On the Use of the `ubcdiss` Template

by

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On the Use of the `ubcdiss` Template

submitted by **Johnny Canuck** in partial fulfillment of the requirements for the degree of **Doctor of Philosophy** in **Basket Weaving**.

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Abstract

This document provides brief instructions for using the `ubcdiss` class to write a **UBC!**-conformant dissertation in LaTeX. This document is itself written using the `ubcdiss` class and is intended to serve as an example of writing a dissertation in LaTeX. This document has embedded **URL!**s (**URL!S**) and is intended to be viewed using a computer-based **PDF!** (**PDF!**) reader.

Note: Abstracts should generally try to avoid using acronyms.

Note: at **UBC!** (**UBC!**), both the **GPS!** (**GPS!**) Ph.D. defence programme and the Library's online submission system restricts abstracts to 350 words.

Lay Summary

The lay or public summary explains the key goals and contributions of the research/ scholarly work in terms that can be understood by the general public. It must not exceed 150 words in length.

Preface

At **UBC!**, a preface may be required. Be sure to check the **GPS!** guidelines as they may have specific content to be included.

Contents

List of Tables

List of Figures

Glossary

This glossary uses the handy `acroynym` package to automatically maintain the glossary. It uses the package's `printonlyused` option to include only those acronyms explicitly referenced in the LaTeX source.

GPS Graduate and Postdoctoral Studies

PDF Portable Document Format

URL Unique Resource Locator, used to describe a means for obtaining some resource on the world wide web

Acknowledgments

Thank those people who helped you.

Don’t forget your parents or loved ones.

You may wish to acknowledge your funding sources.

Chapter 1

Introduction

i mean, the beginning is the part you're not comfortable writing, right? the longer it went on, the better it flowed. at that point you're quoting and weaving results you know well, referencing the little mental web you have woven. it seems cohesive, but also i don't understand it. the beginning bit seems thrown together like Mike told you to include bits about DEs and so you begrudgingly injected something ?? like the very very beginning bit anyway, i'll e-mail you back the tex file and the pdf. i know you're not asking for this advice but it's coming from ozgur and yaniv and they are very smart and i trust them lots:

1. be very careful about whether you're using colloquial language, and how it might be interpreted. e.g. be careful not to insult people's work, and try to not to flip flop on how hand wavey you are being. I think I have a couple of notes in the file pertaining to each of these points
2. when citing work, either use the author names every time or don't. don't mix and match unless appropriate. why would you deny some the respect of appearing in your work, but not others?
3. if you're going to write notes to yourself in your thesis/papers, you must have a way of ensuring that you'll see them later before you send it off. caps lock is not sufficient and yaniv and ozgur can provide examples if you need. I included a little command for you so that you can just Cmd+F (or C-s ??) for all appearances of in the .tex file if you use it. Has the added advantage of making PDF text blue so that everyone reading too knows that it doesn't belong.

also my disclaimer for edits: 1. for some reason my brain is tired today; 2. I don't know the culture of your field nor some of the very elementary things you're presenting 3. Because of 1, I tried to communicate what I wanted to say using the best language I could, but may not have always succeeded at clarity/intent/approachability ?? So basically, remember that it's possible that my edits deserve to be treated with a grain of salt. ??

This start feels outside the realm of where you're going — it seems at once abrupt and off-topic. It would be nice to have an introductory sentence or two to get the reader on track before discussing the "required background" material. A Diophantine equation is a polynomial equation in several variables defined over the integers. The term *Diophantine* refers to the Greek mathematician Diophantus of Alexandria, who studied such equations in the 3rd century A.D. why the history lesson? maybe you could use this as one way of motivating/introducing DEs: "look at these things. look how long they've been studied. here's why, and here are the ways people study them. …or something…

remove separate paragraph if it's the same thought — f is a DE right? If so, then these next lines are providing additional information to what was given above, not starting a new thread. Let $f(x_1,\ldots,x_n)$ be a polynomial with integer coefficients. We wish to study the set of solutions $(x_1,\ldots,x_n)\in\mathbb{Z}^n$ to the equation

$$f(x_1,\ldots,x_n)=0. \tag{1.1}$$

There are several different approaches for doing so, arising from three basic problems concerning Diophantine equations. The first such problem is to determine whether ~~or not~~ (**??**) has any solutions ~~at all~~ (too colloquial imo). Indeed, one of the most famous theorems in mathematics, Fermat's Last Theorem, proven by Wiles in 1995, states that for $f(x,y,z)=x^n+y^n-z^n$, where $n\geq 3$, there are no solutions in the positive integers x,y,z (there are so many commas in this sentence. You can remove at least 2 of 7 by splicing and/or rearranging). Qualitative questions of this type are often studied using algebraic methods.

Suppose now that (**??**) is solvable, that is, has at least one solution. The second basic problem is to determine whether the number of solutions is finite or infinite.

