the velocity (magnitude *and* direction, expressed as the angle with the initial velocity vector of Asteroid A) of the asteroids after the collision.

FNT 16-9

Determine the decrease in total kinetic energy $\Delta KE_{\mathrm{total}}$ of the two asteroids in FNT 16-7 and FNT 16-8 when they collide. If the average specific heat of the material composing the asteroids is assumed to be that of ice (2.05 $^{\mathrm{kJ}}/_{\mathrm{kg\cdot{}^\circ C}}$), by how much does the temperature of the asteroids rise as a result of the collision in each case?

1. In your group, compare your individually created *Momentum Charts* for FNT 16-7 and FNT 16-8. Note that you do not need to complete the entire chart to find the final velocity. That is, you don't need to find the change in momentum of each asteroid separately. However, finding the individual changes is a good opportunity to hone your skills with momentum conservation.

   Note: You may find it useful to choose an x-y coordinate system and fill in the chart for FNT 16-8 with x- and y-components.

2. On the board, put up complete *Momentum Charts* (with equations worked through to numerical values) for the 1-D linear collision and the 2-D collision in these two asteroid collision FNTs.

3. Compare your answers to FNT 16-9, come to a consensus, and put your response on the board.

4. **Extension:** Let's consider the asteroid collision as a "Many Body Problem:" Three asteroids collide and stick together. Create a possible *Momentum Chart*.

Whole Class Discussion

16 B.2 Horizontally Thrown and Dropped Balls

FNT 16-10

Throw an object (like a ball) horizontally and observe its motion. You are now going to analyze this motion using conservation of momentum. We assume there is no air friction on the ball. Consider the motion of the ball just after it has left your hand moving in a horizontal direction.

(a) Draw a force diagram for the ball. What direction is the net force?

(b) Complete the first two rows in the *Momentum Chart* below. The $\vec{p}_i$ of each successive step will be the $\vec{p}_f$ from the previous step. What is the direction of $\Delta\vec{p}$?

Open $\vec{p}$ system	$\vec{p}_i$	$\Delta\vec{p}$	$\vec{p}_f$
Ball Δt_1	$\longrightarrow$		$\searrow$
Ball Δt_2	$\searrow$		

(c) Add three more rows to the *Momentum Chart* (just show the three vectors $\vec{p}_i$, $\Delta\vec{p}$, and $\vec{p}_f$). Assume the same time interval for each change such that $\Delta\vec{p}$ will be $1/5$ the length of the initial momentum, $\vec{p}_i$.

(d) Why does $\Delta\vec{p}$ stay constant for each step?

(e) After you have carefully constructed a series of final momenta, use them and what you know about the relationship of the direction of momentum to the path of an object to construct a sketch of the path of the ball.

FNT 16-11

Repeat what you did in FNT 16-10, but this time do two separate *Momentum Charts*, one for the horizontal and one for the vertical components of the motion. Describe in words how the motion changes in the two directions. Compare your two sketches from part (e) in FNT 16-10 and FNT 16-11. Are the paths you constructed the same?

FNT 16-12

(a) Wrap up what you have done in FNT 16-10 and FNT 16-11 by explaining, in as few words as possible, why the ball moves along the path you sketched.

(b) For any object to be moving in a curved path what is necessary about the relationships between $\vec{p}_i$, $\Delta\vec{p}$, $\vec{p}_f$, and $\sum F$?

(c) If $\Delta\vec{p}$ were always perpendicular to $\vec{p}_i$ what type of path would this result in?

1. In your group, compare your individually created *Momentum Charts* for FNT 16-10 and FNT 16-11 and negotiate a consensus for each one.

2. Put your consensus charts on the board. It is simplest to put one header row, with labels $\vec{p}_i$, $\Delta\vec{p}$, and $\vec{p}_f$, and then start a new row in your chart for each time step.

3. Discuss and be prepared to share with the class your response to the prompt, "Describe in words how the motion changes in the two directions." Compare your diagrams from Part (e) between the two FNTs. Are the paths you constructed the same? Come to a consensus in your small group on your response to Question (a) in FNT 16-12 and be prepared to share it with the whole class.

4. Come to a consensus on Parts (b) and (c) in FNT 16-12 and be prepared to share them with the whole class.

Whole Class Discussion

Unit IV

Forces and Motion: The Newtonian Force Model

Discussion/Lab 17

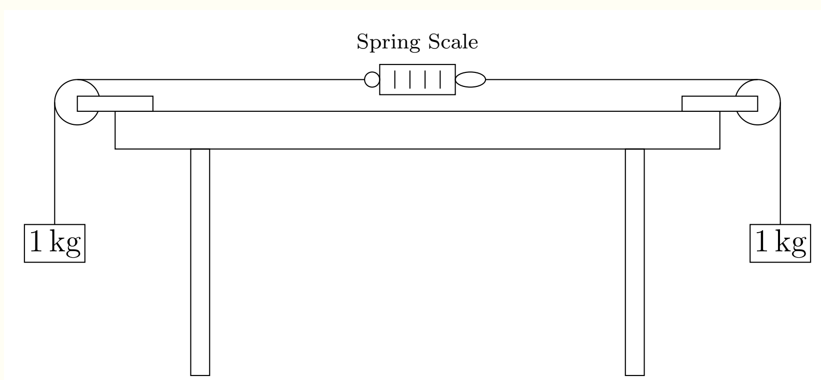
17 A Newton's Laws and Momentum Conservation

Overview: Over the past few weeks, we've dealt a lot with forces, and we know much more about motion in general. We even know that forces cause changes in motion. Sir Isaac Newton, a physicist of the 17th/18th century, formulated three famous laws that powerfully express the relationship between forces and motion. In this next section, we'll discuss these so-called *Newton's Laws*.

