

Statistics

Adejumo Ridwan Suleiman

June 25 2022

Introduction to Statistics and Statistical Measures

What is Statistics?

- ▶ Statistics is the collection, organizing and analysing of data.

Is Data Science Statistics in Disguise?

- ▶ Unlike Statistics, Data Science is an interdisciplinary field consisting of Mathematics, Statistics, Computer Science and Domain Knowledge.

Types of Data

- ▶ Data can be classified into two types
 - ▶ Based on Measurement scale
 - ▶ Based on Time Period

Based on Measurement Scale

- ▶ Qualitative Data
 - ▶ Nominal Data e.g sex
 - ▶ Ordinal Data e.g temperature level; High, Medium and Low
- ▶ Quantitative Data
 - ▶ Ratio e.g weight
 - ▶ Interval e.g temperature in degree celsius

Based on Time Period

- ▶ Cross-Sectional Data e.g number of viewers for different youtube genres in the year 2021
- ▶ Time Series Data e.g number of viewers for Sport channels on youtube from the year 2014-Date.

Types of Data

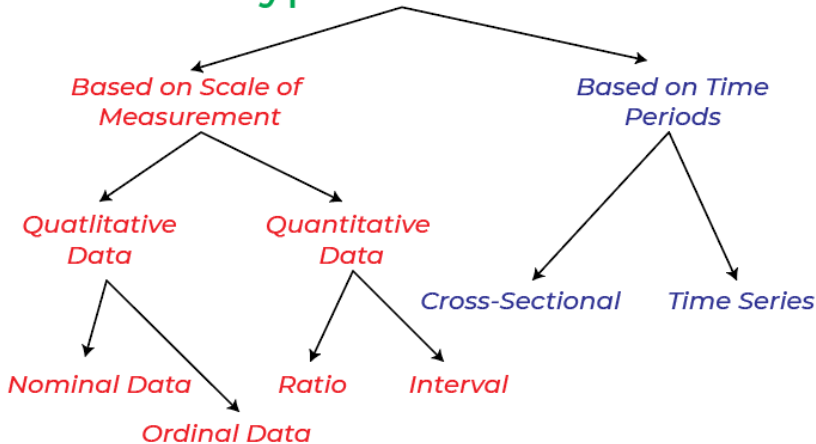


Figure 1: Types of Data

Rectangular or Structured Data

	carat	cut	color	clarity	depth	table	price	x	y	z
1	0.23	Ideal	E	SI2	61.5	55.0	326	3.95	3.98	2.43
2	0.21	Premium	E	SI1	59.8	61.0	326	3.89	3.84	2.31
3	0.23	Good	E	VS1	56.9	65.0	327	4.05	4.07	2.31
4	0.29	Premium	I	VS2	62.4	58.0	334	4.20	4.23	2.63
5	0.31	Good	J	SI2	63.3	58.0	335	4.34	4.35	2.75
6	0.24	Very Good	J	VVS2	62.8	57.0	336	3.94	3.96	2.48
7	0.24	Very Good	I	VVS1	62.3	57.0	336	3.95	3.98	2.47
8	0.26	Very Good	H	SI1	61.9	55.0	337	4.07	4.11	2.53
9	0.22	Fair	E	VS2	65.1	61.0	337	3.87	3.78	2.49
10	0.23	Very Good	H	VS1	59.4	61.0	338	4.00	4.05	2.39
11	0.30	Good	J	SI1	64.0	55.0	339	4.25	4.28	2.73
12	0.23	Ideal	J	VS1	62.8	56.0	340	3.93	3.90	2.46
13	0.22	Premium	F	SI1	60.4	61.0	342	3.88	3.84	2.33
14	0.31	Ideal	J	SI2	62.2	54.0	344	4.35	4.37	2.71
15	0.20	Premium	E	SI2	60.2	62.0	345	3.79	3.75	2.27
16	0.32	Premium	E	I1	60.9	58.0	345	4.38	4.42	2.68
17	0.30	Ideal	I	SI2	62.0	54.0	348	4.31	4.34	2.68
18	0.30	Good	J	SI1	63.4	54.0	351	4.23	4.29	2.70
19	0.30	Good	J	SI1	63.8	56.0	351	4.23	4.26	2.71
20	0.30	Very Good	I	SI1	62.7	59.0	351	4.21	4.27	2.66

Measures of Central Tendency

Mean

- ▶ Sum of all values of observations divided by the number of observations
- ▶ Mathematically denoted as:
$$\bar{a} = \frac{\sum_i^n x_i}{n}$$
- ▶ Sensitive to extreme or high values

Median

- ▶ Center of an ordered observations
- ▶ Also known as the middle of the observations.
- ▶ Not sensitive to extreme values

Mode

- ▶ Observation with the highest number of occurrence.