For example, consider the *Thue equation*,

$$f(x,y) = a, \tag{1.2}$$

where $f(x,y)$ is an integral binary form of degree $n \geq 3$ (feels like you really jump into the language here. you spelled out what a DE was, but now assume the reader knows the definition of an integral binary form. Personally, I knew the former but the latter reads like domain-specific jargon to me) and a is a fixed nonzero rational integer. In 1909, Thue [REF] proved that this equation has only finitely many solutions. This result followed from a sharpening of Liouville's inequality, an observation that algebraic numbers do not admit very strong approximation by rational numbers. That is, if α is a real algebraic number of degree $n \geq 2$ and p, q are integers, Liouville's ([REF]) observation states that

$$\left| \alpha - \frac{p}{q} \right| > \frac{c_1}{q^n}, \tag{1.3}$$

where $c_1 > 0$ is a value depending explicitly on α. The finitude of the number of solutions to (**??**) follows directly from a sharpening of (**??**) of the type

$$\left| \alpha - \frac{p}{q} \right| > \frac{\lambda(q)}{q^n}, \quad \lambda(q) \to \infty. \tag{1.4}$$

what is the limit $\lambda \to \infty$ with respect to? Indeed, if α is a real root of $f(x,1)$ and $\alpha^{(i)}$, $i = 1, \ldots, n$ are its conjugates, it follows from (**??**) that

$$\prod_{i=1}^{n} \left| \alpha^{(i)} - \frac{x}{y} \right| = \frac{a}{|a_0||y|^n}$$

where a_0 is the leading coefficient of the polynomial $f(x,1)$. If the Thue equation has integer solutions with arbitrarily large $|y|$, the product $\prod_{i=1}^{n} |\alpha^{(i)} - x/y|$ must take arbitrarily small values for solutions x, y of (**??**). As all the $\alpha^{(i)}$ are different, x/y must be correspondingly close to one of the real numbers $\alpha^{(i)}$, say α. Thus we obtain

$$\left| \alpha - \frac{x}{y} \right| < \frac{c_2}{|y|^n}$$

where c_2 depends only on a_0, n, and the conjugates $\alpha^{(i)}$. Comparison of this inequality with (**??**) shows that $|y|$ cannot be arbitrarily large, and so the number of solutions of the Thue equation is finite. Using this argument, an explicit bound can be constructed on the solutions of (**??**) provided that an effective (descriptive? explicit? tight? tractable?) inequality (**??**) is known. The sharpening of the Liouville inequality however, especially in effective form, proved to be very difficult. REF? also “very difficult” seems a subjective qualification; is that okay for your audience?

In [REF:THUE], Thue published a proof that

$$\left|\alpha - \frac{p}{q}\right| < \frac{1}{q^{\frac{n}{2}+1+\varepsilon}}$$

has only finitely many solutions in integers $p, q > 0$ for all algebraic numbers α of degree $n \geq 3$ and any $\varepsilon > 0$. In essence, he obtained the inequality (**??**) with $\lambda(q) = c_3 q^{\frac{1}{2}n-1-\varepsilon}$ this function does not match the one appearing above in displaymath. is that supposed to be the case? might have something to do with the $<$ not matching the $>$ in (**??**)? it is not clear to me, but hopefully it will be to typical reader, where $c_3 > 0$ depends on α and ε, thereby confirming that all Thue equations have only finitely many solutions. Unfortunately, Thue’s arguments do not allow one to find the explicit dependence of c_3 on α and ε, and so the bound for the number of solutions of the Thue equation cannot be given in explicit form either. That is, Thue’s proof is ineffective, meaning that it provides no means to ~~actually~~ find the solutions to (**??**). I feel like I would dance more carefully around calling someone’s proof ineffective.

Nonetheless, the investigation of Thue’s equation and its generalizations was central to the development of the theory of Diophantine equations in the early 20th century when it was discovered that many Diophantine equations in two unknowns could be reduced to it. In particular, the thorough development and enrichment of Thue’s method led Siegel to his theorem on the finitude of the number of integral points on an algebraic curve of genus greater than zero [REF?]. However, as Siegel’s result relies on Thue’s rational approximation to algebraic numbers, it too is ineffective in the above sense.

Shortly following Thue’s result, Goormaghtigh conjectured that the only non-trivial integer solutions of the exponential Diophantine equation

$$\frac{x^m - 1}{x - 1} = \frac{y^n - 1}{y - 1} \tag{1.5}$$

satisfying $x > y > 1$ and $n, m > 2$ are

$$31 = \frac{2^5 - 1}{2 - 1} = \frac{5^3 - 1}{5 - 1} \quad \text{and} \quad 8191 = \frac{2^{13} - 1}{2 - 1} = \frac{90^3 - 1}{90 - 1}.$$

These correspond to the known solutions $(x, y, m, n) = (2, 5, 5, 3)$ and $(2, 90, 13, 3)$ to what is nowadays termed *Goormaghtigh's equation*. The Diophantine equation (**??**) asks for integers having all digits equal to one with respect to two distinct bases, yet whether it has finitely many solutions is still unknown. By fixing the exponents m and n however, Davenport, Lewis, and Schinzel ([REF]) were able to prove that (**??**) has only finitely many solutions. Unfortunately, this result rests on Siegel's aforementioned finiteness theorem, and is therefore ineffective.

In 1933, Mahler [REF] published a paper on the investigation of the Diophantine equation

$$f(x, y) = p_1^{z_1} \cdots p_v^{z_v}, \quad (x, y) = 1,$$

in which $S = \{p_1, \ldots, p_v\}$ denotes a fixed set of prime numbers, $x, y, z_i \geq 0$, $i = 1, \ldots, v$ are unknown integers, and $f(x, y)$ is an integral irreducible binary form of degree $n \geq 3$. Generalizing the classical result of Thue, Mahler proved that this equation has only finitely many solutions. Unfortunately, like Thue, Mahler's argument is also ineffective each time I read this, I believe more strongly that a different word should be used to describe their work. ineffective seems like an attack, and a broad stroke that misses the precise critique you're looking to discuss.

