Introduction to Statistics and Statistical Measures

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June 25 2022

Section 1

Introduction to Statistics and Statistical Measures

What is Statistics?

- Statistics is the collection, organizing and analysing of data.

Is Data Science Statistics in Disguise?

- Unlike Statistics, Data Science is an interdisciplinary field consisting of Mathematics, Statistics, Computer Science and Domain Knowledge.

Types of Data

- Data can be classified into two types
  - Based on Measurement scale
  - Based on Time Period

Based on Measurement Scale

- Qualitative Data
  - Nominal Data e.g sex
  - Ordinal Data e.g temperature level; High, Medium and Low
- Quantitative Data
  - Ratio e.g weight
  - Interval e.g temperature in degreee celsius

Based on Time Period

- Cross-Sectional Data e.g number of viewers for different youtube genres in the year 2021
- Time Series Data e.g number of viewers for Sport channels on youtube from the year 2014-Date.

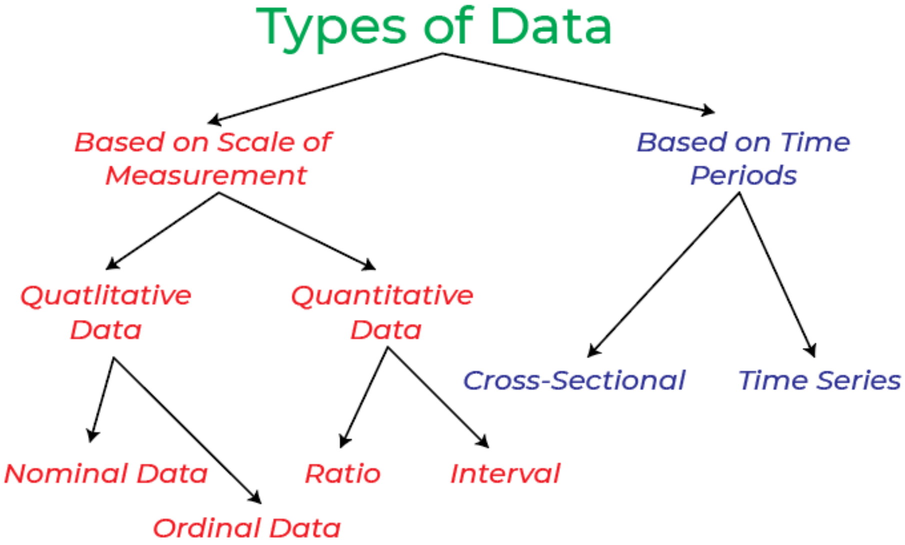


Figure 1: Types of Data

Rectangular or Structured Data

	carat	cut	color	clarity	depth	table	price	x	y	z
1	0.23	Ideal	E	SI2	61.5	55.0	326	3.95	3.98	2.43
2	0.21	Premium	E	SI1	59.8	61.0	326	3.89	3.84	2.31
3	0.23	Good	E	VS1	56.9	65.0	327	4.05	4.07	2.31
4	0.29	Premium	I	VS2	62.4	58.0	334	4.20	4.23	2.63
5	0.31	Good	J	SI2	63.3	58.0	335	4.34	4.35	2.75
6	0.24	Very Good	J	VVS2	62.8	57.0	336	3.94	3.96	2.48
7	0.24	Very Good	I	VVS1	62.3	57.0	336	3.95	3.98	2.47
8	0.26	Very Good	H	SI1	61.9	55.0	337	4.07	4.11	2.53
9	0.22	Fair	E	VS2	65.1	61.0	337	3.87	3.78	2.49
10	0.23	Very Good	H	VS1	59.4	61.0	338	4.00	4.05	2.39
11	0.30	Good	J	SI1	64.0	55.0	339	4.25	4.28	2.73
12	0.23	Ideal	J	VS1	62.8	56.0	340	3.93	3.90	2.46
13	0.22	Premium	F	SI1	60.4	61.0	342	3.88	3.84	2.33
14	0.31	Ideal	J	SI2	62.2	54.0	344	4.35	4.37	2.71
15	0.20	Premium	E	SI2	60.2	62.0	345	3.79	3.75	2.27
16	0.32	Premium	E	I1	60.9	58.0	345	4.38	4.42	2.68
17	0.30	Ideal	I	SI2	62.0	54.0	348	4.31	4.34	2.68
18	0.30	Good	J	SI1	63.4	54.0	351	4.23	4.29	2.70

Measures of Central Tendency

Mean

- Sum of all values of observations divided by the number of observations
- Mathematically denoted as:
  $\bar{a} = \frac{\sum_i^n x_i}{n}$
- Sensitive to extreme or high values

Median

- Center of an ordered observations
- Also known as the middle of the observations.
- Not sensitive to extreme values

Mode

- Observation with the highest number of occurence.

