Question

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In the write-up, we have $n_p - a_p = \sum m_\delta \delta_p$. Throughout the document, we assume that $\sum m_\delta \delta_p > \frac{1}{p-1} - ord_p(\delta_2)$ in order to conclude that a potential solution vector lives in our lattice. Our height bounds on the non-Archimedean places, on the other hand, have $h_p(z) = \log(p)|\sum m_\delta \delta_p|$ and the crux of the argument is in assuming that $h_p(z) > \log(p)\left(\frac{1}{p-1} - ord_p(\delta_2)\right)$, then we obtain that $\sum m_\delta \delta_p > \frac{1}{p-1} - ord_p(\delta_2)$, hence the solution vector would have to live in our lattice.

What happens if $\sum m_\delta \delta_p$ is negative? We would end up with

$$\sum m_\delta \delta_p < -\frac{1}{p-1} + ord_p(\delta_2),$$

and so wouldn't be able to say anything about whether our vector is in the lattice. Maybe we need to have that

$$h_p(z) > \max\left(\log(p)a_p, \log(p)(\frac{1}{p-1} - ord_p(\delta_2))\right).$$

In this case, if $\sum m_\delta \delta_p$ is negative, having $h_p(z) > \log(p)a_p$ would mean that $\sum m_\delta \delta_p < -a_p$, which is impossible since $n_p = \sum m_\delta \delta_p + a_p \geq 0$.

I guess this means that we then need to also search in the range of $h_v(z) \in [0, \log(p)a_p]$, or $\sum m_\delta \delta_p \in [-a_p, 0]$, and in this case, we won't be able to rely on our lattice reduction.

What do you think?