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COMP 212 : Functional Programming, Spring 2020

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## Homework 01

Name: \_\_\_\_\_

Wes Email: \_\_\_\_\_

Question	Points	Score
1	5	
2	14	
3	15	
4	6	
Total:	40	

You *must* print this PDF and write your answers *neatly* by hand. You can hand the assignment in during class or to Dan Licata's office (Exley 633).

**1. Collaboration Policy, Piazza**

Read the collaboration policy on the course website. Then, for each of the following situations, decide whether or not the students' actions are permitted by the policy. Explain your answers.

- (1) (a) Ted and Sam are discussing Problem 3 over Skype. Meanwhile, Ted is writing up his solution to that problem.

**Solution:**

- (1) (b) Lizzie and Max eat lunch (at noon) while talking about their homework, and by the end of lunch, they have covered their napkins with notes and solutions. They throw out all of the napkins and go to class from 1pm-5pm. Then, each individually writes up his solution.

**Solution:**

- (1) (c) Robby and Emily write out a solution to Problem 4 on a whiteboard. Then, they erase the whiteboard and run to the computer cluster. Sitting at opposite sides of the room, each student types up the solution.

**Solution:**

- (1) (d) Ben is working on a problem alone on a whiteboard. He accidentally forgets to erase her solution and goes home to write it up. Later, Justin walks by, reads it, waits 4 hours, and then writes up his solution. Is Ben in violation of the policy? Is Justin?

**Solution:**

We will use a discussion web site called Piazza this semester. You will get an email with a link to sign up for an account.

- (1) (a) One of the nice things about Piazza is that students can collaboratively edit responses to other people's questions, like on a wiki. Add a line to the post titled Story.

## 2. Type Checking and Evaluation

In this section we will explore the step-by-step reasoning of type checking and evaluation to better understand when an SML expression is well-typed, what its type is, how it will evaluate, and what its value will be. We will also do some basic analysis of the number of steps in the evaluation of an expression.

We will start with an example. Consider the expression `intToString 7` and assume that we know `intToString : int -> string`. To determine the type of this expression, we first note that the expression `7` has the type `int`. Now, using this and the fact that `intToString` has the type `int -> string` we conclude that the application of these two expressions has the type `string` since the first expression has a function type and the type of the second expression matches the type of the argument for this function.

- (2) (a) Determine the type of the expression:

`(intToString 3) ^ (intToString 6)`

Describe your reasoning in the same manner as the example.

**Solution:**

- (2) (b) Explain why the expression, `intToString "1"`, is not well-typed.

**Solution:**

- (2) (c) We will now look at an example of reasoning about evaluation. Consider the expression `(intToString 7) ^ "1"` and assume that we know the application, `(intToString 7)`, evaluates to the value `"7"`. Note that the value `"1"` evaluates to itself. Using these two facts, we conclude that the whole expression evaluates to `"71"` since the `^` operator evaluates its two subexpressions and then evaluates to the concatenation of the two strings that result from these evaluations.

Determine the value that results from the evaluation of the expression:

`intToString (4 + 4)`

Describe your reasoning in the same manner as the example.

**Solution:**

(d) Recall, from Lecture 2, the following categories of expressions, each of which is strictly larger than the next:

- syntactically correct expressions
- well-typed expressions
- valuable expressions
- values

For each of the following expressions, state the **most specific** set that the expression belongs to. Briefly explain why your answer is correct.

(2)

i. `f (5 div (1 - 1))` where

```
fun f (x : int) : int = 47
```

**Solution:**

(2)

ii. 6

**Solution:**

(2)