Measures of Variation

Standard Deviation and Variance

- Measures how far an observation is from the mean
- Mathematically defined as:

$$s = \sqrt{\frac{\sum_i^n (x_i - \bar{x})}{n}}$$

- Variance is defined as the square of the standard deviation: $Variance = s^2$

Range

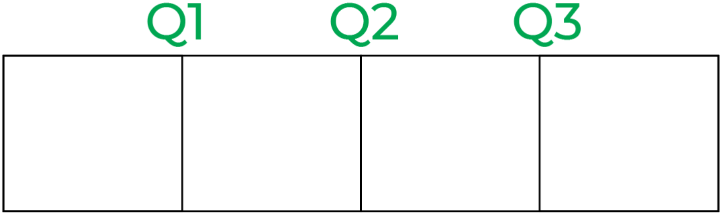
- Difference between the largest and smallest observations.
- Sensitive to extreme values

Percentiles

- Expressing the sorted observations in percentage
- Not sensitive to extreme values

Interquartile Range

- The interquartile range divides the observations into 4 equal part:
  - First Quartile: Q1
  - Second Quratile: Q2 (median)
  - Third Quratile: Q3



Summary Statistics of the Diamond Data Set

```
     carat               cut          color      clarity          depth
 Min.   :0.2000   Fair     : 1610   D: 6775   SI1    :13065   Min.   :43.00
 1st Qu.:0.4000   Good     : 4906   E: 9797   VS2    :12258   1st Qu.:61.00
 Median :0.7000   Very Good:12082   F: 9542   SI2    : 9194   Median :61.80
 Mean   :0.7979   Premium  :13791   G:11292   VS1    : 8171   Mean   :61.75
 3rd Qu.:1.0400   Ideal    :21551   H: 8304   VVS2   : 5066   3rd Qu.:62.50
 Max.   :5.0100                     I: 5422   VVS1   : 3655   Max.   :79.00
                                    J: 2808   (Other): 2531
     table           price             x                y
 Min.   :43.00   Min.   :  326   Min.   : 0.000   Min.   : 0.000
 1st Qu.:56.00   1st Qu.:  950   1st Qu.: 4.710   1st Qu.: 4.720
 Median :57.00   Median : 2401   Median : 5.700   Median : 5.710
 Mean   :57.46   Mean   : 3933   Mean   : 5.731   Mean   : 5.735
 3rd Qu.:59.00   3rd Qu.: 5324   3rd Qu.: 6.540   3rd Qu.: 6.540
 Max.   :95.00   Max.   :18823   Max.   :10.740   Max.   :58.900

       z
 Min.   : 0.000
 1st Qu.: 2.910
 Median : 3.530
 Mean   : 3.539
 3rd Qu.: 4.040
 Max.   :31.800
```