This leads us to the third basic problem regarding Diophantine equations and the main focus of this thesis: given a solvable Diophantine equation, determine all of its solutions. Until long after Thue's work, no method was known for the construction of bounds for the number of solutions of a Thue equation in terms of the parameters of the equation. Only in 1968 was such a method introduced by Baker [REF], based on his theory of bounds for linear forms in the logarithms of algebraic numbers. Generalizing Baker's ground-breaking result to the p-adic case, Sprindžuk and Vinogradov [CITE] and Coates [CITE] proved that the solutions of any *Thue-Mahler equation*,

$$f(x,y) = a p_1^{z_1} \cdots p_v^{z_v}, \quad (x,y) = 1, \tag{1.6}$$

where a is a fixed integer, could, at least in principal, be effectively determined. The first practical method for solving the general Thue-Mahler equation (**??**) over $\mathbb{Z}$ is attributed to Tzanakis and de Weger [CITE], whose ideas were inspired in part by the method of Agrawal, Coates, Hunt, and van der Poorten [CITE] in their work to solve the specific Thue-Mahler equation

$$x^3 - x^2 y + x y^2 + y^3 = \pm 11^{z_1}.$$

Using optimized bounds arising from the theory of linear forms in logarithms, a refined, automated version of this explicit method has since been implemented by Hambrook as a MAGMA package [REF?].

As for Goormaghtigh's equation, when m and n are fixed and

$$\gcd(m-1, n-1) > 1, \tag{1.7}$$

Davenport, Lewis, and Schinzel ([REF]) were able to replace Siegel's result by an effective argument due to Runge. This result was improved by Nesterenko and Shorey ([REF]) and Bugeaud and Shorey ([REF]) using Baker's theory of linear forms in logarithms. In either case, in order to deduce effectively computable bounds (I like this use of effectively) upon the polynomial variables x and y, one must impose the constraints upon m and n that either $m = n+1$, or that the assumption (**??**) holds. In the extensive literature on this problem, there are a number of striking results that go well beyond what we have mentioned here. By way of example, work of Balasubramanian and Shorey ([REF]) shows that equation (**??**) has at most finitely many solutions if we fix only the set of prime divisors of x and y, while Bugeaud and Shorey ([REF]) prove an analogous finiteness result, under the additional assumption of (**??**), provided the quotient $(m-1)/(n-1)$ is bounded above. Additional results on special cases of equation (**??**) are available in, for example, [**?**], [**?**], [**?**] and [**?**]. An excellent overview of results on this problem can be found in the survey of Shorey [**?**].

1.0.1 Statement of the results

The novel contributions of this thesis concern the development and implementation of efficient algorithms to determine all solutions of certain Goormaghtigh equations and Thue-Mahler equations. In particular, we follow [REF: BeGhKr] to prove that, in fact, under assumption (**??**), equation (**??**) has at most finitely many solutions which may be found effectively, even if we fix only a single exponent.

Theorem 1.0.1 (BeGhKr)**.** *If there is a solution in integers x, y, n and m to equation* (**??**)*, satisfying* (**??**)*, then*

$$x < (3d)^{4n/d} \leq 36^n. \tag{1.8}$$

In particular, if n is fixed, there is an effectively computable constant $c = c(n)$ such that $\max\{x, y, m\} < c$.

We note that the latter conclusion here follows immediately from (**??**), in conjunction with, for example, work of Baker ([REF]). The constants present in our upper bound (**??**) may be sharpened somewhat at the cost of increasing the complexity of our argument. By refining our approach, in conjunction with some new results from computational Diophantine approximation, we are able to achieve the complete solution of equation (**??**), subject to condition (**??**), for small fixed values of n.

Theorem 1.0.2 (BeGhKr)**.** *If there is a solution in integers x, y and m to equation* (**??**)*, with $n \in \{3, 4, 5\}$ and satisfying* (**??**)*, then*

$$(x, y, m, n) = (2, 5, 5, 3) \ \textit{and} \ (2, 90, 13, 3).$$

In the case $n = 5$ of Theorem (**??**) “off-the-shelf” techniques for finding integral points on models of elliptic curves or for solving *Ramanujan-Nagell* equations of the shape $F(x) = z^n$ (where F is a polynomial and z a fixed integer) do not apparently permit the full resolution of this problem in a reasonable amount of time. Instead, we sharpen the existing techniques of [TdW] and [Hambrook] for solving Thue-Mahler equations and specialize them to this problem.

A direct consequence and primary motivation for developing an efficient Thue-Mahler algorithm is the computation of elliptic curves over $\mathbb{Q}$. Let S be a finite set of rational primes. In 1963, Shafarevich [CITE] proved that there are at most finitely many $\mathbb{Q}$-isomorphism classes of elliptic curves defined over $\mathbb{Q}$ having good reduction outside S. The first effective proof of this statement was provided by Coates [CITE] in 1970 for the case $K = \mathbb{Q}$ and $S = \{2,3\}$ using bounds for linear forms in p-adic and complex logarithms. Early attempts to make these results explicit for fixed sets of small primes overlap with the arguments of [COATES], in that they reduce the problem to that of solving a number of degree 3 Thue-Mahler equations of the form

$$F(x,y) = au,$$

where u is an integer whose prime factors all lie in S.