Measures of Variation

Standard Deviation and Variance

- ▶ Measures how far an observation is from the mean
- ▶ Mathematically defined as:

$$s = \sqrt{\frac{\sum_i^n (x_i - \bar{x})^2}{n}}$$

- ▶ Variance is defined as the square of the standard deviation:
 $Variance = s^2$

Range

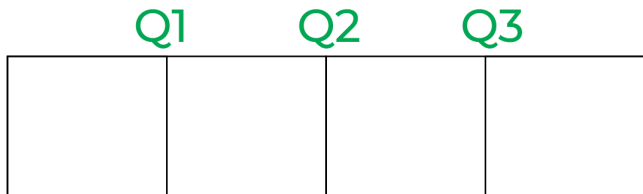
- ▶ Difference between the largest and smallest observations.
- ▶ Sensitive to extreme values

Percentiles

- ▶ Expressing the sorted observations in percentage
- ▶ Not sensitive to extreme values

Interquartile Range

- ▶ The interquartile range divides the observations into 4 equal part:
 - ▶ First Quartile: Q1
 - ▶ Second Quartile: Q2 (median)
 - ▶ Third Quartile: Q3



Summary Statistics of the Diamond Data Set

carat		cut		color		clarity		depth	
Min.	:0.2000	Fair	: 1610	D:	6775	SI1	:13065	Min.	:43.00
1st Qu.	:0.4000	Good	: 4906	E:	9797	VS2	:12258	1st Qu.	:61.00
Median	:0.7000	Very Good	:12082	F:	9542	SI2	: 9194	Median	:61.80
Mean	:0.7979	Premium	:13791	G:	11292	VS1	: 8171	Mean	:61.75
3rd Qu.	:1.0400	Ideal	:21551	H:	8304	VVS2	: 5066	3rd Qu.	:62.50
Max.	:5.0100			I:	5422	VVS1	: 3655	Max.	:79.00
				J:	2808	(Other):	2531		
table		price		x		y			
Min.	:43.00	Min.	: 326	Min.	: 0.000	Min.	: 0.000		
1st Qu.	:56.00	1st Qu.	: 950	1st Qu.	: 4.710	1st Qu.	: 4.720		
Median	:57.00	Median	: 2401	Median	: 5.700	Median	: 5.710		
Mean	:57.46	Mean	: 3933	Mean	: 5.731	Mean	: 5.735		
3rd Qu.	:59.00	3rd Qu.	: 5324	3rd Qu.	: 6.540	3rd Qu.	: 6.540		
Max.	:95.00	Max.	:18823	Max.	:10.740	Max.	:58.900		
z									
Min.	: 0.000								
1st Qu.	: 2.910								
Median	: 3.530								
Mean	: 3.539								
3rd Qu.	: 4.040								
Max.	:31.800								