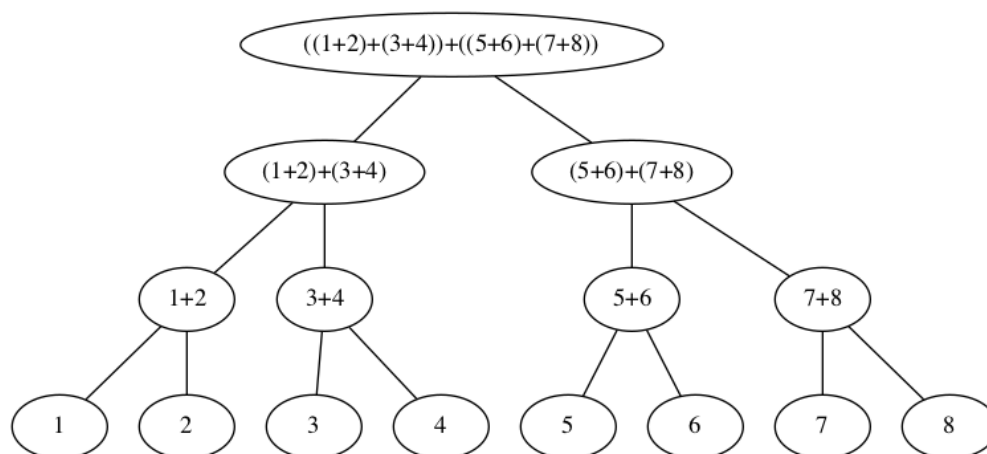
iii. `1+1`

**Solution:**

- (2) iv. `(intToString 5) + 1`

**Solution:**

3. **Parallel Computing** We can draw an expression, such as  $((1 + 2) + (3 + 4)) + ((5 + 6) + (7 + 8))$ , as a tree, such as



Each circle is a *node* of the tree. The leaves (nodes with nothing below them) are values and the non-leaf nodes are operations. The *work*,  $W$ , of the expression tree is the total number of non-leaf nodes, while the *span*,  $S$ , is the number of non-leaf nodes along the longest path from the root to a leaf.

Regardless of the number of processors used to evaluate a compound expression in parallel, the number of steps required to calculate the final result must be at least as great as the span because evaluating one node requires first evaluating the node below it (a *data dependency*). Also, if each of  $P$  processors performs one evaluation step in parallel during each *time cycle*, it would require at least  $W/P$  (rounded up) time cycles to perform all  $W$  operations. Thus, you can't do better than  $W/P$  and you can't do better than  $S$ . Putting these together, this means you cannot run a calculation in time less than  $\max(W/P, S)$ , so we say this is a *lower bound* on the time. However, not all calculations can be done in exactly this lower bound. But, it turns out that you can run a calculation in time *proportional* to this lower bound, which is known as Brent's Theorem:

**Theorem 1** (Brent's Principle). *If an expression,  $e$  has work  $W$  and span  $S$ , then evaluating  $e$  on a  $P$ -processor machine takes time proportional to  $\max(W/P, S)$ .*

- (4) (a) What are the work  $W$  and span  $S$  for the above tree in Figure 3?

**Solution:**

When  $S$  is greater than  $W/P$ , a calculation is *limited by the span*—the running time cannot be less than the span. Conversely, when  $W/P$  is greater than  $S$ , the running time for a calculation is *limited by the work*—there is enough work to do that  $W/P$ , rather than  $S$ , is the overall limiting factor.

- (2) (b) Suppose you have  $P = 2$  processors. Is the calculation limited by the work or the span? Does having 2 processors speed up the calculation relative to having 1 processor? Explain.

**Solution:**

- (2) (c) Suppose you have  $P = 3$  processors. Is the calculation limited by the work or the span? Does having 3 processors speed up the calculation relative to having 2 processors? Explain.

**Solution:**

- (2) (d) Suppose you have  $P = 4$  processors. Is the calculation limited by the work or the span? Does having 4 processors speed up the calculation relative to having 3 processors? Explain.

**Solution:**

- (5) (e) In the first lecture, we acted out the process of counting the number of students in the class who took 211 last semester and analyzed the work and span of this process. Describe a different real-life process in which parallelism occurs, and analyze its work and span.



**Solution:**

#### 4. Interpreting Error Messages

Download the file `hw01.sml` from the assignments page.

You can evaluate the SML declarations in this file using the command

```
use "hw01.sml";
```

at the SML REPL prompt (start `smlnj` in the same directory as where you saved the file). Unfortunately, the file has some errors that must be corrected. The next tasks will guide you through the process of correcting these errors.

- (2) (a) What is the first error message do you see when you evaluate the unmodified `hw01.sml` file? What caused this error and how can it be fixed?<sup>1</sup>

**Solution:**

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<sup>1</sup> *Hint:* Compare the syntax of `double` and `intToString`. What is different?

Correct this one error in the `hw01.sml` file and load it again.

- (2) (b) Now what is the first of the remaining errors? What caused this error?

**Solution:**

Again, correct just this one error in the `hw01.sml` file and reload it.

- (2) (c) What is the final error message? What does this error message mean? How can you fix it?

**Solution:**

When you correct this final error and evaluate the file there should be no more error messages.