In the 1950's and 1960's, Taniyama and Weil asked whether all elliptic curves over $\mathbb{Q}$ of a given conductor N are related to modular functions. While this conjecture is now known as the Modularity Theorem, until its proof in 2001 [**?**], attempts to verify it sparked a large effort to tabulate all elliptic curves over $\mathbb{Q}$ of given conductor N. In 1966, Ogg ([**?**], [**?**]) determined all elliptic curves defined over $\mathbb{Q}$ with conductor of the form 2^a. Coghlan, in his dissertation [**?**], studied the curves of conductor $2^a 3^b$ independently of Ogg, while Setzer [**?**] computed all $\mathbb{Q}$-isomorphism classes of elliptic curves of conductor p for certain small primes p. Each of these examples corresponds, via the [BR] approach, to cases with reducible forms. The first analysis on irreducible forms in (**??**) was carried out by Agrawal, Coates, Hunt and van der Poorten [**?**], who determined all elliptic curves of conductor 11 defined over $\mathbb{Q}$ to verify the (then) conjecture of Taniyama-Weil.

There are very few, if any, subsequent attempts in the literature to find elliptic curves of given conductor via Thue-Mahler equations. Instead, many of the approaches involve a completely different method to the problem, using modular forms. This method relies upon the Modularity Theorem of Breuil, Conrad, Diamond and Taylor [**?**], which was still a conjecture (under various guises) when these ideas were first implemented. Much of the success of this approach can be attributed to Cremona (see e.g. [**?**], [**?**]) and his collaborators, who have devoted decades of work to it. In fact, using this method, all elliptic curves over $\mathbb{Q}$ of conductor N have been determined for values of N as follows

- Antwerp IV (1972): $N \leq 200$
- Tingley (1975): $N \leq 320$
- Cremona (1988): $N \leq 600$
- Cremona (1990): $N \leq 1000$
- Cremona (1997): $N \leq 5077$
- Cremona (2001): $N \leq 10000$
- Cremona (2005): $N \leq 130000$
- Cremona (2014): $N \leq 350000$
- Cremona (2015): $N \leq 364000$
- Cremona (2016): $N \leq 390000$.

In this thesis, we follow [BeGhRe] wherein we return to techniques based upon solving Thue-Mahler equations, using a number of results from classical invariant theory. In particular, we illustrate the connection between elliptic curves over $\mathbb{Q}$ and cubic forms and subsequently describe an effective algorithm for determining all elliptic curves over $\mathbb{Q}$ having good reduction outside S. This result can be summarized as follows. If we wish to find an elliptic curves E of conductor $N = p_1^{a_1} \cdots p_v^{a_v}$ for some $a_i \in \mathbb{N}$, by Theorem 1 of [BeGhRe], there exists an integral binary cubic form F of discriminant $N_0 \mid 12N$ and relatively prime integers u, v satisfying

$$F(u,v) = w_0u^3 + w_1u^2v + w_2uv^2 + w_3v^3 = 2^{\alpha_1}3^{\beta_1} \prod_{p \mid N_0} p^{\kappa_p}$$

for some $\alpha_1, \beta_1, \kappa_p$. Then E is isomorphic over $\mathbb{Q}$ to the elliptic curve $E_{\mathscr{D}}$, where $E_{\mathscr{D}}$ is determined by the form F and (u,v). It is worth noting that Theorem 1 of [BeGhRe] very explicitly describes how to generate $E_{\mathscr{D}}$; once a solution (u,v) to the Thue-Mahler equation F is known, a quick computation of the Hessian and Jacobian discriminant of F evaluated at (u,v) yields the coefficients of $E_{\mathscr{D}}$. Using this theorem, all $E/\mathbb{Q}$ of conductor N may be computed by generating all of the relevant binary cubic forms, solving the corresponding Thue-Mahler equations, and outputting the elliptic curves that arise. The first and last steps of this process are straightforward. Indeed, Bennett and Rechnitzer describe an efficient algorithm for carrying out the first step REF. In fact, they having carried out a one-time computation of all irreducible forms that can arise in Theorem 1 of absolute discriminant bounded by 10^{10}. The bulk of the work is therefore concentrated in step 2, solving a large number of degree 3 Thue-Mahler equations.

Unfortunately, despite many refinements, [Hambrook's] MAGMA implementation of a Thue-Mahler solver encounters a multitude of bottlenecks which often yield unavoidable timing and memory problems, even when parallelization is considered. As our aim is to use the results of [BeGhRe] to generate all elliptic curves over $\mathbb{Q}$ of conductor $N < 10^6$, in its current state, the Hambrook algorithm is inefficient for this task, and in many cases, simply unusable due to its memory requirements. The main novel contribution of this thesis is therefore the efficient resolution of an arbitrary degree 3 Thue-Mahler equation and the implementation of this algorithm as a MAGMA package. This work is based on ideas of Matshke, von Kanel [CITE], and Siksek and is summarized in the following steps.