Figure 3: Summary Statistics of the Diamond Data Set

Graphical Representations of Data

- ▶ Bar Plot
- ▶ Histogram
- ▶ Density Plot
- ▶ Box Plot
- ▶ Scatter Plot

Bar Plot

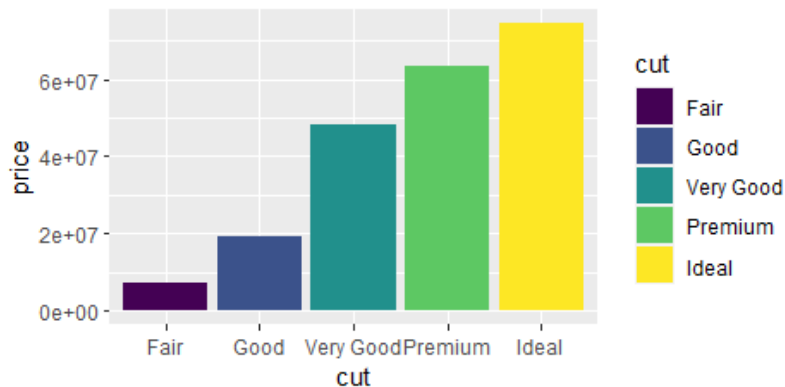


Figure 4: Prices of Various cuts of diamonds

Histogram

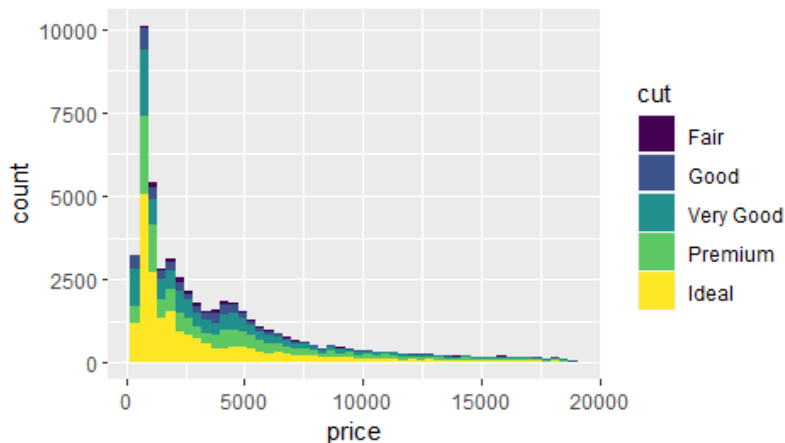


Figure 5: Histogram showing the various cut of diamonds

Density Plot

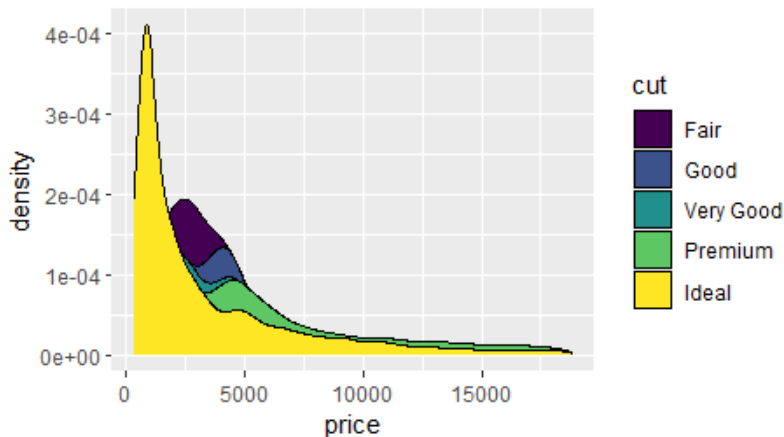


Figure 6: Density Plot of various diamond cut

Box Plot

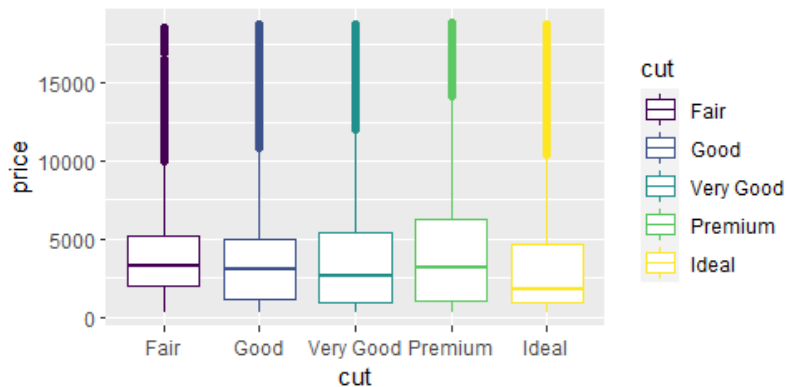


Figure 7: Box plot of various diamond cut

Scatter Plot

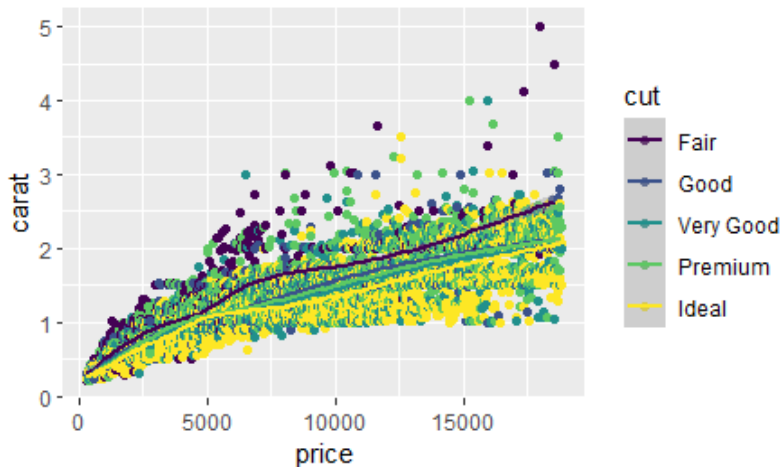


Figure 8: Scatter plot showing the relationship between carat and price

Challenge

Reference

Probability and Probability Distributions

What is probability

- ▶ It is simply the study of uncertainty.
- ▶ Example the possibility of raining, tossing a coin or rolling a die.
- ▶ It is the measuring of how likely an event will occur.
- ▶ Mathematically defined as:

$$Probability = \frac{\text{Number of Required outcomes}}{\text{Number of Possible outcomes}}$$

Terms in Probability

- ▶ Experiment: An uncertain situation e.g tossing a coin
- ▶ Outcome: Result of a trial in an experiment.
- ▶ Event: One or more outcome from a experiment
- ▶ Sample Space: The collection of possible outcomes of an experiment.

