Probaility and Probability Distributions

Adejumo Ridwan Suleiman

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Section 1

Probability and Probability Distributions

What is probability

- It is simply the study of uncertainty.
- Example the possibility of raining, tossing a coin or rolling a die.
- It is the measuring of how likely an event will occur.
- Mathematically defined as:

  $$Probability = \frac{\text{Number of Required outcomes}}{\text{Number of Possible outcomes}}$$

Terms in Probability

- Experiment: An uncertain situation e.g tossing a coin
- Outcome: Result of a trial in an experiment.
- Event: One or more outcome from a experiment
- Sample Space: The collection of possible outcomes of an experiment.

Random Variable

- Outcome of an event expressed in numbers
- For example in the coin toss experiment we can either have a head or tail which can be numerically expressed as 1 or 0 respectively.
- Let's call a set containing these two numbers X where; `X = {1,0}`.
- `X` represents the Random Variable
- What's the random variable of a sixed face die?

The Two Coin Toss Experiment

- S = {HH,HT,TH,TT}

  $$Probability = \frac{\text{Number of Required outcomes}}{\text{Number of Possible outcomes}}$$
- Number of possible outcomes = 4
- Probability of getting a head in both coins is:

  $$= \frac{\text{Number of Required outcomes(HH)}}{\text{Number of Possible outcomes(HH,HT,TH,TT)}} = \frac{1}{4}$$
- Probability of getting a head in the first coin and a tail in the second coin:

  $$= \frac{\text{Number of Required outcomes(HT)}}{\text{Number of Possible outcomes(HH,HT,TH,TT)}} = \frac{1}{4}$$
- Probability of getting a head and a tail in both coins.

  $$= \frac{\text{Number of Required outcomes(HT,TH)}}{\text{Number of Possible outcomes(HH,HT,TH,TT)}} = \frac{2}{4}$$

The Two Die Experiment

		White Die					
		1	2	3	4	5	6
Red Die	1	(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)
	2	(1,2)	(2,2)	(3,2)	(4,2)	(5,2)	(6,2)
	3	(1,3)	(2,3)	(3,3)	(4,3)	(5,3)	(6,3)
	4	(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)
	5	(1,5)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)
	6	(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)

Figure 1: Tabular representation of the sample space of rolling two die

More on the Two Die Experiment

- S = {(1,1),(1,2),(1,3),...,(6,6)}
  $$Probability = \frac{\text{Number of Required outcomes}}{\text{Number of Possible outcomes}}$$
- Number of possible outcomes = 36
- Probability of getting a `one` in both die:
  $$= \frac{\text{Number of Required outcomes(1,1)}}{\text{Number of Possible outcomes}} = \frac{1}{36}$$
- Probability of getting a `one` in the first die and a `two` in the second die: $= \frac{\text{Number of Required outcomes(1,2)}}{\text{Number of Possible outcomes}} = \frac{1}{36}$
- Probability of getting a one and a two:
  $$= \frac{\text{Number of Required outcomes(1,2) or (2,1)}}{\text{Number of Possible outcomes}} = \frac{2}{36}$$

Bernoulli Distribution

- A single trial with only two possible outcomes is called as binomial distribution.
- Example is a coin tossed once or a fight between me and MayWeather where the probability of him winning is 0.9 and I losing is 0.1.

Binomial Distribution

- Unlike the Bernoulli Distribution, the binomial distribution has `n` number of trials.
- A distribution is said to be Binomial if the following are satisfied;
  - A trial with two outcomes and repeated `n` number of trials
  - Each trial is independent
  - A total numbers of `n` trials are conducted
  - The probability of scuccess and failure is same for all trials.

Normal Distribution

- A distribution is said to be normally distributed if it satisfies the following conditions;
  - The mean, median and mode of the distribution are the same.
  - The curve of the distribution is bell shaped
  - Half of the value are left of the center and the other half at the right.

Central Limit Theorem

- Regardless of the distribution of a variable's population, if we have a sufficiently large sample size, the mean and standard deviation of that variable will be normally distributed.

Challenge

References