Step 1. Following [TdW] and [Hambrook], we reduce the problem of solving the given Thue-Mahler equation to the problem of solving a collection of finitely many S-unit equations in a certain algebraic number field K. These are equations of the form

$$\mu_0 y - \lambda_0 x = 1 \tag{1.9}$$

for some $\mu_0, \lambda_0 \in K$ and unknowns x, y. The collection of forms is such that if we know the solutions of each equation in the collection, then we can easily derive all of the solutions of the Thue-Mahler equation. This reduction is performed in two steps. First, (**??**) is reduced to a finite number of ideal equations over K. Here, we employ new results by Siksek [Cite?] to significantly reduce the number of ideal equations to consider. Next, we reduce each ideal equation to a number of certain S-unit equations (**??**) via a finite number of principalization tests. The method of [TdW] reduces (**??**) to $(m/2)h^v$ S-unit equations, where m is the number of roots of unity of K, h is the class number, and v is the number of rational primes $p_1,\dots,p_v$. The method of Siksek that we employ gives only $m/2$ S-unit equations. The principle computational work here consists of computing an integral basis, a system of fundamental units, and a splitting field of K, as well as computing the class group of K and the factorizations of the primes $p_1,\dots,p_v$ into prime ideals in the ring of integers of K.

The remaining steps are performed for each of the S-unit equations in our collection.

Step 2. In place of the logarithmic sieves used in [TdW] to derive a large upper bound, we work with the global logarithmic Weil height

$$h : \mathbb{G}_m(\overline{\mathbb{Q}}) \to \mathbb{R}_{\geq 0}.$$

For a given (**??**), we show that the height $h(1/x)$ admits a decomposition into local heights at each place of K appearing in the S-unit equation. Using [CITE : Matshke, von Kanel], we generate a very large upper bound on the height $h(1/x)$, and subsequently, on the local heights. This step is a straightforward computation, whereas the analogous step in Hambrook and TdW is a complex and lengthy derivation which involves factoring rational primes into prime ideals in a splitting field of K and computing heights of certain elements of the splitting field.

Step 3. For each place of K appearing in (**??**), we drastically reduce the upper bounds derived in Step 2 by using computational Diophantine approximation techniques applied to the intersection of a certain ellipsoid and translated lattice. This technique involves using the Finke-Pohst algorithm to enumerate all short vectors in the intersection. Here, working with the Weil height $h(1/x)$ has the advantage that it leads to ellipsoids whose volumes are smaller than the ellipsoids implicitly used in [TdW] by a factor of $\sim r^{r/2}$ for r the number of places of K appearing in our S-unit equation. In this way, we reduce the number of short vectors appearing from the Fincke-Pohst algorithm, and consequently reduce our running time and memory requirements.

Step 4. Samir's sieve - this may not be done in time as we only just received Samir's writeup and explanation as pertaining to Thue-Mahler equations.

Step 5. Finally, we use a sieving procedure to find all the solutions of the Diophantine equation that live in the box defined by the bounds derived in the previous steps. To carry out this step, we run through all the possible solutions in the box and sieve out the vast majority of non-solutions. This is done via certain low-cost congruence tests. The candidate solutions passing this test are then verified directly against (**??**). Though we expect the bounds defining the box to be small, there can still be a very large number of possible solutions to check, especially if the number of rational primes involved in the Thue-Mahler equation is large. The computations performed on each individual candidate solution are relatively simple, but the sheer number of candidates often makes this step the computational bottleneck of the entire algorithm.

Step 6. Having performed Steps 2-5 for each S-unit equation in our collection, we now have all the solutions of each such equation, and we use this knowledge to determine all the solutions of the Thue-Mahler equation.

The reader will notice several parallels between this refined algorithm and the aforementioned Goormaghtigh equation solver in the case $n = 5$. In particular, both algorithms share the same setup and refinements of the [TdW] and [Hambrook] solver. For (**??**), however, we are left to solve

$$f(y) = x^m,$$

a Thue-Mahler-like equation of degree 4 in explicit values of x and unknown integers y and m. In this case, we are permitted simplifications which allow us to omit the Fincke-Pohst algorithm and final congruence sieves. Instead, for each x, we rely on only a few iterations of the LLL algorithm to reduce our initial bound on the exponents before entering a naive search to complete our computation. Of course, this algorithm can be refined further for efficiency, however, in the context of [BeGhKr], such improvements are not needed.

The outline of this thesis is as follows. ADD

1.1 Suggested Thesis Organization

The UBC! **GPS!** (GPS!) specifies a particular arrangement of the components forming a thesis.[1] This template reflects that arrangement.

In terms of writing your thesis, the recommended best practice for organizing large documents in LaTeX is to place each chapter in a separate file. These chapters are then included from the main file through the use of `\include{file}`. A thesis might be described as six files such as `intro.tex`, `relwork.tex`, `model.tex`, `eval.tex`, `discuss.tex`, and `concl.tex`.

We also encourage you to use macros for separating how something will be typeset (e.g., bold, or italics) from the meaning of that something. For example, if you look at `intro.tex`, you will see repeated uses of a macro `\file{}` to indicate file names. The `\file{}` macro is defined in the file `macros.tex`. The consistent use of `\file{}` throughout the text not only indicates that the argument to the macro represents a file (providing meaning or semantics), but also allows easily changing how file names are typeset simply by changing the definition of the `\file{}` macro. `macros.tex` contains other useful macros for properly typesetting things like the proper uses of the latinate *exempli gratiā* and *id est* (i.e., `\eg` and `\ie`), web references with a footnoted URL! (`\webref{url}{text}`), as well as definitions specific to this documentation (`\latexpackage{}`).