Random Variable

- ▶ Outcome of an event expressed in numbers
- ▶ For example in the coin toss experiment we can either have a head or tail which can be numerically expressed as 1 or 0 respectively.
- ▶ Let's call a set containing these two numbers X where; $X = \{1, 0\}$.
- ▶ X represents the Random Variable
- ▶ What's the random variable of a sixed face die?

The Two Coin Toss Experiment

► $S = \{HH, HT, TH, TT\}$

$$\text{Probability} = \frac{\text{Number of Required outcomes}}{\text{Number of Possible outcomes}}$$

► Number of possible outcomes = 4

► Probability of getting a head in both coins is:

$$= \frac{\text{Number of Required outcomes(HH)}}{\text{Number of Possible outcomes(HH, HT, TH, TT)}} = \frac{1}{4}$$

► Probability of getting a head in the first coin and a tail in the second coin:

$$= \frac{\text{Number of Required outcomes(HT)}}{\text{Number of Possible outcomes(HH, HT, TH, TT)}} = \frac{1}{4}$$

► Probability of getting a head and a tail in both coins.

$$= \frac{\text{Number of Required outcomes(HT, TH)}}{\text{Number of Possible outcomes(HH, HT, TH, TT)}} = \frac{2}{4}$$

The Two Die Experiment

		White Die					
		1	2	3	4	5	6
Red Die	1	(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)
	2	(1,2)	(2,2)	(3,2)	(4,2)	(5,2)	(6,2)
	3	(1,3)	(2,3)	(3,3)	(4,3)	(5,3)	(6,3)
	4	(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)
	5	(1,5)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)
	6	(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)

Figure 9: Tabular representation of the sample space of rolling two die

The Two Die Experiment

► $S = \{(1,1), (1,2), (1,3), \dots, (6,6)\}$

$$\text{Probability} = \frac{\text{Number of Required outcomes}}{\text{Number of Possible outcomes}}$$

► Number of possible outcomes = 36

► Probability of getting a one in both die:

$$= \frac{\text{Number of Required outcomes}(1,1)}{\text{Number of Possible outcomes}} = \frac{1}{36}$$

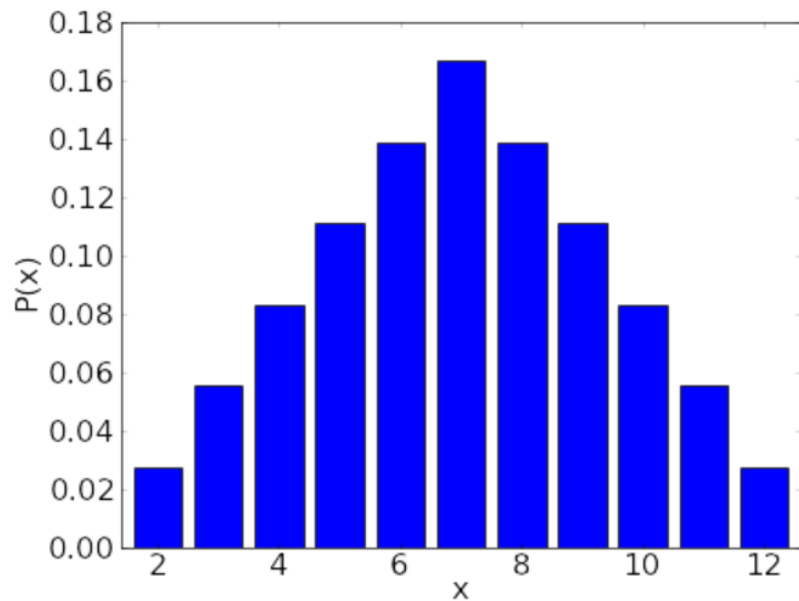
► Probability of getting a one in the first die and a two in the

$$\text{second die:} = \frac{\text{Number of Required outcomes}(1,2)}{\text{Number of Possible outcomes}} = \frac{1}{36}$$

► Probability of getting a one and a two:

$$= \frac{\text{Number of Required outcomes}(1,2) \text{ or } (2,1)}{\text{Number of Possible outcomes}} = \frac{2}{36}$$

The Two Die Experiment



Bernoulli Distribution

- ▶ A single trial with only two possible outcomes is called as binomial distribution.
- ▶ Example is a coin tossed once or a fight between me and Mayowa(DevNet) where the probability of I winning is 0.9 and him losing is 0.1.