17 A.1 Getting started with Newton's Laws

FNT 17-1

A puzzle to think about: Two weights of mass 1 kg hang from strings which go over pulleys (see illustration below). The strings are attached to the two ends of a spring scale which reads the force. Does the scale read 0 N, 9.8 N, or 19.6 N? Why?



Hint: Draw a *Force Diagram* for the scale in this situation and then draw a *Force Diagram* for the scale when it is being used to weigh a hanging object with mass 1 kg.

Whole Class Discussion

17 A.2 Newton's Laws and the Momentum Conservation Model

Please read before moving on:

Newton's Three Laws of Motion

I. Without a net force, there can be no change in the velocity of an object:[a]

$$\boxed{\text{If } \sum \vec{F} = 0, \text{ then } \Delta\vec{v} = 0.}$$

II. An object will be accelerated if there is a net force acting upon it:

$$\boxed{\sum \vec{F} = m\vec{a}}$$

III. If object A exerts a force on object B, object B *simultaneously* exerts an equal and opposite force on object A:[b]

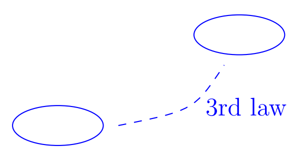
$$\boxed{\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}}$$

[a] This can be expressed as "An object in motion remains in motion in a straight line forever unless acted on by an external force;" and "An object at rest remains at rest forever unless acted on by an external force."

[b] You may have heard of this as "For every action, there is an equal and opposite reaction."

1. Come up with a physical scenario that illustrates each of Newton's three laws. Describe each law in both sentence form and with a concise mathematical expression, specific to your example. Be careful and precise with notation in the algebraic expressions.

2. Make a *Force Diagram* for each object in each of the following situations. For each situation, circle and connect the forces that are in tandem as third law pairs, as shown.



   (a) Refer to the astronaut example from 14 C. Recreate a *Momentum Chart* and follow the instructions for the force diagram above. Explain how each of Newton's Laws can be identified in the *Momentum Charts* and *Force Diagrams*. Suppose we wanted the rocket to move at a constant speed, is a net force necessary to make this happen?

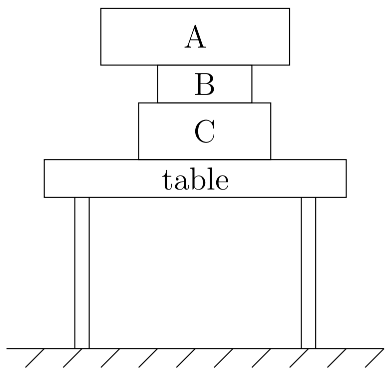
   (b) Refer to the boat example of FNT 14-3. Recreate a *Momentum Chart* and follow the instructions for the force diagram above. Explain how each of Newton's Laws can be identified in the *Momentum Charts* and *Force Diagrams*.

Whole Class Discussion

17 A.3 Use Newton's Three Laws applied to a system of three objects

***Static* Case: All objects sitting still**

1. Refer to the picture below of three books sitting on a table that is standing on a floor. Make a separate *Force Diagram* for each of the four objects: Books A, B, and C, and the table. The masses of the books are 0.8 kg, 0.2 kg, and 0.5 kg for Books A, B, and C, respectively and the mass of the table is 30 kg.



2. Identify all 3rd law pairs of forces appearing in your *Force Diagrams* by circling the two forces and connecting as shown in the diagram on the previous page.

3. Use Newton's 1st and 3rd laws to numerically determine all of the forces acting on the books and the table. If necessary, redraw your *Force Diagrams* so they are more to scale.

***Dynamic* Cases: Objects moving horizontally and vertically**

4. If the books and table together are sliding across the room each at the same constant speed, how would your *Force Diagrams* change? Show precisely how it would change or explain why it would not.

5. Now imagine the table is sitting in an elevator that begins to accelerate upward with an acceleration equal to $2\,{}^{\mathrm{m}}/{}_{\mathrm{s}^2}$.

   (a) Make a new *Force Diagram* for each book and determine the forces acting on it.

   (b) Create a *Momentum Chart* for at least two of the objects.

   (c) Do any of the objects experience a net force? If so, give the magnitude and direction of each.

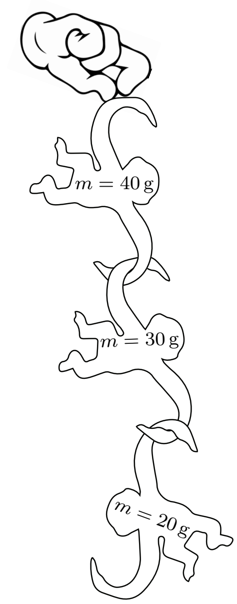
   (d) Is it correct to say a force always equals $m\vec{a}$?