[1] See http://www.grad.ubc.ca/current-students/dissertation-thesis-preparation/order-components

1.2 Making Cross-References

LaTeX make managing cross-references easy, and the `hyperref` package's `\autoref{}` command[2] makes it easier still.

A thing to be cross-referenced, such as a section, figure, or equation, is *labelled* using a unique, user-provided identifier, defined using the `\label{}` command. The thing is referenced elsewhere using the `\autoref{}` command. For example, this section was defined using:

```
\section{Making Cross-References}
\label{sec:CrossReferences}
```

References to this section are made as follows:

```
We then cover the ease of managing cross-references in \LaTeX\
in \autoref{sec:CrossReferences}.
```

`\autoref{}` takes care of determining the *type* of the thing being referenced, so the example above is rendered as

> We then cover the ease of managing cross-references in LaTeX in **??**.

The label is any simple sequence of characters, numbers, digits, and some punctuation marks such as ":" and "–"; there should be no spaces. Try to use a consistent key format: this simplifies remembering how to make references. This document uses a prefix to indicate the type of the thing being referenced, such as `sec` for sections, `fig` for figures, `tbl` for tables, and `eqn` for equations.

For details on defining the text used to describe the type of *thing*, search `diss.tex` and the `hyperref` documentation for `autorefname`.

1.3 Managing Bibliographies with BibTeX

One of the primary benefits of using LaTeX is its companion program, BibTeX, for managing bibliographies and citations. Managing bibliographies has three

[2]The `hyperref` package is included by default in this template.

parts: (i) describing references, (ii) citing references, and (iii) formatting cited references.

1.3.1 Describing References

BibTEX defines a standard format for recording details about a reference. These references are recorded in a file with a `.bib` extension. BibTEX supports a broad range of references, such as books, articles, items in a conference proceedings, chapters, technical reports, manuals, dissertations, and unpublished manuscripts. A reference may include attributes such as the authors, the title, the page numbers, the **DOI!** (**DOI!**), or a **URL!** (**URL!**). A reference can also be augmented with personal attributes, such as a rating, notes, or keywords.

Each reference must be described by a unique *key*.[3] A key is a simple sequence of characters, numbers, digits, and some punctuation marks such as ":" and "–"; there should be no spaces. A consistent key format simiplifies remembering how to make references. For example:

$\boxed{\textit{last-name}}$–$\boxed{\textit{year}}$–$\boxed{\textit{contracted-title}}$

where *last-name* represents the last name for the first author, and *contracted-title* is some meaningful contraction of the title. Then **?** 's seminal article on aspect-oriented programming [**?**] (published in **?**) might be given the key `kiczales-1997-aop`.

An example of a BibTEX `.bib` file is included as `biblio.bib`. A description of the format a `.bib` file is beyond the scope of this document. We instead encourage you to use one of the several reference managers that support the BibTEX format such as JabRef[4] (multiple platforms) or BibDesk[5] (MacOS X only). These front ends are similar to reference manages such as EndNote or RefWorks.

[3] Note that the citation keys are different from the reference identifiers as described in **??**.

[4] http://jabref.sourceforge.net

[5] http://bibdesk.sourceforge.net

1.3.2 Citing References

Having described some references, we then need to cite them. We do this using a form of the `\cite` command. For example:

```
\citet{kiczales-1997-aop} present examples of crosscutting
from programs written in several languages.
```

When processed, the `\citet` will cause the paper's authors and a standardized reference to the paper to be inserted in the document, and will also include a formatted citation for the paper in the bibliography. For example:

> **?**] present examples of crosscutting from programs written in several languages.

There are several forms of the `\cite` command (provided by the `natbib` package), as demonstrated in **??**. Note that the form of the citation (numeric or author-year) depends on the bibliography style (described in the next section). The `\citet` variant is used when the author names form an object in the sentence, whereas the `\citep` variant is used for parenthetic references, more like an end-note. Use `\nocite` to include a citation in the bibliography but without an actual reference.

1.3.3 Formatting Cited References

BibTeX separates the citing of a reference from how the cited reference is formatted for a bibliography, specified with the `\bibliographystyle` command. There are many varieties, such as `plainnat`, `abbrvnat`, `unsrtnat`, and `vancouver`. This document was formatted with `abbrvnat`. Look through your TeX distribution for `.bst` files. Note that use of some `.bst` files do not emit all the information necessary to properly use `\citet{}`, `\citep{}`, `\citeyear{}`, and `\citeauthor{}`.

There are also packages available to place citations on a per-chapter basis (`bibunits`), as footnotes (`footbib`), and inline (`bibentry`). Those who wish to exert maximum control over their bibliography style should see the amazing `custom-bib`

Table 1.1: Available `cite` variants; the exact citation style depends on whether the bibliography style is numeric or author-year.

Variant	Result
`\cite`	Parenthetical citation (e.g., "[**?**]" or "(**? ?**)")
`\citet`	Textual citation: includes author (e.g., "**?**]" or or "**?** (**?**)")
`\citet*`	Textual citation with unabbreviated author list
`\citealt`	Like `\citet` but without parentheses
`\citep`	Parenthetical citation (e.g., "[**?**]" or "(**? ?**)")
`\citep*`	Parenthetical citation with unabbreviated author list
`\citealp`	Like `\citep` but without parentheses
`\citeauthor`	Author only (e.g., "**?** ")
`\citeauthor*`	Unabbreviated authors list (e.g., "**?** ")
`\citeyear`	Year of citation (e.g., "**?** ")

package.