Distribution of a Bernoulli Experiment

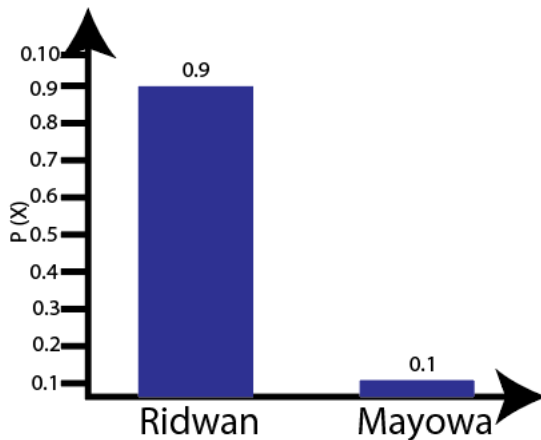


Figure 11: Distribution of a Bernoulli experiment

Binomial Distribution

- ▶ Unlike the Bernoulli Distribution, the binomial distribution has n number of trials.
- ▶ A distribution is said to be Binomial if the following are satisfied;
 - ▶ A trial with two outcomes and repeated n number of trials
 - ▶ Each trial is independent
 - ▶ A total numbers of n trials are conducted
 - ▶ The probability of success and failure is same for all trials.

Distribution of a Binomial Experiment

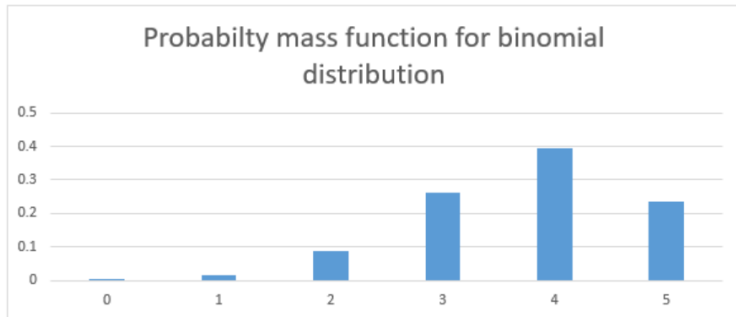


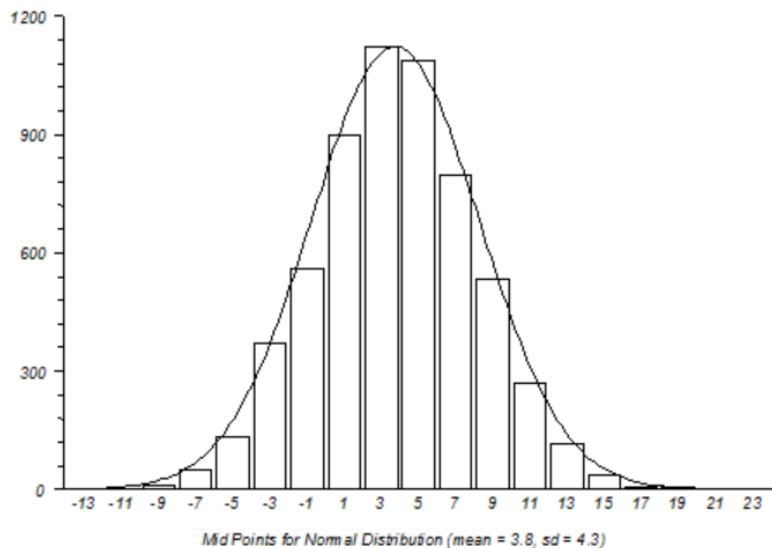
Figure 12: Distribution of a Binomial experiment

Normal Distribution

- ▶ A distribution is said to be normally distributed if it satisfies the following conditions;
 - ▶ The mean, median and mode of the distribution are the same.
 - ▶ The curve of the distribution is bell shaped
 - ▶ Half of the value are left of the center and the other half at the right.

Normal Distribution

Histogram for Normal Distribution (mean = 3.8, sd = 4.3)



Central Limit Theorem

- ▶ Regardless of the distribution of a variable's population, if we have a sufficiently large sample size, the mean and standard deviation of that variable will be normally distributed.

Challenge

References

Testing of Statistical Hypothesis

Regression

Classification