Whole Class Discussion

17 A.4 Applications of Newton's Laws

FNT 17-2

Your little brother is playing with monkeys in a barrel. The mass of each monkey is indicated in the illustration on the right. Note that your brother is holding the monkeys still and his hand weighs 50 N.



(a) Make a separate *Force Diagram* for each of the four objects, the hand, top monkey, middle monkey, bottom monkey.

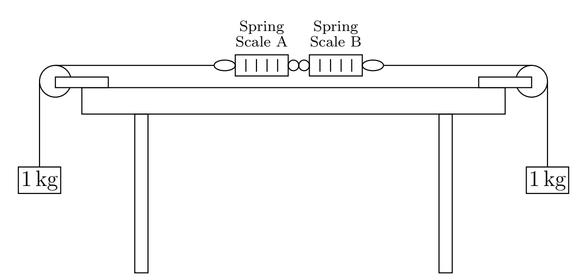
(b) Identify all 3rd law pairs of forces appearing in your *Force Diagrams* by circling the two forces and connecting them with a line.

(c) Use Newton's 1st and 3rd laws to numerically determine all of the forces acting on the monkeys and the hand. If necessary, redraw your *Force Diagrams*, so they are more to scale.

Whole Class Discussion

FNT 17-3

We've already investigated this problem with one spring scale in FNT 17-1. Now, imagine you have two spring scales, A and B, connected at the end of the scale that doesn't move. The end that moves of each spring scale (where you take readings from) is attached to a string that goes over a pulley and connects to a 1 kg mass for both spring scales A and B.



(a) State what you think *each* spring scale will read in this situation.

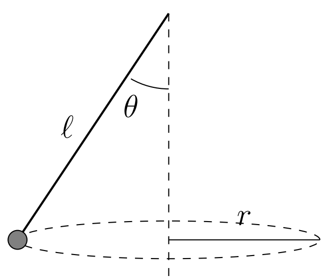
(b) Construct a logical argument that explains why the spring scales read what you reported in Question (a). You should treat this as a quiz/test question and therefore use complete sentences, reference any models you think will strengthen your argument, and provide evidence to support your claim.

Whole Class Discussion

17 B Ball Swinging in a Horizontal Circle

Overview: Remember the Mass Swinging in a Horizontal Circle in Activity 16 A.2? We already started analyzing the forces acting on the ball in that activity. Now that we have another model in our tool bag, we return to this scenario to continue our analysis of the forces on the ball.

Phenomenon: Ball on a string, swinging in a horizontal circle at constant speed. The experiment shows that as the ball moves faster, θ becomes larger, and the tension in the string increases.



What model/approach would you use?

Suppose you are interested in how the tension in the string depends on the angle θ that the string makes with the vertical direction. Which approach do you think would help you make progress with this question: energy conservation, momentum conservation, or Newton's 2nd law?

Discuss in your group what information you specifically need, and which models/approaches can give you that information. Start by thinking about the specific motion and what is required to produce this motion. (Note that rotational velocity – how fast the ball swings around the circle – is assumed constant. What does this imply?)

Use the model

Carry out the analysis using whichever approach/model you have decided on.

1. Put a properly labeled *Force Diagram* for the ball on the board.

2. Use the *Force Diagram* to develop **two** mathematical relationships (one for the vertical direction and one for the horizontal direction). You will need to figure out (or remember) the direction of the acceleration of an object traveling in a circle at constant speed. The magnitude of this acceleration is $a_{\text{centripetal}} = \frac{{v_{\text{tangential}}}^2}{r}$.
(Notice $v_{\text{tangential}}$ refers to the *tangential* speed of the ball).

3. Does the force of the string on the ball equal the force of the Earth on the ball? If not, what does?

Whole Class Discussion

4. Illustrate using vectors what happens to the tension in the string ($\vec{F}_{\text{string on ball}}$) when the tangential speed of the ball increases. Think about how the two relationships in (1) can both be satisfied.

5. Develop a short explanation of why the angle of the string (from the vertical direction) changes when the tangential velocity is increased significantly. Check this out with the real ball and string.

Whole Class Discussion

Appendix

SI Units and Conversions

Table A.1: Common SI Units Relating to Energy

SI Unit	Construct	Abbreviation	Expressed in base units
Joule	energy	J	$^{kg \cdot m^2}/_{s^2} = N \cdot m$
Watt	power	W	$^{J}/_{s} = {}^{kg \cdot m^2}/_{s^3}$
Newton	force	N	$^{kg \cdot m}/_{s^2}$
Pascal	pressure	Pa	$^{J}/_{m^3} = {}^{N}/_{m^2}$

Table A.2: Some common energy units and conversions to SI

$1\,\mathrm{kWh} = 3.6\,\mathrm{MJ}$
$1\,\mathrm{erg} = 10^{-7}\,\mathrm{J}$
$1\,\mathrm{cal} = 4.184\,\mathrm{J}$
1 food Calorie (big “C” calorie) = $1\,\mathrm{kcal} = 4184\,\mathrm{J}$
$1\,\mathrm{ft \cdot lb} = 1.36\,\mathrm{N \cdot m}$
$1\,\mathrm{eV} = 1.602 \times 10^{-19}\,\mathrm{J}$
$1\,\mathrm{BTU} = 778\,\mathrm{ft \cdot lb} = 252\,\mathrm{cal} = 1054\,\mathrm{J}$