1.4 Typesetting Tables

?] made one grievous mistake in LaTeX: his suggested manner for typesetting tables produces typographic abominations. These suggestions have unfortunately been replicated in most LaTeX tutorials. These abominations are easily avoided simply by ignoring his examples illustrating the use of horizontal and vertical rules (specifically the use of `\hline` and `|`) and using the `booktabs` package instead.

The `booktabs` package helps produce tables in the form used by most professionally-edited journals through the use of three new types of dividing lines, or *rules*. Tables **??** and **??** are two examples of tables typeset with the `booktabs` package. The `booktabs` package provides three new commands for producing rules: `\toprule` for the rule to appear at the top of the table, `\midrule` for the middle rule following the table header, and `\bottomrule` for the bottom-most at the end of the table. These rules differ by their weight (thickness) and the spacing before and after. A table is typeset in the following manner:

LaTeX Rocks!

Figure 1.1: Proof of LaTeX's amazing abilities

```
\begin{table}
\caption{The caption for the table}
\label{tbl:label}
\centering
\begin{tabular}{cc}
\toprule
Header & Elements \\
\midrule
Row 1 & Row 1 \\
Row 2 & Row 2 \\
% ... and on and on ...
Row N & Row N \\
\bottomrule
\end{tabular}
\end{table}
```

See the `booktabs` documentation for advice in dealing with special cases, such as subheading rules, introducing extra space for divisions, and interior rules.

1.5 Figures, Graphics, and Special Characters

Most LaTeX beginners find figures to be one of the more challenging topics. In LaTeX, a figure is a *floating element*, to be placed where it best fits. The user is not expected to concern him/herself with the placement of the figure. The figure should instead be labelled, and where the figure is used, the text should use `\autoref` to reference the figure's label. **??** is an example of a figure. A figure is generally included as follows:

```
\begin{figure}
\centering
\includegraphics[width=3in]{file}
\caption{A useful caption}
\label{fig:fig-label}  % label should change
\end{figure}
```

There are three items of note:

1. External files are included using the `\includegraphics` command. This command is defined by the `graphicx` package and can often natively import graphics from a variety of formats. The set of formats supported depends on your TeX command processor. Both `pdflatex` and `xelatex`, for example, can import GIF, JPG, and PDF. The plain version of `latex` only supports EPS files.

2. The `\caption` provides a caption to the figure. This caption is normally listed in the List of Figures; you can provide an alternative caption for the LoF by providing an optional argument to the `\caption` like so:

```
\caption[nice shortened caption for LoF]{%
    longer detailed caption used for the figure}
```

   GPS! generally prefers shortened single-line captions in the LoF: multiple-line captions are a bit unwieldy.

3. The `\label` command provides for associating a unique, user-defined, and descriptive identifier to the figure. The figure can be can be referenced elsewhere in the text with this identifier as described in **??**.

See Keith Reckdahls excellent guide for more details, *Using imported graphics in LaTeX2e*[6].

1.6 Special Characters and Symbols

LaTeX appropriates many common symbols for its own purposes, with some used for commands (i.e., `\ { } & %`) and mathematics (i.e., `$ ^ _`), and others are automagically transformed into typographically-preferred forms (i.e., `- ` '`) or to completely different forms (i.e., <>). **??** presents a list of common symbols and their corresponding LaTeX commands. A much more comprehensive list of symbols and accented characters is available at: http://www.ctan.org/tex-archive/info/symbols/comprehensive/

[6] http://www.ctan.org/tex-archive/info/epslatex.pdf

Table 1.2: Useful LaTeX symbols

LaTeX	Result	LaTeX	Result
`\texttrademark`	™	`\&`	&
`\textcopyright`	©	`\{ \}`	{ }
`\textregistered`	®	`\%`	%
`\textsection`	§	`\verb!~!`	~
`\textdagger`	†	`\$`	$
`\textdaggerdbl`	‡	`\^{}`	^
`\textless`	$<$	`\_`	_
`\textgreater`	$>$		

The `ubcdiss` class is based on the standard LaTeX `book` class [**?**] that selects a line-width to carry approximately 66 characters per line. This character density is claimed to have a pleasing appearance and also supports more rapid reading [**?**]. I would recommend that you not change the line-widths!

1.6.1 The `geometry` Package

Some students are unfortunately saddled with misguided supervisors or committee members whom believe that documents should have the narrowest margins possible. The `geometry` package is helpful in such cases. Using this package is as simple as:

```
\usepackage[margin=1.25in,top=1.25in,bottom=1.25in]{geometry}
```

You should check the package's documentation for more complex uses.