Figure 3: Summary Statistics of the Diamond Data Set

Graphical Repreentations of Data

- Bar Plot
- Histogram
- Density Plot
- Box Plot
- Scatter Plot

Bar Plot

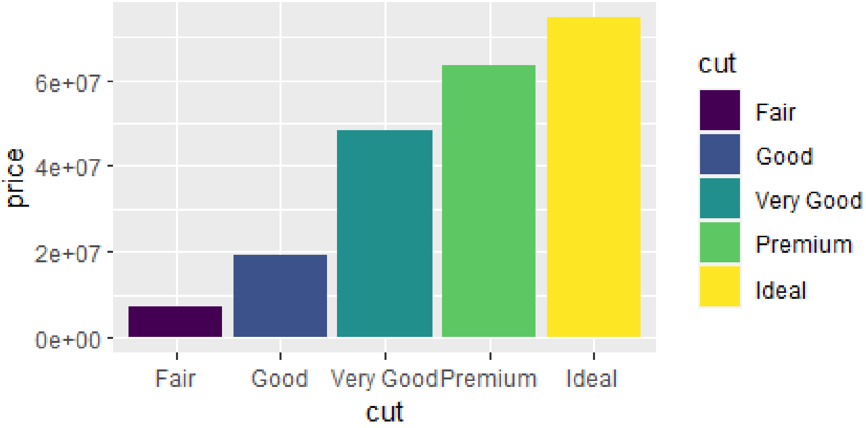


Figure 4: Prices of Various cuts of diamonds

Histogram

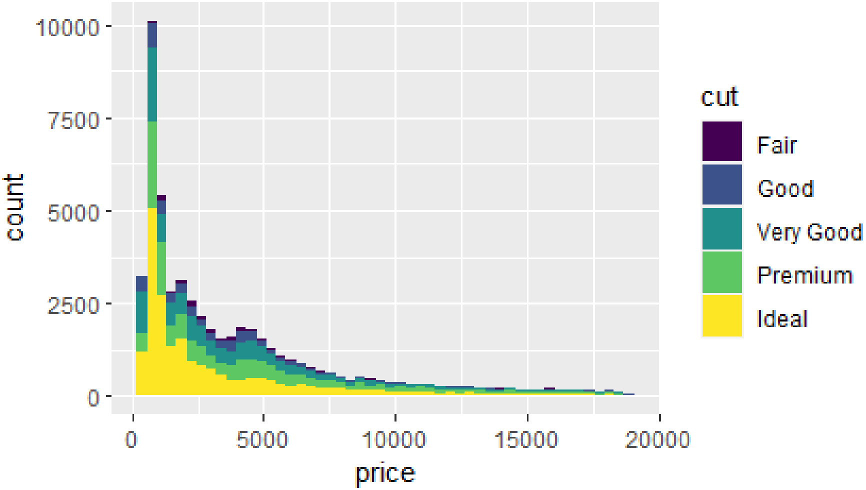


Figure 5: Histogram showing the various cut of diamonds

Density Plot

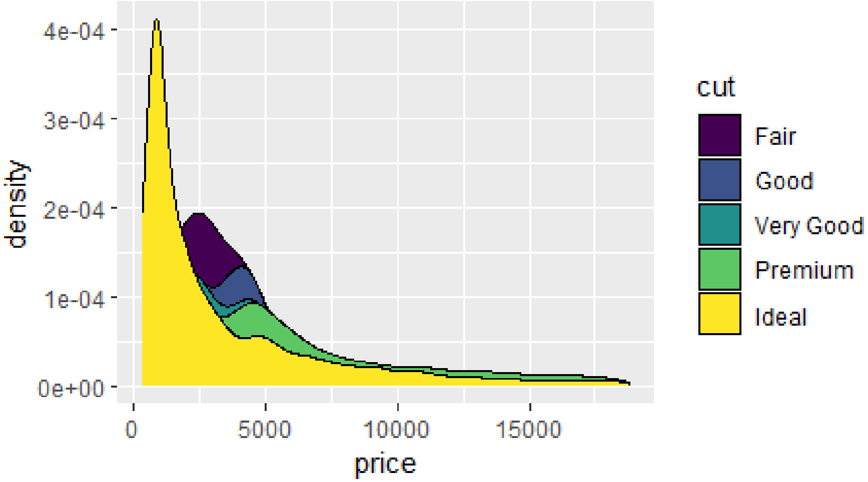


Figure 6: Density Plot of various diamond cut

Box Plot

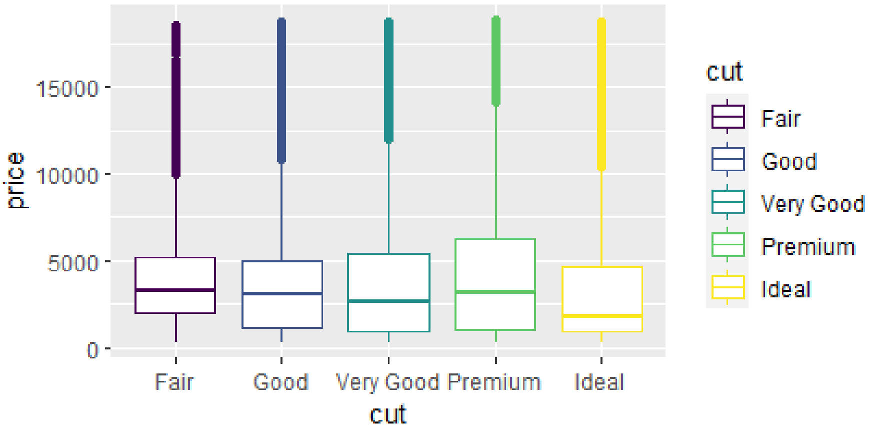


Figure 7: Box plot of various diamond cut

Scatter Plot

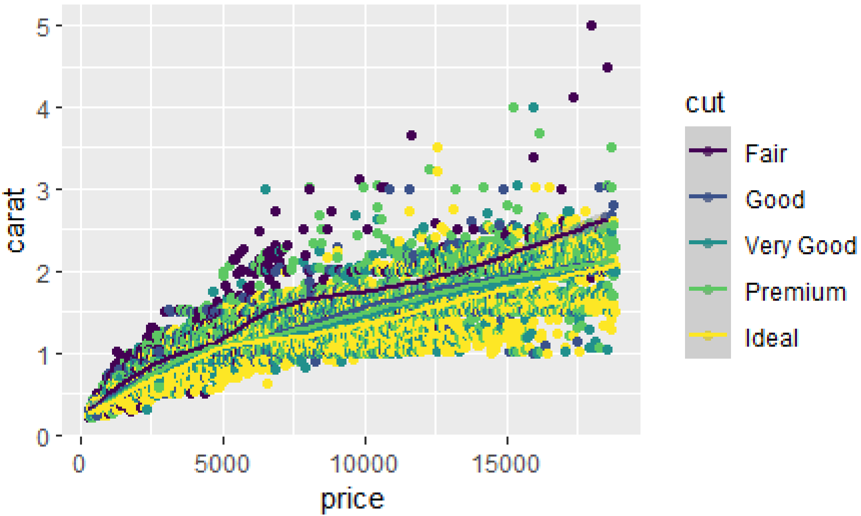


Figure 8: Scatter plot showing the relationship between carat and price

Challenge

Reference

Section 2

Probability and Probability Distributions

What is probability

- It is simply the study of uncertainty.
- Example the possibility of raining, tossing a coin or rolling a die.
- It is the measuring of how likely an event will occur.
- Mathematically defined as:

$$Probability = \frac{\text{Number of Required outcomes}}{\text{Number of Possible outcomes}}$$

Terms in Probability

- Experiment: An uncertain situation e.g tossing a coin
- Outcome: Result of a trial in an experiment.
- Event: One or more outcome from a experiment
- Sample Space: The collection of possible outcomes of an experiment.

Random Variable

- Outcome of an event expressed in numbers
- For example in the coin toss experiment we can either have a head or tail which can be numerically expressed as 1 or 0 respectively.
- Let's call a set containing these two numbers X where; `X = {1,0}`.
- `X` represents the Random Variable
- What's the random variable of a sixed face die?

The Two Coin Toss Experiment

- S = {HH,HT,TH,TT}
  $$Probability = \frac{\text{Number of Required outcomes}}{\text{Number of Possible outcomes}}$$
- Number of possible outcomes = 4
- Probability of getting a head in both coins is:
  $$= \frac{\text{Number of Required outcomes(HH)}}{\text{Number of Possible outcomes(HH,HT,TH,TT)}} = \frac{1}{4}$$
- Probability of getting a head in the first coin and a tail in the second coin:
  $$= \frac{\text{Number of Required outcomes(HT)}}{\text{Number of Possible outcomes(HH,HT,TH,TT)}} = \frac{1}{4}$$
- Probability of getting a head and a tail in both coins.
  $$= \frac{\text{Number of Required outcomes(HT,TH)}}{\text{Number of Possible outcomes(HH,HT,TH,TT)}} = \frac{2}{4}$$