Table A.3: International System (SI) of Units (Metric)

Meter	Distance/Length	m
Centimeter	Distance/Length	cm
Kilogram	mass	kg
Gram	mass	g
Liter	Volume	L
Second	Time	s
Celsius	Temperature	°C
Kelvin	Temperature	K
Mole	Amount of Substance	mol

Table A.4: SI Unit Conversions

Derived Quantity	SI Units in terms of base units	Alternative name for SI Unit
Area	m^2	—
Volume	m^3	—
Density	$^{kg}/_{m^3}$	—
Speed/Velocity	$^{m}/_{s}$	—
Acceleration	$\frac{m/s}{s} = {}^{m}/_{s^2}$	—
Momentum	$^{kg \cdot m}/_{s}$	—
Force	$^{kg \cdot m}/_{s^2}$	Newton, N
Pressure	$^{kg}/_{m \cdot s^2}$	Pascal, Pa
Work, Energy	$^{kg \cdot m^2}/_{s^2}$	Joule, J
Power	$^{kg \cdot m^2}/_{s^3}$	Watt, W

Table A.5: Conversion Factors

Length	Mass/Weight*	Temperature	Volume
$1\,\text{in} = 2.54\,\text{cm}$	$1\,\text{kg} = 2.20\,\text{lbs}$	$\langle °\text{F}\rangle = \frac{9}{5}\langle °\text{C}\rangle + 32$	$1\,\text{L} = 0.26\,\text{gal}$
$1\,\text{cm} = 0.39\,\text{in}$	$1\,\text{lb} = 0.45\,\text{kg}$	$\langle °\text{F}\rangle = \frac{9}{5}\langle K\rangle - 459.67$	$1\,\text{L} = 33.81\,\text{fl oz}$
$1\,\text{m} = 39.37\,\text{in} = 3.28\,\text{ft}$	$1\,\text{g} = 0.035\,\text{oz}$	$\langle °\text{C}\rangle = \frac{5}{9}(\langle °\text{F}\rangle - 32)$	$1\,\text{gal} = 3.79\,\text{L}$
$1\,\text{ft} = 0.31\,\text{m}$	$1\,\text{oz} = 28.35\,\text{g}$	$\langle °\text{C}\rangle = \langle \text{K}\rangle - 273.15$	$1\,\text{gal} = 128\,\text{fl oz}$
$1\,\text{km} = 0.62\,\text{mi}$		$\langle \text{K}\rangle = \langle °\text{C}\rangle + 273.15$	$1\,\text{mL} = 1\,\text{cc} = 1\,\text{cm}^3$
$1\,\text{mi} = 1.61\,\text{km}$		$\langle \text{K}\rangle = \frac{5}{9}(\langle °\text{F}\rangle + 459.67)$	$1\,\text{fl oz} = 29.57\,\text{mL}$
			$1\,\text{mL} = 0.034\,\text{fl oz}$

* Kilograms and grams are units of mass while pounds and ounces are units of weight, which is mass under the influence of gravity. This makes them units of force. These conversions might work on the surface of the earth, but not necessarily everywhere else in the universe.

Table A.6: Table of Melting and Boiling Points, Heats of Melting and Vaporization, and Specific Heats of Some Common Substances (at a constant pressure of one atmosphere)