1.6.2 Changing Page Layout Values By Hand

There are some miserable students with requirements for page layouts that vary throughout the document. Unfortunately the `geometry` can only be specified once, in the document's preamble. Such miserable students must set LaTeX's layout parameters by hand:

```
\setlength{\topmargin}{-.75in}
```

```
\setlength{\headsep}{0.25in}
\setlength{\headheight}{15pt}
\setlength{\textheight}{9in}
\setlength{\footskip}{0.25in}
\setlength{\footheight}{15pt}

% The *sidemargin values are relative to 1in; so the following
% results in a 0.75 inch margin
\setlength{\oddsidemargin}{-0.25in}
\setlength{\evensidemargin}{-0.25in}
\setlength{\textwidth}{7in}      % 1.1in margins (8.5-2*0.75)
```

These settings necessarily require assuming a particular page height and width; in the above, the setting for `\textwidth` assumes a US Letter with an 8.5" width. The `geometry` package simply uses the page height and other specified values to derive the other layout values. The `layout` package provides a handy `\layout` command to show the current page layout parameters.

1.6.3 Making Temporary Changes to Page Layout

There are occasions where it becomes necessary to make temporary changes to the page width, such as to accomodate a larger formula. The `chngpage` package provides an `adjustwidth` environment that does just this. For example:

```
% Expand left and right margins by 0.75in
\begin{adjustwidth}{-0.75in}{-0.75in}
% Must adjust the perceived column width for LaTeX to get with it.
\addtolength{\columnwidth}{1.5in}
\[ an extra long math formula \]
\end{adjustwidth}
```

1.7 Keeping Track of Versions with Revision Control

Software engineers have used **RCS!** (**RCS!**) to track changes to their software systems for decades. These systems record the changes to the source code along with context as to why the change was required. These systems also support examining and reverting to particular revisions from their system's past.

An **RCS!** can be used to keep track of changes to things other than source code, such as your dissertation. For example, it can be useful to know exactly which revision of your dissertation was sent to a particular committee member. Or to recover an accidentally deleted file, or a badly modified image. With a revision control system, you can tag or annotate the revision of your dissertation that was sent to your committee, or when you incorporated changes from your supervisor.

Unfortunately current revision control packages are not yet targetted to non-developers. But the Subversion project's TortoiseSVN[7] has greatly simplified using the Subversion revision control system for Windows users. You should consult your local geek.

A simpler alternative strategy is to create a GoogleMail account and periodically mail yourself zipped copies of your dissertation.

1.8 Recommended Packages

The real strength to LaTeX is found in the myriad of free add-on packages available for handling special formatting requirements. In this section we list some helpful packages.

1.8.1 Typesetting

`enumitem`: Supports pausing and resuming enumerate environments.

`ulem`: Provides two new commands for striking out and crossing out text (`\sout{text}` and `\xout{text}` respectively) The package should likely be used as follows:

```
\usepackage[normalem,normalbf]{ulem}
```

[7] http://tortoisesvn.net/docs/release/TortoiseSVN_en/

to prevent the package from redefining the emphasis and bold fonts.

chngpage: Support changing the page widths on demand.

mhchem: Support for typesetting chemical formulae and reaction equations.

Although not a package, the `latexdiff`[8] command is very useful for creating changebar'd versions of your dissertation.

1.8.2 Figures, Tables, and Document Extracts

pdfpages: Insert pages from other PDF files. Allows referencing the extracted pages in the list of figures, adding labels to reference the page from elsewhere, and add borders to the pages.

subfig: Provides for including subfigures within a figure, and includes being able to separately reference the subfigures. This is a replacement for the older `subfigure` environment.

rotating: Provides two environments, sidewaystable and sidewaysfigure, for typesetting tables and figures in landscape mode.

longtable: Support for long tables that span multiple pages.

tabularx: Provides an enhanced tabular environment with auto-sizing columns.

ragged2e: Provides several new commands for setting ragged text (e.g., forms of centered or flushed text) that can be used in tabular environments and that support hyphenation.

1.8.3 Bibliography Related Packages

bibunits: Support having per-chapter bibliographies.

footbib: Cause cited works to be rendered using footnotes.

[8] http://www.ctan.org/tex-archive/support/latexdiff/

bibentry: Support placing the details of a cited work in-line.

custom-bib: Generate a custom style for your bibliography.

1.9 Moving On

At this point, you should be ready to go. Other handy web resources:

- **CTAN!** (CTAN!)[9] is *the* comprehensive archive site for all things related to TeX and LaTeX. Should you have some particular requirement, somebody else is almost certainly to have had the same requirement before you, and the solution will be found on CTAN!. The links to various packages in this document are all to CTAN!.

- An online reference to LaTeX commands[10] provides a handy quick-reference to the standard LaTeX commands.

- The list of Frequently Asked Questions about TeX and LaTeX[11] can save you a huge amount of time in finding solutions to common problems.

- The teTeX documentation guide[12] features a very handy list of the most useful packages for LaTeX as found in CTAN!.

- The `color`[13] package, part of the graphics bundle, provides handy commands for changing text and background colours. Simply changing text to various levels of grey can have a very dramatic effect.

- If you're really keen, you might want to join the TeX Users Group[14].

[9] http://www.ctan.org
[10] http://www.ctan.org/get/info/latex2e-help-texinfo/latex2e.html
[11] http://www.tex.ac.uk/cgi-bin/texfaq2html?label=interruptlist
[12] http://www.tug.org/tetex/tetex-texmfdist/doc/
[13] http://www.ctan.org/tex-archive/macros/latex/required/graphics/grfguide.pdf
[14] http://www.tug.org

Appendix A

Supporting Materials

This would be any supporting material not central to the dissertation. For example:

- additional details of methodology and/or data;
- diagrams of specialized equipment developed.;
- copies of questionnaires and survey instruments.