The Two Die Experiment

		White Die					
		1	2	3	4	5	6
Red Die	1	(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)
	2	(1,2)	(2,2)	(3,2)	(4,2)	(5,2)	(6,2)
	3	(1,3)	(2,3)	(3,3)	(4,3)	(5,3)	(6,3)
	4	(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)
	5	(1,5)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)
	6	(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)

Figure 9: Tabular representation of the sample space of rolling two die

More on the Two Die Experiment

- S = {(1,1),(1,2),(1,3),...,(6,6)}
  $$Probability = \frac{\text{Number of Required outcomes}}{\text{Number of Possible outcomes}}$$
- Number of possible outcomes = 36
- Probability of getting a one in both die:
  $$= \frac{\text{Number of Required outcomes(1,1)}}{\text{Number of Possible outcomes}} = \frac{1}{36}$$
- Probability of getting a one in the first die and a two in the second die: $= \frac{\text{Number of Required outcomes(1,2)}}{\text{Number of Possible outcomes}} = \frac{1}{36}$
- Probability of getting a one and a two:
  $$= \frac{\text{Number of Required outcomes(1,2) or (2,1)}}{\text{Number of Possible outcomes}} = \frac{2}{36}$$

Bernoulli Distribution

- A single trial with only two possible outcomes is called as binomial distribution.
- Example is a coin tossed once or a fight between me and MayWeather where the probability of him winning is 0.9 and I losing is 0.1.

Binomial Distribution

- Unlike the Bernoulli Distribution, the binomial distribution has n number of trials.
- A distribution is said to be Binomial if the following are satisfied;
  - A trial with two outcomes and repeated n number of trials
  - Each trial is independent
  - A total numbers of n trials are conducted
  - The probability of scuccess and failure is same for all trials.

Normal Distribution

- A distribution is said to be normally distributed if it satisfies the following conditions;
  - The mean, median and mode of the distribution are the same.
  - The curve of the distribution is bell shaped
  - Half of the value are left of the center and the other half at the right.

Central Limit Theorem

- Regardless of the distribution of a variable's population, if we have a sufficiently large sample size, the mean and standard deviation of that variable will be normally distributed.

Challenge

References