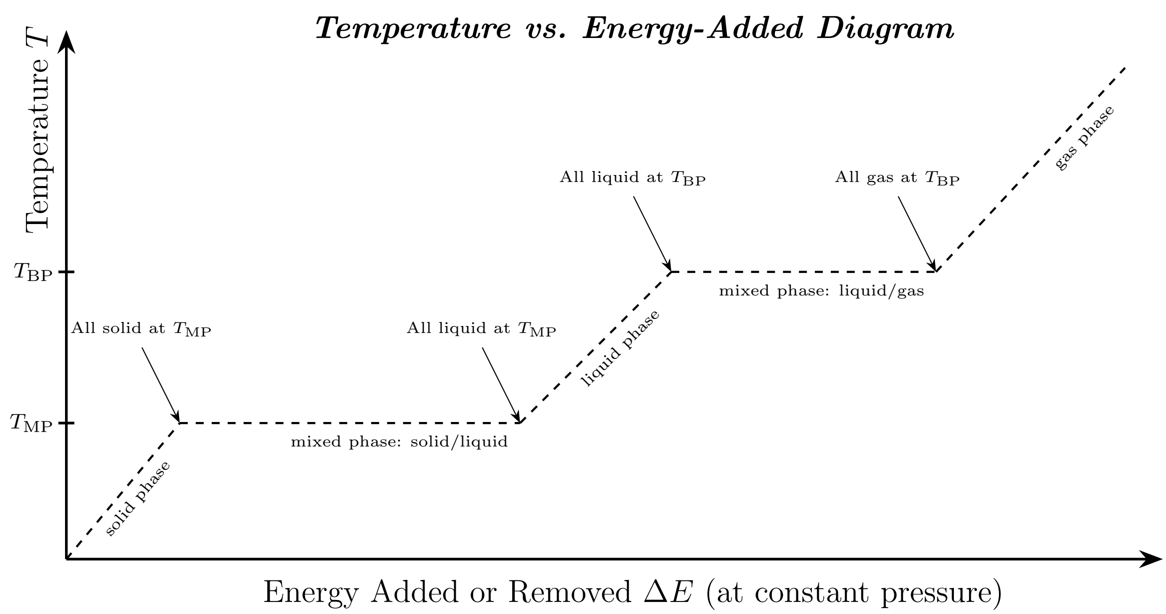
Substance	Formula	Melting/Boiling Points		Heat of Melting		Heat of Vaporization		Specific Heat, C_P	
		(K)	(K)	(kJ/kg)	(kJ/mol)	(kJ/kg)	(kJ/mol)	(J/K·mol)	(kJ/kg·K)
Aluminum	Al(s)	933	2600	389.18	10.5	10790	291	24.3	0.900
Bismuth	Bi(s)	544	1693	52.2	10.9	722.5	151	25.7	0.123
Copper	Cu(s)	1356	2839	205	13	4726	300.3	24.5	0.386
Gold	Au(s)	1336	3081	62.8	1.24	1701	33.5	25.4	0.126
Ice (-10 °C)	H_2O(s)							36.9	2.05
Water	H_2O(l)	273	373	333.5	6.01	2257	40.7	75.2	4.18
Steam	H_2O(g)							33.6	1.866
Lead	Pb(s)	600	2023	24.7	5.12	858	177.78	26.4	0.128
Sodium	Na(s)							28.2	1.23
Sodium	Na(l)	370.82	1154.4	114.8	2.64	4306	99	32.7	1.42
Silver	Ag(s)	1234.93	2436	88.2	9.5	2323	250.6	25.4	0.233
Mercury	Hg(s)							28.3	0.141
Mercury	Hg(l)	234	5630	11.3	2.3	5296	59.1	28	0.140
Mercury	Hg(g)							20.8	0.103
Tungsten	W(s)	3410	5900	184.1	33.86	4812	884.85	24.6	0.134
Nitrogen	N_2(g)	63.14	77	25.7	0.720	199.1	5.58	29.04	1.04
Oxygen	O_2(g)	54.39	90.18	13.9	0.444	213.1	6.82	29.16	0.911
Iron	Fe(s)	1535	3000					25.1	0.449

Table A.7: Metric Prefixes

Prefix	Symbol	10^n	Decimal	English word
yotta	Y	10^{24}	1 000 000 000 000 000 000 000 000	septillion
zetta	Z	10^{21}	1 000 000 000 000 000 000 000	sextillion
exa	E	10^{18}	1 000 000 000 000 000 000	quintillion
peta	P	10^{15}	1 000 000 000 000 000	quadrillion
tera	T	10^{12}	1 000 000 000 000	trillion
giga	G	10^{9}	1 000 000 000	billion
mega	M	10^{6}	1 000 000	million
kilo	k	10^{3}	1 000	thousand
hecto	h	10^{2}	100	hundred
deca	da	10^{1}	10	ten
—	—	10^{0}	1	one
deci	d	10^{-1}	0.1	tenth
centi	c	10^{-2}	0.01	hundredth
milli	m	10^{-3}	0.001	thousandth
micro	μ	10^{-6}	0.000 001	millionth
nano	n	10^{-9}	0.000 000 001	billionth
pico	p	10^{-12}	0.000 000 000 001	trillionth
femto	f	10^{-15}	0.000 000 000 000 001	quadrillionth
atto	a	10^{-18}	0.000 000 000 000 000 001	quintillionth
zepto	z	10^{-21}	0.000 000 000 000 000 000 001	sextillionth
yocto	y	10^{-24}	0.000 000 000 000 000 000 000 001	septillionth

Three-Phase Model of Matter

Graphical Representation



Algebraic Representations

Change in temperature ΔT of an amount m of a substance with specific heat C_p when energy is *added* or *removed* as *heat* Q:

$$\Delta T = \frac{Q}{C_p \cdot m}$$

Amount Δm of a substance with heat of melting/vaporization/sublimation ΔH that changes phase when energy is *added* or *removed* as *heat* Q:

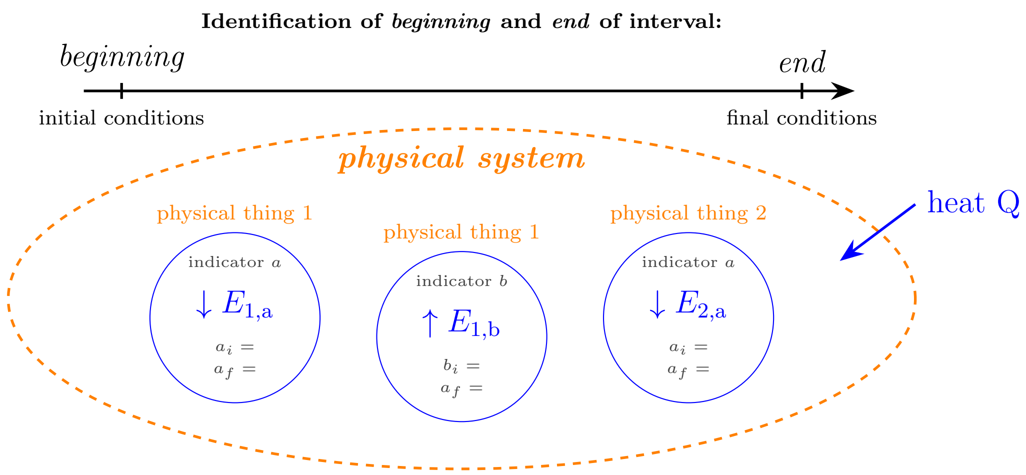
$$|\Delta m| = \frac{Q}{H_p}$$

Constructs	**Relationships**
Pure Substances Three *Phases*: *solid*, *liquid*, *gas*	1. *Pure substances* exist in one of three phases, depending on the *temperature* and *pressure*: solid, liquid, and gas. *Non*-pure substances, e.g., solutions and composites, require more complex models for analysis.
Temperature Energy *transferred* as *Heat* or *Work*	2. To change either the *temperature* or the *phase* of a substance, *energy* must be *added* or *removed*. Often, this energy is *transferred* to or from the substance as *heat* Q but can also be transferred as *work* W.
Phase Change Temperatures: T_{MP}, T_{BP}, T_{SP} *Melting*, *Boiling*, *Sublimation*	3. At constant pressure, changes of *phase* (solid $\Leftrightarrow$ liquid and liquid $\Leftrightarrow$ gas, or at some values of pressure, solid $\Leftrightarrow$ gas) occur at specific temperatures: the *phase change temperatures* T_{MP}, T_{BP}, and T_{SP}. They have particular values for each pure substance. The values of these temperatures are the same when "going through" the respective phase change in "both directions." However, phase change temperatures are *dependent on the pressure*.
Heat of *Melting* Heat of *Vaporization* (Boiling) Heat of *Sublimation*	The amount of *energy added* or *removed* during a particular *phase change* (written as ΔH, indicating constant pressure) is unique to each substance and has been measured and tabulated for most substances.
Thermal Equilibrium Mixed phase	If the substance is in *thermal equilibrium* (i.e., if the entire substance is at the same temperature) at the *phase change temperature*, both phases will *remain* at the phase change temperature as the phase change occurs. *Mixed phases* can exist in thermal equilibrium *only* when the temperature has the value of the phase-change temperature.
	4. Changes of *temperature* of a substance occur when energy is added or removed while the substance is *not* at a phase-change temperature.
Heat Capacity Specific Heat	When the energy added is in the form of heat, the change in temperature ΔT is related to the amount of energy added by a property of the substance called *heat capacity* C. The *specific heat* C_p (heat capacity per unit mass) has a particular value for each substance. This value has been measured and tabulated for most substances (see Table A.6 in Appendix A).

Energy-Interaction Model

Graphical Representation: The *Energy-System Diagram*

Generic Example involving two physical systems, three energy systems, and with energy added to the system as heat. The **physical system** consists of *physical thing 1* and *physical thing 2*.



$$\overset{(-)}{\Delta E_{1,\mathrm{a}}} + \overset{(+)}{\Delta E_{1,\mathrm{b}}} + \overset{(-)}{\Delta E_{2,\mathrm{a}}} = \overset{(+)}{Q}$$

Algebraic Representations

Open system: $$\Delta E_{\text{total}} = \sum \Delta E_i = \Delta E_1 + \Delta E_2 + \Delta E_3 + \ldots = Q + W$$

Closed system: $$\Delta E_{\text{total}} = \sum \Delta E_i = \Delta E_1 + \Delta E_2 + \Delta E_3 + \ldots = 0$$

Power: $$P = \frac{\Delta E}{\Delta t}$$

Constructs	**Relationships**
Energy • Conservation of Energy • Internal Energy (U) • Mechanical Energy • Energy Transfers (Q, W)	1. The heart of the **Energy-Interaction Model** is *energy conservation*, one of a few powerful *conservation principles* used throughout science. One way of expressing a conservation principle is that for an isolated physical system, there are certain physical properties that do not change during an interaction or process. A process or interaction is determined by explicitly expressing the beginning and ending times of the interval characterizing the process.
Physical System • closed • open (with respect to energy transfers) Energy Transfer • Heat, Q • Work, W Energy System • Indicators • Change in Energy (ΔE)	2. The *total energy* of every *physical system* can be expressed as a *sum of the energies* of separately identifiable *energy systems*. This division of the total energy into energy systems can be carried out in multiple ways. The energy associated with a particular energy system can be expressed in terms of *observable* and *measurable properties* of the physical system that we call *indicators*. The *change* in energy of each energy system can be determined from the *observed change in the indicator* that occurs from the beginning to the end of the interval characterizing the interaction or process.
Process or Interaction • Interval • initial (start) Time of Interval • final (end) Time of Interval State of a Physical System • Temperature • Phase Energy Systems related to Thermal & Chemical Processes • E_{thermal} • E_{bond}	3. Conservation of energy in a ***closed*** *physical system* (isolated with respect to energy transfers from other physical systems): The **total energy** of the physical system must remain **constant** during the interaction or process. When internal interactions occur, this conservation principle can be expressed in terms of changes within the energy systems of the physical system: The **changes of the energies of *all* energy systems** associated with the *physical system* in question must **sum to zero**.
Energy Systems related to Mechanical Processes • $PE_{\text{gravitational}}$ • PE_{elastic} • $PE_{\text{mass-spring}}$ • $KE_{\text{translational}}$ • $KE_{\text{rotational}}$	4. Conservation of energy in an ***open*** *physical system*: During an interaction or process during which *energy is added or removed* from the physical system as heat or work, the **changes in energy of *all* energy systems** associated with the physical system in question must **sum to the net energy added (or removed)** as heat and/or work. Equivalently, the **change in the *total energy*** of that physical system must **equal the net energy added (or removed)** as heat and/or work.

Steps Involved in Using the *Energy-System Diagram*

Prior to writing anything down:

1. **Tell a story about what happened!** Be sure you are clear about what the physical phenomenon or process is.

Go back and forth through Steps 2-6 until your *Energy-System Diagram* is complete:

2. **What is the boundary of the physical system you are modeling?** What is inside and what is outside?

3. **Is the physical system in your particular model open or closed?** If open, are there any energy transfers (e.g., heat or work)?

4. **What is the extent of the process or interaction?** What determines the beginning and end?

5. **What energy systems do you include in your diagram?** Which indicators are changing?

6. **What are the values of the indicators at the times corresponding to the ends of the time interval you chose in step 4?**

Only *after the diagram is complete*, move on to Step 7:

7. **Write down an equation expressing energy conservation for your particular *Energy-System Diagram*, in terms of the ΔE's and any Q or W.** Each term in your conservation of energy equation must correspond to an energy system in your diagram.

Energy Systems and their Indicators

Energy Systems related to *Thermal & Chemical Processes*

temperature T
of substance

E_{thermal}

$\Delta T = T_f - T_i$

$$\Delta E_{\text{thermal}} = mc_p\Delta T$$

m: mass of substance
c_p: Specific Heat of substance

mass m
of higher phase

E_{bond}

$\Delta m = m_f - m_i$

$$\Delta E_{\text{bond}} = \pm|\Delta m \Delta H|$$

ΔH: Heat of ⟨Phase Change⟩

Energy Systems related to *Mechanical Processes*

height y
above y=0

$PE_{\text{gravitational}}$

$\Delta y = y_f - y_i$

$$\Delta PE_{\text{gravitational}} = mg\Delta y$$

m: mass of object
g: gravitational constant, 9.8 m/s

distance x
from equilibrium

PE_{elastic}

$\Delta x^2 = x_f^2 - x_i^2$

$$\Delta PE_{\text{elastic}} = \frac{1}{2}k\Delta x^2$$

k: spring constant

distance x
from equilibrium

$PE_{\text{spring-mass}}$

$\Delta x^2 = x_f^2 - x_i^2$

$$\Delta PE_{\text{spring-mass}} = \frac{1}{2}k\Delta x^2$$

k: spring constant

Note that the mass of the object hanging on the spring determines the equilibrium position!

translational
speed v

$KE_{\text{translation}}$

$\Delta v^2 = v_f^2 - v_i^2$

$$\Delta KE_{\text{translation}} = \frac{1}{2}m\Delta v^2$$

m: mass of object

rotational/angular
speed ω

KE_{rotation}

$\Delta \omega^2 = \omega_f^2 - \omega_i^2$

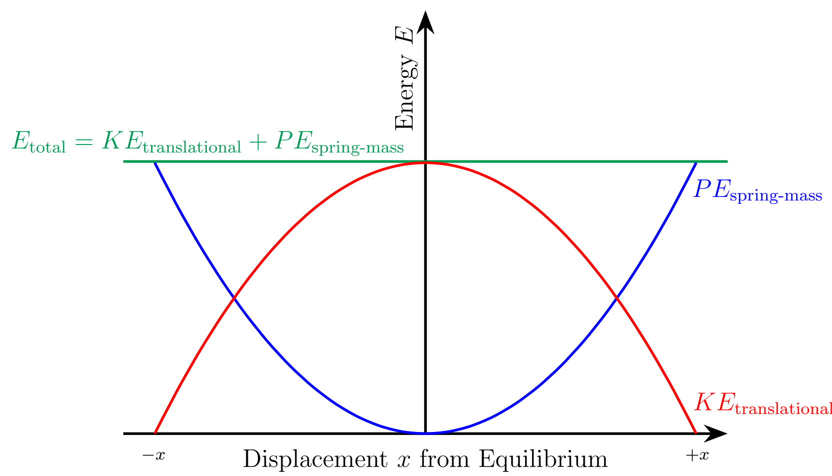
$$\Delta KE_{\text{rotation}} = \frac{1}{2}I\Delta \omega^2$$

I: Moment of Inertia of object

Energy in Mechanical Systems

Graphical Representation: The *Point-to-Point Diagram*

Example: A mass hanging on a spring. The total energy of the system consists of translational kinetic energy and spring-mass potential energy.



While the horizontal axis in the diagram shows (in this example) the displacement x from the equilibrium position of the spring-mass system, each point along the horizontal axis also represents a certain point in time. If you add up the values of $PE_{\text{spring-mass}}$ and $KE_{\text{translational}}$ at this particular point in time, you get the total energy E_{total} for this particular point in time. In a **closed physical system** (no work done on or by the system), the total energy E_{total} is *always conserved*, which means it *does not change over time.*

Algebraic Representations

Total Energy of the System:	$E_{\text{total}} = \sum E_i = E_1 + E_2 + E_3 + \dots$
Gravitational Potential Energy:	$PE_{\text{gravitational}} = mgy$
Elastic Potential Energy:	$PE_{\text{elastic}} = \frac{1}{2}kx^2$
Spring-Mass Potential Energy:	$PE_{\text{spring-mass}} = \frac{1}{2}kx^2$
Translational Kinetic Energy:	$KE_{\text{translational}} = \frac{1}{2}mv^2$
Rotational Kinetic Energy:	$KE_{\text{rotational}} = \frac{1}{2}I\omega^2$

Momentum Conservation Model

Graphical Representation: The *Momentum Chart*

Closed System
Typically used for collisions/interactions involving two or more objects.

Closed $\vec{p}$ system	$\vec{p}_i$	$\Delta\vec{p}$	$\vec{p}_f$
Object 1			
Object 2			
Total System		0	

For total system: $\Delta\vec{p} = 0$
For each object: $\vec{p}_i + \Delta\vec{p} = \vec{p}_f$

Open System
Typically used when the phenomenon involves a net impulse acting on the system.

Open $\vec{p}$ system	$\vec{p}_i$	$\Delta\vec{p}$	$\vec{p}_f$
Total System			

For total system: $\Delta\vec{p} = \Sigma\vec{I}$
$\vec{p}_i + \Delta\vec{p} = \vec{p}_f$

To identify any forces that cause the object's change in momentum (change in motion), it helps to draw a force diagram for each object in the *Momentum Chart*.

Algebraic Representations

Definition of Momentum $\vec{p}$: $\vec{p} = m \cdot \vec{v}$

Net Impulse: $\sum \vec{I} = \sum \vec{F}_{\text{avg. ext.}} \cdot \Delta t$

Conservation of Momentum: $\sum \vec{I} = \sum \vec{F}_{\text{avg. ext.}} \cdot \Delta t = \vec{p}_f - \vec{p}_i = \Delta\vec{p}_{\text{system}} = 0$

Momentum of a Particle System: $\vec{p}_{\text{system}} = \sum_j \vec{p}_j$

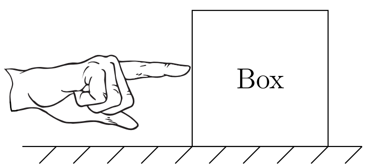
Constructs	**Relationships**
Momentum ($\vec{p}$)	1. *Momentum* is a property of a moving object. Its quantity is defined as the product of the object's mass and velocity
Position ($\vec{r}$), Displacement ($\Delta\vec{r}$), and Velocity ($\vec{v}$) Vectors	
Net Impulse ($\sum \vec{I}$) • by the net force • by a single force	2. The *total* or *net impulse* acting on an object is defined as the net force on the object times the amount of time during which the net force is acting on the object.
	3. The *change in momentum* is equal to the net impulse and is independent of the coordinate system used to express $\vec{F}$, $\sum \vec{I}$, and $\vec{p}$.
Conservation of Momentum	4. *Conservation of Momentum:* If the net external impulse acting on a physical system is zero, then there is no change in the total linear momentum of that system; otherwise, the change in momentum is equal to the net external impulse.
Momentum of a System of Particles	5. The momentum of a system of particles is the vector sum of the individual momenta.
Collisions • Energy Conservation • Momentum Conservation	6. In a collision, the momentum of the system of objects (particles) remains constant if the external impulses are negligible. This is true whether energy is conserved during the collision or not.

Newtonian Force Model

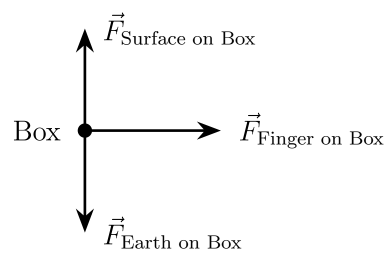
Graphical Representation: The *Force Diagram*

Example: A finger pushing a box over a surface at a constant speed.

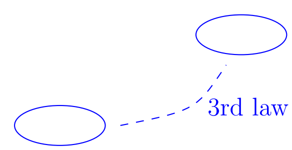
Picture of the situation



Force Diagram



To identify which forces are part of a 3rd law pair, draw a *Force Diagram* for *each* of the two interacting objects, circle the corresponding arrows, and connect the circles with a line:



Algebraic Representations:

Newton's Three Laws of Motion

I. Without a net force, there can be no change in the velocity of an object:

$$\boxed{\text{If } \sum \vec{F} = 0, \text{ then } \Delta\vec{v} = 0.}$$

II. An object will be accelerated if there is a net force acting upon it:

$$\boxed{\sum \vec{F} = m\vec{a}}$$

III. If object A exerts a force on object B, object B *simultaneously* exerts an equal and opposite force on object A:

$$\boxed{\vec{F}_{\text{A on B}} = -\vec{F}_{\text{B on A}}}$$

Relation of a net force to momentum and work:

Net Impulse imparted on a system: $$\sum \vec{I} = \sum \vec{F}_{\text{avg. ext.}} \cdot \Delta t = \Delta \vec{p}$$

Work done on a system: $$W = \sum F_{\text{avg. ext.}} \cdot d_{\text{parallel to net force}}$$